Theory and Application of the Composed Fuzzy Measure of L-Measure and Delta-Measures

Hsiang-Chuan Liu, Chin-Chun Chen, Der-Bang Wu, and Tian-Wei Sheu

Abstract—The well known fuzzy measures, λ-measure and P-measure, have only one formulaic solution. Two multivalent fuzzy measures with infinitely many solutions were proposed by our previous works, called L-measure and δ-measure, but the former do not include the additive measure as the latter and the latter has not so many measure solutions as the former. Due to the above drawbacks, in this paper, an improved fuzzy measure composed of above both, denoted L_δ -measure, is proposed. For evaluating the Choquet integral regression models with our proposed fuzzy measure and other different ones, a real data experiment by using a 5-fold cross-validation mean square error (MSE) is conducted. The performances of Choquet integral regression models with fuzzy measure based L_δ -measure, L-measure, δ-measure, λ-measure, and P-measure, respectively, a ridge regression model, and a multiple linear regression model are compared. Experimental result shows that the Choquet integral regression models with respect to extensional L-measure based on γ-support outperforms others forecasting models.

Keywords—Lambda-measure, P-measure, Delta-measure, Gamma-support, composed fuzzy measure, Choquet integral regression model.

I. INTRODUCTION

When there are interactions among independent variables, traditional multiple linear regression models do not perform well enough. The traditional improved methods exploited ridge regression models [1]. In this paper, we suggest using the Choquet integral regression models [7-15] based on some single or compounded fuzzy measures [2-5, 7-15] to improve this situation. The well-known fuzzy measures, λ-measure [2-4] and P-measure [5] have only one formulaic solution of fuzzy measure, the former is not a closed form, and the latter is not sensitive enough. Two multivalent fuzzy measures with infinitely many solutions were proposed by our previous works, called L-measure [7-9] and δ-measure [10,11], but L-measure do not include the additive measure and δ-measure has not so many measure solutions as L-measure. Due to the above drawbacks, in this paper, an improved fuzzy measure composed of above two multivalent fuzzy measures, denoted L_δ -measure, is proposed. This improved multivalent fuzzy measure is not only including the additive measure, but also having the same infinitely many measure solutions as L-measure. For evaluating the Choquet integral regression models with our proposed fuzzy measure and other different ones, a real data experiment by using a 5-fold cross-validation mean square error (MSE) is conducted. The performances of Choquet integral regression models with fuzzy measure based L_δ -measure, L-measure, δ-measure, λ-measure, and P-measure, respectively, a ridge regression model, and a multiple linear regression model are compared.

This paper is organized as follows: The multiple linear regression and ridge regression [1] are introduced in section II; two well known fuzzy measure, λ-measure [2] and P-measure [5], are introduced in section III; our new measure, δ-measure, is introduced in section IV; the fuzzy support, γ-support [7] is described in section V; the Choquet integral regression model [6-8] based on fuzzy measures are described in section VI; experiment and result are described in section VII; and final section is for conclusions and future works.

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H. C. Liu, is with the Department of Bioinformatics, Asia University, Taichung, 41345, Taiwan (e-mail: lhc@asia.edu.tw).

C. C. Chen, is with National Taichung University, Taichung, 40306, Taiwan. He is now with Department of General Education, Min-Hwei College, Tainan 736 , Taiwan. (e-mail: ccchen@mail.mhchcm.edu.tw).

D. B. Wu, is with National Taichung University, Taichung, 40306, Taiwan. He is now with the Graduate Institute of Educational Measurement and Department of Mathematics Education, National Taichung University, Taichung, 40306 Taiwan. (phone: 886-93673-1088; fax: 886-42218-3500; e-mail:wudb@hotmail.com).

T. W. Sheu, is with National Taichung University, Taichung, Taiwan 40306. He is now with the Graduate Institute of Educational Measurement, National Taichung University, Taichung, 40306 Taiwan. (e-mail: sheu@mail.ntcu.edu.tw).

II. THE MULTIPLE LINEAR REGRESSION, RIDGE REGRESSION

Let $\underline{Y} = X\underline{\beta} + \underline{\varepsilon}$, $\underline{\varepsilon} \sim \mathrm{N}\left(\underline{0}, \sigma^2 I_n\right)$ be a multiple linear model, $\underline{\hat{\beta}} = \left(X'X\right)^{-1} X'Y$ be the estimated regression coefficient vector, and $\underline{\hat{\beta}}_k = \left(X'X + kI_n\right)^{-1} X'Y$ be the estimated ridge regression coefficient vector, Hoerl, Kenard and Baldwin [1] suggested

$$\hat{k} = \frac{n\hat{\sigma}^2}{\underline{\hat{\beta}}'\underline{\hat{\beta}}} \tag{1}$$

III. FUZZY MEASURES

The two well known fuzzy measures, the λ-measure proposed by Sugeno in 1974, and P-measure proposed by Zadah in 1978, are concisely introduced as follows.

