Control of a radiative furnace under precise and uncertainty conditions

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Abstract: - This paper discusses the control of a radiative furnace under precise and imprecise modeling. Regarding nonlinearity nature of the process, in the precise case, feedback linearization technique is applied to the furnace and under uncertainty conditions, sliding mode control is used. The conditions are completely practical and have gained from an industrial process.

Key-Words: Feedback Linearization – Radiation – Furnace – Uncertainty – Sliding Mode Control

1. Introduction

Radiative furnace is the useful equipment at industry. It works with radiation like many other devices [1], [2], [3] and [4], of course by heat radiation.

It is assumed that we encounter with two essential cases. In the first case, we have precise model and we would control it by feedback linearization technique and in the second one, sliding mode control is applied due to uncertainty of the model. Now we explain the basic concepts of two aforementioned control methods and then simulate them.

Feedback linearization is an approach to nonlinear control design which has attracted a great deal of research interest in recent years.

Feedback linearization has been used successfully to address some practical control problems. Design of a nonlinear control system for a Variable Air Volume Air Conditioning (VAVAC) plant through feedback linearization is presented in [5]. For an induction motor the sliding mode controller is combined with an adaptive input–output feedback linearization technique in order to preserve the system robustness with respect to stator and rotor resistances variations and uncertainties [6]. In [7] a decentralized nonlinear controller for large-scale power systems is investigated. The proposed controller design is based on the input–output feedback linearization methodology. In reference [8] the technique of feedback linearization is used to design controllers for displacement, velocity and differential pressure control of a rotational hydraulic drive. Application of a feedback linearization technique for control of a distributed solar collector is described in [9]. Adaptive feedback linearization control technique for chaos suppression in a chaotic system is used in [10]. Disturbance decoupling and trajectory tracking of nonlinear control systems using an observer-based fuzzy feedback linearization control (FLC) is developed in [11]. Application of the feedback linearization to the model of a power system is investigated in [12]. A scheme for temperature control of a greenhouse using the feedback linearization is presented in [13].

2. Feedback linearization review

Consider a system described by

$$\dot{x} = f(x) + ug(x) \tag{1}$$

where f and g are smooth vector fields on some open set $X \subseteq R^n$ containing 0 and f(0) = 0.

There exists smooth functions $q, s \in S(X)$ with $s(x) \neq 0$ for all x in some neighborhood of the origin, and a local diffeomorphism T on R^n with T(0) = 0. We define

$$v = q(x) + s(x) \tag{2}$$

$$z = T(x) \tag{3}$$

The resulting variables z and v satisfy a linear differential equation of the form

$$\dot{z} = Az + bv \tag{4}$$

where the pair (A, b) is controllable. In this case, the system is called feedback linearizable.

$$u = -\frac{q(x)}{s(x)} + \frac{1}{s(x)} v \tag{5}$$

By applying a state transformation $\bar{z} = M^{-1} z$ such that the resulting system is in controllable canonical form

$$\dot{\bar{z}} = M^{-1} A M \bar{z} + M^{-1} b v \tag{6}$$

where

$$M^{-1} A M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1. & 0 \\ .. & .. & . & . \\ -a_0 & -a_1 & -a_2 .. & -a_{n-1} \end{bmatrix} \tag{7}$$

$$M^{-1} b = \begin{bmatrix} 0 \\ 0 \\ : \\ 1 \end{bmatrix} \tag{8}$$

And the a_i 's are the coefficients of the characteristic polynomial

$$|sI - A| = s^n + \sum_{i=0}^{n-1} a_j s_j \tag{9}$$

A further state feedback form

$$v = \bar{v} + [a_0 a_1 ... a_{n-1}] \bar{z} \tag{10}$$

results in the closed loop system

$$\dot{\bar{z}} = \bar{A} \bar{z} + \bar{b} \bar{v} \tag{11}$$

where

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1.. & 0 \\ . & .. & .. & .. \\ 0 & 0 & 0 & 0 \end{bmatrix}, \bar{b} = \begin{bmatrix} 0 \\ 0 \\ : \\ 1 \end{bmatrix} \tag{12}$$

$$\dot{\bar{z}} = M^{-1} T(x) \tag{13}$$

$$\bar{v} = v - a \bar{z} = q(x) - a' M^{-1} T(x) + s(x) u \tag{14}$$

where

$$a' = [a_0 \ a_1 \ ... a_{n-1}] \tag{15}$$

3. Radiative furnace dynamics

Fig. 1 shows a typical horizontal radiative furnace which is used for tempered glass production. Assuming the temperature in the whole of the glass and the upper and lower walls is uniformly changed and the conductive heat transfer is negligible.

