Componentwise Stability Of The Singular Discrete Time System Using The Methodology Of The Drazin Inverse

HABIBA MEJHED, and
Département Génie Informatique
Laboratoire Micro Informatique Systèmes Embarqués Système sur Puce.
Ecole Nationale des sciences appliquées, BP. 575, avenue Abdlekrim Khatabi, Guéliz
Université Cadi Ayyad, Marrakech
MOROCCO
NOUR EL HOUDA MEJHED
Département Génie Informatique,
Ecole Nationale des sciences appliquées de Fès.
Université Sidi Mohammed Ben Abdallah, Fès
MOROCCO
ABDLAZIZ HMAMED
LESSI Département de physique
Faculté des sciences, B.P. 1796, 30 000, FES-Atlas
Université Sidi Mohammed Ben Abdallah, Fès
MOROCCO
mejhed90@gmail.com

Abstract: - The asymptotic (exponential) stability in the sense of componentwise approach is considered for one and two dimensional linear discrete-time singular systems. The two dimensional system is described by Fornasini-Marchesini model. The main motivation for these results is the need, particularly felt in the evaluation in a more detailed manner of the dynamical behaviour of linear discrete-time singular systems. Combining the basic theory of Drazin inverse and the results reported in the case of the discrete time linear systems (see Hmamed, 1997), both necessary and sufficient conditions are established ensuring the componentwise asymptotic (exponential) stability. In addition, numerical examples are proposed to illustrate the correctness of the obtained results.

Key-Words: - Componentwise stability; Singular system; discrete time systems; 2D Fornasini-Marchesini model; Drazin inverse theory.

1 Introduction

During the last few decades, the singular systems, which are also defined as descriptor systems, have been attached much more attention since singular systems can describe better practical dynamical systems than standard state-space systems. Several studies have addressed the stability and stabilization problem of the class of continuous time singular systems with or without delay using various concepts [20], [21], [16]. [18] contributed in the normwise perturbation theory of singular linear structured system with index one. In [26] a parametrized differentiable family of singular regularizable systems is given. [28], [29] discussed the problem of delay-independent and/or delay dependent stability and stabilization for singular systems with multiple time-varying delays, using the continuous-time Markov process sufficient conditions in the linear matrix inequality setting (LMI).

The componentwise stability concept is considered as a special type of asymptotic stability, which combines the positive invariance of time-dependent rectangular sets with respect to the state space trajectories. This concept was first studied by [31] who applied the theory of flow-invariant time_dependent rectangular sets to define and characterize the componentwise asymptotic stability (CWAS) and the componentwise exponential asymptotic stability (CWEAS) for continuous-time linear systems. Further works extended the analysis of componentwise stability to continuous-time with or without time delay [4], [14], [15] and [5], [6], [24], [23].

On the other hand, the Drazin inverse theory is used in several applications such that linear algebra, control and systems modeling theory, were found in the literature, various works focused on this methodology, [19] in Banach algerba, [25], [26] in the control theory and [22], [17] gave some results on the problems of the singular linear system under certain condition. In the work of [27], an oblique projection iterative method is proposed to compute matrix equation AXA=A and to obtain the Drazin inverse for a square matrix A, and in [30] the perturbation of the drazin inverse is considered.

The purpose of this work is to extend the concept of Componentwise asymptotic (exponential) stability of one and two-Dimensional linear discrete-time systems reported in [5] to the componentwise stability of one and two-Dimensional discrete-time singular systems using the methodology of the Drazin inverse.

The paper is organized as follows. Some notations and terminology are given is section 2. Section 3 deals with the main results, giving necessary and sufficient conditions for componentwise asymptotic (exponential) stability of 1D singular systems. This is then extended to the 2D singular system described by Fornasini-Marchesini model in Section 4. Finally, two examples are given to illustrate the developed results.

2 Notations

The main notations of this paper are as follows:

$x = (x_i) \in \Re^n$ (x a real vector);

$H = (h_{ij}) \in \Re^{nxn}$ (H a real matrix);

$\mathrm{Int}\, D$ interior of set D;

δD boundary of set D;

$\det(M)$ determinant of matrix $M \in C^{nxn}$;

H^+ matrix with component $h_{ij}^+ = \sup(h_{ij},0)$, i,j=1,2,..,n;

H^- matrix with components $h_{ij}^- = \sup(-h_{ij},0)$, i,j=1,2,..,n;

$|H|$ matrix with components $|h_{ij}|$, i,j=1,2,..,n; x^+ vector with components;

$x_i^+ = \sup(x_i,0)$, i=1,..,n;

x^- vector with components $x_i^- = \sup(-x_i,0)$, i=1,..,n;

x, y vectors in $\Re^n$; $x \le y$ if $x_i \le y_i$, i=1,2,..,n;

$x < y$ if $x_i < y_i$, i=1,..,n.

3 Discrete Singular Systems

In this section, some results about componentwise asymptotic (exponential) stability of 1D singular systems are established. We focus on discrete-time singular systems, which are described by the implicit form:

$$\begin{cases} Ex(k+1) = Ax(k) \\ x(0) = x_0 \end{cases}, \; k > 0 \tag{1}$$

where $x \in \Re^n$ is the state vector, $E \in \Re^{nxn}$, $\mathrm{rank}(E) = q \le n$, $A \in \Re^{nxn}$.