A. *Axioms of Fuzzy Measures*

Definition 1 fuzzy measure [2-4]
A fuzzy measure μ on a finite set X is a set function $\mu : 2^X \to [0,1]$ satisfying the following axioms:

1) $\mu(\phi)=0$, $\mu(X)=1$ (boundary conditions) (2)
2) $A \subseteq B \Rightarrow \mu(A) \le \mu(B)$ (monotonicity) (3)

B. *Singleton Measures*

Definition 2 singleton measure [2-7]
A singleton measure of a fuzzy measure μ on a finite set X is a function $s : X \to [0,1]$ satisfying:

$$s(x) = \mu(\{x\}),\ x \in X \tag{4}$$

$s(x)$ is called the fuzzy density of singleton x.

C. *λ-measure*

Definition 3 λ-measure [3]
For a given singleton measures s, λ-measure, g_λ, is a fuzzy measure on a finite set X, satisfying:

$$\begin{aligned} &A, B \in 2^X,\ A \cap B = \phi,\ A \cup B \ne X \\ &\Rightarrow g_\lambda(A \cup B) \\ &\quad = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B) \end{aligned} \tag{5}$$

$$\prod_{i=1}^{n}\left[1+\lambda s(x_i)\right] = \lambda + 1 > 0,\ s(x_i) = g_\lambda(\{x_i\}) \tag{6}$$

Where the real number, λ, is also called the determine coefficient of λ-measure.

Note that once the singleton measure is known, we can obtain the values of λ uniquely by using the previous polynomial equation. In other words, λ-measure has a unique solution without closed form. Moreover, for given singleton measures s, $If \sum_{x\in X} s(x) = 1\ then\ g_\lambda(A) = \sum_{x \in A} s(x)$, in other word, if $\sum_{x\in X} s(x) = 1$ then λ-measure is just the additive measure

D. *P-measure*

Definition 4 P-measure [5]
For given a singleton measures s, P-measure, g_P, is a fuzzy measure on a finite set X, satisfying:

$$\begin{aligned} &{}^\forall A \in 2^X \\ &\Rightarrow g_P(A) = \max_{x\in A}\{s(x)\} = \max_{x \in A}\{g_P(\{x\})\} \end{aligned} \tag{7}$$

Note that for any subset of X, A, P-measure considers only the maximum value and will lead to insensitivity.

E. *L-measure*

Definition 5 L-measure [7-9]
For given a singleton measure $s(x)$, L-measure, g_L, is a fuzzy measure on a finite set X, $|X| = n$, satisfying:

1) $L \in [0, \infty)$ (8)
2) ${}^\forall A \subset X, n - |A| + (|A|-1)L > 0 \Rightarrow$

$$g_L(A) = \max_{x\in A}\left[s(x)\right] + \frac{(|A|-1)L\sum_{x\in A} s(x)\left[1-\max_{x\in A}\left[s(x)\right]\right]}{\left[n-|A|+(|A|-1)L\right]\sum_{x\in X} s(x)} \tag{9}$$

Where the real number, L, is also called the determine coefficient of L-measure.

Theorem 1 [7-9]
(i) for each $L \in [0, \infty)$, L-measure is a fuzzy measure, in other words, L-measure has infinitely many solutions of fuzzy measures, for each $L \in [0, \infty)$.
(ii) $L \in [0, \infty)$, L-measure is an increasing function on real number L.
(iii) if $L = 0$ then L-measure is just the P-measure

F. *δ-measure*

Definition 6 δ-measure [10,11]
For given singleton measure $s(x)$, a δ-measure, g_δ, is a fuzzy measure on a finite set X, $|X| = n$, satisfying:

1) $\delta \in [-1, 1],\ \sum_{x\in X} s(x) = 1$ (10)
2) $g_\delta(\phi) = 0, g_\delta(X) = 1$ (11)
3) $\forall A \subset X, A \ne X \Rightarrow$

$$g_\delta(A) = \left[1 + \delta \max_{x\in A} s(x)\right] \frac{(1+\delta)\sum_{x\in A} s(x)}{1 + \delta \sum_{x \in A} s(x)} - \delta \max_{x\in A} s(x) \tag{12}$$

Where the real number, δ, is also called the determine coefficient of δ-measure.

Theorem 2 [11]
(i) $\delta \in [-1, 1]$, δ-measure is an increasing function on δ
(ii) if $\delta = -1$, then δ-measure is just the P-measure
(iii) if $\delta = 0$, then δ-measure is just the additive measure
(iv) if $-1 < \delta < 0$, then δ-measure is a sub-additive measure
(v) if $0 < \delta < 1$, then δ-measure is a supper-additive measure.