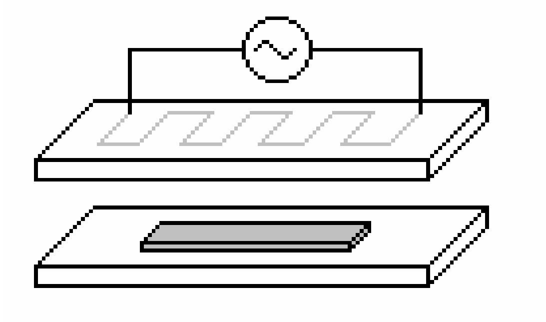

Figure 1. Horizontal radiative furnace

From radiation heat transfer we have

$$\begin{cases} \dot{T}_p = a_1 (T_w^4 - T_p^4) \\ \dot{T}_w = a_2 [T_w^4 - F_{w-p} T_p^4 - (1 - F_{w-p}) T_B^4] + \alpha P \\ \dot{T}_B = a_3 (T_w^4 - T_B^4) \end{cases} \tag{16}$$

where T_P, T_w and T_B are glass plate, upper wall and lower wall temperatures respectively. P is the input electric power and a_1, a_2 and a_3 are as below

$$\begin{aligned} a_1 &= \beta \sigma F_{w-p} A \\ a_2 &= -\alpha \sigma A \\ a_3 &= \sigma \alpha A (1 - F_{w-p}) \end{aligned} \tag{17}$$

where the σ, the Stefan-Boltzmann constant is 5.6697×10^{-8} W/m^2.K^4, A is the area of the wall, F_{w-p} is the shape factor between upper wall and plate and α, β are the warming rate of the walls and plate respectively.

In simulation following practical data have been used.

$F_{w-p} = 0.7$
$\alpha = 8*10^{-5}$
$\beta = 35*10^{-5}$
$A = 1$ (18)

Now apply feedback linearization to furnace system. We define state variables as follows

$T_p = x_1$
$T_w = x_2$
$T_B = x_3$ (19)

Then arrange equations in companion form

$$\dot{\underline{x}} = f(\underline{x}) + g(\underline{x})\underline{u} \tag{20}$$

$$h(\underline{x}) = y = x_1 \tag{21}$$

$$f(x) = \begin{pmatrix} a_1(x_2^4 - x_1^4) \\ a_2[x_2^4 - F_{w-p}x_1^4 - (1-F_{w-p})x_3^4] \\ a_3(x_2^4 - x_3^4) \end{pmatrix} \tag{22}$$

$$g(x) = \begin{pmatrix} 0 \\ \alpha \\ 0 \end{pmatrix} \tag{23}$$

$$L_g h(x) = 0$$
$$L_g L_f h(x) \neq 0 \tag{24}$$

Thus we have relative degree 2 and since the system order is 3, the order of the internal dynamics is 1. Now to investigate stability of zero dynamics, define

$\mu_1 = h(x) = x_1$
$\mu_2 = L_f h(x)$
$\varphi(x) = [\ \mu_1\ \ \mu_2\ \ \psi\ ]$ (24)

$\varphi(x)$ is diffeomorphism so we have

$$L_g \psi(x) = 0 \quad \rightarrow \quad \psi(x) = x_3 \tag{25}$$

$$\mu_1 = \mu_2 = 0 \quad \rightarrow \quad \dot{\psi}(x) = -a_3\psi^4(x) \tag{26}$$

a_3 is a positive value, so zero dynamics is stable and thus we could apply the input-output linearization.

$$y^{(2)} = v \quad \rightarrow \quad x_1^{(2)} = v \tag{27}$$

$$u = m(x) + n(x)v \tag{28}$$

$$m(x) = -\frac{L_f^{\ 2}h}{L_g L_f\ h} \tag{29}$$

$$n(x) = \frac{1}{L_g L_f\ h} \tag{30}$$

From MATLAB symbolic toolbox we obtain

$$m(x) = 4\beta^2\sigma^2F_{w-p}^2x_1^3(x_2^4 - x_1^4) +$$
$$4\alpha\beta\sigma^2F_{w-p}x_2^3[x_2^4 - F_{w-p}x_1^4 - (1-F_{w-p})x_3^4] \tag{31}$$

$$n(x) = 4\sigma\alpha\beta F_{w-p}x_2^4 \tag{32}$$

For tracking control task define desire path x_{1d} and tracking error is

$$e = x_1 - x_{1d} \tag{33}$$

and

$$v = x_{1d}^{(2)} - k_1e^{(1)} - k_0e \tag{34}$$

meanwhile, when F_{w-p} is one, the parameter a_3 is set to zero and since the relative degree remains 2 and system order is decreased to 2, the I.O.L. converts to I.S.L. and there is no internal dynamics appears in this case. This case occurs when the plate and the wall below have the same size and therefore it is a feasible subject.
We use this situation for simplifying the necessary equations under uncertainty condition and applying sliding mode control as below. In section 5 we would explain more the sliding mode control in details.

$$x^{(2)} = f(x) + b(x).u$$

$$f(x) = \sqrt[4]{(\frac{\dot{x}}{a_1} + x)^3}\, 4a_1a_2[\frac{\dot{x}}{a_1} + (1-F_{w-p})x^4] - 4a_1x^3\dot{x}$$

$$b(x) = \sqrt[4]{(\frac{\dot{x}}{a_1} + x)^3}\, 4a_1\alpha \tag{35}$$

4. Simulation results with feedback linearization

Suppose electrical system of the furnace power could vary from zero to 16kW linearly. Also initial conditions for plate and walls are 300°K and 500°K respectively. Set point changes from 700°K to 900°K.