In certain applications, namely in electrical engineering and biology, dynamical systems have to satisfy some additional constraints of the form.

$$x \in \Omega \subset \Re^n \tag{2}$$

where Ω is the set of admissible, states defined by:

$$\Omega = \left\{ x \in \Re^n / -\rho_2(k) \le x(k) \le \rho_1(k); \rho_1(k), \rho_2(k) \in \mathrm{int}\Re_+^n \right\} \tag{3}$$

With

$$\lim_{k \to +\infty} \rho_1(k) = 0, \quad \lim_{k \to +\infty} \rho_2(k) = 0 \tag{4}$$

This is a variant nonsymmetrical polyhedral set, as is generally the case in practical situations.

In certain cases, $\rho_1(k)$ and $\rho_2(k)$ take the form

$$\rho_s(k) = \alpha_s \beta^k \tag{5}$$

with $0 < \beta < 1$ and $\alpha_s > 0$ for s=1,2.

The purpose of this section is to define a special type of asymptotic (exponential) stability of the system (1), namely the componentwise asymptotic (exponential) stability concept characterized by (3) and (4) ((3) and (5)). Necessary and sufficient conditions for componentwise asymptotic (exponential) stability of the system (1) are established.

First, recall some important properties of implicit systems that are assumed intrinsic in the following analysis.

Definition 1 The system (1) is called componentwise asymptotically stable with respect to $\widetilde{\rho}(k) = \left[\rho_1^T(k)\ \rho_2^T(k)\right]^T$ (CWAS $\widetilde{\rho}$) if for every $-\rho_2(0) \le x_0 \le \rho_1(0)$, the response of (1) satisfies:

$$-\rho_2(k) \le x(k) \le \rho_1(k),\ \forall k \ge 0 \tag{6}$$

Definition 2 The system (1) is called componentwise exponential asymptotically (CWEAS) if there exist $\alpha_1 > 0$ and $\alpha_2 > 0$ such that, for every $-\alpha_2 \le x_0 \le \alpha_1$, the response of (1) satisfies

$$-\alpha_2\beta^k \le x(k) \le \alpha_1\beta^k,\ \forall k \ge 0 \tag{7}$$

Definition 3 [1] The system (1) is said to be regular if $\det(sE - A) \ne 0$.

Definition 4 [11] The system (1) is said to be impulse-free if $\deg\det(sE - A) = \operatorname{rank}(E)$ or $(sE - A)^{-1}$ is proper.

Definition 5 [3] A rational function $G(s)$ is said to be proper if $G(\infty)$ is constant matrix, $G(s)$ is said to be strictly proper if $G(\infty) = 0$.

For the convenience of the later statements in this paper, we use the pair (E,A) to represent the system (1).

Theorem 1 [2] Suppose that (E,A) is regular. Then $Ex(k+1) = Ax(k) + f(k)$, $\forall k \ge 0$ is solvable and the general solution is given by:

$$\begin{aligned} x(k) &= \overline{A}^k\overline{P}q + \hat{E}^D\sum_{i=0}^{k-1}\overline{A}^{k-i-1}\hat{f}(i) \\ &\quad - (I - \overline{P})\sum_{i=0}^{\nu-1}\overline{E}^i\hat{A}^D\hat{f}(k+i) \end{aligned} \tag{8}$$

where

$\hat{f}(i) = (\lambda E - A)^{-1}f(i)$

$\hat{E} = (\lambda E - A)^{-1}E$

$\hat{A} = (\lambda E - A)^{-1}A$

$\overline{A} = \hat{E}^D\hat{A}$, $\overline{E} = \hat{E}\hat{A}^D$

$\overline{P} = \hat{E}\hat{E}^D$, $q \in \Re^n$

ν is the index of $\hat{E}$

λ is a scalar such that $\lambda E - A$ is nonsingular.

The projection $\overline{P}$ and $\overline{E}, \overline{A}$, ν are independent of λ.

Matrix $\hat{E}^D$ is the Drazin inverse of $\hat{E}$ and the index of a matrix is the size of the largest nilpotent block in its Jordan canonical form.

We now give necessary and sufficient conditions for componentwise asymptotic (exponential) stability of the system (1).

Theorem 2 Suppose that (E,A) is regular, a necessary and sufficient condition for the system (1) to be CWAS $\widetilde{\rho}$ is

$$\widetilde{\rho}(k+1) \ge \widetilde{H}\widetilde{\rho}(k),\ \forall k \ge 0 \tag{9}$$

with

$$\widetilde{H} = \begin{pmatrix} H^+ & H^- \\ H^- & H^+ \end{pmatrix},\quad \widetilde{\rho}(k) = \left[\rho_1^T(k)\ \rho_2^T(k)\right]^T \tag{10}$$

and

$$H = \hat{E}^D\hat{A} \tag{11}$$

Proof From Theorem 1 the solution of the system (1) is written in the form

$$x(k) = \left(\hat{E}^D\hat{A}\right)^k\hat{E}\hat{E}^Dq = \left(\hat{E}^D\hat{A}\right)^k x_0,\quad q \in \Re^n\ \forall k \ge 0$$

then:

$$x(k+1) = \hat{E}^D\hat{A}x(k) \tag{12}$$

with the initial condition $x_0 = \hat{E}\hat{E}^Dq$, $q \in \Re^n$.

At this step, we can use the proof given in [5] as the proof remains unchanged.

Note that [1] established the continuity proprieties of drazin inverse.

Remark 1

From [1] and [10], we know that the consistent initial conditions $x_0 = x(0)$ of system (1) are defined by:

$$x_0 = \hat{E}\hat{E}^Dx_0$$

and then, there always exists a vector $q \in \Re^n$ such that $x_0 = \hat{E}\hat{E}^Dq$ [13].