(vi) If $\sum_{x\in X} s(x)=1$ and $\delta=0$ then δ-measure is just the λ-measure

(vii) P -measure, additive measure and λ-measure are the special cases of δ-measure

IV. COMPARISON BETWEEN TWO FUZZY MEASURES

Definition 7 $\mu_1-measure \le \mu_2-measure$, $\mu_2-measure \ge \mu_1-measure$ [8,9]

For any given fuzzy density function, $s(x)$, on a finite set, X, If μ_1 and μ_2 are two fuzzy measures, satisfying $g_{\mu_1}(A)\le g_{\mu_2}(A), \forall A\subset X$, then we say that μ_1-measure is not larger than μ_2-measure, or μ_2-measure is not smaller than μ_1-measure, denoted as $\mu_1-measure \le \mu_2-masure$, or $\mu_2-measure \ge \mu_1-measure$

Theorem 3 [8,9]

For any given fuzzy density function, $s(x)$, on a finite set, X, P-measure is not larger than any other fuzzy measure, μ, that is $P-measure \le \mu-measure$

V. COMPOSED MEASURE OF L- METHOD AND-DELTA-MEASURES

A. *Definition of Generalized L-measure*

Definition 8 Generalized L-measure

For given singleton measure $s(x)$, a generalized L-measure based on a fuzzy measure, μ, L_μ, is a fuzzy measure on a finite set X, $|X|=n$, satisfying:

1) $L\in[0,\infty)$ (13)

2) $\forall A\subset X, n-|A|+(|A|-1)L>0 \Rightarrow$

$$g_{L_\mu}(A)=\max_{x\in A}\left[s(x)\right]+\frac{\left[(|A|-1)L\right]\mu(A)\left[1-\max_{x\in A}\left[s(x)\right]\right]}{\left[n-|A|+(|A|-1)L\right]\mu(X)} \tag{14}$$

Where the real number, L, is also called the determine coefficient of L_μ-measure.

Theorem 4

(i) For each $L\in[0,\infty)$, L_μ-measure is a fuzzy measure, In other words, L_μ-measure has infinite many fuzzy measures with determine coefficient L, $L\in[0,\infty)$.

(ii) $L\in[0,\infty)$ L_μ-measure is an increasing function on L,

(iii) if $L=0$ then L_μ-measure is just the μ-measure

(iv) if μ-measure is the P-measure then L_μ-measure is just the L-measure

(v) for each $L\in[0,\infty)$,

P-measure $\le$ L-measure $\le$ L_μ-measure

Proof.

(i) the boundary conditions are trivial, Now to prove the monotonicity.

Let $\forall A,B\in 2^X$, $A\subset B$ to prove $g_{L_\mu}(A)\le g_{L_\mu}(B)$ (15)

If $\max_{x\in A}\left[s(x)\right]=\max_{x\in B}\left[s(x)\right]$,

$$\text{since } \frac{(|B|-1)L\mu(B)}{\left[n-|B|+(|B|-1)L\right]}\ge\frac{(|A|-1)L\mu(A)}{\left[n-|A|+(|A|-1)L\right]} \tag{16}$$

We can obtain $g_{L_\mu}(B)\ge g_{L_\mu}(A)$

If $\max_{x\in B}\left[s(x)\right]=\max_{x\in A}\left[s(x)\right]+a,\ a>0$ (17)

$$g_{L_\mu}(B)-g_{L_\mu}(A)=a\left[1-\frac{(|B|-1)L\mu(B)\left[1-\max_{x\in A}\left[s(x)\right]\right]}{\left[n-|B|+(|B|-1)L\right]\mu(X)}\right]$$

$$+\left[\frac{(|B|-1)L\mu(B)}{\left[n-|B|+(|B|-1)L\right]}-\frac{(|A|-1)L\mu(A)}{\left[n-|A|+(|A|-1)L\right]}\right]\frac{\left[1-\max_{x\in A}\left[s(x)\right]\right]}{\mu(X)} \tag{18}$$

$$\text{Since } 1-\frac{(|B|-1)L\mu(B)\left[1-\max_{x\in A}\left[s(x)\right]\right]}{\left[n-|B|+(|B|-1)L\right]\mu(X)}\ge 0 \tag{19}$$

$$\text{and } \frac{\left[1-\max_{x\in A}\left[s(x)\right]\right]}{\mu(X)}\ge 0 \tag{20}$$

We can also obtain that $g_{L_\mu}(B)\ge g_{L_\mu}(A)$, therefore L_μ-measure is a fuzzy measure.

(ii)

$$f(L)=g_{L_\mu}(A)=\max_{x\in A}\left[s(x)\right]+\frac{\left[(|A|-1)L\right]\mu(A)\left[1-\max_{x\in A}\left[s(x)\right]\right]}{\left[n-|A|+(|A|-1)L\right]\mu(X)}$$

$$\Rightarrow f'(L)=\frac{(|A|-1)\mu(A)\left[1-\max_{x\in A}\left[s(x)\right]\right]\left[n-|A|\right]}{\mu(X)\left[n-|A|+(|A|-1)L\right]^2}\ge 0 \tag{21}$$

Hence L_μ-measure is an increasing function on L.

