# Dynamics and Flight control of the UAV formations

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*Abstract:* - The purpose of this paper is to describe the flight of an Unmanned Aerial Vehicles (UAV) formation by using a 6 degrees of freedom (6 DOF) models. The problem of flight formation will be approached in a simple manner, by using a 3 DOF models, as well as using the complex equations that describe the movement of the 6 DOF for UAV. This theoretical development allows us to define a control structure based on direct commands, which is useful in practical applications. The work will present and analyze the calculus results for each developed solution. The novelty of this paper consists in the definition and testing of some practical solutions for an algorithm that can control an UAV formation. This algorithm allows decrypting the flight of a single UAV, as well as the entire formation. The conclusions will focus the practical possibility of implementing such algorithm on a UAV formation.

*Key-Words: – UAV, Flight, Formation, Dynamic, Control*

## NOMENCLATURE:

$\alpha$ - Attack angle (tangent definition) [8];
$\beta$ - Sideslip angle (tangent definition) [8];
$\beta^*$ - Sideslip angle (sin definition) [8];
$\chi$ - Air -path track angle;
$\gamma$ - Climb angle;
$\mu$ - Aerodynamic bank angle;
$\delta_a$ - Aileron deflection;
$\delta_e$ - Elevator deflection;
$\delta_{e0}$ The balance deflection angle for the elevator;
$\delta_r$ - Rudder deflection;
$\delta_T$ - Thrust command:
$\psi$ - Azimuth angle;
$\theta$ - Inclination angle;
$\phi$ - Bank angle;
$\rho$ - Air density;
$\mathbf{\Omega}$ - Angular velocity;
$\mathbf{\Omega}_V$ Angular velocity in quasi velocity frame;
$\omega_l^*, \omega_m^*, \omega_n^*$ - Rotation velocity components along the axes of the quasi-velocity frame;
$A, B, C, E$ - Inertia moments;
$C_x^A; C_y^A; C_z^A$ - Aerodynamic coefficients of force in the mobile frame;
$C_l^A; C_m^A; C_n^A$, - Aerodynamic coefficients of momentum in the mobile frame;
$C_x^T$ - Axial thrust coefficients in the mobile frame;
$C_{sp}$ - Specific fuel consumption;
Force components in the aerodynamic frame:
$L$ - Lift force; $D$ - Drag force; $N$ - Normal force;
$M$ - Mach number;
$C_D$ -Drag force coefficient;
$C_L$ -Lift force coefficient;

$C_N$ -Normal force coefficient;
$G$ - Gravitational force;
$g$ - Gravitational acceleration;
$\widetilde{q} = \rho V^2/2$ - Dynamic pressure;
$k_V, k_\chi, k_\gamma$ -Trajectory control coefficients;
$k_{dx}, k_{dy}, k_{dz}$ -Position control coefficients;
$k_{Ix}, k_{Iy}, k_{Iz}$ -Integrative position control coefficients;
$l$ - Reference length - body length;
$m$ – UAV mass;
$n$ - Load factor;
$p, q, r$ - Angular velocity components along the axes of mobile frame;
$S$ - Reference area – cross body area;
**T** - Thrust;
$t$ - Time;
**V** - Velocity;
$u, v, w$ - Aircraft velocity components in a mobile frame;
$V_{xp}$ $V_{yp}$ $V_{zp}$ -Velocity components in Earth frame;
$x_p$; $y_p$; $z_p$ - Linear coordinates in Earth frame

# 1 Introduction
The major impact of Unmanned Aerial Vehicles (UAV) consist in its role in the actual aero-space scenarios, where the UAV accomplish the recognition or the saving missions in hostile environments.

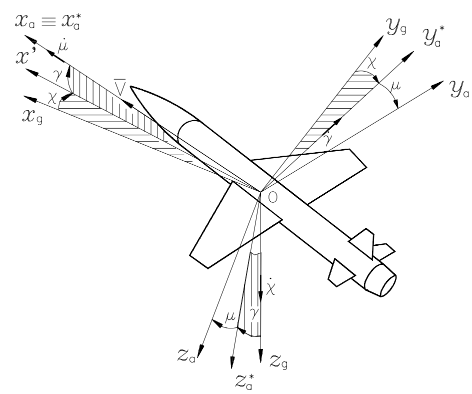
It is well known that the UAV fling formation using sample observation instrument, but in the same time using the data-fusion, is more efficiently then a sophisticate singular UAV or piloted airplane. The purpose of this paper is to describe the flight of an UAV formation. The item of flight formation will be approached in a simple manner, by using 3 DOF models, as well as using the complex equations that describe the movement using a 6 DOF model for each UAV. The control solution adopted can be used also in other domain where we have a body group, which must have a synchronous movement.

The paper will analyze UAV formation structures in a unitary manner using a system of control with an adequate architecture. The approach is inspired by the one proposed in [1] for controlling aircraft formations. An important result consists in the fact that the flight parameters will be presented on a quantitative base and by doing so the objective conclusions will be made available to the designers. In particular, the ratio between the complexity and the efficiency will be highlighted.

All the simulations will be made in a non-linear workspace, without using any simplifying hypothesis. From this point of view, the paper is a novelty, because in all the other papers, the dynamic is treated linearly or the formation is described as a plane model.

# 2 Formation modeling
In order to represent each UAV from the formation a three degree of freedom model was adopted. This simplified model involves only the slow states that correspond to a problem of the trajectory tracking and relative position maintaining by using an autopilot. For this reason, we use the following two reference frames.