Remark 2

In the case where matrix E is non-singular, then system (1) can be written as a classical autonomous linear system defined as

$$x(k+1) = E^{-1}Ax(k) \tag{13}$$

Hence, if we apply the previous result to system (13), then the classical result of the componentwise stability of 1-D linear discrete-time systems is obtained, that is

$$\tilde{\rho}(k+1) \geq \tilde{H}\tilde{\rho}(k) \quad \forall k \geq 0$$

with $\tilde{H} = \begin{pmatrix} (E^{-1}A)^{+} & (E^{-1}A)^{-} \\ (E^{-1}A)^{-} & (E^{-1}A)^{+} \end{pmatrix}$.

For $E = I$, we obtain the result given in [5].

In the symmetrical case $\rho_1(k) = \rho_2(k) = \rho(k)$, we can deduce the following result.

Corollary 1 Suppose that (E,A) is regular , a necessary and sufficient condition for the system (1) to be CWAS ρ is

$$\rho(k+1) \geq \left| H \right| \rho(k) \quad \forall k \geq 0 \tag{14}$$

matrix H is defined by (11).

Proof By observing that $|H| = H^{+} + H^{-}$, the proof follows from Theorem 2.

Using the techniques of [5], we can also extend the results of Theorem 2 to the following Theorem which deals with the componentwise exponential asymptotic stability.

Theorem 3 Suppose that (E,A) is regular , the system (1) (system (12)) is CWEAS if and only if one of the following conditions holds:

i) $\left(\beta I - \tilde{H}\right)\tilde{\alpha} \geq 0$; (15)

$$1 > \beta \geq \max_{i} \max(h_{ii}^{+} + h_{ii}^{-}\frac{\alpha_2^i}{\alpha_1^i} + \sum_{j \neq i} h_{ij}^{+}\frac{\alpha_1^j}{\alpha_1^i} + h_{ij}^{-}\frac{\alpha_2^j}{\alpha_1^j},$$

ii)

$$h_{ii}^{+} + h_{ii}^{-}\frac{\alpha_1^i}{\alpha_2^i} + \sum_{j \neq i} h_{ij}^{+}\frac{\alpha_2^j}{\alpha_2^i} + h_{ij}^{-}\frac{\alpha_1^j}{\alpha_2^j}) \tag{16}$$

with $\tilde{\alpha} = \begin{bmatrix} \alpha_1^T & \alpha_2^T \end{bmatrix}^T$ and $H = \hat{E}^D \hat{A} = (h_{ij})$.

In the symmetrical case $\rho_1(k) = \rho_2(k) = \alpha\beta^k$, we can deduce the following result.

Corollary 2 The regular system (1) is CWEAS if and only if one of the following conditions holds:

(i) $\beta\alpha \geq \left| H \right| \alpha$

(ii) $1 > \beta \geq \max_{i}\left(\sum_{j=1}^{n} \left| h_{ij} \right| \frac{\alpha^j}{\alpha^i} \right)$

with $H = \hat{E}^D \hat{A} = (h_{ij})$.

Remark 3

When $E = I$, we obtain the result given by [5].

4 Two- Dimensional Fornasini-Marchesini model

Applying the results developed in the last section, we extend the notion of componentwise asymptotic (exponential) stability to implicit 2-D Fornasini-Marchesini model described by the following equation:

$$Ex(i+1, j+1) = Ax(i, j+1) + Bx(i+1, j) \tag{17}$$

with the boundary conditions

$$x(i,0) \text{ and } x(0,j) \text{ for } i, j = 0,1,\ldots \tag{18}$$

where $x \in \Re^n$ is the state vector.

Assume then that (E,A) is a regular pencil and impulse-free. A similar discussion applies if (E,B) is regular. Treat j as fixed, If the sequence $x(i,j)$ is considered known, then (17) is a difference equation

$$Ex(i+1, j+1) = Ax(i, j+1) + [Bx(i+1, j)], \quad i \geq 0 \tag{19}$$

for $x(i, j+1)$ with the terms in square brackets known (see [2]).

Since (E,A) is a regular pencil, we may apply Theorem 1, we have:

$$\begin{aligned} x(i, j+1) = {} & \left(\hat{E}^D\hat{A}\right)^i \hat{E}\hat{E}^D q \\ & + \hat{E}^D \sum_{k=0}^{i-1}\left(\hat{E}^D\hat{A}\right)^{i-k-1} \hat{B}x(k+1, j) \\ & - (I - \hat{E}\hat{E}^D)\sum_{r=0}^{\nu-1}(\hat{E}\hat{A}^D)^r \hat{A}^D\hat{B}x(i+r, j) \end{aligned} \tag{20}$$

here ν is the index of $\hat{E}$

$$\begin{aligned} x(i+1, j+1) = {} & \left(\hat{E}^D\hat{A}\right)^{i+1} \hat{E}\hat{E}^D q \\ & + \hat{E}^D \sum_{k=0}^{i}\left(\hat{E}^D\hat{A}\right)^{i-k} \hat{B}x(k+1, j) \\ & - (I - \hat{E}\hat{E}^D)\sum_{r=0}^{\nu-1}(\hat{E}\hat{A}^D)^r \hat{A}^D\hat{B}x(i+1+r, j) \end{aligned}$$

$$= \left(\hat{E}^D\hat{A}\right)\left(\hat{E}^D\hat{A}\right)^i \hat{E}\hat{E}^D q$$

$$+ \left(\hat{E}^D\hat{A}\right)\hat{E}^D \sum_{k=0+}^{i-1}\left(\hat{E}^D\hat{A}\right)^{i-k-1} \hat{B}x(k+1, j)\left(\hat{E}^D\hat{B}\right)x(i+1, j)$$