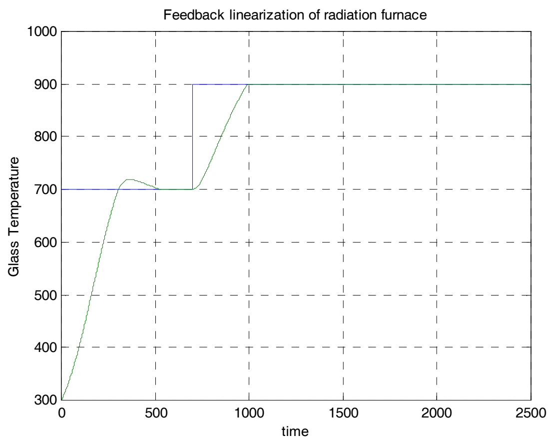

Figure 2. Step response of plate temperature with two pole at s = -0.2

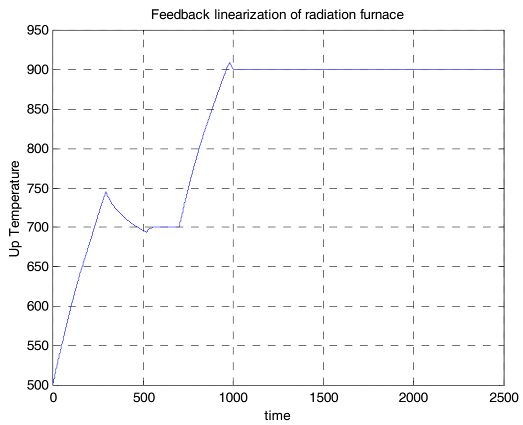

Figure 3. Step response of upper wall temperature with two poles at s = -0.2

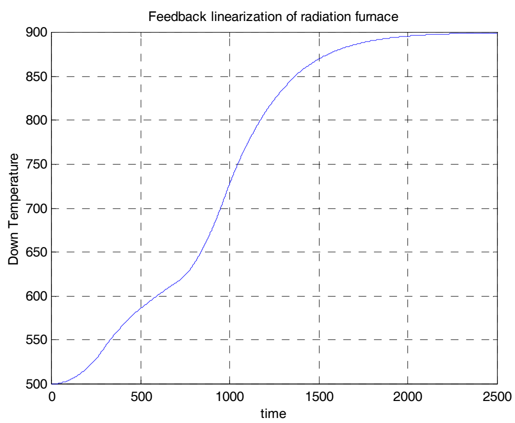

Figure 4. Internal dynamics behavior with two poles at s = -0.2 and step input

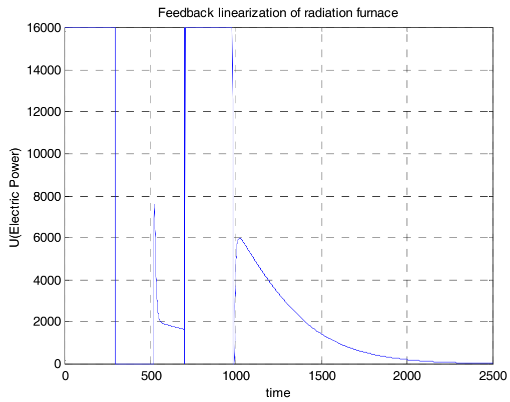

Figure 5. Electric power behavior with two poles at s = -0.2 and step input

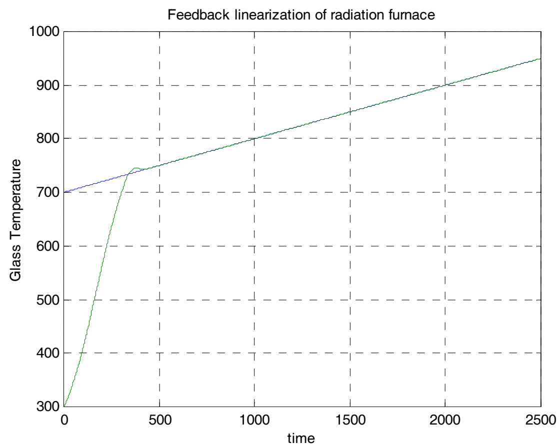

Figure 6. Ramp response of plate temperature with two pole at s = -0.2

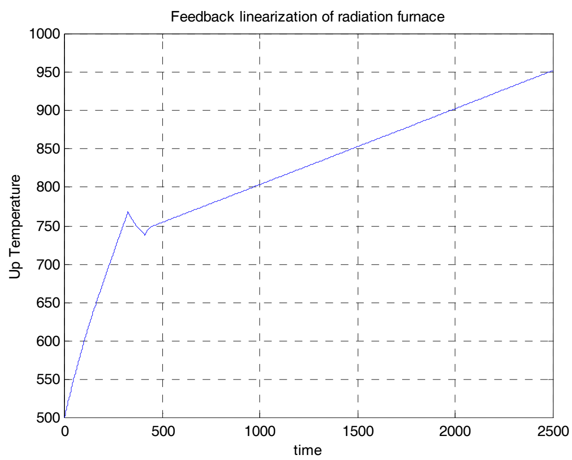

Figure 7. Ramp response of upper wall temperature with two poles at s = -0.2

