Voltage-dependent Parameter Refinement for Single-phase Induction Motors using Genetic Algorithms

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Abstract: - This paper presents a genetic-based approach to correct the parameters for single-phase induction motors in various supply voltage levels. From conventional tests, electrical (resistances and inductances) and mechanical (moment of inertia and damping coefficient) parameters of the stator and the rotor can be estimated. The set of obtained parameters is able to apply for steady-state performance analyses. In transient states, motor responses generated by these parameters are not met the condition of acceptable accuracy. By some efficient search method incorporate with experimental data, the obtained parameters can be refined to yield the best curve fitting in both transient and steady-state responses. A 0.37-kW, 220 V, 50 Hz single-phase induction motor was used for test to verify the effectiveness of the proposed algorithm. Furthermore, with six different motor supply voltages, voltage-dependent parameters of single-phase induction motors can be established. As a result, the voltage-dependent parameters of the induction motors can be satisfactorily improved to represent the motor dynamic in various supply voltages.


1 Introduction
To date, three-phase induction motors have been increasingly important for industrial electric motor applications. It should note that there still exist DC motors in some limited applications, e.g. motors for vehicles. Apart from a large-size electric motor drive, single-phase induction motors are commonly used in household electric motor applications. These applications typically consume the power of a fractional horse power up to around ten horse powers. Although most electric appliances require a few amount of kilo-watt input, minimizing power losses during their operation gives a great benefit resulting in nationwide electric energy used by householders.

In general, single-phase motors are controlled by a thyristor-phase controller or a variable resistor. This is quite simple, but it is not efficient in terms of energy consumption. To achieve this goal, complex control strategy cannot be avoid as long as ac machines are involved. One of widely-used control schemes is variable-voltage, variable-frequency (VVVF) [1]. It can be applied for motor control in many forms. However, this control strategy does not guarantee minimum loss operation. Therefore, adjustable frequency and voltage of the power supply is more flexible and can lead to more economical operation of household electric appliances. To achieve these goals, model-based algorithms must be developed. Motor parameters obtained by conventional test schemes need to be corrected to support the high performance drives.

Over half a century, steady-state analysis of induction motors has become a powerful tool to characterize their performances [2-4] It is fairly good in describing steady-state behaviours. For simple control where accuracy and precision are not that much important, any steady-state model is moderate. However, nowadays, a very accurate torque-speed control of induction motors via the space phasor theory, called vector control [5,6], is increasingly required by industries [7,8]. Thus, accurate parameter identification of induction motors is challenged. Although many methods of parameter identification [8-12] have been proposed within the last decade, so far there is no strong evidence to verify their use.

In this paper, an alternative approach based on some efficient intelligent search techniques was introduced. The space phasor modelling was employed to represent induction motors. Parameters appeared in complex space phasor equations can be adjusted by using a simple tuning procedure proposed in this paper. One efficient intelligent search technique, Genetic Algorithm (GA) [13], was used to illustrate this identification by using parameter information obtained from the
conventional tests. Also, comparisons among results obtained by the use of the intelligent identification technique and the conventional technique were examined. With six different supply voltages that can possibly feed the motor stator by means of reduced voltage operation, sets of voltage-dependent parameters for single-phase induction motors were illustrated.

This paper contains six sections. Modeling of the single-phase induction motor in the space phasor expression was reviewed in the next section. Section 3 gave conventional tests for obtaining those motor’s parameters. Section 4 illustrated methodology for voltage-dependent parameter refinement based on genetic algorithms. Section 5 presented simulation results and Section 6, the last section, was the conclusion.

2 Modeling of Single-phase Induction Motors

Single-phase induction motors can be characterized by several different models. The space-phasor approach [2,4] is the method used in this paper. With this model, motor currents, torque and speed can be observable. The space-phasor model is very complicated and needs more space for explanation. However, in this paper only a brief description is presented as follows.

