Stability and control of the UAV formations flight

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Abstract: - The purpose of this paper is to analyze the flight stability of an Unmanned Aerial Vehicles (UAV) formation by using 3 degrees of freedom (3 DOF) models. The problem of flight formation will be approached in a simple manner, by using 3 DOF nonlinear models, as well as using a linear one. This theoretical development allows us to build stability matrix, command matrix and control matrix and finally to analyze the stability of autonomous flight of the UAV formation. The work will present and analyze the calculus results for developed solution. The novelty of the paper consists in algorithms that allow evaluating the stability of the UAV formation. The results will focus on flight stability of a UAV formation and its answer to regular perturbations.

Key-Words: – Flight, Formation, Automation, Stability, Control, Simulation

NOMENCLATURE:

χ - Air -path track angle;
γ - Climb angle;
μ - Aerodynamic bank angle;
ρ - Air density;
$\mathbf{\Omega}_V$ Angular velocity in quasi velocity frame;
$\omega_l^*, \omega_m^*, \omega_n^*$ - Rotation velocity components along the axes of the quasi-velocity frame;
A - Stability matrix;
B - Command matrix;
K - Controller matrix;

Force components in the aerodynamic frame:
L - Lift force; D - Drag force;
M - Mach number;
C_D -Drag force coefficient;
C_L -Lift force coefficient;
G - Gravitational force;
g - Gravitational acceleration;
$k_u^V, k_u^\gamma, k_u^\chi$ -Trajectory control coefficients;
$k_u^{hx}, k_u^{hy}, k_u^{hz}$ -Position control coefficients;
$k_u^{Ix}, k_u^{Iy}, k_u^{Iz}$ -Integrative position control coefficients;
m – UAV mass;
n - Load factor;
S - Reference area – cross body area;
T - Thrust;
V - Velocity;
z_0 - Altitude.

1 Introduction

The major impact of Unmanned Aerial Vehicles (UAV) consists in its role in the actual aero-space scenarios, where the UAV accomplish the recognition or the rescue missions in hostile environments.

It is well known that a UAV formation using sample observation instrument, but at the same time using the data-fusion, is more efficiently then a sophisticate singular UAV or piloted airplane.

The purpose of this paper is to describe the flight stability of a UAV formation. The problems associated with formation flight will be approached in a simple manner, by using 3 DOF models, unlike paper [2] and [3] where we used the complex equations that describe the movement for each UAV. However, in this paper we develop a linear model of the movement equation that allows us to analyze flight stability of UAV formation.

The paper will analyze UAV formation structures in a unitary manner using a control system with an adequate architecture. The approach proposed for controlling aircraft formations is inspired by reference [1]. All the simulations will be made in a linear workspace, using simplifying hypothesis described in item 6.

The control solution adopted can be used also in other domain where we have a body group, which must have a synchronous movement.

2 Formation modeling

In order to represent each UAV from the formation a 3 DOF model was adopted. This model involves only the slow states that correspond to a problem of the trajectory tracking and the relative position maintaining by using an autopilot. For this reason, we use the following two reference frames.

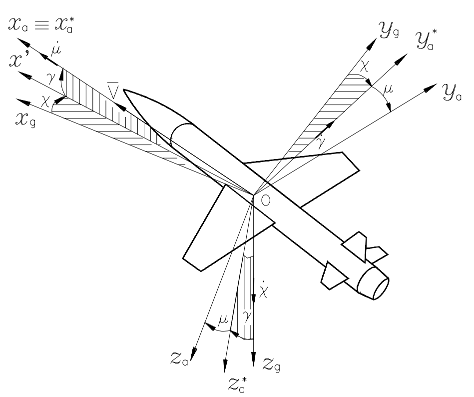

Fig. 1 Quasi velocity frame and rotation angles

A local inertial frame Γ_0, with the origin in the mass centre of the aircraft, with the z axis orientated vertically up. We are assuming that the inertial frame Γ_0 has the axes parallel to the ones of the Earth frame bound to the ground. The second is the quasi-velocity frame Γ_a, connected to the velocity vector, also with the origin in the mass centre of the aircraft, obtained by two successive rotations; by the air-path track angle χ, and the climb angle γ. As usually, the axis x_a^* of the quasi-velocity frame Γ_a is orientated along the velocity vector $\mathbf{V}$, and the y_a^* axis is orientated in the horizontal plane. The transformation between the inertial frame Γ_0 and the quasi - velocity frame Γ_a is given by the matrix:

$$\mathbf{A}_{a0} = \begin{bmatrix} \cos\gamma\cos\chi & -\cos\gamma\sin\chi & \sin\gamma \\ -\sin\chi & -\cos\chi & 0 \\ \sin\gamma\cos\chi & -\sin\gamma\sin\chi & -\cos\gamma \end{bmatrix}. \quad (1)$$

3 Nonlinear equations of motion

If we accept the evolution without sideslip angle $\beta = 0$ then, the lateral force is also void $N = 0$. At the same time if we are assuming the thrust orientated along the velocity vector, and the aerodynamic force components obtained by a rotation with aerodynamic bank angle μ from the velocity frame, the dynamic equations of movement for the UAV named “i” in the quasi-velocity frame are:

$$\dot{V}_i = \frac{T_i - D_i}{m_i} - g\sin\gamma_i;$$

$$\dot{\gamma}_i = \frac{L_i}{m_i V_i}\cos\mu_i - \frac{g}{V_i}\cos\gamma_i; \dot{\chi}_i = \frac{L_i}{m_i V_i}\frac{\sin\mu_i}{\cos\gamma_i}. \quad (2)$$

Writing the load factor:

$$n_i = L_i / m_i g, \quad (3)$$

The equations (2) become:

$$\dot{V}_i = \frac{T_i - D_i}{m_i} - g\sin\gamma_i; \dot{\gamma}_i = \frac{g}{V_i}\left(n_i\cos\mu_i - \cos\gamma_i\right);$$

$$\dot{\chi}_i = \frac{g n_i}{V_i}\frac{\sin\mu_i}{\cos\gamma_i}, \quad (4)$$

where we have denoted:

D_i drag force, L_i lift force, T_i thrust, m_i mass for aircraft "i".