(iii), (iv) and (v) are trivial.

B. *Definition of L_δ-measure*

Definition 9 L_δ-measure

For given singleton measure $s(x)$, the composed measure of L-measure and δ-measure , denoted L_δ-measure, g_{L_δ}, is a fuzzy measure on a finite set X, $|X|=n$, satisfying:

1) $L\in[-1,\infty), \sum_{x\in X} s(x)=1$ (22)

2) $g_{L_\delta}(\phi)=0, g_{L_\delta}(X)=1$ (23)

3) $\forall A \subset X \Rightarrow$

$$g_{L_\delta}(A)=\begin{cases} \max_{x\in A} s(x) & \text{if } L=-1 \\ \dfrac{(1+L)\sum_{x\in A} s(x)\left[1+L\max_{x\in A} s(x)\right]}{1+L\sum_{x\in A} s(x)}-L\max_{x\in A} s(x) & \text{if } L\in(-1,0] \\ \dfrac{L(|A|-1)\sum_{x\in A} s(x)\left[1-\sum_{x\in A} s(x)\right]}{\left[n-|A|+L(|A|-1)\right]\sum_{x\in X} s(x)}+\sum_{x\in A} s(x) & \text{if } L\in(0,\infty) \end{cases} \quad (24)$$

C. *Important Properties of L_δ-measure*

Theorem 5 Important Properties of L_δ-measure

(i) $L\in[-1,\infty)$, L_δ-measure is a fuzzy measure family

(ii) $L\in[-1,\infty)$, L_δ-measure is an increasing function on L

(iii) if $L=-1$ then L_δ-measure is just the P-measure

(iv) if $L=0$ then L_δ-measure is just the additive measure

(v) if $-1<L<0$ then L_δ-measure is a sub-additive measure

(vi) if $0<L<\infty$ then L_δ-measure is a supper-additive measure

(vii) If $\sum_{x\in X} s(x)=1$ and $L=0$ then L_δ-measure is just the λ-measure

(viii) P -measure, additive measure and λ-measure are the special cases of L_δ-measure

Proof.

(i) if $L\in[-1,0)$, then L_δ-measure is a special case of δ-measure, since δ-measure is a fuzzy measure, then L_δ-measure is also a fuzzy measure.

if $L\in[0,\infty)$, then L_δ-measure is a special case of generalized L-measure based on the additive measure, since any generalized L-measure is also a fuzzy measure, then L_δ-measure is also a fuzzy measure.

Therefore, for each $L\in[-1,\infty)$, L_δ-measure is a fuzzy measure.

(ii) if $L\in[-1,0)$, then L_δ-measure is a special case of δ-measure, since δ-measure is an increasing function with upper bound, additive measure , then L_δ-measure is also an increasing function with upper bound, additive measure.

if $L\in[0,\infty)$, then L_δ-measure is a special case of generalized L-measure based on the additive measure, since generalized L-measure based on the additive measure is also an increasing function with lower bound, additive measure, then L_δ-measure is also an increasing function with lower bound, additive measure.

Therefore, for each $L\in[-1,\infty)$, L_δ-measure is also an increasing function on L'

(iii), (iv), (v), (vi), (vii) and (viii) are trivial.

VI. Γ- SUPPORT

Definition 10: γ-support [7]

For given singleton measure s of a fuzzy measure μ on a finite set X, if $\sum_{x\in X} s(x)=1$, then s is called a fuzzy support measure of μ, or a fuzzy support of μ, or a support of μ. One of fuzzy supports is introduced as below.

Let μ be a fuzzy measure on a finite set $X=\{x_1,x_2,...,x_n\}$, y_i be global response of subject i and $f_i(x_j)$ be the evaluation of subject i for singleton x_j, satisfying:

$$0<f_i(x_j)<1,\ i=1,2,...,N,\ j=1,2,...,n \quad (25)$$

$$\gamma(x_j)=\frac{1+r\left(f(x_j)\right)}{\sum_{k=1}^{n}\left[1+r\left(f(x_k)\right)\right]},\ j=1,2,...,n \quad (26)$$

$$\text{where } r\left(f(x_j)\right)=\frac{S_{y,x_j}}{S_y S_{x_j}} \quad (27)$$

$$S_y^2=\frac{1}{N}\sum_{i=1}^{n}\left(y_i-\frac{1}{N}\sum_{i=1}^{N} y_i\right)^2 \quad (28)$$

$$S_{x_j}^2=\frac{1}{N}\sum_{i=1}^{n}\left[f_i(x_j)-\frac{1}{N}\sum_{i=1}^{N} f_i(x_j)\right]^2 \quad (29)$$

$$S_{y,x_j}=\frac{1}{N}\sum_{i=1}^{n}\left(y_i-\frac{1}{N}\sum_{i=1}^{N} y_i\right)\left[f_i(x_j)-\frac{1}{N}\sum_{i=1}^{N} f_i(x_j)\right] \quad (30)$$

$$\text{satisfying } 0\le\gamma(x_j)\le 1 \text{ and } \sum_{j=1}^{n}\gamma(x_j)=1 \quad (31)$$

then the function $\gamma: X\to[0,1]$ satisfying $\mu(\{x\})=\gamma(x)$, $\forall x\in X$ is a fuzzy support of μ, called γ-support of μ.

