# Theoretical and Practical Aspects of Heating Equation 

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#### Abstract

We are interested in a null controllability problem for a class of strongly degenerate heat equations. Heat equation parameters are represented graphically (temperature, heat flux) for a particular situation. Then, first for all $\mathrm{T}>0$, we prove a regional null controllability result at time T at least in the region where the equation is not degenerate. The proof is based on an adequate observability inequality for the homogeneous adjoint problem. This inequality is obtained by application of Carleman estimates combined with the introduction of cut-off functions. Then we improve this result: for all $\mathrm{T}^{\prime \prime}>\mathrm{T}$, we obtain a result of persistent regional null controllability during the time interval [ $\mathrm{T}, \mathrm{T}]$. We give similar results for the (non degenerate) heat equation in unbounded domain. Analysis by numerical simulation of the heating room space has highlighted that the use of composite materials instead of the usual materials of construction is more advantageous in terms of achieving the microclimate conditions, making possible, within certain limits of temperature exterior, a passive air conditioning, which involves reducing energy consumption.


Key-words: - Representation equations, Control, Null controllability, Air conditioning.

## 1 Introduction

In this article, we study first the null controllability of a class of equations of degenerate heat (in limited areas) $(\alpha, \beta) \subset(0,1)$ different from zero and $\delta>0$ (results $\alpha+\delta<1$ ).

If we consider $\alpha \in C^{1}([0,1])$ as positive, $a(x)>0$ for all $x \in[\beta, 1]$.

There is the problem of regional null controllability:

Problem 1. For all $T>0$ and all $u_{0} \in L^{2}(0.1)$, found the solutions:

$$
\left\{\begin{array}{l}
u_{( }(t, x)-\left(a(x) u_{x}(t, x)\right)_{x}=f(t, x) X_{(\alpha, \beta)}(x),(t, x) \in(0, T) \times(0,1)  \tag{1}\\
u(t, 0)=u(t, 1)=0, t \in(0, T) \\
u(0, x)=u_{0}(x), x \in(0,1)
\end{array}\right.
$$

verifies

$$
\begin{equation*}
u(T, x)=0 \text { for } x \in(\alpha+\delta, 1) . \tag{2}
\end{equation*}
$$

## Notes:

a) It can be shown that the problem is well posed (see [1], [7]) within the meaning of the theory of semigroups working in areas with adequate weight.
b) In the not degenerated case (ie $\mathrm{a}>0$ on $[0,1]$ ), the null controllability (overall) is now well known: for all $T>0$, it exists $f \in L^{2}((0, T) \times(0,1))$ so that the solution in (1) verifies $u(T, \cdot)=0$ in all $(0,1)$.

This result is generally obtained via Carleman estimate (see for example [2], [3], [4]).
c) We consider here the case of an equation of degenerate heat, possibly strongly degenerate so that $a(x)$ can be null all through the range $\left[0, \alpha^{`}\right]$ for $0 \leq \alpha<\beta$.

Under this hypothesis, with an $f$ control located in $(\alpha, \beta)$, we can not hope to bring the system to zero in any ( $\alpha, 1$ ). The area of influence of control is necessarily located in the region $(\alpha, 1)$. That is why we focus on the local null controllability [12].
d) Many results are known for non-degenerate parabolic equations. But to our knowledge, no result was known for degenerate equations.

Recently, another result of null regional controllability has been obtained for an linearized Crocco-type equation [5], [9], [13].

This is a degenerate parabolic equation (where there are are phenomena of diffusion and transport) so the type of degeneration is quite different from that studied here.
$e)$ The null controllability (global) at time $T$ is a strong property. Indeed, because of the decrease in energy, it implies that, from time $T$, the system remains indefinitely at rest (by ceasing to apply a control from the time $T$ ): for all $t \geq T, u(t, \cdot)=0$ in $(0,1)$.

On the contrary, the null regional controllability is a property much weaker since, if one ceases to apply a control from the time $T$, it does not imply that the system remains at rest in the region $(\alpha+\delta, 1)$ for $t \geq T$.