Fig. 1 Quasi velocity frame and rotation angles




A local inertial frame $\Gamma_0$, with the origin in the mass center of the aircraft, with the z axe orientated vertically up. We are assuming that the inertial frame $\Gamma_0$ has the axes parallel to the ones of the Earth frame bound to the ground. The second is the quasi-velocity frame, connected to the velocity vector $\Gamma_a$ also with the origin in the mass center of the aircraft, obtained by two successive rotations; with the air-path track angle $\chi$, and with the climb angle $\gamma$. As usually, the axe $x_a^*$ of the quasi-velocity frame $\Gamma_a$ is orientated along the velocity vector **V**, and the $y_a^*$ axe is orientated in the horizontal plane. The transformation between the inertial frame $\Gamma_0$ and the quasi - velocity frame $\Gamma_a$ is given by the matrix:

$$\mathbf{A}_{a0} = \begin{bmatrix} \cos\gamma\cos\chi & -\cos\gamma\sin\chi & \sin\gamma \\ -\sin\chi & -\cos\chi & 0 \\ \sin\gamma\cos\chi & -\sin\gamma\sin\chi & -\cos\gamma \end{bmatrix}. \quad (1)$$

If we consider a $\mathbf{u}$ vector, the transformation relation between the inertial frames and the quasi-velocity frame is:

$$\mathbf{u}_a = \mathbf{A}_{a0}\mathbf{u}_0 \quad (2)$$

## 3 Simplified aircraft moving equations, 3 DOF model

If we accept the evolution without sideslip angle $\beta = 0$ then, the lateral force is also void $N = 0$. At the same time if we are assuming that thrust is orientated along the velocity vector, and the aerodynamic force components are obtained by a rotation with aerodynamic bank angle $\mu$ from the velocity frame, the dynamic equations of movement for the UAV named "i" in the quasi-velocity frame become:

$$\dot{V}_i = \frac{T_i - D_i}{m_i} - g\sin\gamma_i\,;\; \dot{\chi}_i = \frac{L_i}{m_i V_i}\frac{\sin\mu_i}{\cos\gamma_i}\,;$$

$$\dot{\gamma}_i = \frac{L_i}{m_i V_i}\cos\mu_i - \frac{g}{V_i}\cos\gamma_i\,. \quad (3)$$

Writing the load factor:

$$n_i = L_i / m_i g\,, \quad (4)$$

The equations (3) become

$$\dot{V}_i = \frac{T_i - D_i}{m_i} - g\sin\gamma_i\,;\; \dot{\chi}_i = \frac{g n_i}{V_i}\frac{\sin\mu_i}{\cos\gamma_i}\,;$$

$$\dot{\gamma}_i = \frac{g}{V_i}\left(n_i\cos\mu_i - \cos\gamma_i\right) \quad (5)$$

where we have denoted:

$D_i$ drag force, $L_i$ lift force, $T_i$ thrust, $m_i$ mass for aircraft "i".

If we use the aircraft polar coordinate, the drag coefficient is:

$$C_{Di} = C_{D0i} + k_i C_{Li}^2 \quad (6)$$

The drag force is:

$$D_i = F_{0i}C_{D0i} + k_i\frac{\left(n_i m_i g\right)^2}{F_{0i}} \quad (7)$$

where the reference aero dynamical force is:

$$F_{0i} = \frac{1}{2}\rho V_i^2 S_i \quad (8)$$

in witch $\rho$ is the air density at a given altitude and $S_i$ is the reference surface.

Defining the state vector $x_i = \left[V_i \quad \chi_i \quad \gamma_i\right]^T$ and the input vector: $u_i = \left[T_i \quad n_i \quad \mu_i\right]$, the equations (5) can be put under standard form:

$$\dot{x}_i = f_i(x_i, u_i) \quad (9)$$

## 4. Kinematics

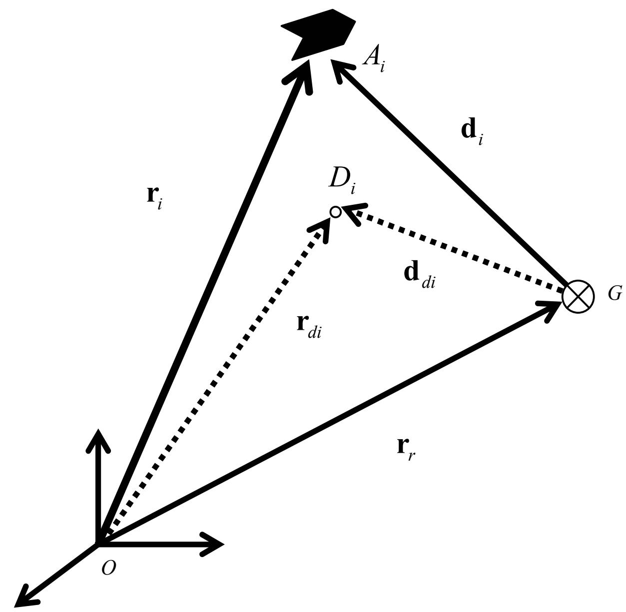



Fig. 2 The defining scheme for the UAV formation

One of the main goal regarding the control system of the formation flight is that every UAV must maintain a certain distance from a reference point denoted in figure 2 as $G$. This point may coincide with the formation leader (real or virtual), or the neighbor UAV (wingman), or a geometrical central point inside the formation.