$$- (I - \hat{E}\hat{E}^D)\sum_{r=0}^{\nu-1}(\hat{E}\hat{A}^D)^r \hat{A}^D\hat{B}x(i+k+1+r, j)$$

$$=\left(\hat{E}^D\hat{A}\right)\left[x(i,j+1)+(I-\hat{E}\hat{E}^D)\sum_{r=0}^{\nu-1}(\hat{E}\hat{A}^D)^r\hat{A}^D\hat{B}x(i+k,j)\right]$$

$$+\left(\hat{E}^D\hat{B}\right)x(i+1,j)$$

$$-(I-\hat{E}\hat{E}^D)\sum_{r=0}^{\nu-1}(\hat{E}\hat{A}^D)^r\hat{A}^D\hat{B}x(i+k+1+r,j)$$

$$\begin{aligned}&=\left(\hat{E}^D\hat{A}\right)x(i,j+1)\\&\quad+\left(\hat{E}^D\hat{B}\right)x(i+1,j)\\&\quad+(I-\hat{E}\hat{E}^D)\times\\&\qquad\sum_{k=0}^{\nu-1}(\hat{E}\hat{A}^D)^k\hat{A}^D\hat{B}(\left(\hat{E}^D\hat{A}\right)x(i+k,j)\\&\quad-\ x(i+k+1,j))\end{aligned}$$

$$\begin{aligned}&=\left(\hat{E}^D\hat{A}\right)x(i,j+1)\\&\quad+\left(\hat{E}^D\hat{B}\right)x(i+1,j)\\&\quad-(I-\hat{E}\hat{E}^D)\sum_{k=0}^{\nu-1}(\hat{E}\hat{A}^D)^k\hat{A}^D\hat{B}x(i+k+1,j)\end{aligned} \tag{21}$$

Definition 6 The regular system (17) is called componentwise asymptotically stable with respect to $\widetilde{\rho}(i,j)=\left[\rho_1^T(i,j)\,\rho_2^T(i,j)\right]^T$ (CWAS $\widetilde{\rho}$) if, for every

$$\begin{cases}-\rho_2(i,0)\le x(i,0)\le\rho_1(i,0)\\ \\-\rho_2(0,j)\le x(0,j)\le\rho_1(0,j)\end{cases}$$

for i,j=0,1,2,.

The response of (17) satisfies

$$-\rho_2(i,j)\le x(i,j)\le\rho_1(i,j)\,,\ \forall(i,j)>(0,0) \tag{22}$$

where

$$\rho_1(i,j)>0,\ \rho_2(i,j)>0\ \forall(i,j)>(0,0)$$

$$\lim_{i\to\infty \text{ and/or } j\to\infty}\rho_1(i,j)=0\ ,\qquad \lim_{i\to\infty \text{ and/or } j\to\infty 2}\rho_1(i,j)=0$$

Definition 7 The regular system (17) is called componentwise exponential asymptotically stable (CWEAS) if there exist $\alpha_1>0$ and $\alpha_2>0$ such that, for every

$$\begin{cases}-\alpha_2\beta^i\le x(i,0)\le\alpha_1\beta^i\\-\alpha_2\gamma^j\le x(0,j)\le\alpha_1\gamma^j\end{cases}\ \text{for i,j=0,1,2,…} \tag{23}$$

The response of (17) satisfies

$$-\alpha_2\beta^i\gamma^i\le x(i,j)\le\alpha_1\beta^i\gamma^i\quad\forall(i,j)>(0,0)$$

where $0<\beta<1$ and $0<\gamma<1$.

We give now necessary and sufficient conditions for componentwise asymptotic (exponential) stability of the system (19) or (17).

Theorem 4 Suppose that (E,A) is regular and impulse-free, a necessary and sufficient condition for the system (17) to be CWAS $\widetilde{\rho}$ is

$$\widetilde{\rho}(i+1,j+1)\ge\widetilde{H}_1\widetilde{\rho}(i,j+1)+\widetilde{H}_2\widetilde{\rho}(i+1,j)\qquad\forall(i,j)>(0,0) \tag{24}$$

with

$$\widetilde{H}_1=\begin{pmatrix}(\hat{E}^D\hat{A})^+ & (\hat{E}^D\hat{A})^-\\(\hat{E}^D\hat{A})^- & (\hat{E}^D\hat{A})^+\end{pmatrix}$$

$$\widetilde{H}_2=\begin{pmatrix}(\hat{E}^D\hat{B})^+ & (\hat{E}^D\hat{B})^-\\(\hat{E}^D\hat{B})^- & (\hat{E}^D\hat{B})^+\end{pmatrix},$$

$$\widetilde{\rho}(i,j)=\left[\rho_1^T(i,j)\quad\rho_2^T(i,j)\right]^T \tag{25}$$

Proof Since system (20) is impulse free, in this case, ν becomes 1 [10].