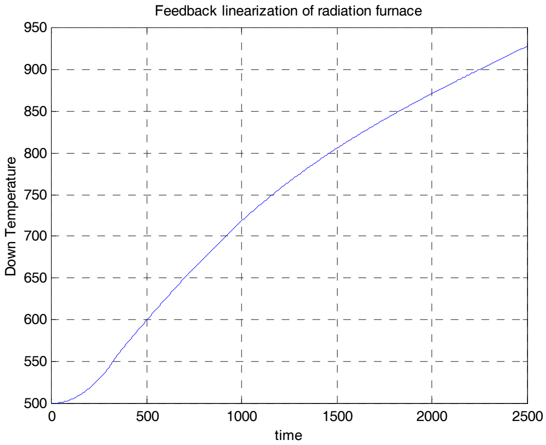

Figure 8. Internal dynamics behavior with two poles at s = -0.2 and ramp input

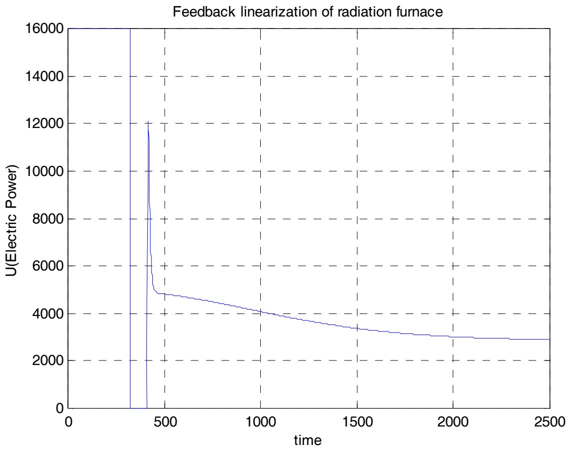

Figure 9. Electric power behavior with two poles at s = -0.2 and ramp input

Now we simulate ramp input response of the furnace system with a step disturbance.

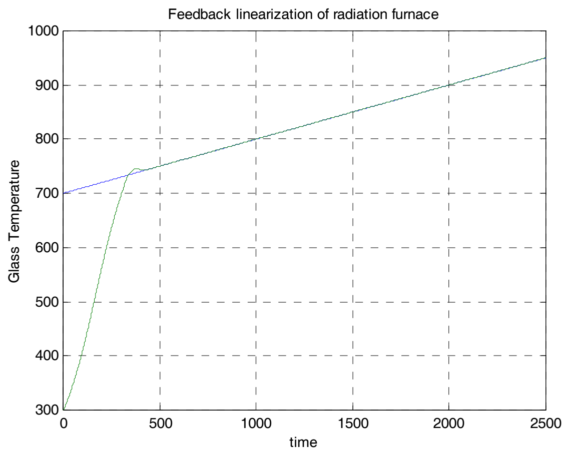

Figure 10. Ramp response of plate temperature with two pole at s = -0.2 and step disturbance

Now we change the one pair poles of the controller form $s = -0.2$ to $s = -0.03$ and investigate responses. We expect the response be slow that the following simulations display this.

