A method and an algorithm for obtaining the Stable Oscillatory Regimes Parameters of the Nonlinear Systems, with two time constants and Relay with Delay and Hysteresis

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Abstract: - The object of this paper is the oscillatory stable regime of a particular nonlinear system. This particular nonlinear system includes a relay, and its linear part is characterized by a transfer function with two time constants. In the beginning, the paper shows a method that can be used in the calculus of the parameters for the limit stable cycle, which is appropriated for these nonlinear systems. After that, the method is particularized for two different commutation lows of the relay nonlinearity: relay with hysteresis and relay with delay time. The different commutation laws induced, for the same linear parts, different shapes and parameters for the oscillatory stable regimes proper to assembly nonlinear system. These differences are presented in a case study.

Key-Words: - nonlinear systems, oscillatory regime, relay, delay time, hysteresis, calculus method, algorithm.

1 Introduction

The systems that include linear and nonlinear parts, in assembly, are nonlinear systems. The figure 1 shows a typical structure for these nonlinear systems.

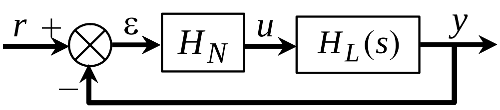

Fig.1. A typical nonlinear system

The previous structure uses the feedback principle, with a unitary reaction. The controlled process (or plant) is described by a linear transfer function, $H_L(s)$, which also includes the linear parts of the actuator. The regulator functionality and the nonlinear actuator characteristic are described by a nonlinear function, H_N. The error signal, ε, is the difference between the system input (or reference), r, and system output, y. Based on the error signal, the nonlinear parts generates the command u, that are the input of the plant.

Following, we will consider that the kind of the nonlinearity is a real relay, with delay time and hysteresis. This is the model of certain actuators, frequently included in many industrial and civilian applications, even in advanced equipments: liquid alimentation parts, air or gas conditioning devices. The main function of theses nonlinear systems is to maintain the values of any system parameters, in specified ranges [1].

Usually, these systems work with a constant input values. According to the system input level, the response of those systems bring to a punctually stationary state or to an oscillatory stable regime. The oscillatory stable regime is exclusively induced by the system nonlinearity. The characteristics of oscillatory stable regime, if it appears, can be analytically expressed [2], [3], [4], [8], and in other situations can be obtained only by simulations.

The relay model is proper for the actuators that work in maximal regime. An ideal relay model of the system nonlinearity can induce a very fast oscillatory regime, which increases the actuators stress and wearing. Nevertheless, in many automatic systems, is not preferred the ideal relay work for the nonlinear part. The relay model can be deliberately depreciated, using delays or hysteresis characteristics, in order to increase the systems oscillatory regime periods. In this way, the commutations frequency of the actuators can be reduced, the actuators wearing decreases and the lifetime increases.

This paper proposes a calculus model for the parameters of the oscillatory stable regime proper to the nonlinear systems that include non-ideal relay nonlinearity. The relay model used includes a delay time and a hysteresis characteristic. Separately, the case of relay with hysteresis characteristic and the case of a relay with delay time was treated in [4] and [8].

2 The study model

Further down, we will consider a nonlinear system with the structure showed in the figure 1.

For the system nonlinearity, described by a nonlinear function H_N, we will consider a relay symmetrical characteristic with unitary gain:

$$\left|u(t)\right|_{\varepsilon=ct}=1, \tag{1}$$

and the real gains of actuators are including in the gain, K, of the linear controlled process model.

The pattern of the linear controlled process (or plant) has two real time constants, without integrator elements, and it is modeled by a linear strictly stable transfer function, $H_L(s)$:

$$H_L(s)=\frac{K}{(s\cdot T_1+1)\cdot(s\cdot T_2+1)}, \tag{2}$$

where:

$$0<T_2<T_1. \tag{3}$$

The nonlinear function H_N can be presented in an analytically form, in order to emphasize the effect of the relay delay time τ:

$$H_N(\varepsilon(t))=H_{N_1}(\varepsilon(t-\tau)). \tag{4}$$

In the relation (4) $H_{N1}(\varepsilon)$ is the relay with hysteresis characteristic. This characteristic, presented in figure 2, will be considered symmetrical and its hysteresis bandwidth will be considered $2K\varepsilon_0$ (in order to simplify the next assumptions).

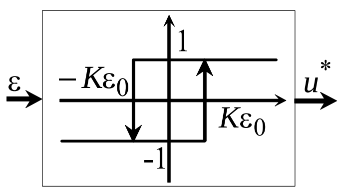

Fig. 2. The nonlinear characteristic $H_{N1}(\varepsilon)$

Using the previously assumptions, the system can be presented as in the figure 3.