VII. CHOQUET INTEGRAL REGRESSION MODELS

A. *Choquet Integral*

Definition 11 Choquet Integral [2-6]

Let μ be a fuzzy measure on a finite set X. The Choquet integral of $f_i: X\to R_+$ with respect to μ for individual i is denoted by

$$\int_C f_i\,d\mu=\sum_{j=1}^{n}\left[f_i\left(x_{(j)}\right)-f_i\left(x_{(j-1)}\right)\right]\mu\left(A_{(j)}^i\right),\ i=1,2,...,N \quad (32)$$

where $f_i\left(x_{(0)}\right)=0$, $f_i\left(x_{(j)}\right)$ indicates that the indices have been permuted so that

$$0 \le f_i\left(x_{(1)}\right) \le f_i\left(x_{(2)}\right) \le \ldots \le f_i\left(x_{(n)}\right) \tag{33}$$

$$A_{(j)} = \left\{x_{(j)}, x_{(j+1)}, \ldots, x_{(n)}\right\} \tag{34}$$

B. *Choquet Integral Regression Models*

Definition 12 Choquet Integral Regression Models [7-15]
Let $y_1, y_2, \ldots, y_N$ be global evaluations of N objects and $f_1\left(x_j\right), f_2\left(x_j\right), \ldots, f_N\left(x_j\right),\ j=1,2,\ldots,n$, be their evaluations of x_j, where $f_i : X \to R_+$, $i=1,2,\ldots,N$.

Let μ be a fuzzy measure, $\alpha, \beta \in R$,

$$y_i = \alpha + \beta \int_C f_i dg_\mu + e_i\ ,\ e_i \sim N\left(0, \sigma^2\right)\ ,\ i=1,2,\ldots,N \tag{35}$$

$$\left(\hat{\alpha}, \hat{\beta}\right) = \arg\min_{\alpha,\beta}\left[\sum_{i=1}^{N}\left(y_i - \alpha - \beta \int_C f_i dg_\mu\right)^2\right] \tag{36}$$

then $\hat{y}_i = \hat{\alpha} + \hat{\beta}\int f_i dg_\mu,\ i=1,2,\ldots,N$ is called the Choquet integral regression equation of μ, where

$$\hat{\beta} = S_{yf} / S_{ff} \tag{37}$$

$$\hat{\alpha} = \frac{1}{N}\sum_{i=1}^{N} y_i - \hat{\beta}\frac{1}{N}\sum_{i=1}^{N}\int f_i dg_\mu \tag{38}$$

$$S_{yf} = \frac{\sum_{i=1}^{N}\left[y_i - \frac{1}{N}\sum_{i=1}^{N} y_i\right]\left[\int f_i dg_{\mu^*} - \frac{1}{N}\sum_{k=1}^{N}\int f_k dg_{\mu^*}\right]}{N-1} \tag{39}$$

$$S_{ff} = \frac{\sum_{i=1}^{N}\left[\int f_i dg_{\mu^*} - \frac{1}{N}\sum_{k=1}^{N}\int f_k dg_{\mu^*}\right]^2}{N-1} \tag{40}$$

VIII. Experiment and Result

A. *Education Data*

The total scores of 60 students from a junior high school in Taiwan are used for this research [9-13]. The examinations of four courses, physics and chemistry, biology, geoscience and mathematics, are used as independent variables, the score of the Basic Competence Test of junior high school is used as a dependent variable.

The data of all variables listed in Table III is applied to evaluate the performances of five Choquet integral regression models with P-measure, λ-measure and δ-measure, L-measure measure and L_δ-measure based on γ-support respectively, a ridge regression model, and a multiple linear regression model by using 5-fold cross validation method to compute the mean square error (MSE) of the dependent variable. The formula of MSE is

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2 \tag{41}$$

The singleton measures, γ-support of the P-measure, λ-measure, δ-measure, L-measure and L_δ-measure are listed as follows which can be obtained by using the formula (26).

$$\{0.2488,\ 0.2525,\ 0.2439,\ 0.2547\} \tag{42}$$

For any fuzzy measure, μ-measures, once the fuzzy support of the μ-measure is given, all event measures of μ can be found, and then, the Choquet integral based on μ and the Choquet integral regression equation based on μ can also be found by using above corresponding formulae.

The experimental results of seven forecasting models are listed in Table I. We find that the Choquet integral regression model with L_δ-measure based on γ-support outperforms other forecasting regression models.