For establishing a suitable mathematical model, similar to [1] we are assuming that $\mathbf{r_i}$ and $\mathbf{r_r}$ are the position vectors of the UAV position $A_i$, and of the reference point $G$ regarding the origin $O$ of the inertial frame. $\mathbf{d_i}$ is the current relative distance vector between $G$ and the UAV position $A_i$. The vector for the desired position of airplane $D_i$ is noted as $\mathbf{r}_{di}$. Also, we are assuming that an orthogonal frame $\Gamma_{ar}$, similar with the quasi - velocity frame, is attached to the $G$ point, whose orientation is defined by the angles $\gamma_r, \chi_r$. At the same time we are defining the velocity vector $\mathbf{V}_r$ as the velocity of the $G$ point. From figure 2 it results:

$$\mathbf{r}_r + \mathbf{d}_i = \mathbf{r}_i \tag{10}$$

and

$$\mathbf{r}_r + \mathbf{d}_{di} = \mathbf{r}_{di} \tag{11}$$

from where we obtain:

$$\mathbf{d}_{di} - \mathbf{d}_i = \mathbf{r}_{di} - \mathbf{r}_i \tag{12}$$

Deriving the relation (12), related to time in the quasi-velocity frame $\Gamma_{ai}$ and assuming that the desired velocity is that of the reference point $\mathbf{V}_r$, we obtain:

$$\dot{\mathbf{d}}_{di} - \dot{\mathbf{d}}_i + \mathbf{\Omega}_{Vi} \times (\mathbf{d}_{di} - \mathbf{d}_i) = \mathbf{V}_r - \mathbf{V}_i \tag{13}$$

where $\mathbf{\Omega}_{Vi}$ is the angular velocity of the quasi-velocity frame $\Gamma_{ai}$, related to the inertial frame $\Gamma_0$. In order to evaluate the vectored relation (13) we can project it along the axes of the quasi-velocity frame of each UAV:

$$\left[\dot{\mathbf{d}}_{di} - \dot{\mathbf{d}}_i\right]_{ai} = \left[\mathbf{V}_r\right]_{ai} - \left[\mathbf{V}_i\right]_{ai} - \mathbf{A}_{\omega i}\left[\mathbf{d}_{di} - \mathbf{d}_i\right]_{ai} \tag{14}$$

where the angular velocity vector has its components along the quasi-velocity frame axes given by:

$$\left[\mathbf{\Omega}_V\right]_{ai} = \left[\omega^*_{li} \quad \omega^*_{mi} \quad \omega^*_{ni}\right]^T. \tag{15}$$

In addition, the anti symmetric matrix associated to the angular velocity vector is given by:

$$\mathbf{A}_{\omega i} = \begin{bmatrix} 0 & -\omega^*_{ni} & \omega^*_{mi} \\ \omega^*_{ni} & 0 & -\omega^*_{li} \\ -\omega^*_{mi} & \omega^*_{li} & 0 \end{bmatrix}. \tag{16}$$

The connection between the orientation angles derivates and the angular velocity vector components is:

$$\begin{bmatrix} \omega^*_{li} \\ \omega^*_{mi} \\ \omega^*_{ni} \end{bmatrix} = \begin{bmatrix} \cos\gamma_i & 0 & -\sin\gamma_i \\ 0 & 1 & 0 \\ \sin\gamma_i & 0_i & \cos\gamma_i \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\gamma}_i \\ \dot{\chi}_i \end{bmatrix}, \tag{17}$$

from where we can obtain the scalar relations:

$$\omega^*_{li} = -\dot{\chi}_i \sin\gamma_i; \quad \omega^*_{mi} = \dot{\gamma}_i; \quad \omega^*_{ni} = \dot{\chi}_i \cos\gamma_i, \tag{18}$$

Note: In the paper [1] the problem is treated in a similar way, but the rotation matrix (1) is constructed by using three successive rotations, which represent the passing to the velocity frame, operation that makes difficult defining the relations (17) because it does not clearly point out the method of obtaining the derivates for the rolling angle $\dot{\mu}$. We are assuming that the orientation of the "i" UAV coincides with that of the $\Gamma_{ar}$ frame. In this case we define the rotation matrixes.

$$\mathbf{A}_{ai0} = \mathbf{A}_{a0}(\gamma_i, \chi_i) \tag{19}$$

and

$$\mathbf{A}_{ar0} = \mathbf{A}_{a0}(\gamma_r, \chi_r) \tag{20}$$

where $A_{a0}$ is obtained from equation (1). Starting from the defined matrixes we can write:

$$\left[\mathbf{V}_r\right]_{ai} = \mathbf{A}_{ai0}\left[\mathbf{V}_r\right]_0 \tag{21}$$

$$\left[\mathbf{V}_r\right]_{ar} = \mathbf{A}_{ar0}\left[\mathbf{V}_r\right]_0 \tag{22}$$

By using (21) and (22) we find the desired velocity in the reference frame $\Gamma_{ai}$:

$$\left[\mathbf{V}_r\right]_{ai} = \mathbf{B}\left[\mathbf{V}_r\right]_{ar} \tag{23}$$

where we have made the denotations:

$$\mathbf{B} = \mathbf{A}_{ai0}\mathbf{A}^T_{ar0} \tag{24}$$

$$\left[\mathbf{V}_r\right]_{ar} = \left[V_r \quad 0 \quad 0\right]^T \tag{25}$$

By introducing the relation (22) into (14) we obtain:

$$\left[\dot{\mathbf{h}}_{\mathbf{i}}\right]_{ai} = \mathbf{B}\left[\mathbf{V}_r\right]_{ar} - \left[\mathbf{V}_i\right]_{ai} - \mathbf{A}_{\omega i}\left[\mathbf{h}_i\right]_{ai} \tag{26}$$

where we denoted:

$$\mathbf{h}_i = \mathbf{d}_{di} - \mathbf{d}_i, \tag{27}$$

with the components along reference frame $\Gamma_{ai}$:

$$\mathbf{h}_i = \left[h_{ix} \quad h_{iy} \quad h_{iz}\right]^T \tag{28}$$

If we introduce the reference vector:

$$\mathbf{x}_r = \left[V_r, \quad \chi_r, \quad \gamma_r\right], \tag{29}$$

and the position of the UAV regarding the reference point between the states of the UAV then the equation can also be written in the compact form like this:

$$\dot{\mathbf{h}}_i = g(\mathbf{x}_i, \mathbf{x}_r, \mathbf{h}_i, \mathbf{u}_i) \quad (30)$$