Fig. 1 describes winding alignment of a single-phase induction motor consisting of main and auxiliary windings with their induced voltages and currents. As shown in the figure, a stationary reference frame which is along the axis of the main stator winding is defined and used for mathematical analysis throughout this paper. It is essential to inform that all quantities especially on the rotor need to be transferred to the stator axis. This can be performed by using the following transform matrix.

\[
\begin{bmatrix}
V^s_{qr} \\
V^s_{dr}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_r & \sin \theta_r \\
-sin \theta_r & \cos \theta_r
\end{bmatrix}
\begin{bmatrix}
V^r_{qr} \\
V^r_{dr}
\end{bmatrix}
\] (1)

The superscripts \(s\) and \(r\) indicate the reference axis in which the variable belongs to. The dynamic machine model for a single-phase induction motor with squirrel-cage rotor in a stationary reference can be expressed as follows [15].

\[
\frac{d}{dt} \lambda_{qr} = -i_{qr} \omega_r + v_{qr}
\] (2)

\[
\frac{d}{dt} \lambda_{dr} = -i_{dr} \omega_r + v_{dr}
\] (3)

\[
\frac{d}{dt} \lambda_{qr}' = -i_{qr}' \omega_r + \omega_r \lambda_{qr}^-
\] (4)

\[
\frac{d}{dt} \lambda_{dr}' = -i_{dr}' \omega_r + \omega_r \lambda_{dr}^-
\] (5)

By rearranging (2) – (5) based on flux linkage and current relations of the single-phase induction motor, the state-space model, in which stator and rotor currents, and rotor speed are state variables, can be formed.

\[
\frac{d}{dt} [x] = [A][x] + [B][u]
\] (6)

\[
[y] = [C][x]
\] (7)

Where

\[
[x] = [i_{qr}, i_{dr}, i_{qr}', i_{dr}', \omega_r, \omega_r^-]^T
\]

\[
[A] = [G]^{-1}[M]
\]

\[
[B] = [G]^{-1}[N]
\]

\[
[M] =
\begin{bmatrix}
-r_{qr} & 0 & 0 & 0 & 0 \\
0 & -r_{dr} & 0 & 0 & 0 \\
0 & X_m & -r' & X'_r & 0 \\
0 & -X_m' & X'_m & 0 & 0 \\
\alpha i_{qr}' & -\alpha i_{dr}' & 0 & 0 & \frac{B_m}{J_m}
\end{bmatrix}
\]

\[
[N] =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & \frac{J_m}{P} & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
X_m = \omega_r L_{mqr}, X'_m = \omega_r (L_{mqr}' + L_{mqr}), \alpha = \frac{P \cdot L_{mqr}}{2J_m}
\]
The current and flux linkage of the stator and rotor windings. \( L_{q_s}, L_{d_s}, L_{q_r}, L_{d_r} \) are the leakage inductances of the stator and rotor windings. \( L_m \) is the stator-rotor mutual inductance. \( r_s, r_{d}, r_{r} \) are the stator and rotor resistances. All variables and parameters are referred to the stationary main-winding reference. \( J_m \) is motor’s moment of inertia and \( B_m \) is damping coefficient.

Applying a numerical time-stepping method to solve a set of differential equations, motor currents, angular speed and position can be calculated numerically.

3 Conventional Tests of the Single-phase Induction Motor

In practice, winding resistances, main and auxiliary (if any) windings, of the single-phase induction motor can be determined accurately. Leakage and magnetizing inductances are practical problems. These values vary considerably according to the motor operating state, transient or steady-state. The inductances calculated from the conventional tests can be fairly used when the steady-state analysis is involved. It is because these conventional tests are performed in the steady-state operation. In this paper, detail of the conventional tests is not necessarily discussed because readers can find this associated information from several research articles or even some well-known textbooks [11,15]. Briefly, the conventional tests for the electrical parameters can be categorized in two procedures, the blocked-rotor and the no-load tests. Each of which is also divided by winding configuration to be i) auxiliary winding opened and ii) main winding opened.

3.1 Auxiliary winding opened

This test performed by disconnecting the auxiliary winding from the main winding. For blocked-rotor test, an equivalent circuit representing the circuit connection is shown in Fig. 2.

From this test, input impedance, total resistance and total reactance of the circuit can be determined by (8) – (10). With stator resistance information from the winding resistance tests e.g. volt-ampere method or direct ohmic measurement, effective resistance and leakage reactance of the rotor bars can be calculated by using (11) – (12), respectively.

\[
\begin{align*}
Z_{bn} &= \frac{V_{bn}}{I_{bn}} \\
R_{bn} &= \frac{P_{bn}}{I_{bn}^2} \\
X_{bn} &= \sqrt{Z_{bn}^2 - R_{bn}^2} \\
R_{bn} &= R_l + R_t \\
X_{bn} &= X_1 + X_2
\end{align*}
\]

Fig. 2. Blocked-rotor test of auxiliary winding opened

For the no-load test, an equivalent circuit representing the schematic connection is shown in Fig. 3.