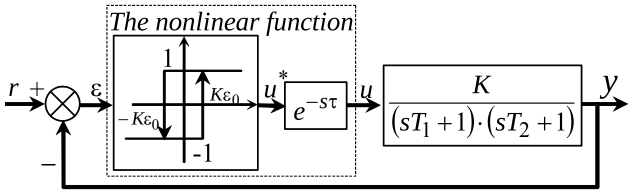

Fig. 3. The initial study model

In order to simplify the following considerations, we will use an equivalent structure of this system. This structure is presented in the figure 4.

In the figure (4) we can remark that, the central parts of the equivalent structure are a simplified nonlinear system. This simplified nonlinear system has a unitary linear part with unitary gain, and for the equivalent relay the hysteresis bandwidth is $2\varepsilon_0$.

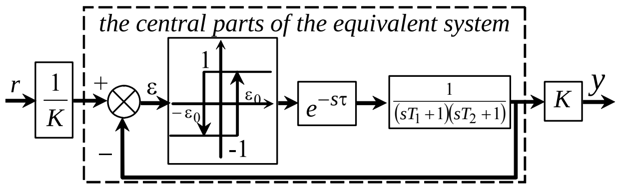

Fig. 4. The equivalent model

The following considerations will refer to a nonlinear system like the central parts of the equivalent system, presented in figure 4.

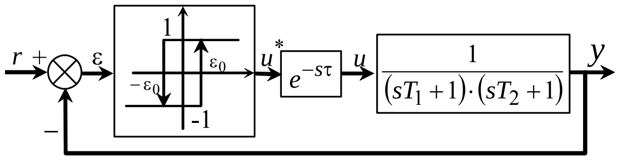

Fig. 5. The study model

The study system is presented in the figure 5, and for this system, the transfer function $H_L(s)$ is:

$$H_L(s)=\frac{1}{(s\cdot T_1+1)\cdot(s\cdot T_2+1)}. \tag{2'}$$

The differences between the initial system and the study model consist in the scales of its inputs and outputs and the hysteresis relay bandwidth. The dynamics of internal parameters for the two systems will have identically shapes at different scales.

3 The conditions for apparition of the oscillatory stable regime

All the following considerations will be based on the assumption that the system input are constant. Only in this case we can speak about a stationary regime.

Starting to the structural representation of the study model, we can emphasize an important aspect: because the linear part of study system is strictly stable, when the error signal is constant, the system output tends to a limit value:

$$y(t)\big|_{\varepsilon=ct} \to y_{st} = sign(\varepsilon)\,. \qquad (5)$$

If the system input level is high enough or low enough, for any initial condition, the error signal will not change its sign. In all cases, the system outputs are limited, and we can conclude that the assembly nonlinear system is globally stable, ***“in great”***.

In any initial conditions, for reduced values of references [4], [8], the error signal can change its sign, the system evolution tends to an ***oscillatory stable regime***, and its outputs tend alternately toward values 1 or -1. In these cases, we can conclude that the system is punctually instable, ***“in small”***. We are interested to evaluate the parameters of the stable oscillatory regime, because it emphasizes the effectiveness of this sort of automatic system. The principal parameter of the oscillatory stable regime is the oscillations period.

The apparition of the oscillatory stable regime is possible if the reference module achieves the next condition:

$$\left|r(t)\right| < r_{\lim} = 1 - \varepsilon_0\,. \qquad (6)$$

The previous condition is applicable and for the systems presented in figure 3 and it do not depend on the delay time.

4 The oscillatory stable regime study method

Further, we will use a particularization for the phases-plane method (the phases plane will be noted ***PF***). Will consider the linear system part as a parallel connection of two order-one elements, as it is presented in figure 6.

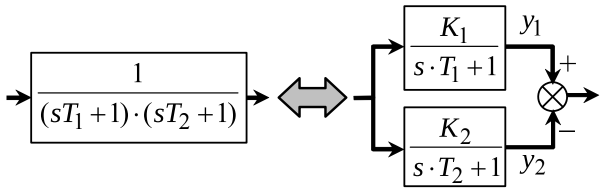

Fig. 6. The equivalent parallel connection

The gains of the two order one elements, noted k_1 and k_2, are the following:

$$\begin{cases} k_1 = T_1/(T_1 - T_2), \\ k_2 = T_2/(T_1 - T_2). \end{cases} \qquad (7)$$

The gains are positive and fulfill the condition $k_1 - k_2 = 1$. This is the reason to express the gain k_2 trough k_1. The state variables associated to each element of connection are noted y_1 and y_2.