Express $x(i,j+1)$ by using (20) with $\nu=1$ as

$$x(i,j+1)=\left(\hat{E}^D\hat{A}\right)^i\hat{E}\hat{E}^Dq+\hat{E}^D\sum_{k=0}^{i-1}\left(\hat{E}^D\hat{A}\right)^{i-k-1}\hat{B}x(k+1,j) \tag{26}$$

then

$$x(i+1,j+1)=\left(\hat{E}^D\hat{A}\right)^{i+1}\hat{E}\hat{E}^Dq+\hat{E}^D\hat{B}x(i+1,j)+\hat{E}^D(\hat{E}^D\hat{A})\sum_{k=0}^{i-1}\left(\hat{E}^D\hat{A}\right)^{i-k-1}\hat{B}x(k+1,j) \tag{27}$$

From the Drazin inverse theory used in [1], we know that

$$\hat{E}\hat{A}=\hat{A}\hat{E}\ ,\ \hat{E}^D\hat{A}=\hat{A}\hat{E}^D\ \text{and}\ \hat{E}^D\hat{A}^D=\hat{A}^D\hat{E}^D$$

then

$$x(i+1,j+1)=H_1\,x(i,j+1)+H_2\,x(i+1,j) \tag{28}$$

with

$$H_1=\hat{E}^D\hat{A}\,,\ H_2=\hat{E}^D\hat{B} \tag{29}$$

and boundary conditions $x(i,0)=x_{i,0}$ and $x(0,j)=\hat{E}^D\hat{E}q$, $q\in\Re^n$, $j\ge1$.

At this step, this follows similar lines to the proof of Theorem 3.3 in [5].

Remark 4
The boundary values $x(i,0)$ may be taken arbitrary and the boundary values $\hat{E}^D\hat{E}x(0,j),\ j \geq 1$, are arbitrary [2]. Then, we can apply the results given in the last section, implying the existence of a vector $q \in \Re^n$ such that:

$$x(0,j) = \hat{E}^D\hat{E}q = x_{0,j},\ j \geq 1$$

By analogy with section 3, we give some definitions.

Remark 5
If E is nonsingular square matrix, then equation (24) is the classical condition given in [5].

In the symmetrical case $\rho_1(i,j) = \rho_2(i,j) = \rho(i,j)$, we can deduce the following result.

Corollary 3 Suppose that (E,A) is regular and impulse-free, a necessary and sufficient condition for the system (17) to be CWAS ρ is

$$\rho(i+1,j+1) \geq |\ H_1\ |\rho(i,j+1) + |\ H_2\ |\rho(i+1,j) \tag{30}$$

matrices H_1 and H_2 are defined by (29).

Proof On observing that $|H_1| = H_1{}^+ + H_2{}^-$ and $|H_2| = H_2{}^+ + H_2{}^-$, the proof follows from Theorem 1.

Theorem 5 Suppose that (E,A) is regular and impulse-free, the system (17) (system (28)) is CWEAS if and only if one of the following conditions holds:

i) $$\beta\gamma\tilde{\alpha} \geq (\tilde{H}_1\gamma - \tilde{H}_2\beta)\tilde{\alpha}\,; \tag{31}$$

ii) $$1 > \beta\gamma \geq \max_i \max \left\{ [\sum_{j=1}^{n}(h_{ij}^{1+}\gamma + h_{ij}^{2+}\beta)\alpha_1^j + (h_{ij}^{1-}\gamma + h_{ij}^{2-}\beta)\alpha_2^j]/\alpha_1^i,\ [\sum_{j=1}^{n}(h_{ij}^{1-}\gamma + h_{ij}^{2-}\beta)\alpha_1^j + (h_{ij}^{1+}\gamma + h_{ij}^{2+}\beta)]/\alpha_2^j \right\} \tag{32}$$

with $\tilde{\alpha} = [\alpha_1\ \ \alpha_2]^T$.

In the symmetrical case $\rho_1(i,j) = \rho_2(i,j) = \tilde{\alpha}\beta^i\gamma^j$, we can deduce the following result.

Corollary 4 Suppose that (E,A) is regular and impulse-free, the system (17) is CWEAS if and only if one of the following conditions holds:

i) $$\beta\gamma\alpha \geq (|\ H_1|\gamma + |\ H_2|\beta)\alpha \tag{33}$$

ii) $$1 > \beta\gamma \geq \left\{ \sum_{j=1}^{n}(|h_{ij}^1|\ \gamma + |h_{ij}^2|\ \beta)\frac{\alpha^j}{\alpha^i} \right\} \tag{34}$$

matrices H_1 and H_2 are defined by (29).

Remark 6
We can extend the results of this section to the Roesser two dimensional model given in [8] and [11], [12] by:

$$\begin{bmatrix} E_1 & E_2 \\ E_3 & E_4 \end{bmatrix}\begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}\begin{bmatrix} x_{i,j}^h \\ x_{i,1}^v \end{bmatrix} \tag{35}$$

with the boundary conditions
$x^h(0,j)$ and $x^v(i,0)$ for $i,j = 0,1$.

Several techniques may be used to show that the implicit Roesser and implicit FM model are equivalent [8]. Indeed, in the Roesser model define

$$F_1 = \begin{bmatrix} E_1 & 0 \\ E_2 & 0 \end{bmatrix},\ F_2 = \begin{bmatrix} 0 & E_3 \\ 0 & E_4 \end{bmatrix} \tag{36}$$

and similar quantities with respect to $A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$.

Then (35) may be written as

$$F_1\ x(i+1,j) + F_2\ x(i,j+1) = Ax(i,j) \tag{37}$$

Consequently all the results derived in this section still hold for the Roesser model (35) on taking into account the relation (36).