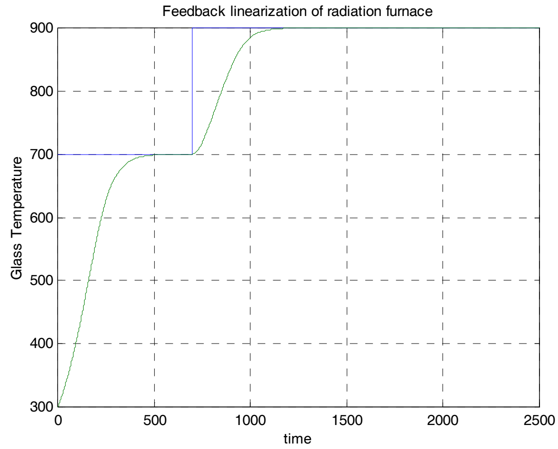

Figure 11. Step response of plate temperature with two pole at s = -0.03

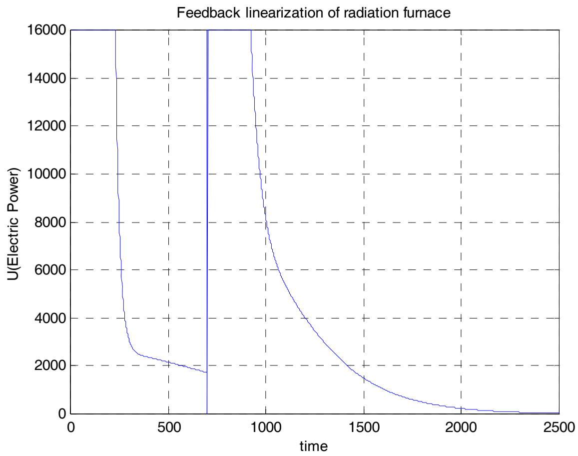

Figure 12. Electric power behavior with two poles at s = -0.03 and step input

5. Sliding Mode Control review

Consider the single input dynamic system as the following dynamics

$$x^{(n)} = f(X) + b(X)u \tag{36}$$

Where the scalar x is the output of interest, the scalar u is the control input and X is the state vector as below

$$X = \begin{bmatrix} x \\ \dot{x} \\ \vdots \\ x^{(n-1)} \end{bmatrix} \tag{37}$$

In equation (36) the function *f(X)* (in general nonlinear) is not exactly known, but the extent of the imprecision on *f(X)* is upper bounded by a known continuous function of *X* ; similarly, the control gain *b(X)* is not exactly known, but is of known sign and is bounded by known, continuous function of *X*.

The control problem is to get the state *X* to track a specific time-varying state X_d in the presence of model imprecision on *f(X)* and *b(X)* as the following definition

$$X_d = \begin{bmatrix} x_d \\ \dot{x}_d \\ \vdots \\ x_d^{(n-1)} \end{bmatrix} \tag{38}$$

For the tracking task to be achievable using a finite control *u*, the initial desired state $X_d(0)$ must be such that

$$X_d(0) = X(0) \tag{39}$$

Let $\tilde{x}$ be the tracking error in the variable x as below

$$\tilde{x} = x - x_d \tag{40}$$

And let

$$\tilde{X} = X - X_d = \begin{bmatrix} \tilde{x} \\ \dot{\tilde{x}} \\ \vdots \\ \tilde{x}^{(n-1)} \end{bmatrix} \tag{41}$$

Be the tracking error vector. Furthermore, let us define a time-varying surface *S(t)* in the state-space $R^{(n)}$ by the scalar equation $s(X;t) = 0$, where

$$s(X;t) = (\frac{d}{dt} + \lambda)^{n-1}\tilde{x} \tag{42}$$

And λ is a strictly positive constant, whose choice we shall interpret later. For instance, if $n = 2$,

$$s = \dot{\tilde{x}} + \lambda\tilde{x} \tag{43}$$

Satisfying condition or sliding condition for remaining system trajectories on the surface is

$$\frac{1}{2}\frac{d}{dt}s^2 \le -\eta|s| \tag{44}$$

Where η is a strictly positive constant. Essentially, equation (44) states that the squared "distance" to the surface, as measured by s^2, decreases along all system trajectories.

6. Simulation results with sliding mode control

Uncertainty in variable α causes an uncertainty in *f(X)* and *g(X)*. The practical range for α as the following

$$6\times10^{-5} \le \alpha \le 9\times10^{-5} \tag{45}$$

$$\tilde{X} = X - X_d = \begin{bmatrix} \tilde{x} \\ \dot{\tilde{x}} \end{bmatrix} \tag{46}$$

$$S(x,t) = (\frac{d}{dt} + \lambda)\tilde{x} \tag{47}$$

$$\dot{S} = \ddot{\tilde{x}} + \lambda\dot{\tilde{x}} = 0 \Rightarrow \hat{u} = -\hat{f} + \ddot{x}_d - \lambda\dot{\tilde{x}} \tag{48}$$

$$\hat{f} = -3\times10^{-4}a_1\sqrt[4]{(\frac{\dot{x}}{a_1}+x)^3}\times \\ \times[\frac{\dot{x}}{a_1}+(1-F_{w-p})x^4] - 4a_1x^3\dot{x} \tag{49}$$

$$|\hat{f} - f| \le F \tag{50}$$

$$F = 6\times10^{-5}a_1\sqrt[4]{(\frac{\dot{x}}{a_1}+x)^3}\times \\ \times\left|\frac{\dot{x}}{a_1}+(1-F_{w-p})x^4\right| \tag{51}$$