Fig. 1: Variation of heat equation parameters [11]:
a) temperature variation across the $x$ axis;
b) the varioation of the heat flow in time;
c) the heating variation in time.

We therefore seek to improve this by addressing the second problem of persistent regional null controllability:

Problem 2. For all $T^{`}>T>0$ and all $u_{0} \in L^{2}(0,1)$, find $f \in L^{2}\left(\left(0, T^{`}\right) \times(0,1)\right)$ so that the solution of

$$
\begin{cases}u_{t}(t, x)-\left(a(x) u_{x}(t, x)\right)_{x}=f(t, x) X_{(\alpha, \beta)}(x),(t, x) \in(0, T) \times(0,1)  \tag{3}\\ u(t, 0)=u(t, 1)=0, & t \in(0, T) \\ u(0, x)=u_{0}(x), & x \in(0,1)\end{cases}
$$

verifies

$$
\begin{equation*}
u(t, x)=0 \text { for }(t, x) \in\left(T, T^{\prime}\right) \times(\alpha+\delta, 1) \tag{4}
\end{equation*}
$$

## 2 Equation of degenerate heat

Problem 1. It shows first the observability of the inequality for the secondary problem associated:

$$
\left\{\begin{array}{l}
v_{t}(t, x)-\left(a(x) v_{x}(t, x)\right)_{x}=0, \quad(t, x) \in\left(0, T^{\prime}\right) \times(0,1)  \tag{5}\\
v(t, 0)=v(t, 1)=0, \quad t \in\left(0, T^{\prime}\right)
\end{array}\right.
$$

Theorem 1 ([1]). For all $\delta>0$ (so that $\alpha+\delta<1$ ), $C>$ 0 exists, so that any solution $v$ of (5) verifies.

$$
\begin{equation*}
\int_{0}^{1} v(0, x)^{2} d x \leq C \int_{0}^{T} \int_{\alpha}^{\beta} v(t, x)^{2} d x d t+C \int_{0}^{\alpha+\beta} v(T, x)^{2} d x \tag{6}
\end{equation*}
$$

we deduce the
Theorem 2 ([1]). Under the previous assumptions, it exists $f \in L^{2}\left(\left(0, T^{`}\right) \times(0,1)\right)$ so that the solution to (1) verifies (2).

Note. If $\alpha=0$, there is actually (global) null controllability throughout $(0,1)$ (and not only for $(\delta, 1)$ ).

Principle of evidence. For Theorem 1, we used truncation functions. In particular, in the region whre it doesn't become null, it boils down to a nondegenerate equation to which we apply the Carleman estimates. The planned Theorem 2 is based on the introduction of the penalized problem.

$$
\inf _{f \in L^{2}((0, T) \times(0,1))}\left(\frac{1}{2} \int_{0}^{T} \int_{\alpha}^{\beta} f(t, x)^{2} d x d t+\frac{1}{2 \varepsilon} \int_{\alpha+\delta}^{1} u^{f}(T, x)^{2} d x\right),
$$

where $u^{f}$ is the solution of (1) associated to $f \in L^{2}\left(\left(0, T^{\prime}\right) \times(0,1)\right)$.

Problem 2. For all $t \in(0, T)$ we consider the family of adjoint problems:

$$
\left\{\begin{array}{l}
v^{t}{ }_{s}(s, x)-\left(a(x) v^{t} x(s, x)\right)_{x}=0, \quad(s, x) \in(0, t) \times(0,1)  \tag{7}\\
v^{t}(s, 0)=v^{t}(s, 1)=0, \quad s \in\left(0, T^{\prime}\right)
\end{array}\right.
$$

Theorem 3 ([1]). For all $\delta>0$ (so that $\alpha+\delta<1$ ), it exists $C>0$ so that

$$
\begin{gather*}
\int_{0}^{1}\left(\int_{T}^{T^{\prime}} v^{t}(0, x) d t\right)^{2} d x  \tag{8}\\
\leq C \int_{0}^{T} \int_{\alpha}^{\beta}\left(\int_{\max (\beta, T)}^{T^{\prime}} v^{t}(s, x) d t\right)^{2} d x d s+C \int_{T}^{T^{\prime}} \int_{0}^{\alpha+\beta} v^{s}(s, x)^{2} d x d s .
\end{gather*}
$$

we deduce the
Theorem 4 ([1]). Under the previous assumptions, there is $f \in L^{2}\left(\left(0, T^{\prime}\right) \times(0,1)\right)$ so that the solution to (3) verifies (4).