Following the same ideas, those results can easily be extended to the following general two dimensional system model:

$$Ex(i+1,j+1) = A_0x(i,j) + A_1x(i,j+1) + A_2x(i+1,j)\ . \tag{38}$$

Example 1
To illustrate the application of Theorem 2, we consider the system (1), where,

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} 0.2 & 1 \\ 0 & 1 \end{bmatrix}$$

Since $(\lambda E - A)^{-1}$ exists for $\lambda = 0$, then we can choose $\lambda = 0$ so that, $\hat{E} = A^{-1}E$, $\hat{A} = -I_2$.
we get:

$$\hat{E} = \begin{bmatrix} -1/0.2 & 0 \\ 0 & 0 \end{bmatrix}, \ \hat{E}^D = \begin{bmatrix} -0.2 & 0 \\ 0 & 0 \end{bmatrix}$$

The consistence conditions are given in the following form:

$$x(0) = \hat{E}\hat{E}^D x(0) = \begin{bmatrix} x_1(0) \\ 0 \end{bmatrix}$$

It is not hard to see that the system (1) takes the form:

$$x(k+1) = \hat{E}^D \hat{A} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix} x(k)$$

Let $\rho_1(k) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} 2^{-k} \log(k + 1.01)$

and $\rho_2(k) = \begin{bmatrix} 5 \\ 6 \end{bmatrix} 2^{-k} \log(k + 1.01)$.

It is obvious that condition $\tilde{\rho}(k+1) \geq \tilde{H}\tilde{\rho}(k)$ holds where $H = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix}$ that is

$$\tilde{\rho}(k+1) = \begin{pmatrix} 1 \\ 1.5 \\ 2.5 \\ 3 \end{pmatrix} 2^{-2} \log(k + 2.01)$$

$$\geq \begin{pmatrix} 0.4 \\ 0 \\ 1 \\ 0 \end{pmatrix} 2^{-k} \log(k + 1.01)$$

As it is illustrated by Fig. 1, it is easy to see that this system is CWAS $\tilde{\rho}$.
where $x(k) = [x_1(k) \quad 0]^T$

Example2
Now, we consider the 2-D Fornasini- Marchesini model described by (17) where,

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \ A = \begin{bmatrix} 0.1 & 1 \\ 0 & 0.2 \end{bmatrix}, \ B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(E, A) is regular [2] if we choose $\lambda = 0$ then,

$$\hat{E} = -A^{-1}E$$

and $\rho_1(k) = [\rho_{11}(k) \quad \rho_{12}(k)]^T$.

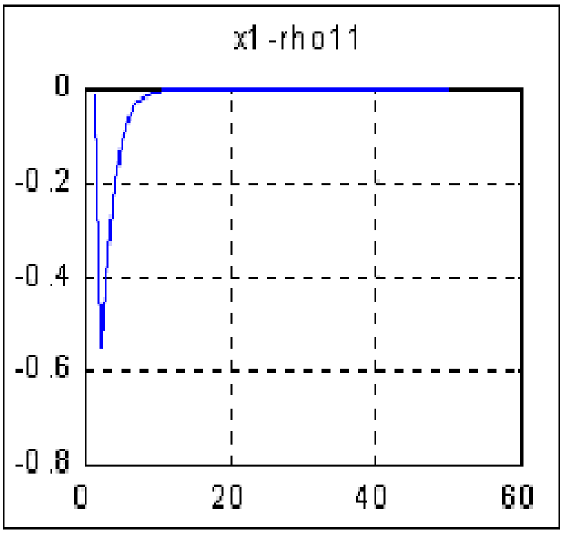

Fig .1: The trajectory of $\mathbf{x_1(k)}$ _ $\rho_{11}\mathbf{(k)}$

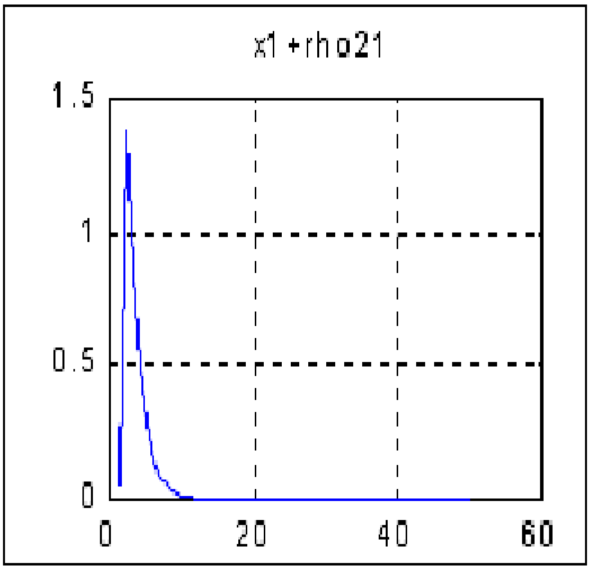

Fig . 2: The trajectory of $\mathbf{x_1(k)}$ _ $\rho_{12}\mathbf{(k)}$

$$\hat{A} = -I$$
$$\hat{B} = -A^{-1}B$$

then we obtain:

$$\hat{E} = \begin{bmatrix} -1/0.1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} 0 & -1/0.1 \\ 0 & 0 \end{bmatrix}$$

and then, $H_0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $H_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

Let,

$$\rho_1(i,j) = \begin{pmatrix} 3 \\ 0.5 \end{pmatrix} 2^{-i} 3^{-j} \log(i+1.01)\log(j+1.01)$$

And

$$\rho_2(i,j) = \begin{pmatrix} 6 \\ 1 \end{pmatrix} 2^{-i} 3^{-j} \log(i+1.01)\log(j+1.01)$$