If the plant input has a constant value $u = 1$, the evolutions of the state variables can be calculated [6], [7], depending on the initial conditions, (y_{10}, y_{20}) and the time, t:

$$\begin{cases} y_1(t) = (y_{10} - k_1)\cdot e^{-t/T_1} + k_1; \\ y_2(t) = (y_{20} - k_2)\cdot e^{-t/T_2} + k_2. \end{cases} \qquad (8)$$

In the case $u = -1$, the evolutions of the state variables are described by equations:

$$\begin{cases} y_1(t) = (y_{10} + k_1)\cdot e^{-t/T_1} - k_1; \\ y_2(t) = (y_{20} + k_2)\cdot e^{-t/T_2} - k_2. \end{cases} \qquad (8')$$

Based on (8), we will obtain, in ***PF***, the equations of trajectories, which are supra-unitary power curves $(T_1/T_2 > 1)$:

$$\begin{cases} \left|\dfrac{k_2 - y_2(t)}{k_2 - y_{20}}\right| = \left|\dfrac{k_1 - y_1(t)}{k_1 - y_{10}}\right|^{T_1/T_2}, u = 1; \\[2ex] \left|\dfrac{k_2 + y_2(t)}{k_2 + y_{20}}\right| = \left|\dfrac{k_1 + y_1(t)}{k_1 + y_{10}}\right|^{T_1/T_2}, u = -1. \end{cases} \qquad (9)$$

In the case $u=1$, the trajectories converge to an ***accumulation point***, (k_1, k_2), placed in the first quadrant and noted **PAP**. The figure 7 presents this point and this trajectory curves family.

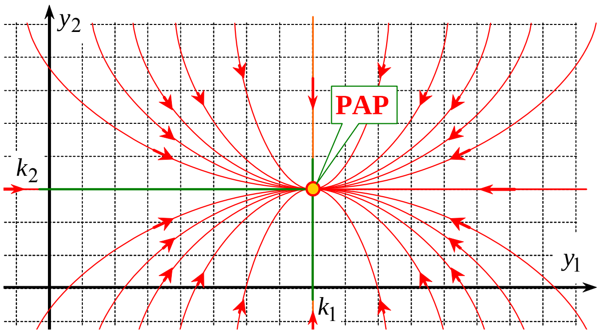

Fig. 7 The PAP add the family of trajectory curves in case of $u=1$

If $u=-1$, all trajectories tend to another accumulation point, $(-k_1, -k_2)$, placed in the third quadrant and noted **PAN**.

The points PAP and PAN and the two family of trajectory curves are symmetrically positions and shapes in the PF plane.

If the conditions (6) are achieved, the system state trajectory tends to a ***stable limit cycle***, ***CLS***, this situation are presented in the figure 8.

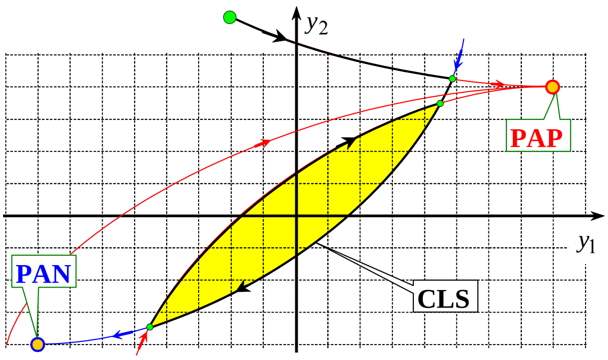

Fig. 8 The stable limit cycle

Starting from any arbitrary initial state of the system (y_{10}, y_{20}), after some steps, the state system trajectory bring the CLS. In order to find the parameters of the oscillatory stable regime, will use the next figure.

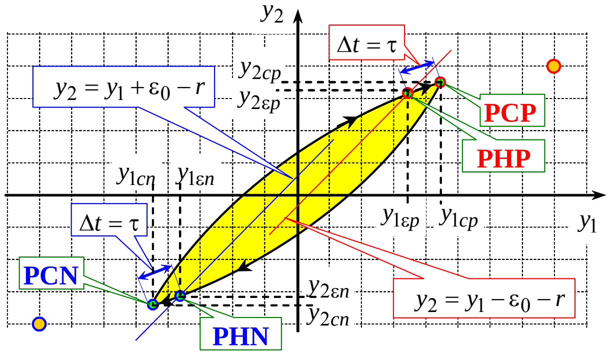

Fig. 9 The main points of the stable limit cycle

The main points of the CLS, presented in the previous figure are:

- ***the commutations points***, PCP and PCN. In these points, the nonlinear part of system effectively commutes. The coordinates of this point are: (y_{1cp}, y_{2cp}), for the PCP and (y_{1cn}, y_{2cn}) for the PCN;
- ***the hysteresis commutations points***, PHP and PHN. In these points, the conditions for nonlinear part commutation are achieved, excepting the delay. The coordinates of this point are: $(y_{1\varepsilon p}, y_{2\varepsilon p})$, for the PCP and $(y_{1\varepsilon n}, y_{2\varepsilon n})$ for the PCN.

