Design of Control Systems Based on Vector Error

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Abstract: a new approach to the design of control systems is offered. This approach based on definition the vector error allowing combining in real time a stage of identification of unknown plant and calculation of control. The measurement method of a vector error with use of Hilbert transform also it is shown.

Keywords: Vector error, identification, nonlinear system, Hilbert transform, stability.

1. INTRODUCTION

Development of control systems is directed on design adaptive and robust controllers allowing providing required quality of control for unknown, nonlinear plants at various conditions and restrictions. The review of the literature on control systems shows, that alongside with modern directions adaptive and robust controllers [1-9], such as fuzzy control, artificial neural networks, genetic algorithms, H^2, H^∞, μ-synthesis, etc. for plant control in the industry, power systems, transport and other still remains to the most widespread classical PID controller structures [10-16]. It is caused by simplicity of its design and tuning. However for PID control cannot be taken into account the basic features of nonlinear systems:

- The superposition principle is not carried out. The research of nonlinear systems at several influences cannot be reduced to research at the sum of single influence;
- The plant stability to depend at initial deviation from position of balance;
- At the fixed external influences some positions of balance are possible.

Development of control systems both on the basis of modern control algorithms and with use of the PID control demands an exact model of plant. For this purpose it is necessary to execute identification of plant: defined its dynamic behaviors - change of an outputs ***y*** at all possible changes of an inputs ***u***, noises ***n*** and disturbances ***d.*** Performance of the given problem theoretical is possible, however in practice in all cases is impracticable on restrictions of technological character and impossibility to predict ***n*** and ***d***.

Therefore control systems development, providing quality, stability and robust control achievable on the basis of last modern algorithms and having simplicity of a design inherent in the PID control, is perspective. Performance of the given requirements probably if identification of plant to carry out continuously and changing parameters or even structure of controller depending at received results. Taking into account complexity of sensors installation for measurement state variables of plant, identification is preferable to carry out used information about output change of plant ***Δy*** depending on an input change of plant ***Δu*** at real parameters ***n*** and ***d***. For this purpose it is necessary to measure not only ***y*** and ***u***, but also time (dynamic) relations between changes of an input and an output under influence ***n*** and ***d***.

The decision of the given scientific problem can be achieved if an error of control system to present as a vector [17, 18]. The ***Vector Error*** of control system is a difference vector between an input vector of controller (vector of reference ***r***) and an output vector of plant that allows identifying control system (controller + plant) at influences of noises ***n*** and disturbances ***d*** and to take into account delays and the unknown order of plant. On each step of control change of the real component of vector error defines intensity of a control while the argument or a phase of a vector error defines character of a feedback in control system - positive, negative or its absence on some steps of control - i.e. feedforward control.

In paper is considered the opportunity application a Hilbert transform for measurement vector error of control system. All input and an output of control system are represented as analytical signals. It is allows to apply the Hilbert transform. Using the given transform, differences of instant phases for all required combinations of inputs - outputs of plant can be determined. On the basis of the received differences of instant phases of vector errors and their components are calculated the optimum control for each of inputs.

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The paper includes the following sections. In the second section definition of a vector error of control system and the technical method of its measurement is shown. This method based on representation of inputs - outputs of plant as analytical signals and use of Hilbert transform for calculation of differences of their instant phases. In the third section the algorithm and block diagram of a vector controller and definition stability of control system with a vector controller is considered. Examples of application an offered vector controller for control of the first order plant with a transport delay and also more complex example as an automatic voltage regulator (AVR) of the synchronous generator with use of SIMULINK are resulted in the fourth section. Conclusions are presented in Sec.5

2. THE VECTOR ERROR OF CONTROL SYSTEM

Modern control systems for maintenance their adaptive and robust properties should be created on the basis of the algorithms which are not demanding the detailed aprioristic information about plant, capable to make identification, structural and parametrical optimization based on continuous measurement inputs and outputs of plant. The control system with disturbance ***d*** and noise ***n*** is given in Fig. 1.

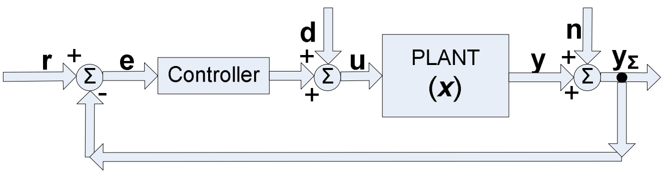

Fig. 1. Control system with disturbance ***d*** and noise ***n***.

where ***r*** - reference, $\boldsymbol{e = r - y_\Sigma}$ - a control error, ***u*** - a control input, ***y*** – a plant output, dependent at state variables ***x***, ***d*** - disturbance, ***n*** - noise, $\boldsymbol{y_\Sigma}$ - a measured output of plant. For identification based on the information about inputs and outputs it is determined the vector error ***e*** which is function not only difference of the modules reference ***r*** and a measured output of plant $\boldsymbol{y_\Sigma}$, but also the difference phases between them $\boldsymbol{e = r - y_\Sigma} = e_R + je_I$. Thus the module e and argument $\Delta\varphi$ a vector error, its real e_R and imaginary e_I components will be defined according to geometrical ratio:

$$\begin{aligned} &e^2 = r^2 + y_\Sigma^2 - 2 \cdot r \cdot y_\Sigma \cdot \cos(\Delta\phi); \\ &\Delta\phi = arctg\left(\frac{e_I}{r + e_R}\right) = \arcsin\left(\frac{r + e_R}{y_\Sigma}\right), \\ &e_R = r - y_\Sigma \cdot \cos(\Delta\phi); \\ &e_I = y_\Sigma \cdot \sin(\Delta\phi) \end{aligned} \qquad (1)$$

where r - the module of a reference vector concerning which displacement of an output of plant is determined, y_Σ - the module of an output of plant, $\Delta\varphi$ - difference phase between a reference vector and an output vector. Thus, the real part of a vector error e_R takes into account dynamics of plant $y_\Sigma cos\Delta\varphi$. In a case $\Delta\varphi = 0$ value of the real part of a vector error coincides with traditional definition of control error with a negative feedback $e_R = r - y_\Sigma$. In a case $\Delta\varphi = \pm \pi/2$ the real component of a vector error $e_R = r$ and it is feedforward control. In a case $\Delta\varphi = \pm \pi$ value of the real part of a vector error coincides with traditional definition of control error for positive feedback $e_R = r + y_\Sigma$. Graphic interpretation of a vector error of control system shown in Fig. 2:

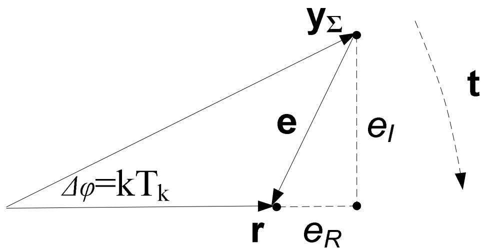

Fig. 2. Graphic interpretation of a vector error.

The difference of instant phases between an output and an input can be determined with use of Hilbert transform, representing an input and an output as analytical signals. As is known [19, 20], the analytical signal represents the sum of two orthogonal signals with components are shifted on 90º to each other. For analytical signal can be determined the instant phase and instant frequency. The imaginary part of an analytical signal $Z_s(t)$ is analytically connected with its real part $Re[Z_s(t)] = s(t)$ through Hilbert transform HT: $Im[Z_s(t)] = s'(t) = HT[s(t)]$ and accordingly the analytical signal is represented as: $Z_s(t) = s(t) + j\, s'(t)$. The difference of instant phases of two signals $s_1(t)$ and $s_2(t)$ can be determined with use of Hilbert transform as [21]:

$$\Delta\phi_{12} = \phi_1(t) - \phi_2(t) = \arctan\frac{\tilde{s}_1(t)\cdot s_2(t) - s_1(t)\cdot \tilde{s}_2(t)}{s_1(t)\cdot s_2(t) + \tilde{s}_1(t)\cdot \tilde{s}_2(t)}. \qquad (2)$$

For control system with single input and single output (SISO) a difference of their instant phases:

$$\Delta\phi_{yr} = \phi_y(t) - \phi_r(t) = \arctan\frac{\tilde{y}(t)\cdot r(t) - y(t)\cdot \tilde{r}(t)}{y(t)\cdot r(t) + \tilde{y}(t)\cdot \tilde{r}(t)}, \qquad (3)$$

In case of control system with a reference vector of k dimensions $\boldsymbol{r_k}$ and an output vector of m dimensions $\boldsymbol{y_m}$ (MIMO) the difference of instant phases can be determined similarly:

$$\Delta\phi_{mk} = \phi_m(t) - \phi_k(t) = \arctan\frac{\tilde{y}_m(t)\cdot r_k(t) - y_m(t)\cdot \tilde{r}_k(t)}{y_m(t)\cdot r_k(t) + \tilde{y}_m(t)\cdot \tilde{r}_k(t)}, \ (4)$$

The calculated differences of instant phases (3), (4) can be used for identification of plant, calculation of a vector error and its components according to (1) and control of plant.

3. CONTROL SYSTEM WITH VECTOR CONTROLLER

Control system with a vector controller based on the Hilbert transform shown in Fig. 3:

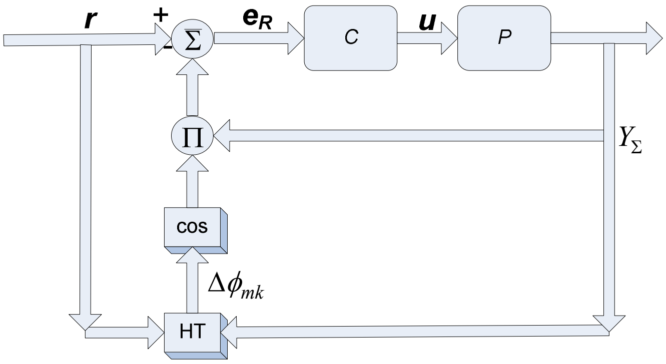

Fig. 3. Control system with a vector controller based on the Hilbert transform.

where C - controller, P - plant, HT – block the Hilbert transform and calculate of a difference instant phases between input and output, $\boldsymbol{r}$ – a reference vector, $\boldsymbol{e_R}$ – a real component of the vector error, $\boldsymbol{u}$ – a control inputs, $\boldsymbol{y_\Sigma}$ – an output, $\Delta\varphi_{mk}$ - a differences of instant phases. The algorithm of vector controller given in Fig. 4:

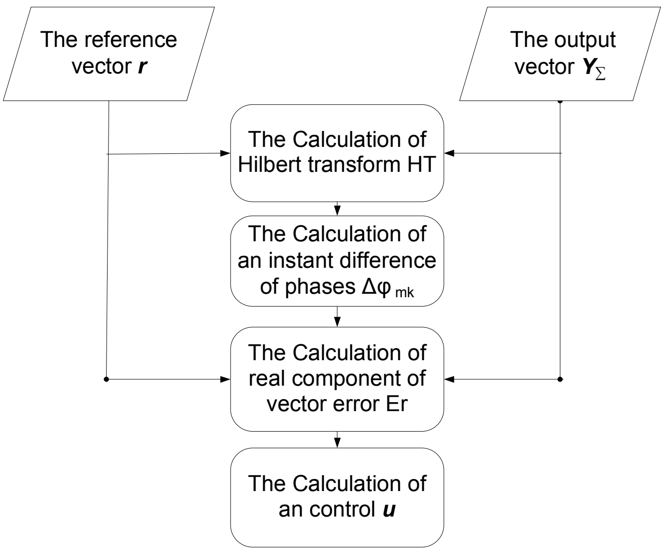

Fig. 4. Algorithm of vector controller

The control with the account (1), can be submitted as:

$$u(t) = K_V e_R(t) = K_V\left(r - y_\Sigma \cdot \cos(\Delta\phi)\right), \qquad (5)$$

where K_V – the gain of vector controller.