It is obvious that condition (26) holds. The following figure shows clearly that this system is CWAS $\tilde{\rho}$

$$x(i,j) = [x_1(i,j) \quad x_2(i,j)]^T$$

$$\rho_1(i,j) = [\rho_{11}(i,j) \quad \rho_{12}(i,j)]^T$$

$$\rho_2(i,j) = [\rho_{21}(i,j) \quad \rho_{22}(i,j)]^T$$

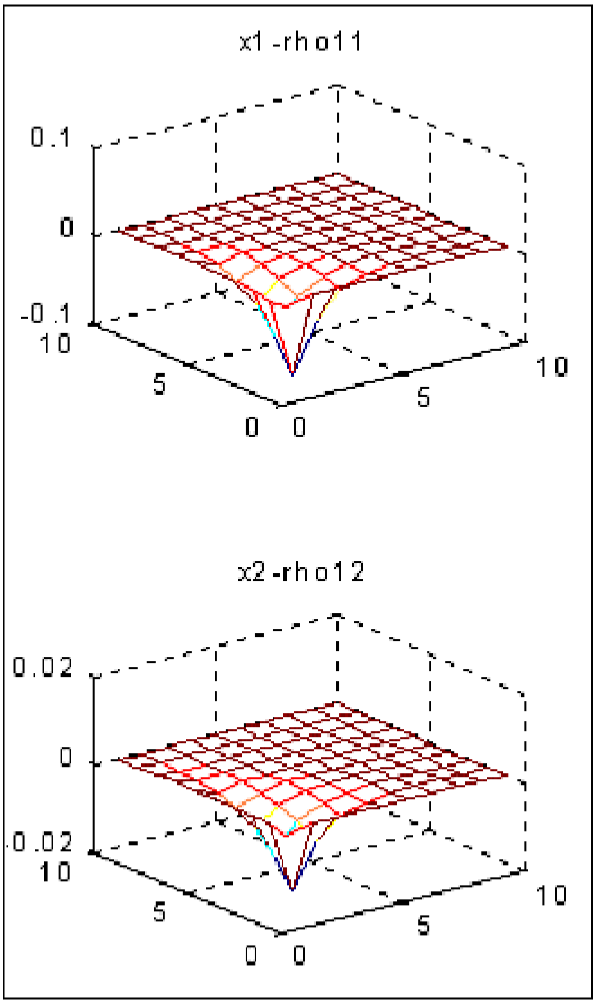

Fig .3: Trajectories of **x(i, j)** _ ρ_1**(i, j)**

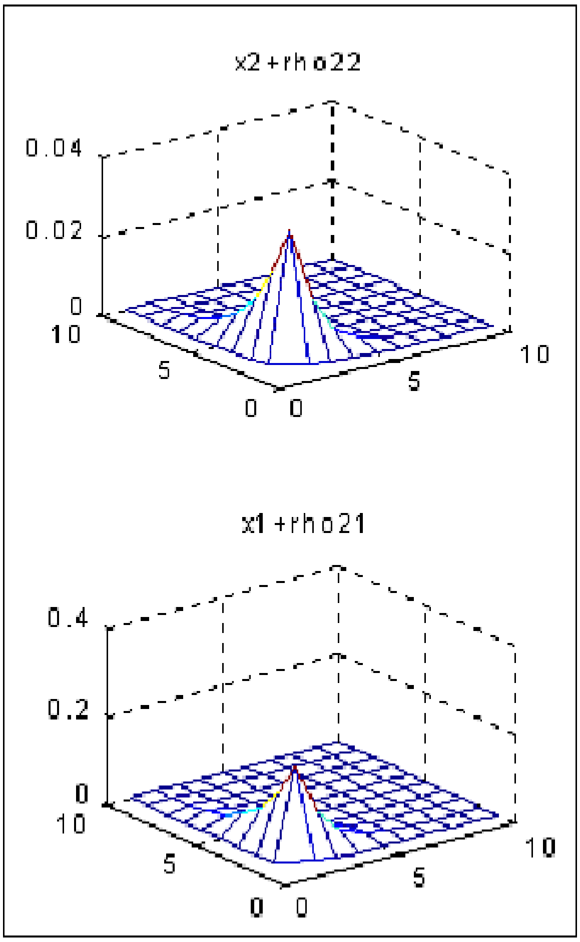

Fig .4: Trajectories of **x(i, j)** + ρ_2**(i, j)**

Conclusion

In this paper, we have given an extension of the componentwise asymptotic (exponential) stability concept for singular 1D and 2D discrete linear singular systems. Necessary and sufficient conditions for componentwise asymptotic (exponential) stability have been established. We considered also the symmetrical particular case, results have been obtained by applying the same approach. To show the validness of the theoretical results two numerical examples have been given.

References:

[1] Campbell, S. L., C.D. Meyer and N.J. Rose, *Applications of the Drazin inverse to linear systems of differential equations with singular constant coefficients*. SIAM J. Appl. Math. 31(3) 411-425. 1976.

[2] Campbell, S. L., *Comment on 2-D Descriptor systems.* Automatica, vol. 27, no.1,pp.189-192. 1991.