Supplementary, for control command we need integral terms, which can be define by differential equations:

$$\dot{\mathbf{I}}_i = \mathbf{h}_i \quad (31)$$

where:

$$\mathbf{I}_i = \begin{bmatrix} I_{ix} & I_{iy} & I_{iz} \end{bmatrix}^T \quad (32)$$

## 5 Controlling the formation with pseudo-commands

This section of the paper will sketch the control system design. We assume the existence of a reaction loop with the standard autopilot, which will maintain the UAV in formation. Our intention is to define a law of formation control, capable of simultaneously manage the trajectory tracing and the formation position maintaining. As shown in [1] we can start from the command functions along the three axes of the quasi-velocity frame:

$$\begin{aligned} f_{1i} &= k_{dx} h_{ix} + k_{Ix} I_{ix} + k_V \tilde{V}_i \\ f_{2i} &= -\frac{V_i}{g}\left(k_{dz} h_{zi} + k_{Iz} I_{iz} - k_\gamma \tilde{\gamma}_i\right) + \cos\gamma_i \\ f_{3i} &= \frac{V_i}{g}\left(k_{dy} h_{yi} + k_{Iy} I_{yi} + k_\chi \tilde{\chi}_i\right)\cos\gamma_i \end{aligned} \quad (33)$$

where we denoted:

$$\tilde{V}_i = V_r - V_i\,;\ \tilde{\chi}_i = \chi_r - \chi_i\,;\ \tilde{\gamma}_i = \gamma_r - \gamma_i \quad (34)$$

and the relations (33) coefficients are:

$k_{dx} = 0.3$; $k_{dy} = 0.05$; $k_{dz} = 0.09$;
$k_{Ix} = 0.01$; $k_{Iy} = 0.006$; $k_{Iz} = 0.006$;
$k_V = 0.8$; $k_\gamma = 3.5$; $k_\chi = 3.5$

The trajectory control coefficients and the position control coefficients can be obtained by using an optimization procedure, which can be the subject of the future paper.

With the help of these three functions previously defined the pseudo-commands regarding each aircraft are formed. In this manner, the velocity-rolling angle is given by:

$$\mu_i = \arctan\left(\frac{f_{3i}}{f_{2i}}\right), \quad (35)$$

and the square sum of the last two functions (33) gives the necessary load factor:

$$n_i = \sqrt{f_{3i}^2 + f_{2i}^2} \quad (36)$$

In the end, we obtain thrust from the relation:

$$T_i = f_{1i} mg \quad (37)$$

## 6 Complex movement equations of an UAV formation, a 6 DOF model

In order to describe an UAV formation in a complex form we must keep the idea previously developed of using a simplified model as reference, which should define the velocity vector of each aircraft in the formation. Supplementary, for each aircraft in particular we must write the complete system of equations including those of the movement around the center of mass, which will allow us to define the real commands; the surface's deflection (ailerons, elevator and rudder) and the thrust modulation. For the moment equations we will use a mobile frame $\Gamma$, connected to the UAV, whose axes are the main inertia axes of the aircraft, the $Oxz$ being the symmetry plane.

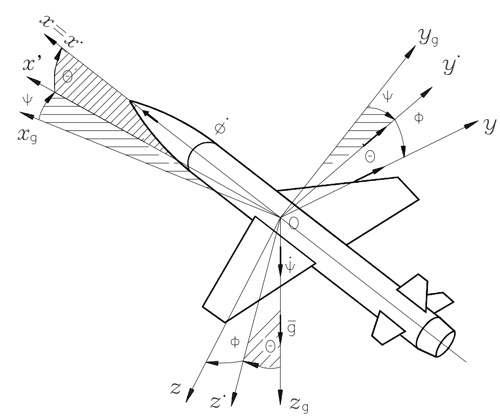


Fig. 3 Body frame and orientation angles

Therefore, for each UAV of the formation, in order to write the dynamic equations we define the aerodynamic coefficients in the mobile frame:

$$\begin{aligned} C_x^A &= \frac{X^A}{F_0}; C_y^A = \frac{Y^A}{F_0}; C_z^A = \frac{Z^A}{F_0}; \\ C_l^A &= \frac{L^A}{H_0^A}; C_m^A = \frac{M^A}{H_0^A}; C_n^A = \frac{N^A}{H_0^A}. \end{aligned} \quad (38)$$

where:

$$F_0 = \rho\frac{V^2}{2}S; \quad H_0^T = F_o l.$$

Similarly, if we consider the thrust $T$ and the nominal thrust as reference: $T_0$, we can define axial thrust coefficient:

$$C_x^T = T/T_0 \tag{39}$$

As shown in the papers [2] and [3] the UAV's dynamic equations are the projection equations of the force, equations that achieve from the impulse theorem, written in the Earth's frame and the momentum equations, which come from the kinetic momentum theorem, equations written in the mobile frame. Therefore, the force equations in the ground frame are:

$$\begin{bmatrix} \dot{V}_{xp} \\ \dot{V}_{yp} \\ \dot{V}_{zp} \end{bmatrix} = \frac{1}{m}\mathbf{B}_p \left\{ F_0 \begin{bmatrix} C_x^A \\ C_y^A \\ C_z^A \end{bmatrix} + T_0 \begin{bmatrix} C_x^T \\ 0 \\ 0 \end{bmatrix} \right\} + \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}, \tag{40}$$

where the matrix $\mathbf{B}_P$ is defined using the Euler's angles:

$$\mathbf{B}_p = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi s\theta c\phi \\ -s\psi c\theta & -c\psi c\phi - s\psi s\theta s\phi & c\psi s\phi - s\psi s\theta c\phi \\ s\theta & -c\theta s\theta & -c\theta c\phi \end{bmatrix}, \tag{41}$$

with:

$c\psi \equiv \cos\psi;\quad s\psi \equiv \sin\psi$

$c\theta \equiv \cos\theta;\quad s\theta \equiv \sin\theta$

$c\phi \equiv \cos\phi;\quad s\phi \equiv \sin\phi$

The momentum equations around the center of the mass of the UAV, written in the mobile frame are:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \mathbf{J}^{-1} H_c^A \begin{bmatrix} C_l^A \\ C_m^A \\ C_n^A \end{bmatrix} + \mathbf{J}^{-1} \begin{bmatrix} (B-C)qr + Epq \\ (C-A)rp + E(r^2 - p^2) \\ (A-B)pq - Eqr \end{bmatrix}, \tag{42}$$

where the inverse matrix for the inertia momentum is given by:

$$\mathbf{J}^{-1} = \frac{1}{AC - E^2} \begin{bmatrix} C & 0 & E \\ 0 & (AC - E^2)/B & 0 \\ E & 0 & A \end{bmatrix} \tag{43}$$

The kinematical equations are additional equations, which allow us to obtain the linear coordinates in the inertial frame:

$$\begin{bmatrix} \dot{x}_p & \dot{y}_p & \dot{z}_p \end{bmatrix}^T = \begin{bmatrix} V_{xp} & V_{yp} & V_{zp} \end{bmatrix}^T, \tag{44}$$

and Euler's angle when the rotation velocity components are known:

$$\begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T = \mathbf{W}_A \begin{bmatrix} p & q & r \end{bmatrix}^T, \tag{45}$$

where

$$\mathbf{W}_A = \begin{bmatrix} 1 & \sin\phi\,\mathrm{tg}\,\theta & \cos\phi\,\mathrm{tg}\,\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix}. \tag{46}$$

Supplementary we have mass equation which describes mass UAV's modification during the fly:

$$\dot{m} = -C_{sp} T \tag{47}$$

where $C_{sp}$ is specific fuel consumption.

The ordinary differential equations (40),(47) fully described one UAV's movement, the rest of the parameters such as the aerodynamic angles $\alpha$ $\beta^*$ or the angles that define the velocity frame $\gamma, \chi, \mu$ are achieved by using direct analytical relations from the system's state variables.

Therefore, for attack and sideslip angles, which have low values that do not require a reduction to the first quadrant of the trigonometric functions, we can use the relations indicated in [3], [2]:

$$\alpha = -\arctan(w/u);\quad \beta^* = \arcsin(v/V) \tag{48}$$

In order to obtain the velocity roll angle we can use the relation indicated in papers [2], [3]:

$$\mathrm{tg}\,\mu = \frac{\mathrm{c}\,\gamma[\mathrm{t}\,\phi\,\mathrm{t}\,\gamma\,\mathrm{c}(\chi-\psi)\,\mathrm{s}\,\theta + \mathrm{t}\,\gamma\,\mathrm{s}(\chi-\psi) + \mathrm{t}\,\phi\,\mathrm{c}\,\theta]}{\mathrm{c}(\chi-\psi)[1 - \mathrm{t}\,\phi\,\mathrm{t}(\chi-\psi)\,\mathrm{s}\,\theta]} \tag{49}$$

with:

$\mathrm{c}\,\gamma \equiv \cos\gamma;\quad \mathrm{s}\,\gamma \equiv \sin\gamma;\quad \mathrm{t}\,\gamma \equiv \tan\gamma$

$\mathrm{c}\,\chi \equiv \cos\chi;\quad \mathrm{s}\,\chi \equiv \sin\chi;\quad \mathrm{t}\,\chi = \tan\chi$

$\mathrm{c}(\chi-\psi) \equiv \cos(\chi-\psi);\quad \mathrm{s}(\chi-\psi) \equiv \sin(\chi-\psi);$

$\mathrm{t}(\chi-\psi) = \tan(\chi-\psi)$

Regarding the track angle $\chi$ and the climb angle $\gamma$, although we cannot obtain them from analytical relations starting from the velocity projections in the ground frame, regarding the fact that they take values that can exceed the first quadrant, the direct expression in differential form is preferred:

$$\dot{\chi} = \frac{F_o}{mV}\left(C_N\frac{\cos\mu}{\cos\gamma} - C_L\frac{\sin\mu}{\cos\gamma}\right) + $$
$$+\frac{T_0}{mV}\frac{C_x^T}{\cos\gamma}(\sin\mu\sin\alpha + \cos\mu\cos\alpha\sin\beta^*)$$
$$\dot{\gamma} = -\frac{F_0}{mV}\left(C_N\sin\mu + C_L\cos\mu\right) + $$
$$+\frac{T_0C_x^T}{mV}(\cos\mu\sin\alpha - \sin\mu\cos\alpha\sin\beta^*) - \frac{g}{V}\cos\gamma$$
. (50)

This form, unlike the equations (5) indicated in [1] are written for a non-zero sideslip angle $\beta^*$.