Principle of evidence. For the 3rd Theorem, we appply the 1st Theorem to the function [21]:

$$
w(s, x):=\int_{\max (s, R)}^{T} v^{t}(s, x) d t
$$

The planned Theorem 4 based on the introduction of the penalized problem

$$
\inf _{f \in L^{2}((0, T) \times(0,1))}\left(\frac{1}{2} \int_{0}^{T} \int_{\alpha}^{\beta} f(t, x)^{2} d x d t+\frac{1}{2 \varepsilon} \int_{\alpha+\delta}^{1} u^{f}(t, x)^{2} d x\right)
$$

A different result. The concept of persistent regional null controllability also gives interesting results even in the equation of not degenerate heat.

For example, if we are given a null starting value on a part of $(0,1)$, we can maintain (cheaper) the solution to be zero on that part (instead of bringing it to zero on the whole domain).
Results: $0 \leq \alpha<\gamma<\alpha+\delta<\beta<1$.
Let's suppose that $a \in C^{1}([0,1])$ so that $a(x)>0$ for all $x \in[0,1]$.

For all $T>0$ and all $u_{0} L^{2}(0,1)$ so that $u_{0}(x)=0$ for $x \in(\gamma, 1), \quad f \in L^{2}((0, T) \times(0,1))$ exists, so that the solu-tion of (1) verifies $u(t, x)=0$ for $(t, x) \in$ $(0, T) \times(\alpha+\delta, 1)$.

In [1], we give another result of persistent regional null controllability for the heat equation subject to a localized source term.

## 3 Heat equation in an unbounded domain

We consider now the equation of heat (nondegenerate) on the positive half-axis for $T>0$ and $u_{0} \in L^{2}(0+\infty)$

$$
\left\{\begin{array}{l}
u_{t}(t, x)-u_{x x}(t, x)=f(t, x) X_{(\alpha, \beta)}(x), \quad(t, x) \in(0, T) \times(0, \infty)  \tag{9}\\
u(t, 0)=0, \quad t \in(0, T) \\
u(0, x)=u_{0}(x),: x \in(0,1)
\end{array}\right.
$$

Micu, Zuazua [10] and then Popescu [14], [15] have shown that there is no data to support initial regular compact that can be brought to zero in finite time. This negative result is that one seeks to control the heat equation in unbounded domain by a control

$$
f X_{(\alpha, \beta)} \in L^{2}((0, T) \times(\alpha, \beta))
$$

located in a bounded domain $(\alpha, \beta)$.
Cabanillas, De Menezes and Zuazua [1] have subsequently obtained a positive result of null controllability, with the condition to use an $f$ control located in an unbounded domain like $(\alpha,+\infty)$.

With the techniques previously used, we have the following intermediate result (see [1]): with an $f$ control located in the bound domain $(\alpha, \beta)$, there is a null regional controllability at time $T$ in the region ( $0, \beta-\delta$ ) (for all $\delta>0$ ).
In particular, if $\beta=+\infty$, we find the result of [7].
We can also improve this result to obtain the persistent regional null controllability in the region ( $0, \beta-\delta$ ). During the whole time range $(T, T)$.

Notes:
1.We have proven in [8] another intermediate result for the null (global) controllability by a control $f$ located in an unbounded domain, but of finite extent
2.The techniques of evidence used are not specific to the dimension 1. The results presented here can be a statement in dimension N .


Fig. 2: Represantion of heat equation [16].