If we use the aircraft polar coordinate, the drag coefficient is:

$$C_{Di} = C_{D0i} + k_i C_{Li}^2 \quad (5)$$

moreover, the drag becomes:

$$D_i = F_{0i} C_{D0i} + k_i \left(n_i m_i g\right)^2 / F_{0i} \quad (6)$$

where the reference aerodynamical force is:

$$F_{0i} = \frac{1}{2} \rho V_i^2 S_i \quad (7)$$

where ρ is the air density at a given altitude and S_i is the reference surface.

Defining the state vector:

$$\mathbf{x}_i = \begin{bmatrix} V_i & \gamma_i & \chi_i \end{bmatrix}^T$$

and the input vector:

$$\mathbf{u}_i = \begin{bmatrix} T_i & n_i & \mu_i \end{bmatrix},$$

the equations (4) can be put under standard form:

$$\dot{\mathbf{x}}_i = f_i(\mathbf{x}_i, \mathbf{u}_i) \quad (8)$$

4 Kinematics

One of the main goal regarding the control system of the formation is that every UAV must maintain a certain position D_i related to a reference point denoted in figure 2 as G. This point may coincide with the formation leader (real or virtual), or the neighbour UAV (wingman), or a geometrical central point inside the formation.

For establishing a suitable mathematical model, similar to [1] we are assuming that $\mathbf{r_i}$ and $\mathbf{r_r}$ are the vectors of the UAV position A_i, and of the reference point G regarding the origin O of the inertial frame. $\mathbf{d_i}$ is the current relative distance between G and the UAV position A_i.

The vector for the desired position of airplane D_i is denoted as $\mathbf{r}_{di}$. Also, we are assuming that an orthogonal frame Γ_{ar}, similar to the quasi - velocity frame, is attached to the G point, whose orientation is defined by the angles γ_r, χ_r.

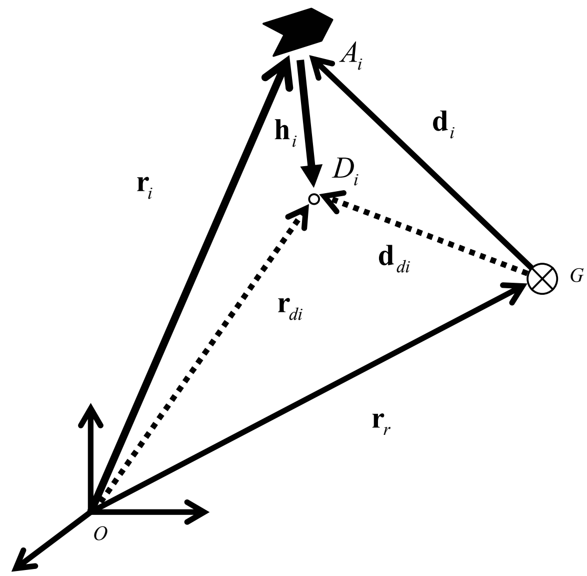

Fig. 2 The defining scheme for the UAV formation

At the same time, we are defining the velocity vector $\mathbf{V}_r$ as the velocity of the G point. From figure 2 results:

$$\mathbf{r}_r + \mathbf{d}_i = \mathbf{r}_i, \quad (9)$$

and

$$\mathbf{r}_r + \mathbf{d}_{di} = \mathbf{r}_{di}, \quad (10)$$

from where we obtain:

$$\mathbf{h}_i = \mathbf{d}_{di} - \mathbf{d}_i = \mathbf{r}_{di} - \mathbf{r}_i \quad (11)$$

where we denoted $\mathbf{h}_i$ the guidance parameter.

Deriving the relation (11), related to time in the quasi-velocity frame Γ_{ai} and assuming that the desired velocity is that of the reference point $\mathbf{V}_r$, we obtain:

$$\dot{\mathbf{h}}_i + \mathbf{\Omega}_{Vi} \times \dot{\mathbf{h}}_i = \mathbf{V}_r - \mathbf{V}_i \quad (12)$$

where $\mathbf{\Omega}_{Vi}$ is the angular velocity of the quasi-velocity frame Γ_{ai}, related to the inertial frame Γ_0. In order to evaluate the vectored relation (12) we can project it along the axes of the quasi-velocity frame of each UAV:

$$[\dot{\mathbf{h}}_i]_{ai} = [\mathbf{V}_r]_{ai} - [\mathbf{V}_i]_{ai} - \mathbf{A}_{\omega i}[\mathbf{h}_i]_{ai} \quad (13)$$

where the angular velocity vector has its components along the quasi-velocity frame axes given by:

$$[\mathbf{\Omega}_V]_{ai} = [\omega^*_{li} \quad \omega^*_{mi} \quad \omega^*_{ni}]^T. \quad (14)$$