For defining the necessary commands of the flight formation, we must take into account that the command signals (28) are obtained in the quasi-velocity frame, and that the actual commands are formed in the mobile frame. In order to do this we define a matrix that will successfully accomplish 3 rotations with the angles $\mu$, $\beta$, $\alpha^*$, matrix that can be written:

$$\mathbf{A}_{\alpha\beta\mu} = \begin{bmatrix} c\alpha c\beta & c\mu c\alpha s\beta + s\mu s\alpha & s\mu c\alpha s\beta - c\mu s\alpha \\ -s\beta & c\mu c\beta & s\mu c\beta \\ s\alpha c\beta & c\mu s\alpha s\beta - s\mu c\alpha & s\mu s\alpha s\beta + c\mu c\alpha \end{bmatrix} \quad (51)$$

where:

$$\mathrm{c}\,\alpha \equiv \cos\alpha;\ \mathrm{s}\,\alpha \equiv \sin\alpha;$$
$$\mathrm{c}\,\beta \equiv \cos\beta^*;\ \mathrm{s}\,\beta \equiv \sin\beta^*;$$
$$\mathrm{c}\,\mu \equiv \cos\mu;\ \mathrm{s}\,\mu \equiv \sin\mu\ .$$

With the transpose of the matrix (51) in particular case when the aerodynamic bank angle is null ($\mu = 0$), we can obtain the aerodynamic coefficients in aerodynamic frame starting from coefficient in body frame:

$$\begin{bmatrix} C_D \\ C_N \\ C_L \end{bmatrix} = \begin{bmatrix} c\alpha c\beta & -s\beta & s\alpha c\beta \\ c\alpha s\beta & c\beta & s\alpha s\beta \\ -s\alpha & 0 & c\alpha \end{bmatrix}\begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} \quad (52)$$

where the aerodynamic coefficients in aerodynamic frame are defining by:

$$C_D = \frac{D}{F_0}; C_N = \frac{N}{F_0}; C_D = \frac{L}{F_0}; \quad (53)$$

With the help of the same matrix (51) we can rewrite the specific command functions for each UAV in the formation:

$$\begin{bmatrix} g_{1i} & g_{2i} & g_{3i} \end{bmatrix}^T = A_{\alpha i\beta i\mu i}\begin{bmatrix} f_{1i} & f_{2i} & f_{3i} \end{bmatrix}^T . \quad (54)$$

Starting from the command system components in the mobile frame, we can rewrite the pseudo-commands, using the mobile frame. This way, the roll angle, necessary for a non-drifting steering is:

$$\phi_{id} = \arctan\left(\frac{g_{3i}}{g_{2i}}\right) . \quad (55)$$

And the load factor along the z axis of the mobile frame becomes:

$$n_{zi} = \sqrt{g_{3i}^2 + g_{2i}^2} . \quad (56)$$

Starting from these equations, we can define the UAV's commands, which can be written in a simplified form:

$$\delta_{ai} = 1.5(\phi_{id} - \phi_i) - 0.5p_i \ \ \delta_{ei} = \delta_{eoi}n_{zi} - 0.5q_i$$
$$\delta_{ri} = -1.5\beta_i - 0.5r_i \ \ \delta_{Ti} = g_{1i} \quad (57)$$

The $\delta_{e0i}$ is the balance deflection angle for the elevator, corresponding to a given evolution.

# 7 Input data, airplane performances, results for formation fly

## 7.1 Input data for the model

### 7.1.1 Geometrical data

In Figure 4 are show the main geometrical data of the UAV. All data are in meters.

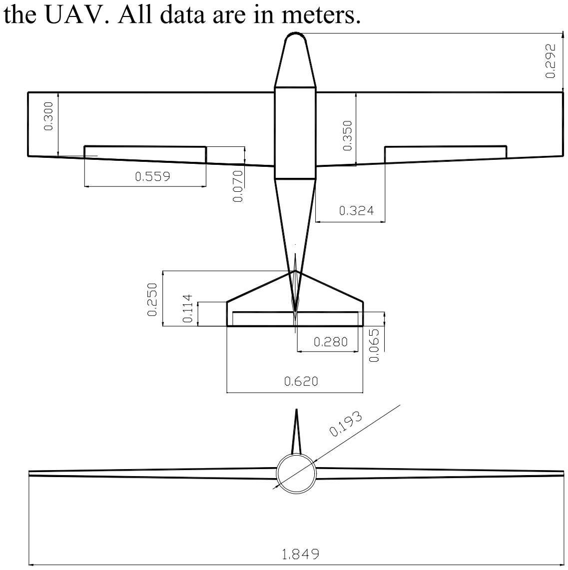



Fig. 4 –a Geometrical data

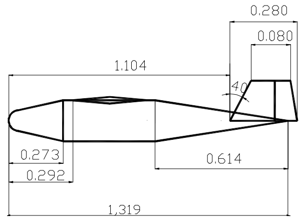



Fig. 4 –b Geometrical data

Geometrical reference data for the model are:
Reference length – body length:

$$l = 1.319\ m;$$

Reference area – cross body area:

$$S = 0.02925\ m^2$$

### 7.1.2 Mechanical data

As input data we use the geometrical elements of the UAV from figure 4. Mass characteristics of the model are: Initial mass: $m_i = 11 kg$;
Corresponding to initial mass, we have:
Center of mass position: $x_{cm} = 0.35 m$;
Inertial moments: $A = 2.8\ kgm^2$; $B = 0.86 kgm^2$; $C = 3.63 kgm^2$; $E = 0.006 kgm^2$.