Using the hysteresis characteristic, for the PHP and PHN points we can obtain two relations between its coordinates:

$$\begin{cases} y_{2\varepsilon p} = y_{1\varepsilon p} - r - \varepsilon_0; \\ y_{2\varepsilon n} = y_{1\varepsilon n} - r + \varepsilon_0. \end{cases} \tag{10}$$

Using the state trajectories equations, we can obtain other relations:

$$\begin{cases} \dfrac{k_2 - y_{2cp}}{k_2 - y_{2\varepsilon p}} = \left(\dfrac{k_1 - y_{1cp}}{k_1 - y_{1\varepsilon p}}\right)^{T_1/T_2}; \\ \dfrac{k_2 + y_{2cn}}{k_2 + y_{2\varepsilon n}} = \left(\dfrac{k_1 + y_{1cn}}{k_1 + y_{1\varepsilon n}}\right)^{T_1/T_2}, \end{cases} \tag{11}$$

and:

$$\begin{cases} \dfrac{k_2 - y_{2cp}}{k_2 - y_{2cn}} = \left(\dfrac{k_1 - y_{1cp}}{k_1 - y_{1cn}}\right)^{T_1/T_2}; \\ \dfrac{k_2 + y_{2cn}}{k_2 + y_{2cp}} = \left(\dfrac{k_1 + y_{1cn}}{k_1 + y_{1cp}}\right)^{T_1/T_2}. \end{cases} \tag{12}$$

Based on the equations (8) and the delay time, it results another other relations:

$$\begin{cases} y_{1cp} = \left(y_{1\varepsilon p} - k_1\right) \cdot e^{-\tau / T_1} + k_1; \\ y_{1cn} = \left(y_{1\varepsilon n} + k_1\right) \cdot e^{-\tau / T_1} - k_1. \end{cases} \quad (13)$$

By adding of relations (10), (11), (12) and (13), we obtain a nonlinear equation system, with eight equations:

$$\begin{cases} y_{2\varepsilon p} = y_{1\varepsilon p} - r - \varepsilon_0; \\ y_{2\varepsilon n} = y_{1\varepsilon n} - r + \varepsilon_0; \\ \dfrac{k_2 - y_{2cp}}{k_2 - y_{2\varepsilon p}} = \left(\dfrac{k_1 - y_{1cp}}{k_1 - y_{1\varepsilon p}}\right)^{T_1 / T_2}; \\ \dfrac{k_2 + y_{2cn}}{k_2 + y_{2\varepsilon n}} = \left(\dfrac{k_1 + y_{1cn}}{k_1 + y_{1\varepsilon n}}\right)^{T_1 / T_2}; \\ \dfrac{k_2 - y_{2cp}}{k_2 - y_{2cn}} = \left(\dfrac{k_1 - y_{1cp}}{k_1 - y_{1cn}}\right)^{T_1 / T_2}; \\ \dfrac{k_2 + y_{2cn}}{k_2 + y_{2cp}} = \left(\dfrac{k_1 + y_{1cn}}{k_1 + y_{1cp}}\right)^{T_1 / T_2}; \\ y_{1cp} = \left(y_{1\varepsilon p} - k_1\right) \cdot e^{-\tau / T_1} + k_1; \\ y_{1cn} = \left(y_{1\varepsilon n} + k_1\right) \cdot e^{-\tau / T_1} - k_1. \end{cases} \quad (14)$$

The solving of the nonlinear equation system (14) permit to obtain the elements of the CLS. The main parameters are the coordinates of PCP and PCN, (y_{1cp}, y_{2cp}) and (y_{1cn}, y_{2cn}), and the times between the two commutations. We will use the notations:

- t_n, the time of transition between PCP and PHN;
- t_p, the time of transition between PCN and PHP.