From this test, magnetizing reactance of the circuit can be determined by (13) – (16).

\[
\begin{align*}
Z_{nl} &= \frac{V_{nl}}{I_{nl}} \\
R_{nl} &= \frac{P_{nl}}{I_{nl}^2} \\
X_{nl} &= \sqrt{Z_{nl}^2 - R_{nl}^2} \\
X_M &= 2X_{nl} - 1.5X_{bn}
\end{align*}
\]
3.2 Main winding opened

This test performed by disconnecting the main winding from the main winding. For blocked-rotor test, an equivalent circuit representing the circuit connection is shown in Fig. 4.

\[
\begin{align*}
R_{ba} &= \frac{P_{ba}}{I_{ba}^2} \quad (17) \\
R_{2a} &= R_{ba} - R_a \quad (18) \\
\alpha &= \frac{R_{2a}}{R_a} \quad (19)
\end{align*}
\]

Fig. 4. Blocked-rotor test of main winding opened

This test is employed in order to determine the effective turn ratio of the auxiliary winding to the main winding by executing (17) – (19).

3.3 Retardation test

Mechanical parameters, the rotor inertia and the viscous friction coefficient, can be determined by the test of retardation. This test performed by applying the rated stator voltage. The rotor is accelerating to its no-load speed. Without any connection of additional inertia mass coupling to the rotor shaft, the rotor will be decelerated from the no-load speed down to zero dependent on its rotor inertia only. By observing this deceleration, the rotor moment of inertia can be approximated.

![Fig. 5. Rotor deceleration curve for machine’s retardation](image)

4 GAs for Parameter Refinement

There exist many different approaches to adjust the motor parameters. The GAs is well-known [16,17], there exist a hundred of works employing the GAs technique to identify system parameters in various forms. The GAs is a stochastic search technique that leads a set of population in solution space evolved using the principles of genetic evolution and natural selection, called genetic operators e.g. crossover, mutation, etc. With successive updating new generation, a set of updated solutions gradually converges to the real solution. Because the GAs is very popular and widely used in most research areas [16-21] where an intelligent search technique is applied, it can be summarized briefly as shown in the flowchart of Fig. 6 [18].

In this paper, the GAs is selected to build up an algorithm to refine all motor parameters (all resistances and inductances, the rotor inertia and the friction coefficient). To reduce programming complication, the Genetic Algorithms (GADS TOOLBOX in MATLAB [17]) is employed to generate a set of initial random parameters. With the searching process, the parameters are adjusted to give response best fitting close to the desired response based on the minimum least square error.

![Fig. 6. Flowchart of the GAs procedure](image)

5 Voltage-dependent Parameters

The key assumption made in this research is that motor’s parameters are not constant for all operating conditions. They can be varied from one operating condition to others. However, some parameters can be considered as fixed parameters. These are the rotor inertia and the damping coefficient. They are mechanical parameters and their values do not change with electrical excitation. The electrical excitation can be categorized into current and voltage excitations. The current excitation depends on a mechanical loading condition. This excitation strongly results in change of winding resistances due to material thermal characteristics and leakage inductances due to magnetic saturation of magnetic cores.

In this paper, only voltage-dependent parameters are emphasized. The tests can be conducted by varying the 50-Hz single-phase supply voltage source of 220 V, 200 V, 180 V, 160 V, 140 V and
120 V. Neglecting all loading conditions, no-load tests for the six supply voltages are performed. No-load speeds are recorded for each case and used for parameter refinement. With the help of genetic algorithms, accurate parameters for each supply voltage can be obtained.

6 Results and Discussion
To examine the effectiveness of the proposed intelligent identification, a 0.37-kW, 220-V, 50-Hz, single-phase, squirrel-cage induction motor was used for test as shown in Fig. 7.

With the conventional resistance, no-load, blocked-rotor and retardation tests, the parameters of the single-phase induction motor can be obtained as follows.

Resistance Test:
Main winding resistance = 7.3 Ω
Auxiliary winding resistance = 1.3 Ω

No-load and blocked-rotor tests:

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Current</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>220 V</td>
<td>2.8 A</td>
<td>120.6 W</td>
</tr>
<tr>
<td>99.6 V</td>
<td>3.47 A</td>
<td>194.5 W</td>
</tr>
</tbody>
</table>

Retardation test: (Result shown in Fig. 8)
The estimated rotor inertia is 0.784 kg·m².

With the parameters obtained by using the conventional tests, the simulation result of the motor speed at the rated voltage can be compared against the experimental result as shown in Fig. 9.

As can be seen, the response of the simulation result is able to satisfactorily fit the experimental data in the steady-state condition only. This parameter set is not suitable to be used for the parameters of the space phasor model where the transient behavior is typically an important issue.