### 7.1.3 Aerodynamic data

For the configuration of figure 4, aircraft type, considering a Taylor series expanding around the origin, taking into account the parity of the terms, we obtain the following polynomial form of the aerodynamic coefficients in a body frame of the aircraft:

$$\begin{aligned}
C_x^A &= a_1 + a_9\alpha + a_{21}\alpha^2 + a_{22}\beta^2 + a_6\delta_e^2 + a_7\delta_a^2 + a_8\delta_r^2 \\
&\quad + a_{10}\hat{q}\delta_e \\
C_y^A &= b_{12}\beta + b_{42}\hat{r} + b_{52}\delta_r + b_6\delta_a + b_{92}\hat{\beta} + b_{10}\hat{p} \\
C_z^A &= b_0 + b_{11}\alpha + b_{41}\hat{q} + b_{51}\delta_e + b_{91}\hat{\alpha} \\
C_l^A &= c_3\hat{p} + c_5\hat{r} + c_6\delta_a + c_7\delta_r + c_{13}\beta \\
C_m^A &= d_0 + d_{11}\alpha + d_{41}\hat{q} + d_{51}\delta_e + d_{91}\hat{\alpha} \\
C_n^A &= d_{12}\beta + d_{42}\hat{r} + d_{52}\delta_r + d_6\delta_a + d_{92}\hat{\beta} + d_{10}\hat{p},
\end{aligned} \tag{58}$$

where the coefficients $a_1$, $a_{21}$ … generally are depending by Mach number and, by definition [8]

$$\alpha = -\arctan(w/u);\ \beta = \arctan(v/u).$$

In addition, following notations are used for the non-dimensional angular velocities:

$$\hat{p} = p\frac{l}{V};\ \hat{q} = q\frac{l}{V};\ \hat{r} = r\frac{l}{V}; \tag{59}$$

and for the non dimensional and non stationary attack and sideslip angles:

$$\hat{\alpha} = \dot{\alpha}\frac{l}{V};\ \hat{\beta} = \dot{\beta}\frac{l}{V} \tag{60}$$

where $l$ is reference length – body length.

In our case, for low subsonic flow, the coefficients $a_1$, $a_{21}$ … $d_{10}$ are practically constant, they having the following values:
$a_{11} = -1.44$; $a_{21} = 116.4$; $a_{22} = -2.22$;
$a_6 = -8.24$; $a_7 = -35.1$; $a_8 = -1.38$; $a_9 = 7.6$;
$a_{10} = 5.03$; $b_{12} = 2.94$; $b_{42} = 0.81$; $b_{52} = -0.81$;
$b_6 = 0.003$; $b_{92} = 0.$; $b_{10} = -0.33$; $b_0 = -2.65$;
$b_{11} = -144.6$; $b_{41} = -11.19$; $b_{51} = 12.37$;
$b_{91} = -0.02$; $c_3 = -27.12$; $c_5 = 0.06$;
$c_6 = 1.62$; $c_{13} = 0.054$; $c_7 = -0.03$;
$d_0 = -0.0272$; $d_{11} = -11.28$; $d_{41} = -6.14$;
$d_{51} = 10.63$; $d_{91} = -0.01$; $d_{12} = -0.42$;
$d_{42} = -0.60$; $d_{52} = 0.51$; $d_6 = 0.$;
$d_{92} = 0.$; $d_{10} = 0.21$.

On the other hand, if we use the coefficients in aerodynamic frame like in the equations (3), (4), (5), the polar relation between drag coefficient and lift coefficient is given by (6) where: $C_{D0} = 1.44$, $k = 0.000121$. Similarly with the body frame coefficients, in this case, the reference area is the cross area of the body.

### 7.1.4 Thrust

The propeller thrust is determined by the relation:

$$T = T_0 C_x^T \tag{61}$$

Where $T_0$ is the nominal value at ground, a fix point, and $C_x^T$ and axial gas -dynamic coefficient.
Fashion experimental results indicate in work [9] we obtain the following approximate relation:

$$C_x^T = k_1(M)k_2(z_p)k_3(\delta_T) \tag{62}$$

where was separated the influence of the main parameters:
Mach number $M$:

$$k_1(M) = 1 - 0.4M - 0.6M^2; \tag{63}$$

Altitude $z_p$:

$$k_2(z_p) = 1 - 8.8 \cdot 10^{-5} z_p \,; \qquad (64)$$

Thrust command $\delta_T$ :

$$k_3(\delta_T) = a_T \delta_T + b_T \,. \qquad (65)$$

where $a_T = 1.667$ ; $b_T = -0{,}667$

Thrust command is limited to: $0.5 < \delta_T < 1$. It is obvious that for the null velocity (fixed point flight) at ground level with maximum command the thrust takes the nominal value: $T_0 = 55[N]$

Similarly, we can obtain specific fuel consumption, by using relation:

$$C_{sp} = C_{sp0} k_4(M) \qquad (66)$$

where the Mach dependence is:

$$k_4(M) = \frac{1 - 0.5M + M^2}{1 - M} \,. \qquad (67)$$

and the nominal specific consume at ground level with maximum command, corresponding with $T_0$ has the value $C_{sp0} = 1 \times 10^{-5} [Kg / N / s]$

Using specific fuel consumption, we can evaluate fuel debit that coincides with mass variation, as we see in the equation (47).

## 7.2 Airplane performances and flight parameters

In order to define test cases for the formation flight it is important for our application to obtain some information regarding performances of our aircraft without guidance control.

To this goal, following work [2], we can start from the balance movement in the longitudinal plane, which is describe by the equilibrium conditions:

$$F_0 C_x^A + T_0 C_x^T - G \sin\theta = 0 \,; \quad C_z^A + \frac{G}{F_0} \cos\theta = 0 \,;$$

$$C_m^A = 0 \,, \qquad (68)$$

where, for vertical evolution we have:

$$C_x^A = a_1 + a_9 \alpha + a_{21} \alpha^2 + a_6 \delta_e^2 \,;$$

$$C_z^A = b_0 + b_{11} \alpha + b_{51} \delta_e \,;$$

$$C_m^A = d_0 + d_{11} \alpha + d_{51} \delta_e \,;$$

$$C_x^T = k_1(M) k_2(z_p)(a_T \delta_T + b_T) \qquad (69)$$

Further, starting from relations (68), (69), we will obtain a few flight parameters and performance of the aircraft.