Let's consider nonlinear control system, Fig.5 ($y_\Sigma = y$):

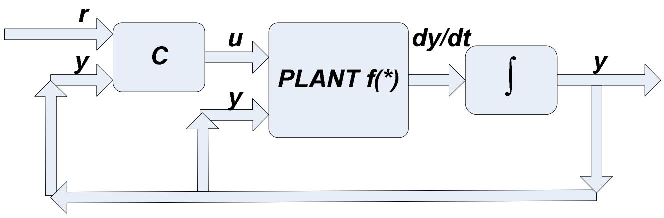

Fig. 5. The nonlinear control system.

$$\dot{y}(t) = y(t) \cdot f(*) + u(t). \qquad (6)$$

With the account (5) dynamics of nonlinear control system can be submitted as:

$$\dot{y}(t) = y(t) \cdot f(*) + K_V\left(r - Y_\Sigma \cdot \cos(\Delta\phi)\right). \qquad (7)$$

The gain K_V is determined from the stability analysis of control system. For this purpose we shall define the Lyapunov's function in the square-law form concerning the real component of vector error and a control input, Fig. 6:

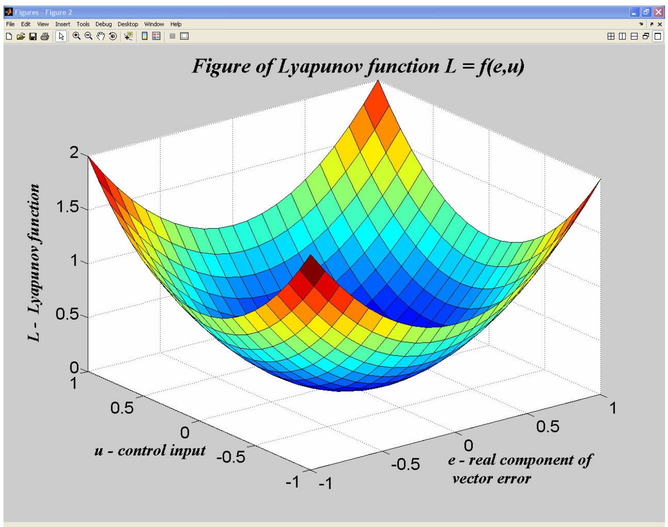

Fig.6. The Lyapunov's function determined by values of the real component of vector error and an control input.

$$L(e_R, u) = e_R^{\,2} + u^2 = \left(1 + K_V^2\right)e_R^{\,2} = \left(1 + K_V^2\right) \cdot \left(r^2 + y^2\cos^2(\Delta\phi) - 2yr\cos(\Delta\phi)\right) \qquad (8)$$

The Lyapunov's function is positively determined. We shall calculate a derivative of Lyapunov's function (8) for control system (6) along trajectories of change the real component of vector error e_R and an control input u:

$$\dot{L}(e_R,u)=2\cdot\left(1+K_V^2\right)\cdot \cdot\begin{pmatrix} r\cdot y\cdot\sin(\Delta\phi)-r\cdot\dot{y}\cdot\cos(\Delta\phi)+ \\ +y\cdot\cos^2(\Delta\phi)-y^2\cdot\cos(\Delta\phi)\cdot\sin(\Delta\phi)\end{pmatrix} \quad (9)$$

According to Lyapunov's theorem [1,2] stability of control system is provided, if the derivative of function (9) will be negatively determined:

$$\begin{pmatrix} r\cdot y\cdot\sin(\Delta\phi)-r\cdot\dot{y}\cdot\cos(\Delta\phi)+ \\ +y\cdot\cos^2(\Delta\phi)-y^2\cdot\cos(\Delta\phi)\cdot\sin(\Delta\phi)\end{pmatrix}<0$$

Substituted value (7), we shall receive a stability condition of nonlinear control system with a vector controller:

$$K_V<\frac{y}{r}\cdot\frac{(r\cdot f(*)+y\cdot\sin\Delta\phi)-(\cos\Delta\phi+r\cdot tg\Delta\phi)}{r+y\cdot\cos\Delta\phi} \quad (10)$$

We research an inequality (10) at change of a difference of phases *Δφ*:

$$\Delta\phi\to 0\Rightarrow K_V<\frac{y}{r}\cdot\frac{r\cdot f(*)-1}{r+y},$$
$$\Delta\phi\to\pm\frac{\pi}{2}\Rightarrow K_V<\infty, \quad (11)$$
$$\Delta\phi\to\pm\pi\Rightarrow K_V<\frac{y}{r}\cdot\frac{r\cdot f(*)+1}{r-y}$$

In case $r f(*) >> 1$ inequalities (11) determining stability of nonlinear control system can be simplified:

$$\Delta\phi\to 0\Rightarrow K_V<\frac{y\cdot f(*)}{r+y},$$
$$\Delta\phi\to\pm\frac{\pi}{2}\Rightarrow K_V<\infty, \quad (12)$$
$$\Delta\phi\to\pm\pi\Rightarrow K_V<\frac{y\cdot f(*)}{r-y}$$

Inequalities (10) - (12) allow defining allowable area of values the gain of vector controller K_V for nonlinear control system proceeding only from the measured values of inputs - outputs and a difference of phases between them.

Further are considered an example of application a vector controller for control of the first order plant with a transport delay, and also more complex example of application as an automatic voltage regulator (AVR) of the synchronous generator.