To correct the parameters for appropriate use with the space phasor model, some refinement technique to adjust those obtained parameters from the conventional tests must be applied. For intelligent identification, the GAs is employed. The followings describe parameter setting and search space (set arbitrarily) for the GAs used in this paper.

Number of population = 100
Crossover probability = 70%
Mutation probability = 4.7%

From the parameter setting given above with initial values from the conventional tests, the rotor speed can be simulated within the GAs loop through the space phasor equations as described in Section 2. When one of the termination criteria is met, the search process is terminated. Hence, a set of the best parameters, as shown in Table 1, that fit to the experimental data can be found. Fig. 10 shows the convergence of the search process performed by using the GAs.

Table 1. Variable limits (search space):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{qs} )</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>( r_r )</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>( r_{ds} )</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>( L_{lqs} )</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>( L_{lr} )</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>( L_{lds} )</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>( L_{mqs} )</td>
<td>0.1</td>
<td>0.6</td>
</tr>
<tr>
<td>( J_m )</td>
<td>0.003</td>
<td>0.02</td>
</tr>
<tr>
<td>( B_m )</td>
<td>0</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Table 2. Optimal parameters for 220-V case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Optimal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{qs} )</td>
<td>5.5219 Ω</td>
</tr>
<tr>
<td>( r_r )</td>
<td>11.94 Ω</td>
</tr>
<tr>
<td>( r_{ds} )</td>
<td>26.937 Ω</td>
</tr>
<tr>
<td>( L_{lqs} )</td>
<td>0.030599 H</td>
</tr>
<tr>
<td>( L_{lr} )</td>
<td>0.048808 H</td>
</tr>
<tr>
<td>( L_{lds} )</td>
<td>0.041793 H</td>
</tr>
<tr>
<td>( L_{mqs} )</td>
<td>0.37862 H</td>
</tr>
<tr>
<td>( J_m )</td>
<td>0.008788 kg-m²</td>
</tr>
<tr>
<td>( B_m )</td>
<td>0.000552 kg-m/s</td>
</tr>
</tbody>
</table>

With the parameters obtained by using the search, the simulation result of the motor speed can be compared against the experimental result as shown in Fig. 11.

Fig. 11. Simulation result using the set of parameters from the GAs (220-V case)

In the same manner, parameters for each voltage case can be obtained and summarized in Table 3-7. Fig. 12, 14, 16, 18 and 20 are solution convergences of GA search results for 200-V, 180-V, 160-V, 140-V and 120-V cases, respectively. Fig. 13, 15, 17, 19 and 21 are comparison of the test result and the best-fit of the simulated motor speed for other five respective cases.

Table 3. Optimal parameters for 200-V case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Optimal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{qs} )</td>
<td>6.0995 Ω</td>
</tr>
<tr>
<td>( r_r )</td>
<td>8.3755 Ω</td>
</tr>
<tr>
<td>( r_{ds} )</td>
<td>24.595 Ω</td>
</tr>
<tr>
<td>( L_{lqs} )</td>
<td>0.037003 H</td>
</tr>
<tr>
<td>( L_{lr} )</td>
<td>0.024528 H</td>
</tr>
<tr>
<td>( L_{lds} )</td>
<td>0.025479 H</td>
</tr>
<tr>
<td>( L_{mqs} )</td>
<td>0.40243 H</td>
</tr>
<tr>
<td>( J_m )</td>
<td>0.008788 kg-m²</td>
</tr>
<tr>
<td>( B_m )</td>
<td>0.000552 kg-m/s</td>
</tr>
</tbody>
</table>

Table 4. Optimal parameters for 180-V case

<table>
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<tr>
<td>( r_{qs} )</td>
<td>6.2126 Ω</td>
</tr>
<tr>
<td>( r_r )</td>
<td>7.4612 Ω</td>
</tr>
<tr>
<td>( r_{ds} )</td>
<td>36.18 Ω</td>
</tr>
<tr>
<td>( L_{lqs} )</td>
<td>0.028312 H</td>
</tr>
<tr>
<td>( L_{lr} )</td>
<td>0.024079 H</td>
</tr>
<tr>
<td>( L_{lds} )</td>
<td>0.051837 H</td>
</tr>
<tr>
<td>( L_{mqs} )</td>
<td>0.44792 H</td>
</tr>
<tr>
<td>( J_m )</td>
<td>0.008788 kg-m²</td>
</tr>
<tr>
<td>( B_m )</td>
<td>0.000552 kg-m/s</td>
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Table 5. Optimal parameters for 160-V case