The previous two times can be calculate using the relations:

$$\begin{cases} t_p = T_1 \cdot \ln \dfrac{k_1 - y_{1cn}}{k_1 - y_{1cp}} - \tau; \\ t_n = T_1 \cdot \ln \dfrac{k_1 + y_{1cp}}{k_1 + y_{1cn}} - \tau. \end{cases} \quad (15)$$

Because the times of transitions between the points PHP and PCP and between the points PHN and PCN are τ, the CLS period, T, will be:

$$T = t_n + t_p + 2\tau \quad (16)$$

or:

$$T = T_1 \cdot \ln\left(\frac{k_1 + y_{1cp}}{k_1 - y_{1cp}} \cdot \frac{k_1 - y_{1cn}}{k_1 + y_{1cn}}\right). \quad (17)$$

5 The calculus of output system characteristics

The equations of system (14), completed with (15) and (16) permit to find out the main parameters of the system oscillatory stable regime. Additionally to all that were presented, we need to determine the system output characteristics: the mean value, the extreme values and the output bandwidth.

The outputs mean value can be calculated using an integral relation:

$$\bar{y} = \frac{1}{T} \cdot \int_0^T \left(y_1(t) - y_2(t)\right) \cdot dt. \quad (18)$$

Using the CLS arcs equations, after same calculus and transformations we obtain a short result:

$$\bar{y} = \frac{t_p - t_n}{T}. \quad (19)$$

For the calculus of the extreme output values, we will begin with same observations:

- the slope of a tangent of the CLS arc which converge to the PAP from the point PCN, can be express such as:

$$\frac{dy_2}{dy_1} = \frac{T_1}{T_2} \cdot \frac{k_2 - y_2}{k_1 - y_1}; \quad (20)$$

  and after some transformation:

$$\frac{dy_2}{dy_1} = \frac{T_1}{T_2} \cdot \frac{k_2 - y_{cn}}{\left(k_1 - y_{1cn}\right)^{T_1 / T_2}} \cdot \left(k_1 - y_1\right)^{1 / k_2}. \quad (21)$$

  In the previous relations (y_1, y_2) are the coordinate of the tangency point. Because the value k_2 is positive, it is obviously that the tangent slope continuously decreases; this conclusion will be also obtained for the other CLS arc;
- the extremes system output values are characterized by a extremes values of the difference $y_1 - y_2$;
- for the points where CLS cross the line between the points PAP and PAN, the coordinates satisfied the condition:

$$y_2 = \frac{k_2}{k_1} \cdot y_1 = \frac{T_2}{T_1} \cdot y_1, \tag{22}$$

and the slope of the tangent of the CLS are unitary:

$$y_2 = \frac{T_2}{T_1} y_1 \Leftrightarrow \frac{dy_2}{dy_1} = 1. \tag{23}$$

Using the previous observations, we can conclude that the extremes output values correspond to the points where CLS cross the line between the points PAP and PAN. The figure 10 presents theses crossing points.

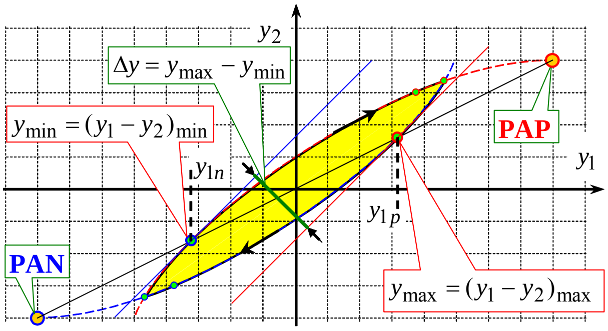

Fig. 10 The points of the CLS where the system output has the extreme values

We will use the notation y_{1p} and y_{1n} for the values of the variable y_1 which corresponds to y_{max} and y_{min}, the system output extreme values. Using the last conclusion and the CLS parameters, we will obtain the following relations for the calculus of the values y_{1p} and y_{1n}:

$$\begin{cases} y_{1n} = k1 - \left(\dfrac{k_2}{k_1}\right)^{k_2} \cdot \dfrac{(k_1 - y_{1cp})^{k_1}}{(k_2 - y_{2cp})^{k_2}}; \\ y_{1p} = \left(\dfrac{k_2}{k_1}\right)^{k_2} \cdot \dfrac{(k_1 + y_{1cp})^{k_1}}{(k_2 + y_{2cp})^{k_2}} - k_1. \end{cases} \tag{24}$$

The maximum value of the system output will be:

$$y_{max} = y_{1p} / k_1, \tag{25}$$

and the minimum of the system output:

$$y_{min} = y_{1n} / k_1, \tag{26}$$

Finally, the output bandwidth can be calculated by using the relation:

$$\Delta y = (y_{1p} - y_{1n}) / k_1. \tag{27}$$

The relations (15), (16), (19), (24), (25), (26) and (27) will complete the system equations (14). In this way, we obtain a complete equations system, which can be used for the calculus of the CLS parameters and of the oscillatory stable regime parameters.