In addition, the anti symmetric matrix associated to the angular velocity vector is given by:

$$\mathbf{A}_{\omega i} = \begin{bmatrix} 0 & -\omega^*_{ni} & \omega^*_{mi} \\ \omega^*_{ni} & 0 & -\omega^*_{li} \\ -\omega^*_{mi} & \omega^*_{li} & 0 \end{bmatrix}. \quad (15)$$

The connection between the orientation angles derivates and the angular velocity vector components is:

$$\begin{bmatrix} \omega^*_{li} \\ \omega^*_{mi} \\ \omega^*_{ni} \end{bmatrix} = \begin{bmatrix} \cos\gamma_i & 0 & -\sin\gamma_i \\ 0 & 1 & 0 \\ \sin\gamma_i & 0_i & \cos\gamma_i \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\gamma}_i \\ \dot{\chi}_i \end{bmatrix}, \quad (16)$$

from where we can obtain the scalar relations:

$$\omega^*_{li} = -\dot{\chi}_i \sin\gamma_i; \quad \omega^*_{mi} = \dot{\gamma}_i; \quad \omega^*_{ni} = \dot{\chi}_i \cos\gamma_i, \quad (17)$$

Note: In the paper [1] the problem is treated in a similar way, but the rotation matrix (1) is constructed by using three successive rotations, which represent the passing to the velocity frame, operation that makes difficult defining the relations (16) because it does not clearly point out the method of obtaining the derivate for the rolling angle: $\dot{\mu}$. We are assuming that the orientation of the "i" UAV coincides with that of the Γ_{ar} frame. In this case we define the rotation matrixes:

$$\mathbf{A}_{ai0} = \mathbf{A}_{a0}(\gamma_i, \chi_i) \quad (18)$$

and

$$\mathbf{A}_{ar0} = \mathbf{A}_{a0}(\gamma_r, \chi_r) \quad (19)$$

where A_{a0} is obtained from equation (1). Starting from the defined matrixes we can write:

$$[\mathbf{V}_r]_{ai} = \mathbf{A}_{ai0}[\mathbf{V}_r]_0 \quad (20)$$

$$[\mathbf{V}_r]_{ar} = \mathbf{A}_{ar0}[\mathbf{V}_r]_0 \quad (21)$$

By using (20) and (21) we find the desired velocity in the reference frame Γ_{ai}:

$$[\mathbf{V}_r]_{ai} = \mathbf{A}_{ai0}\mathbf{A}^T_{ar0}[\mathbf{V}_r]_{ar} \quad (22)$$

with the following denotation:

$$[\mathbf{V}_r]_{ar} = [V_r \quad 0 \quad 0]^T \quad (23)$$

By introducing the relation (22) into (13) we obtain:

$$[\dot{\mathbf{h}}_\mathbf{i}]_{ai} = \mathbf{A}_{ai0}\mathbf{A}^T_{ar0}[\mathbf{V}_r]_{ar} - [\mathbf{V}_i]_{ai} - \mathbf{A}_{\omega i}[\mathbf{h}_i]_{ai} \quad (24)$$

where the guidance parameter has the components along reference frame Γ_{ai}:

$$\mathbf{h}_i = [h_{ix} \quad h_{iy} \quad h_{iz}]^T \quad (25)$$

If we introduce the reference vector:

$$\mathbf{x}_r = [V_r, \quad \gamma_r, \quad \chi_r], \quad (26)$$

and the position of the UAV regarding the reference point between the states of the UAV then the equation can also be written in the compact form like this:

$$\dot{\mathbf{h}}_i = g(\mathbf{x}_i, \mathbf{x}_r, \mathbf{h}_i, \mathbf{u}_i) \quad (27)$$

Supplementary, for control command we need integral terms, which can be defined by differential equation:

$$\dot{\mathbf{I}}_i = \mathbf{h}_i \quad (28)$$

5 Controlling the formation with pseudo-commands

This section of the paper will sketch the control system design. We assume the existence of a reaction loop with the standard autopilot, which will maintain the UAV in formation. Our intention is to define a formation control law, capable of simultaneously managing the trajectory tracing and maintaining the formation position. As shown in [1] we can start from the pseudo-command signals along the three axes of the quasi-velocity frame:

$$\begin{aligned} u_{Vi} &= k_u^V \widetilde{V}_i + k_u^{hx} h_{ix} + k_u^{Ix} I_{ix} \\ u_{\gamma i} &= \frac{V_i}{g}\left(k_u^{\gamma}\widetilde{\gamma}_i - k_u^{hz} h_{zi} - k_u^{Iz} I_{iz}\right) + \cos\gamma_i \\ u_{\chi i} &= \frac{V_i}{g}\left(k_u^{\chi}\widetilde{\chi}_i + k_u^{hy} h_{yi} + k_u^{Iy} I_{yi}\right)\cos\gamma_i \end{aligned} \qquad (29)$$

where we denoted:

$$\widetilde{V}_i = V_r - V_i \, ; \; \widetilde{\gamma}_i = \gamma_r - \gamma_i \, ; \; \widetilde{\chi}_i = \chi_r - \chi_i \, . \qquad (30)$$