## 4 Experimental result

In the modelling space of room, difficult issues arise in determining intakes heat introducing outside non stationary regime due to heat transmission. In literature this problem is resolved differently; there are a number of approximate models [14], [15], [16]. Consider a room with space monolayer structure and low surface without glass surfaces.
$Q_{c d}$ noting the amount of heat that passes through the wall into the hall of time, and assuming that heat transfer will be conduction from the inside out, then the thermal resistance equivalent to the construction of $R_{t}$, the resulting regime stationary thermal (Law Fourier's) [14]:

$$
\begin{align*}
& Q_{c d}=\frac{\theta_{e}-\theta_{i}}{R_{f}}+\frac{\theta_{e}-\theta_{i}}{R_{z}}=\frac{\left(\theta_{e}-\theta_{i}\right)\left(R_{f}+R_{z}\right)}{R_{f} R_{z}}  \tag{10}\\
& =\frac{\theta_{i}-\theta_{e}}{R_{t}},[W]
\end{align*}
$$

where:

- $\theta_{\mathrm{e}}, \theta_{\mathrm{I}}$ are air temperatures outside and inside;
- $R_{f}$ and $R_{z}$ represent thermal resistances of the doors and walls, data relations

$$
\begin{equation*}
R_{f}=\frac{z_{f}}{\lambda_{f} S_{f}}, \quad R_{z}=\frac{z_{z}}{\lambda_{z} S_{z}}, \quad\left[{ }^{0} \mathrm{C} / \mathrm{W}\right] \tag{11}
\end{equation*}
$$

where:
$-z_{f}, z_{z}$ are thickness doors and walls of built space, [m];

- $\lambda_{\delta}, \lambda_{2}$ are the thermal conductivity of material $\left[\mathrm{W} / \mathrm{m} \cdot{ }^{0} \mathrm{C}\right]$;
- $S_{f}, S_{z}$ areas are sections of the interior doors and walls, the normal direction of heat flow $\left[\mathrm{m}^{2}\right]$.
Expressing $C_{t}$ heat capacity of air, as the product between the mass $m$ of the existing room air and specific heat of indoor air $c_{s i}$, then mass balance equation, assuming the existence of consideration heat from the rooms attached $Q_{l}$, the release produced by thermal evaporation $Q_{2}$, or warm air $Q_{3}$, or from other sources is [13]:

$$
\begin{gather*}
C_{t} \frac{d \theta_{i}}{d t}+Q_{i}+Q_{c d}=0 \Leftrightarrow \\
R_{t} C_{t} \frac{d \theta_{i}}{d t}+\theta_{i}-\theta_{e}+R_{t} Q_{i}=0 \tag{12}
\end{gather*}
$$

the report $\frac{d \theta_{i}}{d t}$, temperature gradient is the direction
of heat flow, and $Q_{i}$ is the quantity of heat released in unit time incubation space expressed as:

$$
Q_{i}=Q_{1}+Q_{2}+\ldots
$$

In the absence of this release of heat ( $Q_{i}=0$ ), and considering $\theta_{e}$ outside air temperature is constant, equation (12), integrated on the time interval $\left[t_{0}, t\right]$, corresponding temperatures and $\theta_{\mathrm{i} 0}$ and $\theta_{i}$ the ends of time [15]

$$
\begin{equation*}
\frac{1}{T_{\theta}} \int_{t_{0}}^{t} d t=\int_{\theta_{i 0}}^{\theta_{i}} \frac{d \theta}{\theta_{i}-\theta_{e}} \Rightarrow \frac{t-t_{0}}{T_{\theta}}=\ln \left(\frac{\theta_{i}-\theta_{e}}{\theta_{i 0}-\theta_{e}}\right) \tag{13}
\end{equation*}
$$

final solution will

$$
\begin{align*}
& \theta_{i}=\theta_{e}+\left(\theta_{i 0}-\theta_{e}\right) \exp \left(\frac{t-t_{0}}{T_{\theta}}\right)= \\
& \theta_{e}\left[1-\exp \left(\frac{t-t_{0}}{T_{\theta}}\right)\right]+\theta_{i 0} \exp \left(\frac{t-t_{0}}{T_{\theta}}\right)^{\prime} \tag{14}
\end{align*}
$$

where $R_{t} C_{t}=T_{\theta}$, is the constant heat of the system.
If the heat transfer by conduction takes place and the transfer of heat by convection (natural or artificial) $Q_{c v}$, then mass balance equations in the room will be:

- for natural convection,

$$
\begin{equation*}
C_{t} \frac{d \theta_{i}}{d t}+Q_{i}+\frac{\theta_{e}-\theta_{i}}{R_{t}}=D_{m} \cdot c_{s e}\left(\theta_{e}-\theta_{i}\right) \tag{15}
\end{equation*}
$$

- for all air conditioning systems (heating, ventilation and moistening)

$$
\begin{equation*}
C_{t} \frac{d \theta_{i}}{d t}+Q_{i}+\frac{\theta_{i}-\theta_{e}}{R_{t}}=D_{m} \cdot c_{s e}\left(\theta_{f f}-\theta_{i f}\right)_{\text {inst. }}, \tag{16}
\end{equation*}
$$

where:

- $D_{m}$ is the mass flow of fluid circulating in the hall (air) or plant (water) $[\mathrm{kg} / \mathrm{s}]$;
$-\theta_{e}$ - initial temperature of the air outside, $\left[{ }^{0} \mathrm{C}\right]$
- $\theta_{f f}$ - final temperature of the fluid (water) from the heat, $\left[{ }^{0} \mathrm{C}\right]$
- $\theta_{i f}$ - initial temperature of the fluid from the plant, [ $\left.{ }^{0} \mathrm{C}\right]$;
- $\boldsymbol{c}_{s e}$ is specific heat of the fluid circulated in the hall (air), or the heating (water), $\left[\mathrm{J} / \mathrm{kg} .{ }^{0} \mathrm{C}\right]$.
In the absence of heat release $Q_{i}=0$, and considering $\theta_{e}$ outside air temperature is constant, equation (14),
integrated on the time interval $\left[t_{\theta} t\right]$, corresponding temperatures $\theta_{\mathrm{i} 0}$ and $\theta_{\mathrm{i}}$, [12]

$$
\begin{align*}
& \frac{1}{T_{\theta}} \int_{t_{0}}^{t} d t=-\int_{\theta_{i 0}}^{\theta_{i}} \frac{d \theta_{i}}{\left(\theta_{i}-\theta_{e}\right)\left(D_{m} R_{t} c_{s e}-1\right)} \Leftrightarrow  \tag{17}\\
& \frac{1}{D_{m} R_{t} c_{s e}-1} \ln \left(\frac{\theta_{i}-\theta_{e}}{\theta_{i 0}-\theta_{e}}\right)
\end{align*}
$$

final solution will

$$
\begin{align*}
& \theta_{i}=\theta_{e}\left(1-\exp \left(-\frac{\left(D_{m} R_{t} c_{s e}-1\right)\left(t-t_{0}\right)}{T_{\theta}}\right)\right)+ \\
& \theta_{i 0} \exp \left(-\frac{\left(D_{m} R_{t} c_{s e}-1\right)\left(t-t_{0}\right)}{T_{\theta}}\right) \tag{18}
\end{align*}
$$

In most practical applications, the amount of heat is expressed in [W], reporting the quantity in [J], the unit of time.
For the amount of heat coming through natural convection $D_{m} c_{s e}\left(\theta_{\mathrm{e}}-\theta_{\mathrm{i}}\right),[\mathrm{J}]$, using the relationship

$$
n V \rho_{\mathrm{e}} c_{s e}\left(\theta_{\mathrm{e}}-\theta_{\mathrm{j}}\right)[\mathrm{W}],
$$

where:
$V$ - hall volume $\left[\mathrm{m}^{3}\right]$,
$\rho_{\mathrm{e}}$ - outside air density $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$,
$n$ - number of hours of air exchanges per hour [1/s].
For conditions that have made inquiries, Incubation space is bordered with a desktop, in the natural temperature remains constant. During the exothermic phase Incubation is $\theta_{\mathrm{e}}<\theta_{\mathrm{i}}$ and the question of disposal of excess heat. Temperature variation was examined for heat transfer by conduction (Fig. 3, Fig. 4) for heat transfer by conduction and convection artificial (Fig. 5, Fig. 6) assuming that $\theta_{\mathrm{i}}>\theta_{\mathrm{e}}$ and the temperature difference is $1^{\circ} \mathrm{C}$.