The complete equations system is too complex and nonlinear. The solving of the complete equation system is possible, using an iterative algorithm. This algorithm reproduces the system evolution toward the CLS.

6 The calculus algorithm

In order to find all the parameters of CLS and of the oscillatory stable regime, the next steps will be passed:

a. choose an arbitrary value $y_{1\varepsilon p}$;

b. calculate the value $y_{2\varepsilon p}$, based on (10):

$$y_{2\varepsilon p} = y_{1\varepsilon p} - r - \varepsilon_0;$$

c. calculate the values y_{1cp} and y_{2cp} using the state variable evolution:

$$\begin{cases} y_{1cp} = k_1 + \left(y_{1\varepsilon p} - k_1\right) \cdot e^{-\tau / T_1}; \\ y_{2cp} = k_2 + \left(y_{2\varepsilon p} - k_2\right) \cdot e^{-\tau / T_2}, \end{cases}$$

d. calculate the value t_n, like solution of equation:

$$\frac{y_{2cp} + k_2}{\exp(t_n / T_2)} = \frac{y_{1cp} + k_1}{\exp(t_n / T_1)} - 1 - r + \varepsilon_0;$$

e. calculate the values y_{1cn} and y_{2cn}:

$$\begin{cases} y_{1cn} = -k_1 + (y_{1cp} + k_1) \cdot e^{-(\tau + t_n)/T_1}; \\ y_{2cn} = -k_2 + (y_{2cp} + k_2) \cdot e^{-(\tau + t_n)/T_2}, \end{cases}$$

f. calculate the value t_p, like solution of equation:

$$\frac{y_{2cn} - k_2}{\exp(t_p / T_2)} = \frac{y_{1cn} - k_1}{\exp(t_p / T_1)} + 1 - r - \varepsilon_0;$$

g. calculate the values y_{1cp} and y_{2cp}:

$$\begin{cases} y_{1cp} = \dfrac{y_{1cp} - k_1}{\exp((\tau + t_p)/T_1)} + k_1; \\ y_{2cp} = \dfrac{y_{2cp} - k_2}{\exp((\tau + t_p)/T_2)} + k_2, \end{cases}$$

h. if the differences between the successive values of y_{1cp} and y_{2cp} are not acceptable **go to step d**;

i. calculate the CLS period:

$$T = t_n + t_p + 2\tau ,$$

j. calculate the output parameters:

$$\begin{cases} \overline{y} = (t_p - t_n)/T ; \\ \Delta y = (y_{1p} - y_{1n})/k_1 ; \\ y_{max} = y_{1p}/k_1 ; \\ y_{min} = y_{1n}/k_1 . \end{cases} ,$$

k. **STOP**.

The presented algorithm strongly converges (the steps ***d-h***) to the solution; the next example emphasizes this fact.

Example 1. *The calculus of CLS parameters by using the above presented algorithm, in case of a nonlinear system parameterized by the next values:* T_1=5s, T_2=3s, ε_0=0.15, τ=0.2s.

For the reference, we will consider the values: r_1=0, r_2=0.22, r_3=0.5, *and the results are presented in the next tables.*

Table 1. The results in case $r = r_1$.

parameters	Iteration 1	Iteration 2	Iteration 3
$y_{1\varepsilon p}$	2.5	--	--
$y_{2\varepsilon p}$	2.3	--	--
y_{1cp}	2.5	1.38436	1.38232
y_{2cp}	2.26098	1.16743	1.16544
$t_n+\tau$	7.55779	6.07892	6.07591
y_{1cn}	-1.42976	-1.38241	-1.38232
y_{2cn}	-1.21193	-1.16552	-1.16543
$t_p+\tau$	6.14576	6.07603	6.07590
y_{1cp}	1.38436	1.38232	1.38232
y_{2cp}	1.16743	1.16544	1.16543
Δy_{1cp}	**1.11564**	**0.00204**	**0.0000**
Δy_{2cp}	**1.09355**	**0.00199**	**0.00001**
T			**12.15181**

Table 2. The results in case $r = r_2$.

parameters	Iteration 1	Iteration 2	Iteration 3
$y_{1\varepsilon p}$	2.5	--	--
$y_{2\varepsilon p}$	2.08	--	--
y_{1cp}	2.5	1.73626	1.73499
y_{2cp}	2.05171	1.30310	1.30185
$t_n+\tau$	6.03762	5.06660	5.06489
y_{1cn}	-1.04949	-1.00763	-1.00757
y_{2cn}	-1.04846	-1.00744	-1.00737
$t_p+\tau$	7.53171	7.46407	7.46397
y_{1cp}	1.73626	1.73499	1.73499
y_{2cp}	1.30310	1.30185	1.30185
Δy_{1cp}	**0.76364**	**0.00127**	**0.00000**
Δy_{2cp}	**0.74861**	**0.00125**	**0.00000**
T			**12.52886**