The trajectory control coefficients and the position control coefficients can be obtained by using a synthesis procedure described in item 8. With the help of these three functions previously defined the pseudo-commands regarding each aircraft are formed. In this manner, we obtain thrust from the relation:

$$T_i = G u_{Vi} \, , \qquad (31)$$

and the square sum of the last two functions (29) gives the necessary load factor:

$$n_i = \sqrt{u_{\gamma i}^2 + u_{\chi i}^2} \; . \qquad (32)$$

In the end, the velocity-rolling angle is given by:

$$\mu_i = \arctan\left(u_{\chi i} / u_{\gamma i}\right). \qquad (33)$$

6 Balance movement

The study of formation fly stability will be made accordingly to Liapunov theory, considering the system of movement equations perturbed around the balanced movement. This involves a disturbance shortly applied on the balance movement, which will produce deviation of the state variables. Developing in series the perturbed movement equations in relation to status variables and taking into account the first order terms of the detention, we will get linear equations which can be use to analyze the stability in the first approximation, as we proceed in most dynamic non linear problems.

In order to obtain linear form of the movement equations, we consider a balance movement in vertical plane, without manoeuvre in lateral plane. Without loosing the generality, we can considerate the air -path track angle null

$$\chi = 0 \, .$$

In addition, we consider the state vector of each UAV is identical with reference values. That means:

$$V_i = V_r \, ; \; \gamma_i = \gamma_r \, ; \; \chi_i = \chi_r \, , \qquad (34)$$

and for balance movement we denote these parameters V, γ, χ.

In this situation, the guidance parameter and its integrative also, will have all components null:

$$\mathbf{h} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \quad \mathbf{I} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T ; \qquad (35)$$

In this case pseudo-command functions indicated by relation (29) are:

$$u_V = 0; \;\; u_{\gamma} = \cos\gamma \, ; \; u_{\chi} = 0 \;\; , \qquad (36)$$

and the command parameters for the balance movement became:

$$T = T_0 \, ; n = \cos\gamma \, ; \; \mu = 0 \, . \qquad (37)$$

Starting from these parameters of the base movement in the next item, we will obtain linear form of movement equations, guidance parameter and command law.

7 Linear movement equations

Starting from relations (4) we obtain:

$$\Delta\dot{V}_i = \frac{2F_0}{V}\left(2kmg^2n^2F_0^{-3} - \frac{C_{D0}}{m}\right)\Delta V_i - g\cos\gamma\Delta\gamma_i + \frac{\Delta T_i}{m} - 2kmg^2F_0^{-2}n\Delta n_i$$

$$\Delta\dot{\gamma}_i = \frac{g}{V}\sin\gamma\Delta\gamma_i + \frac{g}{V}\Delta n_i;\ \Delta\dot{\chi}_i = \frac{g}{V}\Delta\mu_i \quad (38)$$

Next, starting from relation (26) we will obtain the linear form of the guidance parameter equation. Starting from equation (26), taking into account the balance movement, we can write:

$$\left[\Delta\dot{\mathbf{h}}_{\mathbf{i}}\right]_{ai} = \Delta\left(\mathbf{A}_{ai0}\mathbf{A}_{ar0}^T\left[\mathbf{V}_r\right]_{ar}\right) - \left[\Delta\mathbf{V}_i\right]_{ai} - \mathbf{A}_{\omega i}\left[\Delta\mathbf{h}_i\right]_{ai}$$

or:

$$\left[\Delta\dot{\mathbf{h}}_{\mathbf{i}}\right]_{ai} = \mathbf{A}_{auv}\left[\Delta\mathbf{x}_i\right]_{ai} - \mathbf{A}_{\omega i}\left[\Delta\mathbf{h}_i\right]_{ai} \quad (39)$$

where the matrix $\mathbf{A}_{auv}$ is:

$$\mathbf{A}_{auv} = \left[\begin{array}{c|c|c} \begin{matrix}-1\\0\\0\end{matrix} & \left[\dfrac{\partial\mathbf{A}_{ao}}{\partial\gamma}\right. & \left.\dfrac{\partial\mathbf{A}_{ao}}{\partial\chi}\right] \end{array}\begin{bmatrix}\mathbf{A}_{a0}^T[\mathbf{V}]_a & 0\\ 0 & \mathbf{A}_{a0}^T[\mathbf{V}]_a\end{bmatrix}\right]_{\chi=0}$$

If we take into consideration that:

$$\left.\frac{\partial\mathbf{A}_{a0}}{\partial\gamma}\right|_{\chi=0} = \begin{bmatrix}-\sin\gamma & 0 & \cos\gamma\\ 0 & 0 & 0\\ \cos\gamma & 0 & \sin\gamma\end{bmatrix};$$

$$\left.\frac{\partial\mathbf{A}_{a0}}{\partial\chi}\right|_{\chi=0} = \begin{bmatrix}0 & -\cos\gamma & 0\\ -1 & 0 & 0\\ 0 & -\sin\gamma & 0\end{bmatrix};$$

$$\mathbf{A}_{a0}\big|_{\chi=0} = \begin{bmatrix}\cos\gamma & 0 & \sin\gamma\\ 0 & -1 & 0\\ \sin\gamma & 0 & -\cos\gamma\end{bmatrix} \quad (40)$$

the matrix $\mathbf{A}_{auv}$ becomes:

$$\mathbf{A}_{auv} = \begin{bmatrix}-1 & 0 & 0\\ 0 & 0 & -V\cos\gamma\\ 0 & V & 0\end{bmatrix} \quad (41)$$