<table>
<thead>
<tr>
<th>Parameter</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$r_{qs}$</td>
<td>6.7585 Ω</td>
</tr>
<tr>
<td>$r_{rr}$</td>
<td>6.1975 Ω</td>
</tr>
<tr>
<td>$r_{ds}$</td>
<td>36.626 Ω</td>
</tr>
<tr>
<td>$L_{lsq}$</td>
<td>0.02674 H</td>
</tr>
<tr>
<td>$L_{lr}$</td>
<td>0.020242 H</td>
</tr>
<tr>
<td>$L_{ldd}$</td>
<td>0.04168 H</td>
</tr>
<tr>
<td>$L_{mqqs}$</td>
<td>0.5004 H</td>
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<tr>
<td>$J_m$</td>
<td>0.008788 kg-m$^2$</td>
</tr>
<tr>
<td>$B_m$</td>
<td>0.000552 kg-m/s</td>
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</table>

Table 6. Optimal parameters for 140-V case

<table>
<thead>
<tr>
<th>Parameter</th>
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<tbody>
<tr>
<td>$r_{qs}$</td>
<td>6.2702 Ω</td>
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<tr>
<td>$r_{rr}$</td>
<td>7.1434 Ω</td>
</tr>
<tr>
<td>$r_{ds}$</td>
<td>36.305 Ω</td>
</tr>
<tr>
<td>$L_{lsq}$</td>
<td>0.036223 H</td>
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<tr>
<td>$L_{lr}$</td>
<td>0.018638 H</td>
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<tr>
<td>$L_{ldd}$</td>
<td>0.049867 H</td>
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<td>$L_{mqqs}$</td>
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<td>$J_m$</td>
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<tr>
<td>$B_m$</td>
<td>0.000552 kg-m/s</td>
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Table 7. Optimal parameters for 120-V case

<table>
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<tr>
<th>Parameter</th>
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<tbody>
<tr>
<td>$r_{qs}$</td>
<td>7.4503 Ω</td>
</tr>
<tr>
<td>$r_{rr}$</td>
<td>5.6556 Ω</td>
</tr>
<tr>
<td>$r_{ds}$</td>
<td>45.809 Ω</td>
</tr>
<tr>
<td>$L_{lsq}$</td>
<td>0.032499 H</td>
</tr>
<tr>
<td>$L_{lr}$</td>
<td>0.015448 H</td>
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<tr>
<td>$L_{ldd}$</td>
<td>0.031344 H</td>
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<tr>
<td>$L_{mqqs}$</td>
<td>0.55269 H</td>
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<tr>
<td>$J_m$</td>
<td>0.008788 kg-m$^2$</td>
</tr>
<tr>
<td>$B_m$</td>
<td>0.000552 kg-m/s</td>
</tr>
</tbody>
</table>
Fig. 15. Simulation result using the set of parameters from the GAs (180-V case)

Fig. 16. Solution convergence for 160-V case

Fig. 17. Simulation result using the set of parameters from the GAs (160-V case)

Fig. 18. Solution convergence for 140-V case

Fig. 19. Simulation result using the set of parameters from the GAs (140-V case)

Fig. 20. Solution convergence for 120-V case
With the refined parameters obtained in this paper, control and drive of single-phase induction motors can be performed more accurately. Parameter-based or model-reference approaches can be used extensively to a wide range of applications. Fig. 20 concludes the simulated speed response of all cases in comparative with their corresponding experimental result.

Fig. 20. Comparison among simulation and test results for six voltage cases

7 Conclusion
This paper illustrates an intelligent approach to estimate dynamic parameters of a single-phase induction motor. Due to the complication of the space phasor equations describing dynamic behaviors of the single-phase induction motor, the parameters can be roughly estimated through conventional tests (no-load, blocked-rotor and retardation tests). These tests are based on the steady-state analysis. Therefore, they may cause inaccurate estimation, especially when transient characteristics are interested. In this paper, some efficient intelligent search technique, Genetic Algorithms (GAs), is employed to demonstrate this intelligent identification. With the additional key assumption made in this paper, some parameters are voltage-dependent and can be varied due to the supply voltage. In single-phase motor applications, reduced voltage control is typical for domestic-based electric appliances due to its lower cost. As a result, the speed response simulated from the proposed method satisfactorily fits to that obtained from the experiment. In comparison with the conventional parameter tests, the effectiveness of the proposed scheme is confirmed.

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