4. THE APPLICATIONS OF VECTOR CONTROLLER

4.1 The control system for first order plant with a transport delay.

Let's consider the first order plant with a transport delay:

$$W(s)=\frac{K_p}{1+sT}e^{-sL}, \quad (13)$$

where s - complex frequency, $K_P = 1$ – gain of plant, $T = 1$ - time constant of plant , $L = 0.3$ - transport delay of plant. The optimum values of the PID controller for this plant have been determined used method CHR (Chien, Hrones, Reswick) [22] and the manual adjustment: K = 3, $T_i = 1$, $T_d = 0.0452$. The control system with the plant (13) it has been simulated with use of SIMULINK and it is shown on Fig. 7:

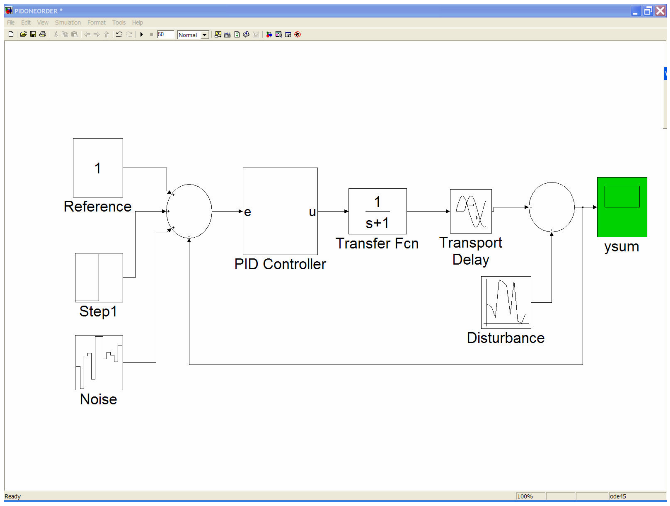

Fig. 7. The control system for first order plant with a transport delay.

The measurement noise of control system is submitted as block "Band-Limited White Noise" at: *Noise power = 0.001, Sample time = 0.01, Seed = 23341*. Disturbance of control system is submitted as block "Random Number" with parameters: *Mean = 0, Variance = 0.001, Initial seed = 0*. The PID controller is shown on Fig. 8:

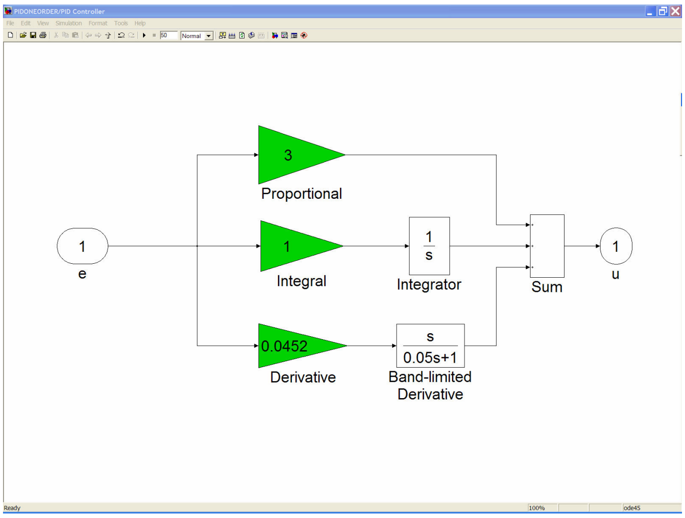

Fig. 8. PID controller for plant (13)

Properties of the control system were investigated at submission of measurement noise and disturbance at reference r = 1 and the submission a step signal at the time t = 15s. Received transient for plant with the nominal parameters is shown on Fig. 9:

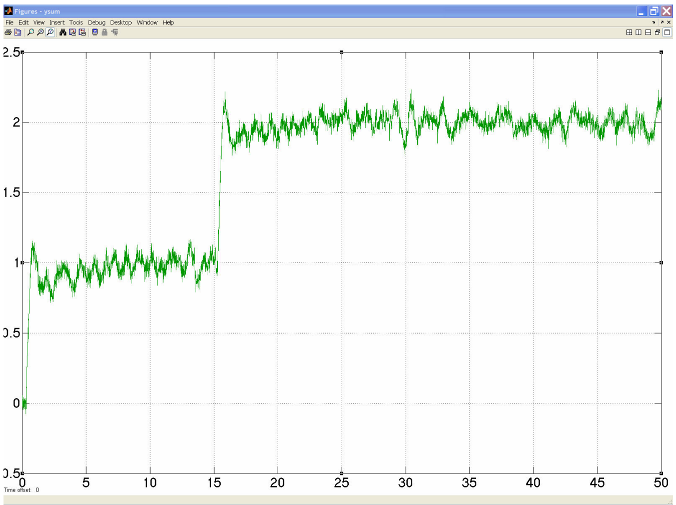

Fig. 9. The transient for plant with the nominal parameters.

The properties of control system were investigated at change parameters of plant (16) in range $K_P = 0.5$ - 2, $T = 0.5$ - 2, $L = 0.3$ - 1. The received results, Figs. 10-14, shown low adaptive and robust properties of control system with a PID controller adjusted for plant with nominal parameters:

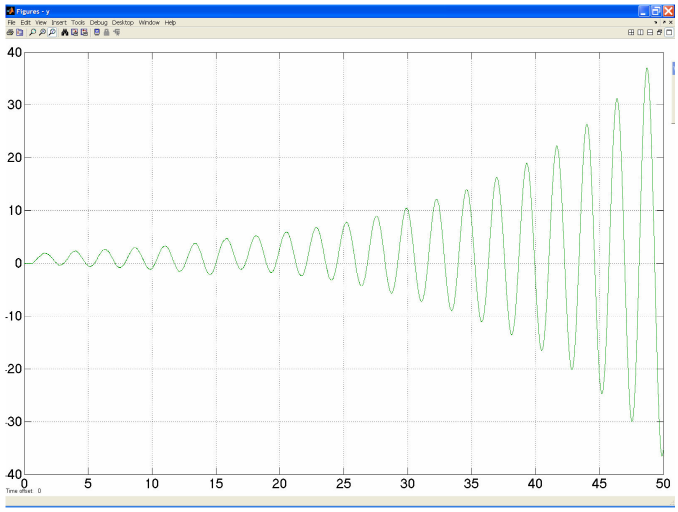

Fig. 10. The transient for plant (13) at $L = 0.7$.