Finally, we can put relation (39) in scalar form:

$$\Delta\dot{h}_{xi} = -\Delta V_i + \dot{\gamma}\Delta h_{zi} \quad \Delta\dot{h}_{yi} = -V\cos\gamma\Delta\chi_i$$

$$\Delta\dot{h}_{zi} = V\Delta\gamma_i - \dot{\gamma}\Delta h_x \quad (42)$$

In order to add up the equation system we attach linear form of the integral equation (28):

$$\Delta\dot{\mathbf{I}}_i = \Delta\mathbf{h}_i\ . \quad (43)$$

Next, it is necessary to obtain the linear form of the command relations. Starting from (31), (32) and (33) and taken into consideration (37) we obtain:

$$\Delta T_i = G\Delta u_{Vi};\ \Delta n_i = \Delta u_{\gamma i};\ \Delta\mu_i = \Delta u_{\chi i}/\cos\gamma; \quad (44)$$

Developing equations (44) results:

$$\Delta T_i = -Gk_u^V\Delta V_i + Gk_u^{hx}\Delta h_{ix} + Gk_u^{Ix}\Delta I_{ix} + Gk_u^V V_r$$

$$\Delta n_i = -\left(\frac{V}{g}k_u^\gamma + \sin\gamma\right)\Delta\gamma_i - \frac{V}{g}k_u^{hz}\Delta h_{zi} + -\frac{V}{g}k_u^{Iz}\Delta I_{iz\,i} + \frac{V}{g}k_u^\gamma\Delta\gamma_r$$

$$\Delta\mu_i = -\frac{V}{g}k_u^\chi\Delta\chi_i + \frac{V}{g}k_u^{hy}\Delta h_{yi} + \frac{V}{g}k_u^{Iy}\Delta I_{yi} + +\frac{V}{g}k_u^\chi\Delta\chi_r \quad (45)$$

If we consider the extended state vector:

$$\widetilde{\mathbf{x}}_i = \left[\Delta\mathbf{x}_i \quad \Delta\mathbf{h}_i \quad \Delta\mathbf{I}_i\right]^T \quad (46)$$

with the command:

$$\mathbf{u}_i = \left[\Delta T_i \quad \Delta n_i \quad \Delta\mu_i\right]^T \quad (47)$$

we can concatenate equations (38), (42) and (45) in a linear extended equation system heaving a regular form:

$$\dot{\widetilde{\mathbf{x}}}_i = \mathbf{A}\widetilde{\mathbf{x}}_i + \mathbf{B}\mathbf{u}_i;\ \mathbf{u}_i = -\mathbf{K}\widetilde{\mathbf{x}}_i, \quad (48)$$

where $\mathbf{A}$ elements are:

$$a_{1,1} = \frac{2F_0}{V}\left(2kmg^2\cos^2\gamma F_0^{-3} - \frac{C_{D0}}{m}\right);$$

$$a_{1,2} = -g\cos\gamma\,;\, a_{2,2} = \frac{g}{V}\sin\gamma\,;\, a_{4,1} = -1\,;\; a_{4,6} = \dot{\gamma}\,;$$

$$a_{5,3} = -V\cos\gamma\,;\, a_{6,2} = V\,;\; a_{6,4} = -\dot{\gamma}\,;$$

$$a_{7,4} = 1\,;\, a_{8,5} = 1\,;\, a_{9,6} = 1\,; \qquad (49)$$

B elements are:

$$b_{1,1} = \frac{1}{m};\; b_{1,2} = -2kmg^2F_0^{-2}\cos\gamma\,;$$

$$b_{2,2} = g/V\;;\; b_{3,3} = g/V\,; \qquad (50)$$

K elements are:

$$k_{1,1} = Gk_u^V\,;\; k_{1,4} = -Gk_u^{hx}\,;\; k_{1,7} = -Gk_u^{Ix}\,;$$

$$k_{2,2} = Vk_u^\gamma/g + \sin\gamma\,;\; k_{2,6} = Vk_u^{hz}/g\,;$$

$$k_{2,9} = Vk_u^{Iz}/g\,;\, k_{3,3} = Vk_u^\chi/g\,;\; k_{3,5} = -Vk_u^{hy}/g\,;$$

$$k_{3,8} = -Vk_u^{Iy}/g\,. \qquad (51)$$

8 Guidance law synthesis using state control

In order to obtain command law coefficients we suppose a horizontal evolution with $\gamma = 0$; $\dot{\gamma} = 0$ and neglect terms that contains negative power of F_0. In these hypothesis, derivating twice linear equations (38) we obtain.