For security of the sliding condition, we add a discontinuous around surface *s=0* as below

$$u = \hat{b}^{-1}\left[\hat{u} - k \cdot \mathrm{sgn}(s)\right] \tag{52}$$

Where

$$\hat{b} = 4\sqrt{54} \times 10^{-5} a_1 \sqrt[4]{\left(\frac{\dot{x}}{a_1} + x\right)^3} \tag{53}$$

$$k \geq \beta(F + \eta) + (\beta - 1)|\hat{u}| \tag{54}$$

$$\beta = \left(b_{max}/b_{min}\right)^{1/2} = \sqrt{1.5} \tag{55}$$

To choose an appropriate value for k , one may be use a constant value of k that satisfies equation (54) at all times. In case of absence of this fulfillment, it should be select a greater k for satisfaction.

Also to decrease input chattering with high frequency switches, one can uses $sat\left(\frac{s}{\varphi}\right)$ instead of $\mathrm{sgn}(s)$ in equation (52). *sat* is saturation element and sgn means sign function or element in MATLAB.

In addition, if $x_d(0) \neq x(0)$, then reach time to the sliding surface is related to the following inequality

$$t_{reach} \leq \frac{|s(t=0)|}{\eta} \tag{56}$$

In reach time, state path reaches to the sliding surface and then tends to x_d with time constant $\frac{n-1}{\lambda}$ that for our problem with $n = 2$ is $1/\lambda$.

Now regarding the above mentioned equations, we can simulate responses in MATLAB environment. First we consider the saturation element between 0 and 16000 according to power limitations. Also initial conditions are assumed 300 for plate and 500 for the wall. We want to examine different cases in simulation.

First suppose $\eta = 1$, $\lambda = 1000$, $\varphi = 50$ and $k = 100000$. Then check satisfaction of the sliding condition for simulation times as shown in Fig. 13. Positive values of the plotted curve displays the fulfillment of sliding condition and therefore the simulation results are valid.