Table 3. The results in case $r = r_3$.

parameters	Iteration 1	Iteration 2	Iteration 3
$y_{1\varepsilon p}$	2.5		
$y_{2\varepsilon p}$	1.8		
y_{1cp}	2.5	2.14188	2.14164
y_{2cp}	1.78536	1.43434	1.43410
$t_n+\tau$	4.49866	4.07684	4.07655
y_{1cn}	-0.52670	-0.50677	-0.50676
y_{2cn}	-0.80237	-0.78284	-0.78283
$t_p+\tau$	10.52189	10.48547	10.48545
y_{1cp}	2.14188	2.14164	2.14164
y_{2cp}	1.43434	1.43410	1.43410
Δy_{1cp}	**0.35812**	**0.00024**	**0.00000**
Δy_{2cp}	**0.35102**	**0.00024**	**0.00000**
T			**14.562**

7 A case study

In this example we will consider a nonlinear system, with the structure presented in figure 11.

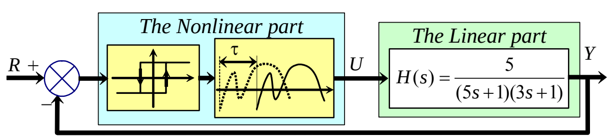

Fig. 11 Example of a nonlinear system

The parameters of the linear part are: T_1=5s, T_2=3s and the gain K_L=5. The stationary output value of the nonlinear part is $U = 2$.

We will use the presented algorithm to determinate the characteristics of its oscillatory stable regime, for any values of the delay τ and of the input hysteresis bandwidth, ΔH.

In order to apply the presented calculus method, we will use an equivalent system, as in figure 12.

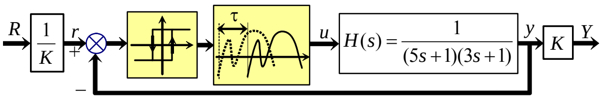

Fig. 12 The equivalent shape for the exemplified nonlinear system

The open loop gain for the initial system is:

$$K = K_L \cdot U_{st} = 10 .$$

The central part of the equivalent system fulfills the conditions of the study model. The interval of hysteresis bandwidth values for the equivalent system is:

$$2\varepsilon_0 = \frac{\Delta H}{K} .$$

For the central part of the equivalent system, the reference r and the output y represent normed values:

$$r = R / K, \quad y = Y / K .$$

If the parameters of the output of the equivalent system central part y are $\bar{y}$, Δy, $y_{\min}$ and $y_{\max}$, the parameters of the initial system output Y will be:

$$\bar{Y} = K \cdot \bar{y}, \quad \Delta Y = K \cdot \Delta y ,$$

$$Y_{\min} = K \cdot y_{\min} , \; Y_{\max} = K \cdot y_{\max} .$$

The oscillatory regime appears if the next condition is achieved:

$$|r| < 1 - \varepsilon_0 .$$

For the initial system, the previous condition becomes:

$$|R| < K - \frac{\Delta H}{2} .$$

The gains of the two elements, which decompose the linear parts of the equivalent system, are:

$$k_1 = 5/2 = 2.5 , \; k_2 = 3/2 = 1.5 .$$

Based on the presented algorithm, in any possible cases (τ, ε_0), for differed reference values it was obtained the parameters of the CLS and of the oscillatory stable regime. The next tables and figures present the results.

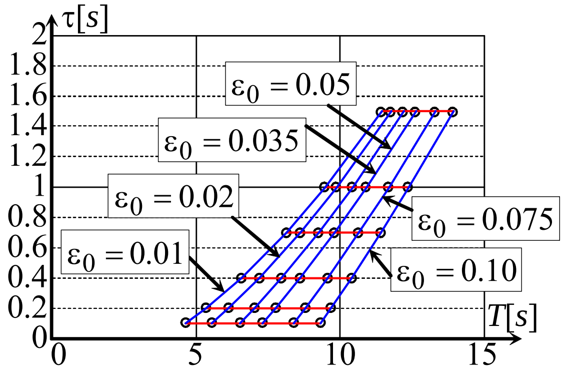

Fig. 13 The oscillatory stable regime period depending on relay delay time for a null input