$$\Delta\dddot{V}_i \cong a_V^V\Delta\ddot{V}_i - g\Delta\ddot{\gamma}_i + \frac{\Delta\ddot{T}_i}{m}$$

$$\Delta\dddot{\gamma}_i \cong \frac{g}{V}\Delta\ddot{n}_i$$

$$\Delta\dddot{\chi}_i = \frac{g}{V}\Delta\ddot{\mu}_i \qquad (52)$$

where:

$$a_V^V = -\frac{\rho SVC_{D0}}{m} \qquad (53)$$

In the other hand, starting from pseudo-command equations (45), if we insert guidance parameter relations (42) and taking into account the integral relations (43), and derivate twice relations (45) , we will obtain:

$$\Delta\ddot{T}_i = -Gk_u^V\Delta\ddot{V}_i - Gk_u^{hx}\Delta\dot{V}_i + Gk_u^{Ix}\Delta V_i + Gk_u^V\ddot{V}_r$$

$$\Delta\ddot{n}_i = -\frac{V}{g}k_u^\gamma\Delta\ddot{\gamma}_i - \frac{V^2}{g}k_u^{hz}\Delta\dot{\gamma}_i - \frac{V^2}{g}k_u^{Iz}\Delta\gamma_i + \frac{V}{g}k_u^\gamma\Delta\ddot{\gamma}_r$$

$$\Delta\ddot{\mu}_i = -\frac{V}{g}k_u^\chi\Delta\ddot{\chi}_i - \frac{V^2}{g}k_u^{hy}\Delta\dot{\chi}_i - \frac{V^2}{g}k_u^{Iy}\Delta\chi_i + \frac{V}{g}k_u^\chi\Delta\ddot{\chi}_r \qquad (54)$$

Inserting pseudo-commands (54) in linear equations (52), we obtain:

$$\Delta\dddot{V}_i + \left(gk_u^V - a_V^V\right)\Delta\ddot{V}_i + gk_u^{hx}\Delta\dot{V}_i + gk_u^{Ix}\Delta V_i = gk_u^V\Delta\ddot{V}_r - g\Delta\ddot{\gamma}_i$$

$$\Delta\dddot{\gamma}_i + k_u^\gamma\Delta\ddot{\gamma}_i + Vk_u^{hz}\Delta\dot{\gamma}_i + Vk_u^{Iz}\Delta\gamma_i = k_u^\gamma\Delta\ddot{\gamma}_r$$

$$\Delta\dddot{\chi}_i + k_u^\chi\Delta\ddot{\chi}_i + Vk_u^{hy}\Delta\dot{\chi}_i + Vk_u^{Iy}\Delta\chi_i = k_u^\chi\Delta\ddot{\chi}_r \qquad (55)$$

We can observe that for the hypothesis related to horizontal flight the last two equations are symmetrical, so we can expect to have the same coefficients in the command law. For the first equations (55) in order to simplify the problem we can neglect the second input and we will obtain a relation with a single input and a single output (SISO). In this case, after Laplace transformation, the system (55) becomes:

$$H_V(s) = \frac{gk_u^V s^2}{s^3 + (gk_u^V - a_V^V)s^2 + gk_u^{hx}s + gk_u^{Ix}}$$

$$H_\gamma(s) = \frac{k_u^\gamma s^2}{s^3 + k_u^\gamma s^2 + Vk_u^{hz}s + Vk_u^{Iz}}$$

$$H_\chi(s) = \frac{k_u^\chi s^2}{s^3 + k_u^\chi s^2 + Vk_u^{hy}s + Vk_u^{Iy}} \qquad (56)$$

We consider an optimal allocation for poles of this kind of function indicate in [4]:

$$P(s) = s^3 + 6.7\Omega_0 s^2 + 6.7\Omega_0^2 s + \Omega_0^3\;, \qquad (57)$$

where the pulsation is given by:

$$\Omega_0 = 1.5/t_r\,, \qquad (58)$$

and t_r is settling time.

In this case, identifying between (56) and

(57) we obtain:

$$k_u^V = \left(6.7\Omega_0 + a_V^V\right)/\,g\,;\; k_u^{hx} = 6.7\Omega_o^2\,/\,g\,;$$
$$k_u^{Ix} = \Omega_o^3\,/\,g\,; k_u^{\gamma} = k_u^{\chi} = 6.7\Omega_0\,;$$
$$k_u^{hz} = k_u^{hy} = 6.7\Omega_o^2\,/\,V\,;\; k_u^{Iz} = k_u^{Iy} = \Omega_o^3\,/\,V \tag{59}$$

9 Optimal control

If we suppose to have access to extended state vector $\widetilde{\mathbf{x}}_i$, we can obtain directly the controller $\mathbf{K}$ for optimal command:

$$\mathbf{u} = -\mathbf{K}\mathbf{x} \tag{60}$$

in order to satisfy the linear quadratic performance index (cost function):

$$\min J = \int_0^{\infty} (\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u})\,\mathrm{d}\,t\,, \tag{61}$$

where the pair $(\mathbf{A},\mathbf{B})$ is controllable and the state weighting matrix $\mathbf{Q}$ is symmetric and quasi positive:

$$\mathbf{Q} \geq 0;\; \mathbf{Q} = \mathbf{Q}^T\,. \tag{62}$$

while the control weighting matrix $\mathbf{R}$ is symmetric and positive:

$$\mathbf{R} > 0;\; \mathbf{R} = \mathbf{R}^T\,; \tag{63}$$

In this case, the following relation gives the optimal controller:

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \tag{64}$$

where the matrix $\mathbf{P}$ is the solution of the algebraic Riccati equation:

$$\mathbf{A}^{\mathbf{T}}\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{65}$$

10 Input data and results

10.1 Input data for the model

10.1.1 Geometrical and mechanical data

In figure 3 are shown the main geometrical data of the UAV. All data are in meters.