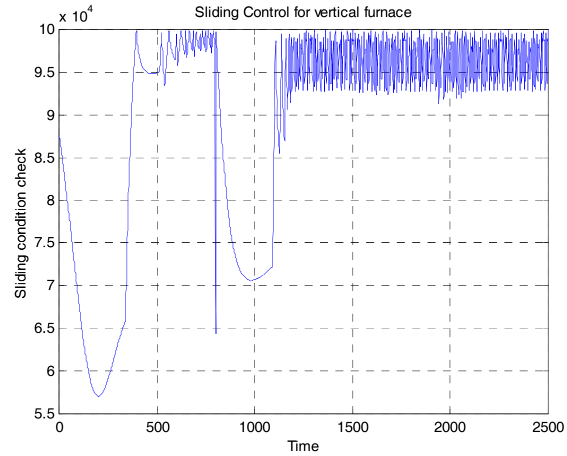

Figure 13. Sliding condition check

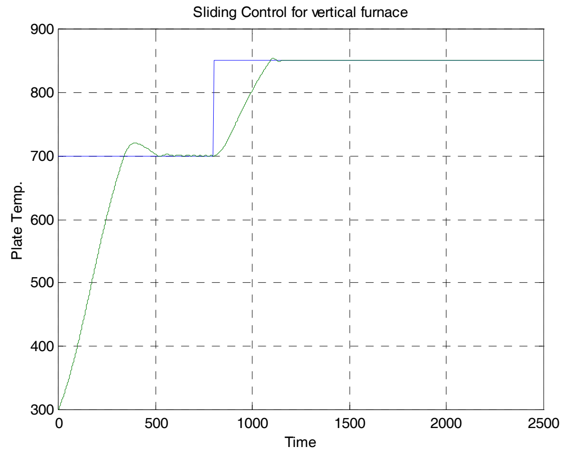

Figure 14. Step input response with SMC

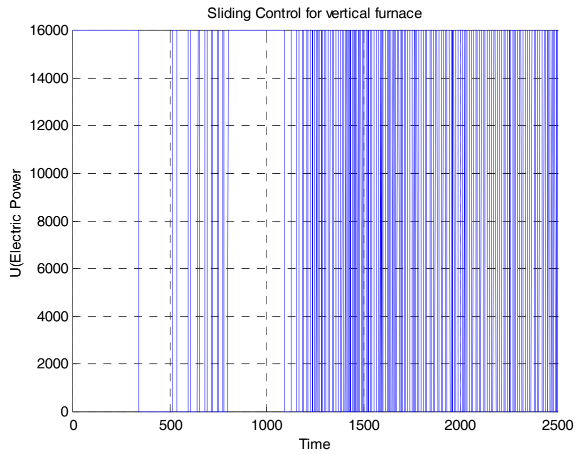

Figure 15. Electric power behavior with SMC and step input

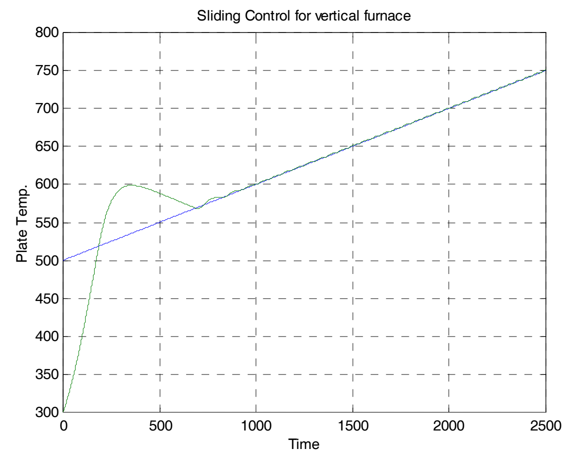

Figure 16. Ramp input response with SMC

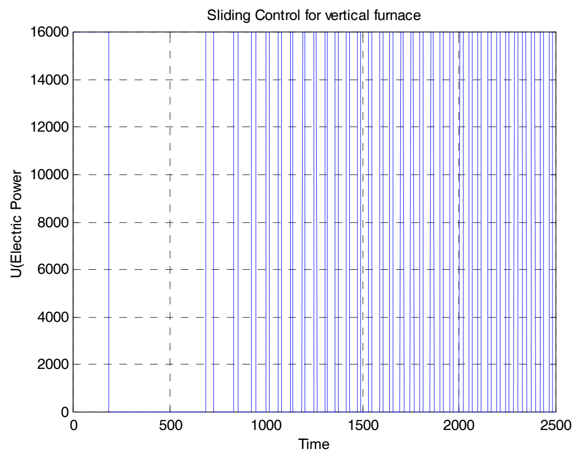

Figure 17. Electric power behavior with SMC and ramp input

Now we change the $\varphi = 50$ to $\varphi = 0.00001$ for investigating the chattering of the input.

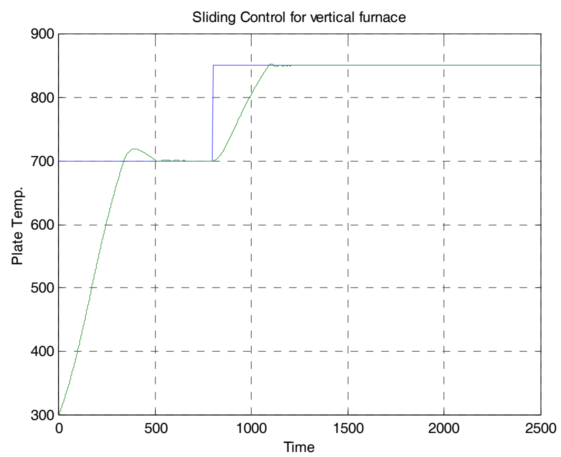

Figure 18. Step input response with SMC and selecting $\varphi = 0.00001$

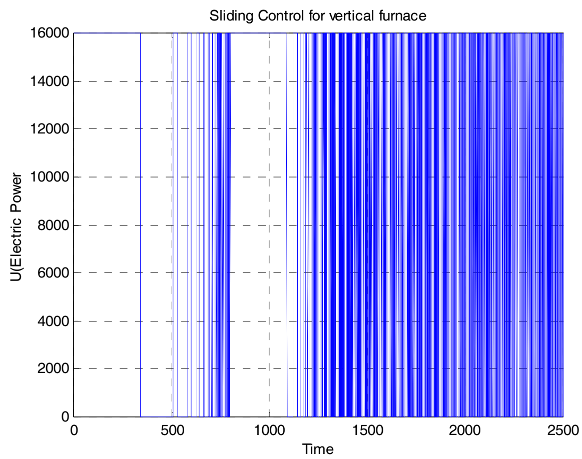

Figure 19. Input signal chattering occurs with $\varphi = 0.00001$

Now examine the case $k = 40000$. In this case the sliding condition is contravened as shown in Fig. 20.

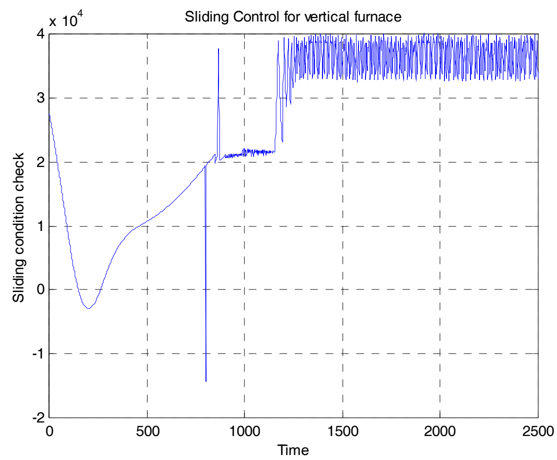

Figure 20. Sliding condition contravening

We can see that in some simulation times, the plotted curve is negative and this means contravening the sliding condition.