Fig. 3: Increase of $1^{\circ} \mathrm{C}$, the temperature in the room where the heat transfer by conduction.

It is noted that if ventilation (Fig. 5) decrease in
temperature is faster. The result of this analysis was verified experimentally as follows:

- in the first round of experiments we have used 4 fans with a total flow of $640 \mathrm{~m}^{3} / \mathrm{h}$;
- in the second phase of experiments we have used two fans with the flow $1700 \mathrm{~m}^{3} / \mathrm{h}$, one mounted on the inlet pipe and one mounted on the exhaust pipe.


Fig. 4: The block structure of the phenomenon of heat transfer by conduction in the room.

Since buildings may be constructed and under conditions other than those specific to our research, continued analysis of heat transfer for a concrete wall and a composite material, subject to the same average external temperature for 24 hours, using Schedule block in Figure 7, were obtained curves of Figure 8, [19], [20]


Fig. 5: Increase of $1^{\circ} \mathrm{C}$, the temperature in the room where the heat transfer by conduction and convection artificial.


Fig. 6: The block structure of the phenomenon of transfer of heat into the room by conduction and convection natural or artificial (air-conditioning system air only).
$R_{Z}$ changing thermal resistance of wall, by altering the coefficient of thermal conductivity $\lambda$ dateclim.dat [17] in the MATLAB can see the
behaviour of the two materials under simulation namely a variation of a concrete wall temperature compared to that of wall material composite.

In the first phase of the Incubation stage, phase which takes about 2 days, is required for space heating for Incubation temperature should be maintained between 19 and $23{ }^{\circ} \mathrm{C}$. Using the block diagram in Figure 9, can simulate temperature regulation inside the hall from room temperature inside a reference in a spread, by modifying the block parameters "Thermostat".


Fig. 7: The thermodynamic [102] room of space.


Fig. 8: Changes in room temperature for 24 hours for concrete $\theta_{\text {i beton }}\left(\lambda=1,74 \mathrm{~W} / \mathrm{m}^{0} \mathrm{C}\right)$, and composite material and composite $\theta_{\mathrm{i} \text { compozit }}\left(\lambda=0,21 \mathrm{~W} / \mathrm{m}^{0} \mathrm{C}\right)$, for

$$
\theta_{\mathrm{em}}=28^{\circ} \mathrm{C}
$$



Fig. 9: The block structure for regulating the temperature within the space of room.

If the room no major release of heat, the only
disturbance being outside temperature $\theta_{\mathrm{e}}$, which varies between $10^{\circ} \mathrm{C}$ and $26^{\circ} \mathrm{C}$ (Fig. 10) and is intended to help maintain the temperature of an air conditioning part [17], observed that the operation of the heating is necessary only in the time the outside temperature is lower than the values prescribed for the inside temperature, $21^{\circ} \mathrm{C}$ respectively, as only the mass flow of fluid circulating in $D_{m}$ plant. Figure 10.a shows the process of adjustment during the 24 hours, and in Figure 10.b are detailing this process during a cycle "switched-off". Results of the simulations serve to determine service operation [11] electric motor to drive the fan. To further analyze the phenomenon of heat transfer from the room if it is not built into the ground for periods of autumn spring. Considered that (Fig. 11) $\theta_{e}$ is the outside temperature varies between 4 and $20^{\circ} \mathrm{C}$ (curve 1 ).


Fig. 10: Variation of the temperature outside (curve 1) and inside (curve 2 ) room space for $\theta_{\mathrm{em}}=18^{\circ} \mathrm{C}$.