TABLE I MSE OF REGRESSION MODELS

Regression model	measure	5-fold CV MSE
Choquet Integral Regression model	L_δ	47.5722
	L	48.4610
	δ	48.7672
	λ	49.1832
	p	53.9582
Ridge regression		59.1329
Multiple linear regression		65.0664

B. *Fat Data*

In this study, anthropometric dimensions were measured following a standard protocol [11, 16]. High was measured to the nearest 0.1 cm using anthropometers. Body weight was measured to the nearest 0.1 kg at the same time the bioelectric impedance was measured using a body fat analyzer (TBF310; Tanita, Tokyo, Japan) to estimate the percentage of body fat (%fat). Skinfold thicknesses at biceps, triceps, subscapular, and suprailiac of the right side of body were measured with GMP skinfold calipers (Siber Hegener and Co. Ltd, Switzerland). The measurements were performed by one experienced operator that took two repeated measurements at the test site of the same subject. The mean of the two readings from each site was used to calculate body composition.

A real data set with 128 samples from a elementary school in Taiwan including the independent variables, 4 Skinfold determination values, and the dependent variable, the measurements of the BIA of each student listed in Table IV is applied to evaluate the performances of three Choquet integral regression models with P-measure, λ-measure, δ-measure, L-measure and L_δ-measure based on γ-support respectively, a ridge regression model, and a multiple linear regression model by using 5-fold cross validation method to compute the mean square error (MSE) of the dependent variable.

The singleton measures, γ-support of the P-measure, λ-measure, δ-measure, L-measure and L_δ-measure are listed as follows which can be obtained by using the formula (26).

$$\{0.2396, 0.2466, 0.254, 0.2596\} \tag{43}$$

The formulas of MSE is by using 5-fold cross validation method to compute the mean square error (MSE) of the depen dent variable.

TABLE II MSE OF REGRESSION MODELS

Regression model	measure	5-fold CV MSE
Choquet Integral Regression model	L_δ	13.7136
	L	14.2344
	δ	14.4228
	λ	14.9218
	p	18.3846
Ridge regression		15.7434
Multiple linear regression		16.1122

The experimental results of seven forecasting models are listed in Table II. We also find that the Choquet integral regression model with L_δ-measure based on γ-support outperforms other forecasting regression models.

IX. CONCLUSION

In this paper, a multivalent composed fuzzy measure of L-measure and δ-measure, called L_δ-measure, is proposed. This new measure is proved that it is of closed form with infinitely many solutions, and it can be considered as an extension of the two well known fuzzy measures, λ-measure and P-measure. Furthermore, this improved multivalent fuzzy measure is not only including the additive measure, but also having the same infinitely many measure solutions as L-measure. By using 5-fold cross-validation MSE, two experiments are conducted for comparing the performances of a multiple linear regression model, a ridge regression model, and the Choquet integral regression model with respect to P-measure, λ-measure, δ-measure, L-measure and our proposed L_δ-measure -measure based on γ-support respectively. The result shows that the Choquet integral regression models with respect to the proposed L_δ-measure based on γ-support outperforms other forecasting models.

In the future, we will apply the proposed Choquet integral regression model with the new fuzzy measure based on γ-support to develop multiple classifier system.

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Hsiang-Chuan Liu received the Ph.D. degree in Statistics from National Taiwan University, Taiwan. He is a professor at the Department of Bioinformatics, Asia University, Taiwan since August 2001 and also a honored professor at the Graduate Institute of Educational Measurement and Statistics, National Taichung University, Taiwan. He was the President of National Taichung University, Taiwan from 1993 to 2000. Dr. Liu is a member of IEEE since 2007. He has funded research and published articles in the areas of Biostatistics, Bioinformatics, Fuzzy Theory, Educational Measurement, and E-Learning.

Chin-Chun Chen is an Associate Professor and the President of Min-Hwei College, Taiwan. He is now also a doctoral student of Graduate Institute of Educational Measurement and Statistics, Taichung University since 2006. His studies focus in Fuzzy Measure, Grey System, etc.

Der-bang Wu, Ph.D. (M'07) became a Member (M) of **NAUN** in 2007. He was born in Yun-Lin County, Taiwan on July, 1958. He earned his first doctoral degree (Ph.D.) from Department of Mathematics, Graduate School of University of Northern Colorado, Greeley, Colorado, USA in 1994. Also, in order to earn his second doctoral degree, he is a doctoral student of Graduate Institute of Educational Measurement and Statistics, Taichung University since 2007. His studies focus in Fuzzy c-mean, Fuzzy Measure, Grey System, Rough set, Educational Measurement, and e-Learning, etc.

Wu is an Associate Professor of Graduate Institute of Mathematics, Education, Taichung University, Taichung, Taiwan. He was a teacher of Elementary School for 4 years (1979-1983) and work in Taichung University for 24 years (1985-2009), mainly engaged in the design and the development of Mathematics Textbooks, Research Instruments, van Hiele Geometrical Thinking, etc.