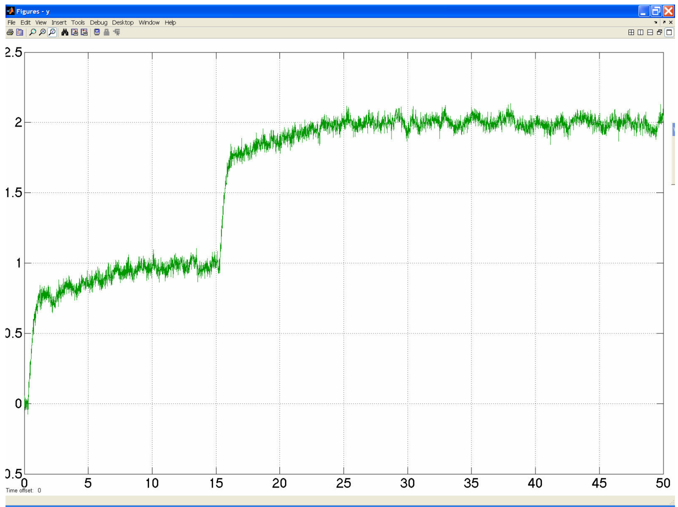

Fig. 11. The transient for plant (13) at $K_P = 0.5$.

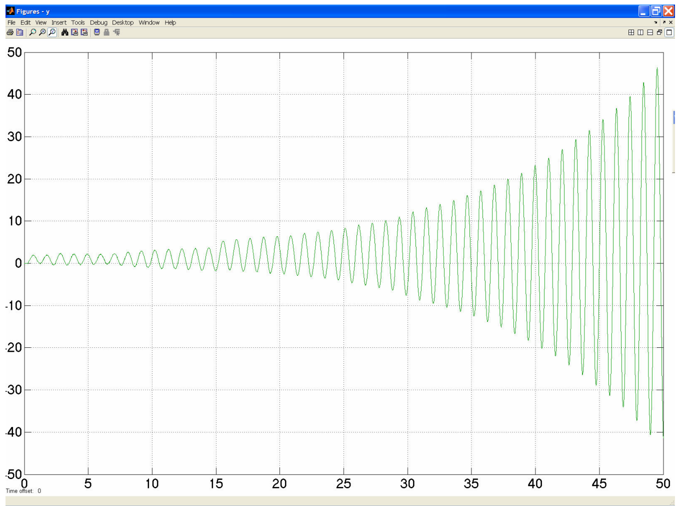

Fig. 12. The transient for plant (13) at $K_P = 2$.

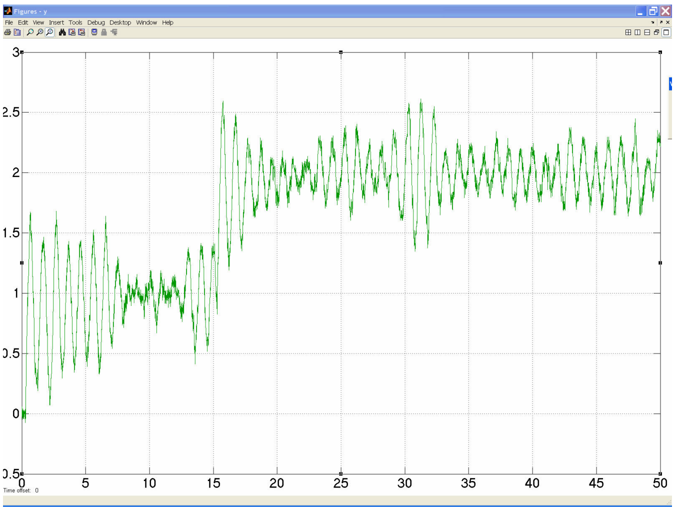

Fig. 13. The transient for plant (13) at $T = 0.5$.

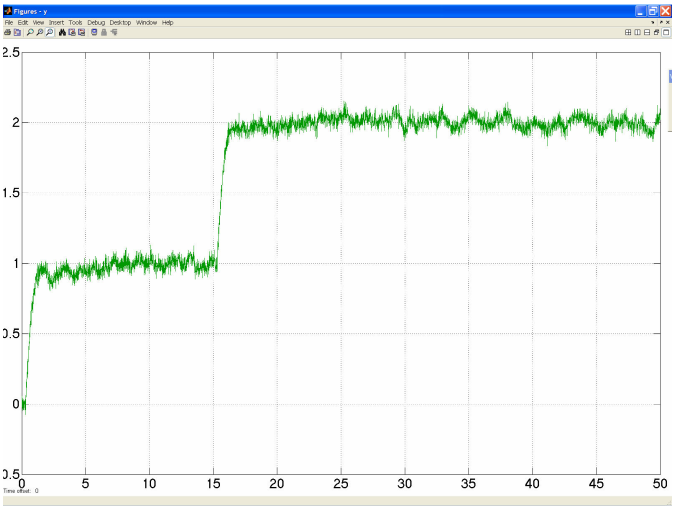

Fig. 14. The transient for plant (13) at $T = 2$.

Further it is considered the control system for plant (13) and the vector controller (5), shown on Fig. 15, 16:

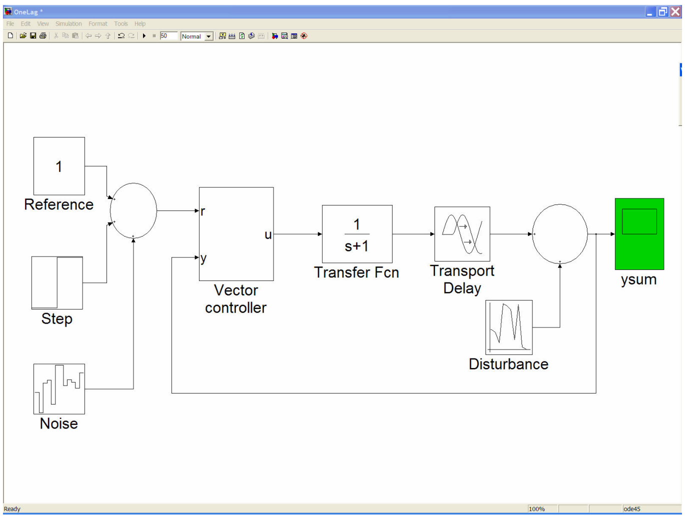

Fig. 15. The control system with a vector controller.