It is noted that the air must operate throughout the period considered. Results of the simulations serve to analyze energy consumption and production scheduling cycle, so that electricity consumption should be reduced as much as possible. It is noted that a facility is required of air, to alter the air temperature and suppress default air temperature of
the room.
Modeling the phenomenon recirculation air in the event of release of heat, on the one hand, highlights interdependeting between climatic variables (pressure-temperature), on the other hand, allows a consistent determination of ventilation used in describing the effectiveness of a ventilated cavity.


Fig. 11: Variation of the temperature outside (curve 1) and inside (curve 2 ) room space for $\theta_{\mathrm{em}}=12^{\circ} \mathrm{C}$.

In conclusion, this constant will be taken into account when choosing the elements of the implementation of the automation process.

Also demonstrated that the use of composite materials instead of the usual materials of construction is more advantageous in terms of achieving the microclimate conditions, making possible - within certain limits on the temperature outside - a passive air-conditioning (justified according to climatic conditions outside a nonadjustable temperature and, therefore, an air conditioning installation, which involves reducing energy consumption).

In conclusion, the design of rooms must be an economic analysis of investments related to construction materials, on the one hand and the cost of equipment insurance climate, on the other. On the
other hand, for an elementary volume of fluid, in motion mono-dimensional the $x$ direction (along the pipe interior), the second of the motion and Newton's equation of continuity is written [19], as

$$
\frac{\partial H}{\partial x}+\frac{1}{g S} \frac{\partial D}{\partial t}+\frac{k_{f} D|D|}{2 g Z S^{2}}=0
$$

$$
\begin{equation*}
\frac{\partial D}{\partial x}+\frac{g S}{v^{2}} \frac{\partial H}{\partial t}=0 . \tag{19}
\end{equation*}
$$

where:
$v$ - air velocity $[\mathrm{m} / \mathrm{s}]$;
$D$ - volume flow [m $\left.\mathrm{m}^{3} \mathrm{~s}\right]$;
$t$ - time [s];
$Z$ - pipe diameter [m];
$S$ - area of pipe cross-section $\left[\mathrm{m}^{2}\right]$;
$H$ - total pressure (static and dynamic), expressed in [m], given by the relationship [18]

$$
\begin{equation*}
H=\frac{p}{\rho g} \tag{20}
\end{equation*}
$$

where $p$ is the total pressure $\left[\mathrm{N} / \mathrm{m}^{2}\right]$.
$k_{f}$ - coefficient of friction is dependent on the flow
(thus, the Reynolds number Re) and is calculated as follows [18]:

- for the laminar ( $\operatorname{Re}<2000$ ), with the formula of Poiseuille's:

$$
k_{f}=\frac{16}{\mathrm{Re}},
$$

- for the turbulent regime ( $\mathrm{Re}>3000$ ), with the formula of Colebrook-White:

$$
k_{f}=\frac{0,25}{4 \lg \left(\frac{k_{s}}{3,7 Z}+\frac{5,74}{\operatorname{Re}^{0,9}}\right)},
$$

where $k_{S}$ is the relative roughness parameter.
Noting with $H_{0}$ and $D_{0}$ values of absolute total pressure and flow on the outer portion of the conduit, and passing the coordinates relative, $h=\frac{H}{H_{0}}$, $d=\frac{D}{D_{0}}$, will be achieved

$$
\begin{gather*}
\frac{\partial h}{\partial x}+\frac{D_{0}}{H_{0} g S} \frac{\partial d}{\partial t}+\frac{k_{f} D_{0}^{2}}{2 g Z S^{2} H_{0}} d|d|=0 \\
\frac{\partial d}{\partial x}+\frac{H_{0} g S}{D_{0} v^{2}} \frac{\partial h}{\partial t}=0 \tag{21}
\end{gather*}
$$

and then into equations,

$$
\frac{\partial h}{\partial x} \rightarrow \frac{\Delta h}{\Delta x}, \frac{\partial h}{\partial t} \rightarrow \frac{d h}{d t}, \frac{\partial d}{\partial x} \rightarrow \frac{\Delta d}{\Delta x} \text { and } \frac{\partial d}{\partial t} \rightarrow \frac{d(d)}{d t}
$$

resulting

$$
\begin{gather*}
\Delta h=-\frac{D_{0} \Delta x}{g H_{0} S} \frac{d(d)}{d t}-\frac{k_{f} \Delta x D_{0}^{2}}{2 g Z S^{2} H_{0}} d|d| \\
\Delta d=-\frac{H_{0} g S \Delta x}{D_{0} v^{2}} \frac{d h}{d t} \tag{22}
\end{gather*}
$$