Reference area for the model – cross body area is: $S = 0.02925\ m^2$

Initial mass for the model is: $m_i = 11kg$;

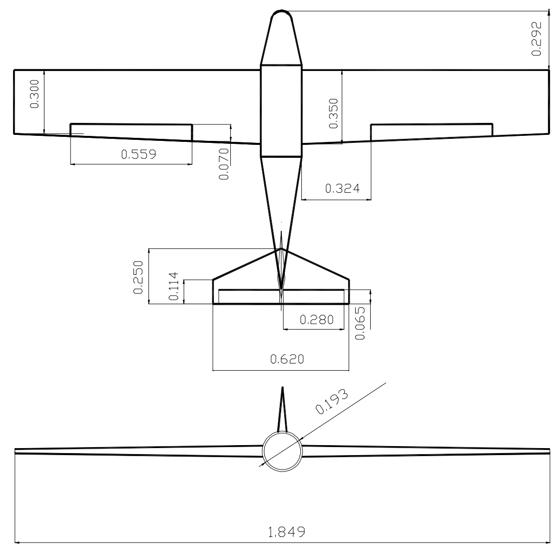

Fig. 3 Geometrical data

10.1.2 Aerodynamic data

If we use the coefficients in aerodynamic frame like in equations (3), (4), (5), the polar relation between drag coefficient and lift coefficient is given by (6) where: $C_{D0} = 1.44$, $k = 0.000121$. The reference area is the cross area of the body.

10.1.3 Guidance data

If we choose settling time for all parameters $t_r = 3s$, the pulsation from relation (58) is $\Omega_0 = 0.5s^{-1}$. The velocity is $V = 20\,m/s$ the gains from the relations (59) became:

$$k_u^V = 0.33\,;\; k_u^{hx} = 0.17\,; k_u^{Ix} = 0.013$$
$$k_u^{\gamma} = k_u^{\chi} = 3.35\,;$$
$$k_u^{hy} = k_u^{hz} = 0.084\,;\; k_u^{Iy} = k_u^{Iz} = 0.0063\,,$$

and the state controller using relations (51) becomes:

Table 1 State controller

35.928	0.	0.	-18.425	0.	0.	-1.375	0.	0.
0.	6.83	0.	0.	0.	0.17	0.	0.	0.0127
0.	0.	6.83	0.	-0.17	0.	0.	-0.0127	0.

If we use optimal control (64), the controller matrix becomes directly:

Table 2 Optimal controller

6.775	-0.346	0.	-2.84	0.	3.2096	-0.538	0.	0.843
-2.24	11.91	0.	2.237	0.	0.6379	0.843	0.	0.537
0.	0.	10.904	0.	-1.45	0.	0.	-1.	0.

10. 2 Results

Test case for which we obtained the results is described by the following parameters:

$$V = 20\,m/s\,;\ z_0 = 400\,m\,;\ \gamma = 0\,;\ \dot{\gamma} = 0\,.$$

In table 1 there are presented eigenvalues for the stability matrix. It can be observe that the real parts of the eigenvalues for the close loop system are negatives, which prove the good stability of it.

Table 3 Eigenvalues for the stability matrix

A		**A – BK State control**		**A – BK Optimal control**	
Re	**Im**	**Re**	**Im**	**Re**	**Im**
-0.084	0	-2.76	0	-2.3	2.38
0	0.69×10^{-8}	-0.5	0	-2.3	-2.38
0	-0.69×10^{-8}	-0.09	0	-0.26	0.31
0	0	-2.76	0	-0.26	-0.31
0	0	-0.5	0	-0.58	0
0	0	-0.09	0	-0.85	0
0	0	-2.76	0	-2.17	2.26
0	0	-0.5	0	-2.17	2.26
0	0	-0.09	0	-0.99	0

Figures 4, 6 and 8 present the system answer using a state controller in different perturbation cases.

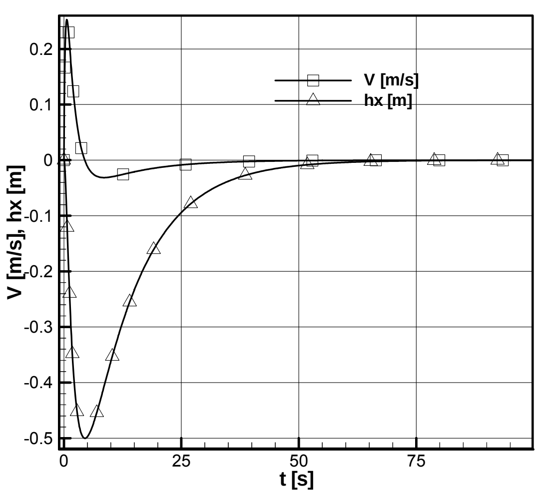

Fig. 4 UAV formation answer for velocity perturbation, state controller

Figures 5, 7 and 9 also present the formation answer, but in these cases using an optimal controller.

Figures 4 and 5 present the formation answer for velocity perturbation. It can see that the answer is similar in the two control cases.

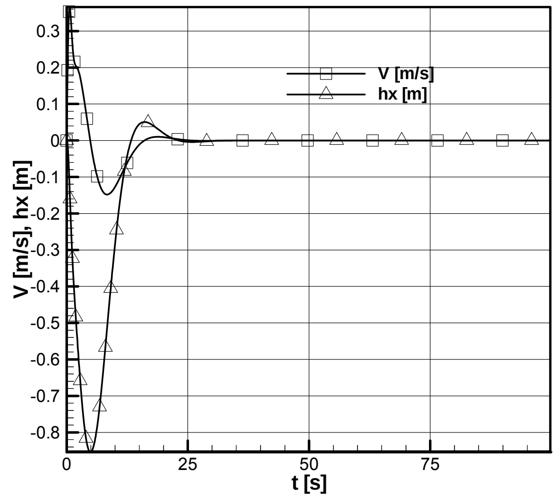

Fig. 5 UAV formation answer for velocity perturbation, optimal controller

Figures 6 and 7 show the formation answer for climb angle perturbation in both control cases. The answers are also similar.