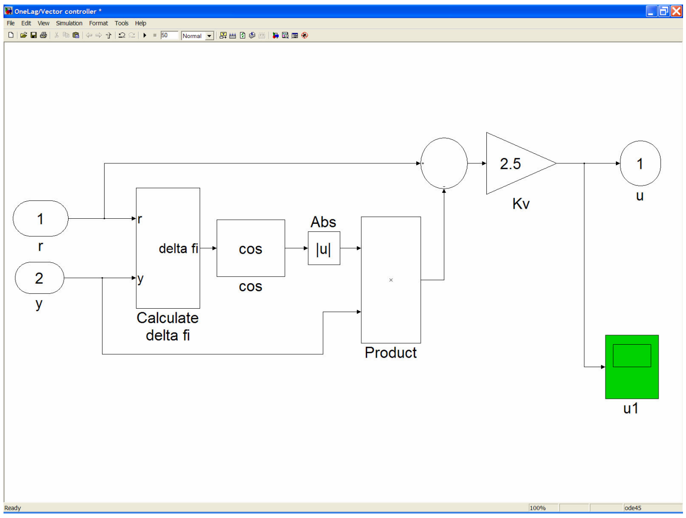

Fig. 16. The vector controller for plant (13)

For measurement a difference of phases between a reference and an output according to (1) and (3) the circuit shown on Fig. 17 has been developed:

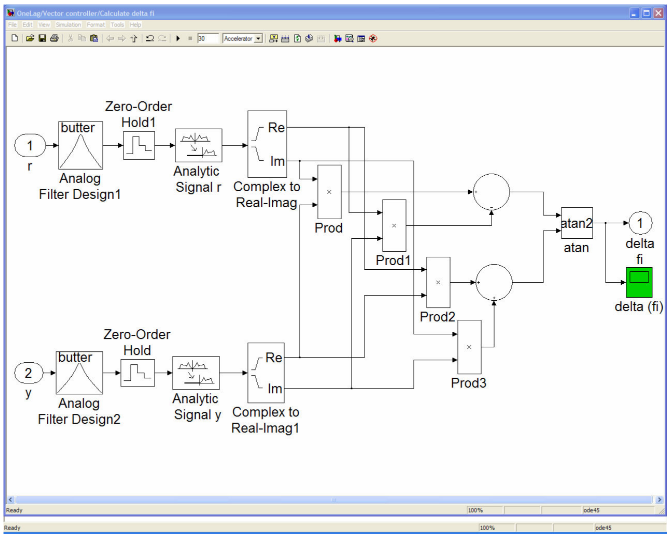

Fig. 17. The circuit for measurement a difference of phases based on the Hilbert transforms.

Properties of control system with a vector controller were investigated under the same conditions as for control system with the adjusted PID controller. Value of gain $K_V = 2.5$ was accepted proceeding from (10) - (12). The transient for plant with nominal parameters is shown on fig. 18:

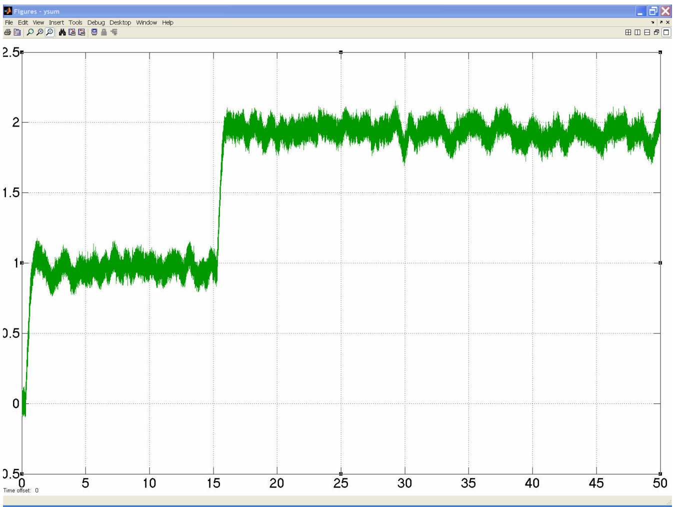

Fig. 18. The transient for control system with vector controller and plant with the nominal parameters.

Properties the control system with a vector controller was investigated at similar changes of plant (13). The received results, Figs. 19-23, have shown high adaptive and robust properties control system with the vector controller (5):

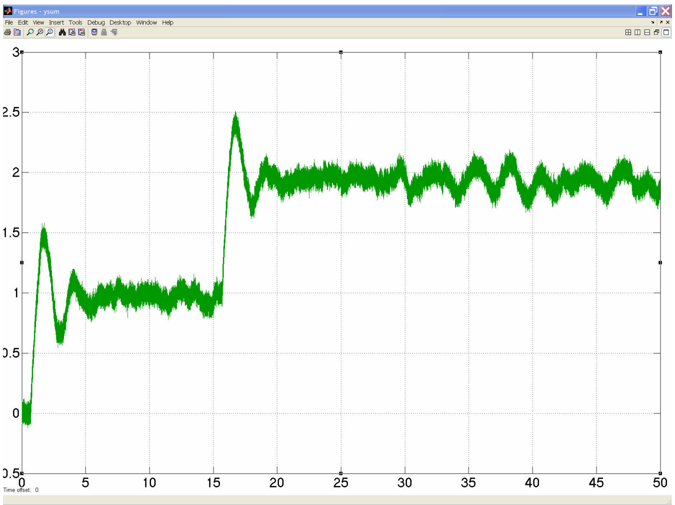

Fig. 19. The transient for plant (13) at L = 0.7 with vector controller.