Dr. Wu is a member of IEEE, Taiwan Association of Mathematics Education, Taiwan Association of Science Education, and International Group of Psychology of Mathematics Education, etc.

Tian-Wei Sheu received the Ph.D. degree in Mathematics from National Osaka University, Japan in 1990. He is the Dean of College of Education and a professor of Graduate Institute of Educational Measurement, National Taichung University, Taichung, 40306 Taiwan. His studies focus in IRT, Educational Measurement, and e-Learning, etc.

TABLE III THE DATA SET WITH FOUR COURSES AND SCIENCE SCORES OF THE BCT

No.	C1	C2	C3	C4	BCT	No.	C1	C2	C3	C4	BCT
1	72	66	78	72	19	31	66	68	75	74	25
2	86	80	82	81	35	32	68	70	74	76	40
3	56	63	69	75	21	33	57	65	75	70	24
4	78	86	86	86	33	34	74	70	80	75	35
5	66	72	80	76	23	35	49	60	69	64	13
6	68	74	77	80	28	36	51	60	63	64	18
7	74	86	87	88	44	37	58	64	68	66	32
8	54	56	62	68	7	38	73	78	84	81	39
9	71	74	80	77	26	39	56	56	65	61	6
10	68	70	80	75	33	40	61	62	70	70	25
11	53	56	70	63	22	41	57	60	68	64	23
12	67	70	80	75	35	42	57	64	67	70	26
13	70	66	70	74	13	43	50	52	68	60	7
14	60	65	75	70	23	44	84	80	76	72	49
15	68	68	78	76	35	45	62	66	76	71	22
16	58	66	76	71	37	46	70	74	78	82	32
17	61	66	72	78	33	47	69	70	80	75	26
18	68	68	80	74	26	48	63	74	74	74	42
19	56	66	76	71	21	49	66	78	80	82	39
20	59	62	70	78	29	50	67	70	80	75	31
21	62	64	76	70	36	51	56	65	75	70	23
22	71	72	78	75	26	52	50	54	66	60	18
23	74	63	69	75	12	53	71	75	85	80	41
24	59	70	80	76	37	54	74	77	80	85	26
25	75	75	85	80	39	55	71	72	76	80	31
26	73	78	84	81	24	56	60	65	75	70	21
27	62	68	72	74	29	57	59	57	70	68	17
28	77	74	80	76	42	58	50	56	65	68	13
29	63	60	68	69	17	59	72	76	80	78	38
30	56	61	75	68	22	60	81	76	78	80	33

C1 : physics and chemistry
C2 : biology
C3 : geoscience
C4 : mathematics
BCT : Basic Competence Test of nature science

Table IV Measurements of BIA and four skinfold determinations of percent body fat