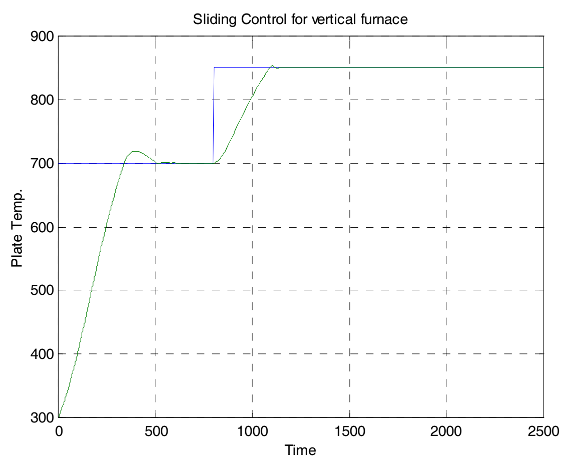

Figure 21. Step input response with SMC and selecting $\varphi = 0.00002$ and $\lambda = 1$

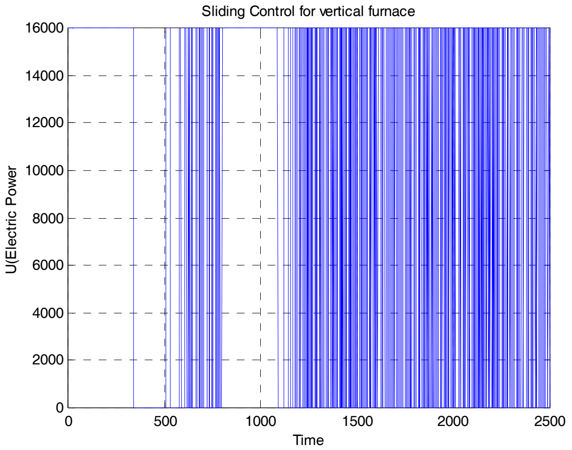

Figure 22. Input signal chattering occurs with $\varphi = 0.00002$ and $\lambda = 1$

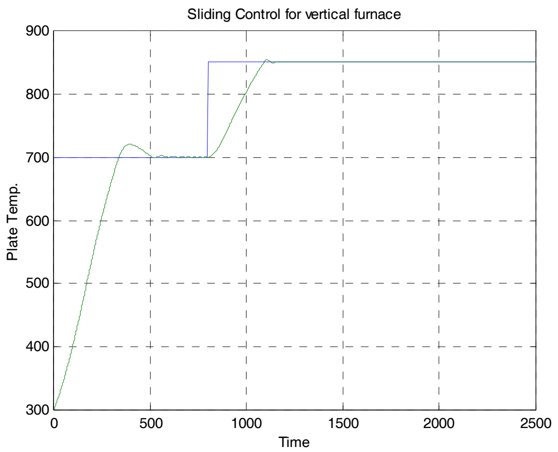

Figure 22. Step response of plate temperature with step disturbance and $\varphi = 0.00002$ and $\lambda = 1$

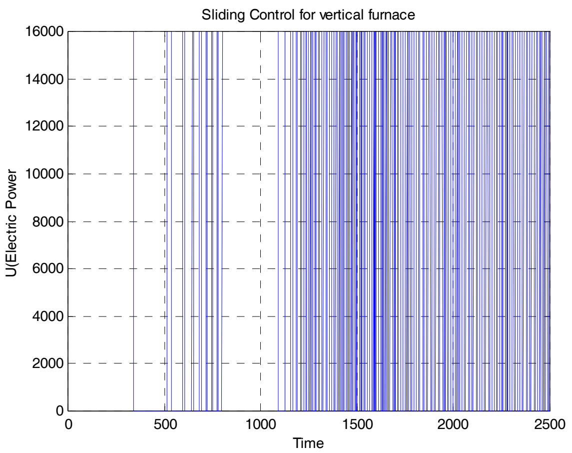

Figure 23. Input signal behavior with step disturbance and $\varphi = 0.00002$ and $\lambda = 1$

7. Conclusion

In this paper an industrial furnace as a nonlinear plant investigated and two different controller designed for it, as well as a linear system by feedback linearization technique in precise modeling case and sliding mode control in uncertainty of modeling parameters.

In feedback linearization control, stability and response speed adjusted by changing k_0 and k_1 in spite of nonlinearity and large dynamic range of the furnace temperature and totally two pairs of poles is selected and the various outputs behaviors were investigated.

In sliding mode control, response speed adjusted by changing the combination of λ, φ. Also for softening the control signal and avoiding chattering used an saturation element with large value of φ. Parameter η used to adjust the time to reaching sliding surface. Increasing the parameter k also helps for retaining satisfying condition.

In both of the aforementioned controllers, a step disturbance signal applied and the behavior of various parts of the furnace studied.

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