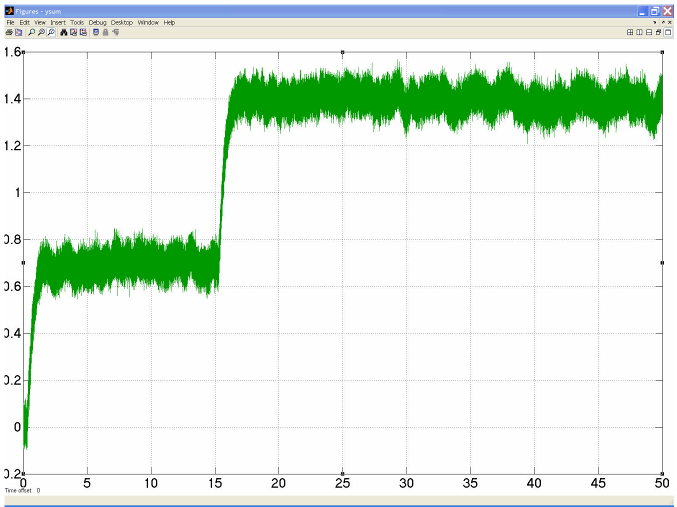

Fig. 20. The transient for plant (13) at K_P = 0.5 with vector controller.

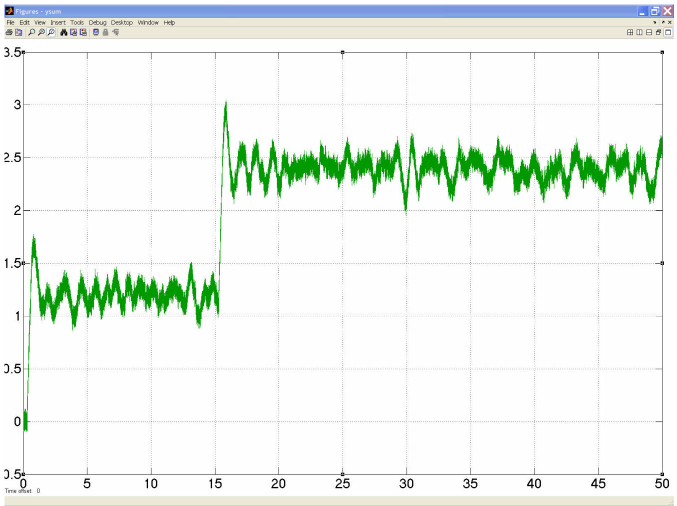

Fig. 21. The transient for plant (13) at K_P = 2 with vector controller.

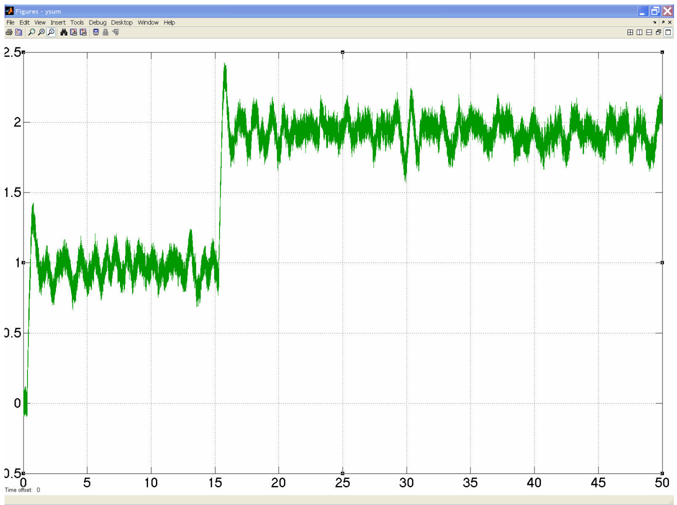

Fig. 22. The transient for plant (13) at T = 0.5 with vector controller.

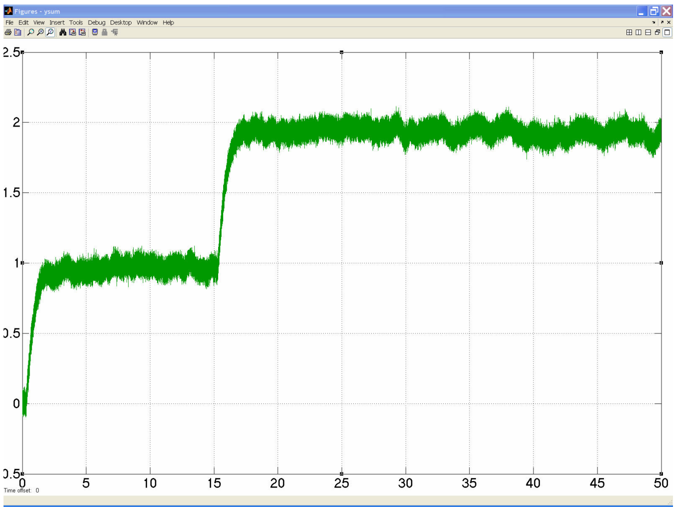

Fig. 23. The transient for plant (13) at $T = 2$ with vector controller.

The control system with vector controller for first order plant with a transport delay provides comprehensible quality of control and stability at change of plant parameters in significant ranges. The divergence of plant output from required at change of gain plant K_P can be eliminated by introduction the adaptive linear gain K_V from an plant output as the given divergence has linear character.