[3] Chen, C. T ., *Introduction to linear system Theory. Holt, Rinehart and Winston, New York. (1984) Linear System Theory and Design.* Holt, Rinehart and Winston, New York. 1970.
[4] Hmamed, A. , *Componentwise stability of continuous time delay linear systems*. Automatica. 32, 651_653. 1996.
[5] Hmamed A. , *Componentwise stability of 1-D and 2-D linear discrete-time systems*, Automatica, vol. 33, no 9, pp, 1759-1762. 1997.
[6] Hmamed, A. and Benzaouia, A., *Componentwise stability of linear systems: a nonsymmetrical case*. Int. J. Robust Nonlinear Control, vol. 7, pp. 1023-1028. 1997.
[7] Kaczorek, T. , *General response formula for two-dimensial linear systems with variable coefficients.* IEEE Trans. Aurom. Control Ac-31, 278_280. 1986.
[8] Kaczorek, T. , *Singular Roesser model and reduction to its canonical form*. Bulletin polish Academy of Sciences, Technical Sciences, vol.35, pp.645-652. 1987.
[9] Kaczorek, T., Equivalence of singular 2-D linear models. Bull. Polish Academy Sci., Electr. Electrotechnics, 37. 1989.
[10] Lewis, F.L., *A survey of linear singular systems.* J. Circuit, Syst., Sig. Process., Vol. 5, No1, pp. 3-36. 1986.
[11] Lewis F.L., *Recent work in singular systems*. Proc. Int. Symp. Singular systems, pp. 20-24, Atlanta, GA. 1987.
[12] Lewis, F.L., *A review of 2-D implicit system*s. Automatica. Vol. 28. No. 2, pp. 345-354, 1992.
[13] Tarboureich, S. and E.B. Castelan, *Positively invariant sets for singular discrete-time systems*, Int. J. Systems SCI., Vol. 24, No. 9, 1687-1705. 1993.
[14] Voicu, M., *Componentwise asymptotic stability of linear constant dynamical systems*, IEEE Trans. Autom. Control AC-29, 937-939. 1984.
[15] Voicu, M. , *On the application of the flow-invariance method in control theory and design.* In Proc. 10th IFAC World Congress, Munich, pp. 364-369. 1987.
[16] Huaian Diao, Yimin Wei, *Structured perturbations of group inverse and singular linear system with index one*, Journal of Computational and Applied Mathematics, v.173 n.1, p.93-113, 1 January 2005.
[17] Hua Xiang , Yimin Wei, *Structured mixed and componentwise condition numbers of some structured matrice*s, Journal of Computational and Applied Mathematics, v.202 n.2, p.217-229, May, 2007 .
[18] Yimin Wei , Huaian Diao, *Condition number for the Drazin inverse and the Drazin-inverse solution of singular linear system with their condition number*s. Journal of Computational and Applied Mathematics, v.182 n.2, p.270-289, 15 October 2005.
[19] N. Castro-González and M.F. Martínez-Serrano , *Expressions for the g-Drazin inverse of additive perturbed elements in a Banach algebra*. Submitted by L. Verde. Available online 5 March 2009.
[20] Debeljkovic, D.L., Stojanovic, S.B.; Visnjic, N.S.; Milinkovic, S.A.. *Singular Time Delayed System Stability Theory in the sense of Lyapunov: A Quite New Approach*. American Control Conference, 2007. ACC apos;07. Volume , Issue , 9-13 July 2007 Page(s):4939 – 4944. 2007.
[21] Debeljkovic, D.L.J., Stojanovic, S.B.; Jovanovic, M.B.; Milinkovic, S.A.. *Further results on singular time delayed system stability.* American Control Conference, 2005. Proceedings of the 2005 Volume, Issue , 8-10, Page(s): 4271 - 4276 vol. 6. June 2005.
[22] Yimin Wei and all, *Condition number for the Drazin inverse and the Drazin-inverse solution of singular linear system with their condition numbers.* Journal of Computational and Applied Mathematics, Volume 182, Issue 2 , Pp : 270 - 289 , 2005.
[23] Hua Xiang and All, *Structured mixed and componentwise condition numbers of some structured matrices*, Journal of Computational and Applied Mathematics, Volume 202, Issue 2, pp: 217-229 , 2007.
[24] N.H. Mejhed, A. Hmamed and A. Benzaouia, *Generalized componentwise stability of non-stationary continuous-time systems with constrained control.* Proceedings of the 9th WSEAS International Conference on Systems. Athens, Greece , Article No. 44 , Year of Publication: 2005 , ISBN:960-8457-29-7. 2005.
[25] M. Isabel Garcia-Planas, *Bifurcation diagrams of generic families of singular systems under proportional and derivative feedback.* WSEAS TRANSACTIONS on MATHEMATICS. ISSN: 1109-2769. Issue 2, Volume 7, pages 48 _58 February 2008.
[26] M. Isabel Garcia-Planas, *Holomorphic canonical forms for holomorphic families of singular systems*. Proceedings of the WSEAS International Conference on Applied Computing Conference Science And Engineering WSEAS. Istanbul, Turkey. Pages 24-29. Year of Publication: 2008. ISBN ~ ISSN: 1790-2769 , 960-6766-67-1. 2008.

[27] Xingping Sheng and Guoliang Chen, *An oblique projection iterative method to compute Drazin inverse and group inverse*. Applied Mathematics and Computation. Volume 211, Issue 2, Pages 417-421. 15 May 2009.

[28] Haidara H. and Boukas E.K., *Exponential stability of singular systems with multiple time-varying delay*s, Automatica, Volume 45, Issue 2, Pages 539-545. 2009.

[29] BOUKAS E. K. *On stability and stabilisation of continuous-time singular Markovian switching systems*, IET control theory & applications, ISSN 1751-8644 , vol. 2, n°10, pp. 884-894 , 2008.

[30] Chi-Yewu and Ting-Zhu Huang, *The Perturbation For The Drazin Inverse*, J. Appl. Math. & Informatics. Vol. 27, No. 1 - 2, pp. 267 – 273, 2009.

[31] Voicu, M., *Componentwise asymptotic stability of linear constant dynamical syste*ms, IEEE Transaction on Automatic Control, **29(**10), 937-939, 1984.