Noting reports

$$
\frac{D_{0} \Delta x}{g H_{0} S}=L, \quad \frac{k_{f} D_{0}^{2} \Delta x}{2 g Z S^{2} H_{0}}=R, \quad \frac{H_{0} g S \Delta x}{D_{0} v^{2}}=C
$$

equations (22) for a discrete volume of fluid in motion become:

$$
\begin{equation*}
\Delta h=-L \frac{d(d)}{d t}-R d|d|, \quad \Delta d=-C \frac{d h}{d t} \tag{23}
\end{equation*}
$$

Based on these equations to establish a model analog of the heating of the incubation space in the form of an electrical circuit in a "T" in Figure 12.


Fig. 12: The electrical circuit analog of a recirculation of fluid volume.


Fig. 13: Scheme of structural a recicle air block in Simulink.

Structural block scheme for the analysis of heating, during the liberation of heat by the mushrooms is depicted in Figure 13 and corresponding electrical equivalent circuit in Figure 12. The airing of the room space is equivalent to connecting the electrical circuit in a "T" to an alternative source of power (Fig. 14) [20]. Efficiency room space ventilation can be established with constant ventilation

$$
T_{C}=\frac{H_{0} S_{C}}{D_{0}}
$$

here $S_{c}$ is cross-sectional area covered by discrete volume of fluid.


Fig. 14: Electrical circuit analog room ventilation by natural convection.

In technical air conditioning the premises shall be deemed recirculation processes are isobar. However during the research, noted that the start ventilation fans for the exothermic stage, the pressure variations occur. It is believed that research can be continued in the direction of pressure on the analysis of the influence of heating process of the room space, where the use of fans supplied by static converters. For this purpose it is proposed the model in Fig. 15.


Fig. 15: Electrical circuit analog room with a ventilation fan ordered.

## 6 Conclusion

In this paper, we study the null controllability of a class of degenerate parabolic equations in a bounded domain.

Null controllability of non degenerate parabolic equations has been recently widely studied, using

Carleman estimates. Roughly speaking, in the non degenerate case, the following result, there exists a control that drives the solution to zero on the holds: given $T>0$ and an initial condition $\mathrm{u}_{0}$ whole domain at time $T$. However many physical problems are described by degenerate parabolic equations.

The main difficulty in the study of degenerate parabolic equations comes from the fact that in general it is not possible to find a control that drives the solution to zero on the whole domain. This is why we will study regional null controllability properties: the problem is to find a control that drives the solution to zero on some part of the domain. The proof follows from the observability inequality (8), obtained via Carleman estimates and well hosencutoff functions.

This notion of persistent regional null controllability allows us also to extend classical results on the (non degenerate) heat equation in a bounded domain when the initial condition is compactly supported in the domain, and to extend also some results on the null controllability of the heat equation in an unbounded domain, for which proved that global null controllability does not hold if the control region is bounded.

Of section through the experimental results the following conclusions:

Analysis of microclimate in the event of release of heat, on the one hand, highlights interdependeting between climatic variables (pressure-temperature), on the other hand, allows a consistent determination of ventilation used in describing the effectiveness of ventilation. In selecting items for the implementation of the automation process of ventilation to take account of this constant.

Analysis by numerical simulation of the heating room space has highlighted that the use of composite materials instead of the usual materials of construction is more advantageous in terms of achieving the microclimate conditions, making possible, within certain limits of temperature exterior, a passive air conditioning, which involves reducing energy consumption. The results of this simulation, and analysis used in energy consumption and production scheduling cycle, so that electricity consumption should be reduced as much as possible. Therefore, the design of rooms, should be done and an economic analysis of investments related to construction materials, on the one hand, and equipment costs of providing microclimate, on the other.

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