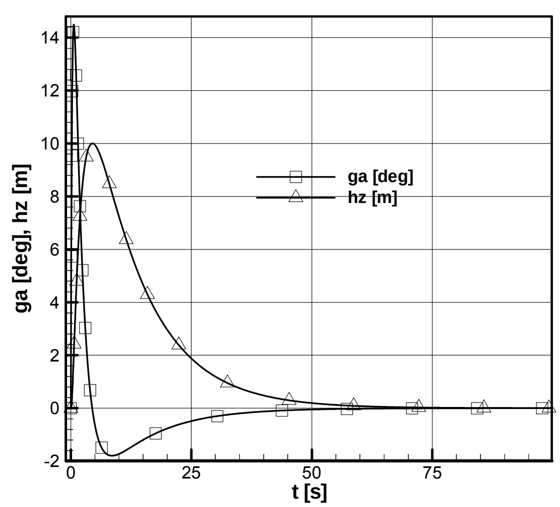

Fig. 6 UAV formation answer for climb angle perturbation, state controller

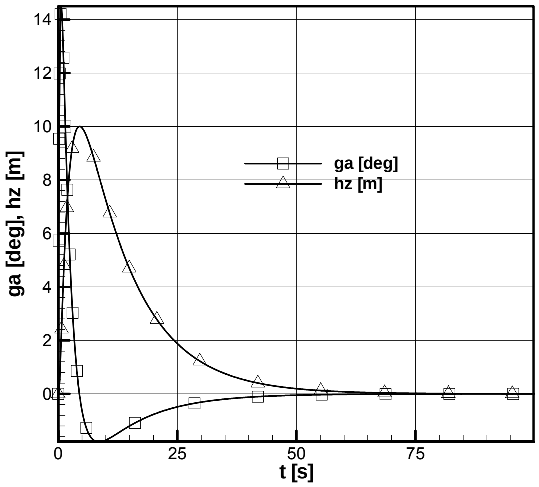

Fig. 7 UAV formation answer for climb angle perturbation, optimal controller

Finally, figures 8 and 9 show the formation answer for air-path angle perturbation. The answer using optimal control is better, having a shorter settling time.

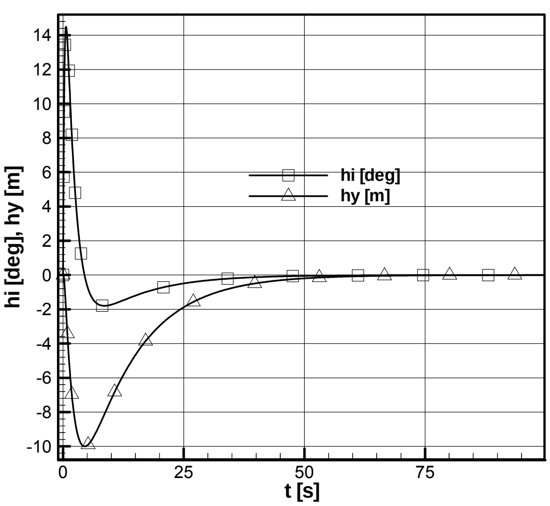

Fig. 8 UAV formation answer for air-path angle perturbation, state controller

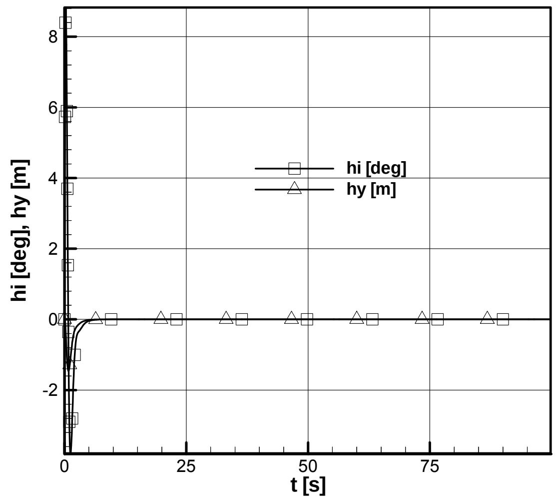

Fig. 9 UAV formation answer for air-path angle perturbation, optimal controller

11 Conclusions

To approach the UAV's formation stability problem we developed a linear model with 3 degrees of freedom starting from paper [1], [2] and [3]. Analyzing the stability matrix, we obtained the eigenvalues with real parts negative, which prove the stability of the close loop system. Further, the guidance law adopted allows the system to have a correct answer for the common perturbation, as we can see in figures 4, 6, and 8. Figures 5,7 and 9 show that the optimal control used provide good answers for common perturbations, especially for lateral evolution when the answer is better than that obtained by state controller. This preliminary results show that it can be possible to control UAV formation, which can be stable at the same time. In addition, the approach adopted in work [1], [2] and [3] characterized by a model of a generic "i" UAV from the formation, can finally solve the problem of the entire formation.

In the next work, we will try to build a complex linear model in order to analyze the stability of the complex system as we have described in paper [3]

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