4.2 The vector automatic voltage regulator of the synchronous generator.

The offered vector controller is applied to model « synchronous machine – infinite bus» (SMIB) as an automatic voltage regulator (AVR) of the synchronous generator. The model "power_turbine.mdl" SIMULINK represents work of the three-phase synchronous hydrogenerator with nominal parameters: full capacity 200 MVA, rated voltage 13.8 кV, nominal frequency 112.5 min^{-1} connected to a power system by power 10000 MVA and voltage 220 кV through the block transformer and a long transmission line with a ratio of inductive and active resistance X/R = 10. A three-phase short circuit in a network 220 кV by duration 0.2s and also submission a step signals on inputs references of terminal voltage V_t and active power P_{ref} was simulated. The size of step signals varied in a range of 0.1-0.3 p.u. for reference of terminal voltage and 0.05-0.15 p.u. for active power. AVR represents a vector controller with input $\boldsymbol{\Delta V} = \boldsymbol{V_r} - \boldsymbol{V_t}$, where $\boldsymbol{V_t}$ - a vector of a terminal voltage, $\boldsymbol{V_r}$ - a vector of reference. For measurement of phase difference $\Delta\varphi$ between of this vectors simulated the equation (3), shown on Fig. 17. The given block allows to develop vector AVR integrated it in structure of excitation system standard IEEE type 1 [23]. The excitation system shown on Fig. 24.

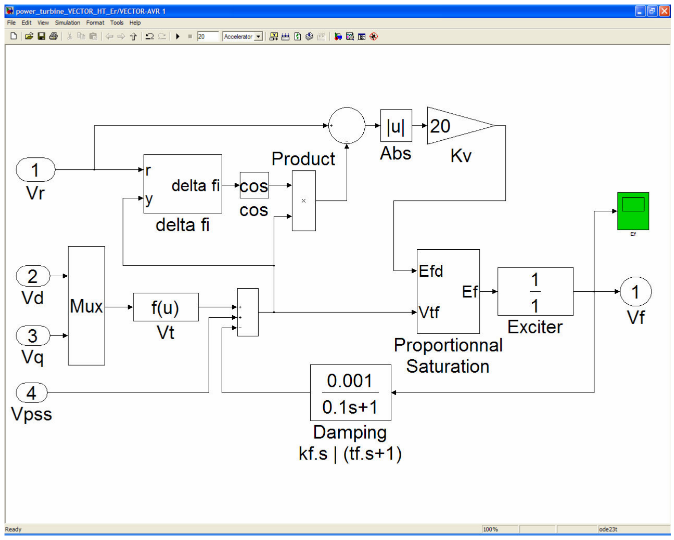

Fig. 24. The excitation system with vector AVR.

The experiments with given model have shown, that stability of the synchronous hydrogenerator is provided at three-phase short circuit with duration 0.25s for a wide range of changes a model parameters: reference a terminal voltage $V_r = 0.7\text{-}1.1$ *p.u.*, active power $P = 0.8\text{-}1.3$ *p.u.*, frequency $f = 0.9\text{-}1.1$ *p.u.*, power of system $S = 2000\text{-}10000$ *MVA*, ratio $X/R = 1\text{-}100$. Changes frequency and power of the hydrogenerator in a transient shown in Figs 25-26:

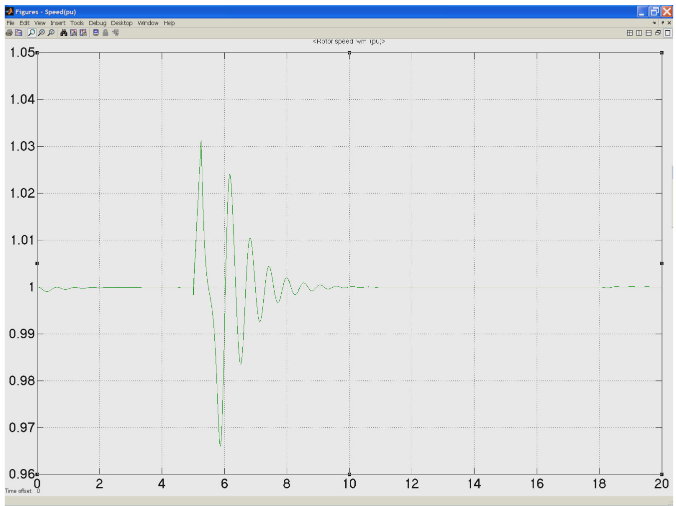

Fig. 25. Change frequency of the hydrogenerator.

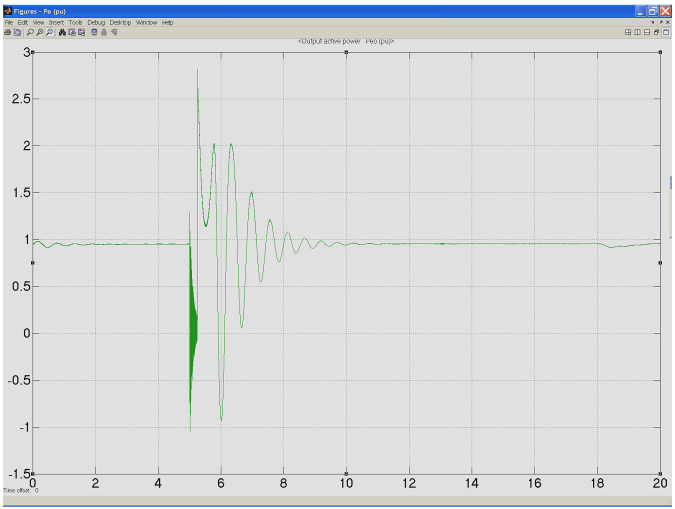

Fig. 26. Change power of the hydrogenerator.

In comparison with AVR standard IEEE type 1, a vector controller possesses the greater robust to change of parameters.

The problem of a vector controller it is an adaptation of gain K_V to change of parameters plant. The given question defines a direction of the further researches.

5. CONCLUSIONS

1. The new approach for development of control system with use of a vector controller is considered. Vector controller based on Hilbert transform and allows executing identification and control of plant not only depending on value, but also difference phases between its input (reference) and an output.
2. Conditions of stability nonlinear control system with a vector controller are determined.
3. Applications of a vector controller for control the first order plant with a transport delay and as AVR of the synchronous generator shown robust properties of the offered algorithm.

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