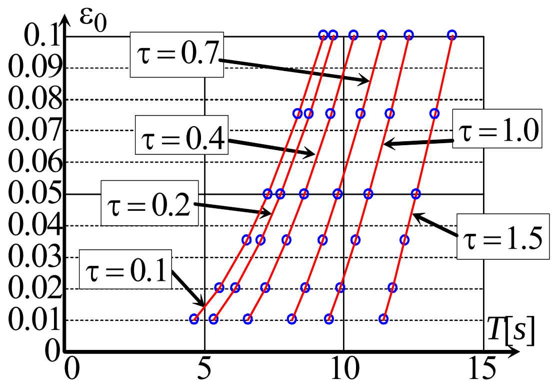

Fig. 14 The oscillatory stable regime period depending on relay input bandwidth for a null input

Table 4. The period of the oscillatory stable regime for a null input

		ε_0					
		0.01	**0.02**	**0.035**	**0.05**	**0.075**	**0.10**
$\tau[s]$	**0.1**	4.62	5.52	6.51	7.30	8.38	9.30
	0.2	5.32	6.11	7.02	7.75	8.78	9.67
	0.4	6.57	7.19	7.96	8.62	9.56	10.39
	0.7	8.14	8.62	9.26	9.82	10.66	11.42
	1.0	9.50	9.90	10.44	10.94	11.70	12.40
	1.5	11.46	11.78	12.23	12.65	13.32	13.95

Table 5. The output bandwidth, Δy[%], of the oscillatory stable regime for a null input

		ε_0					
		0.01	**0.02**	**0.035**	**0.05**	**0.075**	**0.10**
τ[s]	**0.1**	8.7	12.3	16.8	20.8	26.9	32.5
	0.2	11.4	14.9	19.3	23.3	29.3	34.8
	0.4	17.0	20.2	24.4	28.3	34.1	39.5
	0.7	25.5	28.3	32.2	35.8	41.3	46.5
	1.0	33.7	36.3	39.8	43.2	48.4	53.4
	1.5	46.8	49.0	52.1	55.2	59.9	64.5

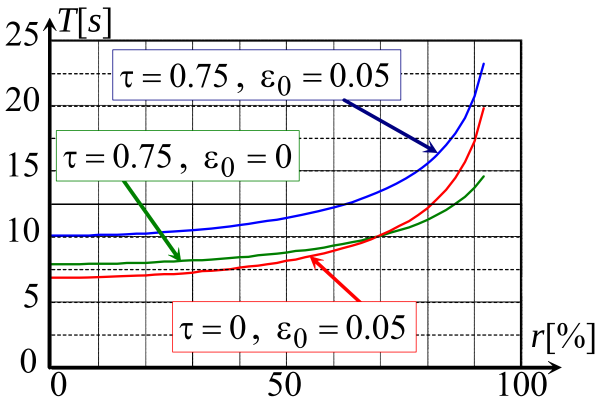

Fig. 15 The oscillatory stable regime period depending on the input

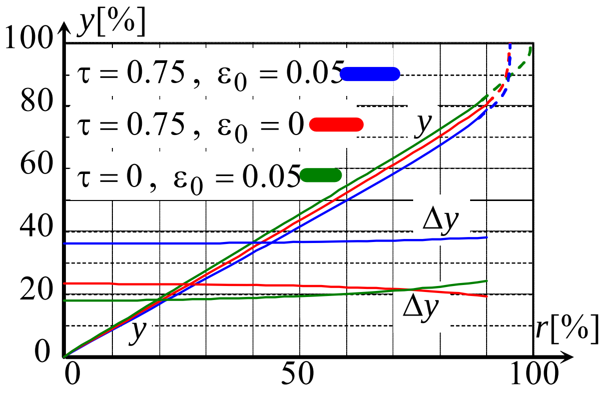

Fig. 16 The output and the output bandwidth depending on the input

8 Conclusions

For the type of nonlinear systems taken into consideration, the paper offers certain instruments useful for evaluating the performances of the limit stable cycle which characterize its working.

Compared with the use of the numerical simulation methods, the use of the calculus relations is more advantageous, being able to offer a more comprehensive view on the limits of these automatic structures.

Regarding to the results, some conclusions have to be underlined:

- the period of the oscillatory regime growing with the delay and with the hysteresis relay bandwidth;
- the most reduced periods of the oscillatory regime correspond to a null references;
- the mean output of value deviates from reference; this deviation can be considered linear and it can be eventually compensated using a gain correction in the reaction loop;
- for the delay values comparable with the major time constant of the system and for high values of hysteresis relay bandwidth, the system's performances deteriorate considerably;
- the output bandwidths has a very low variation with the input increase, but it increase with the delay and with the hysteresis relay bandwidth.

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