No	BIA	biceps	triceps	Sub-scapular	Sup-railiac	No	BIA	biceps	triceps	Sub-scapular	Sup-railiac
1	11.8	15.8	16.2	27.6	16.3	65	22.8	27.4	32.2	33.6	24.5
2	17.8	20.4	12.8	35.6	23.0	66	5.4	14.4	8.0	17.2	17.5
3	6.2	10.4	9.4	12.6	8.2	67	8.6	9.6	11.4	14.4	14.8
4	7.0	10.2	5.6	10.6	13.6	68	7.4	9.6	8.0	10.2	13.4
5	5.2	10.8	12.0	13.8	11.8	69	20.2	29.2	28.2	40.0	23.9
6	20.2	24.6	39.8	40.0	34.7	70	28.4	30.6	38.6	39.8	39.5
7	14.8	18.8	22.4	24.8	18.6	71	6.8	10.4	8.8	14.6	15.2
8	12.4	18.2	20.2	27.6	18.4	72	11.0	12.4	10.4	14.4	11.7
9	17.0	20.6	21.2	27.8	21.6	73	20.8	30.8	38.8	40.0	37.7
10	16.6	22.6	18.8	32.6	16.6	74	25.0	27.6	40.0	40.0	29.6
11	10.2	13.4	8.4	12.8	15.1	75	4.8	6.2	5.6	7.0	9.7
12	21.0	26.7	36.2	40.0	23.9	76	8.4	7.0	8.0	11.8	15.6
13	10.0	13.4	17.4	17.8	18.1	77	11.0	14.0	10.0	12.4	17.7
14	9.8	13.8	11.6	19.2	15.5	78	10.6	11.6	7.0	14.4	12.9
15	8.6	10.8	11.4	14.4	8.5	79	12.4	16.8	17.0	25.8	14.9
16	21.8	25.6	36.6	36.8	31.7	80	11.4	14.0	20.8	26.4	25.1
17	25.2	29.4	30.2	30.6	31.2	81	12.8	15.8	20.2	19.8	16.4
18	10.2	11.0	10.2	14.8	12.2	82	12.0	15.6	16.0	22.8	17.3
19	10.4	15.8	13.0	25.6	16.6	83	13.0	20.4	24.0	27.0	26.2
20	17.6	22.6	23.2	34.6	23.2	84	14.2	15.4	22.4	22.8	18.9
21	12.6	12.4	14.2	16.0	14.2	85	11.0	16.4	14.2	15.8	17.9
22	9.0	11.2	9.4	11.8	9.3	86	22.4	29.8	35.0	36.2	28.5
23	12.2	19.2	17.4	27.8	19.7	87	6.4	7.6	8.6	11.4	12.1
24	4.6	7.0	8.8	11.2	7.2	88	6.8	10.6	9.6	14.6	12.2
25	6.4	8.8	11.0	12.6	10.8	89	16.2	18.4	27.2	27.4	24.9
26	23.8	29.0	37.0	35.0	30.7	90	22.4	26.8	25.4	33.4	30.4
27	8.4	15.8	17.8	23.0	21.6	91	9.6	11.2	10.4	18.0	11.7
28	12.2	16.6	16.4	20.6	18.7	92	10.8	17.2	24.0	24.8	21.9
29	7.2	12.8	8.6	18.6	15.2	93	13.0	16.2	12.4	18.4	14.2
30	21.4	31.2	31.4	39.4	28.6	94	5.6	12.4	11.4	15.6	14.5
31	18.2	23.0	40.0	40.0	28.2	95	19.4	25.0	36.2	39.0	29.9
32	9.2	12.6	40.0	17.8	16.0	96	14.4	22.4	29.8	35.0	24.8
33	10.2	18.8	17.8	20.8	18.4	97	25.4	29.4	37.0	40.0	24.6
34	19.2	24.4	35.2	35.0	34.1	98	9.4	11.2	11.4	12.4	8.9
35	6.8	12.0	8.0	14.4	16.1	99	17.4	22.6	19.4	31.6	22.7
36	16.8	20.8	25.6	27.8	20.7	100	24.0	30.8	40.0	40.0	29.4
37	35.8	38.6	40.0	40.0	30.1	101	3.8	6.0	6.4	6.8	10.8
38	10.0	11.6	10.4	18.6	8.3	102	11.0	19.4	11.6	18.4	13.7
39	5.4	12.2	12.4	21.4	19.2	103	22.6	24.4	40.0	40.0	33.3
40	11.2	18.0	23.6	30.8	22.1	104	9.2	10.0	11.0	19.2	13.4
41	5.4	11.2	6.8	11.6	11.9	105	18.2	19.0	31.0	29.4	24.5
42	7.6	8.4	9.4	13.6	12.8	106	6.8	12.4	14.0	17.8	14.1
43	6.6	9.8	9.6	12.0	9.3	107	7.4	11.6	10.0	16.0	11.0
44	32.4	37.2	40.0	40.0	18.2	108	9.2	10.6	12.4	14.4	12.7
45	7.8	14.0	11.0	17.8	31.3	109	29.4	23.6	39.8	40.0	37.4
46	17.8	26.6	34.2	40.0	22.8	110	6.8	7.8	9.8	12.8	10.9
47	22.0	27.8	38.2	39.4	23.6	111	12.4	14.6	15.8	19.8	16.9
48	14.4	15.8	18.8	23.8	14.9	112	8.2	9.8	9.2	16.0	14.5
49	15.8	18.4	21.4	24.0	24.9	113	16.4	20.8	25.2	30.4	24.9
50	7.4	12.8	10.2	17.0	14.3	114	9.4	11.4	12.0	21.8	14.3
51	16.2	29.0	21.6	29.8	24.1	115	16.4	22.4	33.2	36.8	25.1
52	6.0	7.4	7.6	9.8	8.6	116	7.0	11.4	13.8	17.4	11.2
53	12.2	15.4	16.2	18.8	17.8	117	10.4	12.6	14.8	23.8	18.0
54	11.6	12.0	9.8	13.0	8.9	118	5.6	8.2	10.2	8.6	7.7
55	17.8	22.6	38.0	31.0	24.8	119	10.8	11.8	17.8	21.2	19.9
56	13.2	16.8	18.6	23.4	20.7	120	9.6	15.8	14.4	19.4	18.6
57	4.4	7.2	8.2	9.8	14.3	121	5.0	6.8	7.4	9.4	6.0

58	16.2	21.8	28.2	32.6	27.2	122	9.8	12.2	12.4	15.4	13.5
59	11.4	19.4	28.8	32.8	22.3	123	13.8	18.0	16.4	21.0	19.3
60	11.2	13.0	18.8	22.6	21.9	124	8.8	12.8	9.8	11.8	13.3
61	8.6	11.4	7.2	10.2	7.5	125	15.8	21.0	35.4	39.8	27.3
61	20.4	26.2	31.0	32.8	25.8	126	10.8	16.6	15.6	23.2	16.5
63	7.0	8.8	11.6	9.4	12.0	127	9.0	10.6	10.0	16.8	11.9
64	14.6	17.4	12.8	16.8	14.7	128	8.8	12.4	10.0	10.8	11.3