### 7.2.1 Horizontal flight with maximum velocity

In order to obtain maximum velocity in horizontal flight for an assignment altitude, we started from following initial data:

$$\gamma = 0 \;;\; \delta_T = 1; \quad z_p = ct; \quad \theta = \alpha \qquad (70)$$

Next, we built an iterative algorithm over all relation (68) because aerodynamic and gas-dynamic coefficients, theoretical, are dependent on the Mach number.

$$F_0 = -\frac{T_0 k_1 k_2 (a_T \delta_T + b_T) - G \sin\alpha}{C_x^A} \;;\; V = \sqrt{\frac{2F_0}{\rho S}} \;;$$

$$M = V / a \,;$$

$$\alpha = \frac{b_{51} \dfrac{d_0}{d_{51}} - \dfrac{G}{F_0} \cos\alpha - b_0}{b_{11} - b_{51} \dfrac{d_{11}}{d_{51}}} \;;\; \delta_e = -\frac{d_{11}}{d_{51}} \alpha - \frac{d_0}{d_{51}} \,.$$

$$(71)$$

The solution of the system (71) must check the conditions:

$$\delta_e < \delta_{e\max} \,;\; \alpha < \alpha_{\max} \,. \qquad (72)$$

### 7.2.2 Horizontal flight with minimum velocity

In order to obtain minimum velocity in horizontal flight for an assignment altitude, we started from following initial data:

$$\gamma = 0 \;;\; \delta_e = \delta_{e\max} \,; \quad z_p = ct; \; \theta = \alpha \qquad (73)$$

Further, we build an iterative algorithm over all relation (68) because aerodynamic and gas-dynamic coefficients, theoretical, depend by Mach number

$$\alpha = -\frac{d_{51}}{d_{11}} \delta_e - \frac{d_0}{d_{11}} \,;$$

$$F_0 = -\frac{G \cos\alpha}{C_z^A} ; V = \sqrt{\frac{2F_0}{\rho S}} \; ; M = V / a \,;$$

$$\delta_T = -\frac{F_0 C_x^A - G \sin\alpha}{T_0 k_1 k_2 a_T} - \frac{b_T}{a_T} \,. \qquad (74)$$

The solution of the system (74) must check the conditions:

$$\delta_T < 1 \,;\; \alpha < \alpha_{\max} \,. \qquad (75)$$

For our model, applying previous established relations for maximum and minimum velocity, we obtain flight level envelope shown in figure 5.

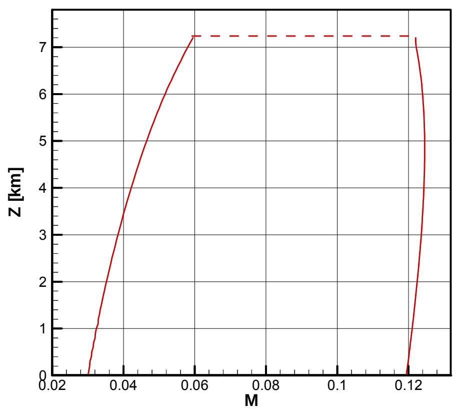



Fig. 5 Level fly envelope

Using this diagram, we choose the velocity for the next test cases in our application $V = 20[m/s]$, which correspond to $M \cong 0.06$.

### 7.2.3 Basic movement for assignment velocity

If we choose a velocity between maximum and minimum values, for an assignment altitude, we can solve iterative system (68) related attack angle and the deflection angle of the elevator, the angles obtained being the balance angles. For this, we initiate an iterative algorithm in attack angle that allow obtaining pitch angle $\theta$.

Input data are:

$$z_p = ct; \quad \gamma = ct. \tag{76}$$

The algorithm is initiated by the iterative relations:

$$\theta = \gamma + \alpha \,; \alpha = \frac{b_{51}\dfrac{d_0}{d_{51}} - \dfrac{G}{F_0}\cos\theta - b_0}{b_{11} - b_{51}\dfrac{d_{11}}{d_{51}}}; \tag{77}$$

which are completed with the final relations:

$$\delta_e = -\frac{d_{11}}{d_{51}}\alpha - \frac{d_0}{d_{51}};$$

$$\delta_T = -\frac{F_0 C_x^A}{T_0 a_T k_1 k_2} + \frac{G\sin\theta}{T_0 a_T k_1 k_2} - \frac{b_T}{a_T}. \tag{78}$$

The solution of the system (78) must check the conditions:

$$\delta_e < \delta_{e\max}\,; \ \alpha < \alpha_{\max} \,.\, \delta_x < 1\,; \tag{79}$$

For our model, applying previous established relations for an assignment velocity, allows us to obtain balance attack angle and balance deflection angle for the elevator, shown in figure 6.

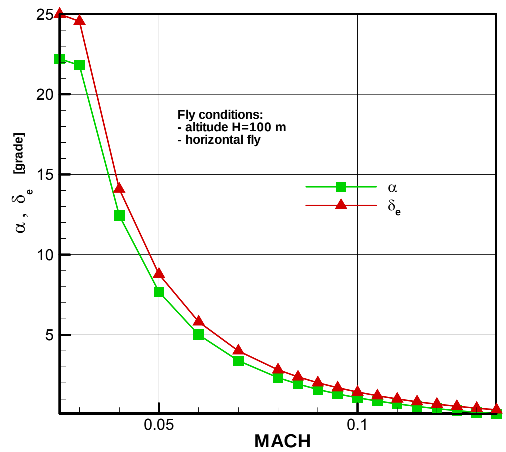



Fig. 6 Balance attack angle and balance deflection angle for the elevator

The values of the balance deflection angle for the elevator $\delta_{e0}$, corresponding to a chosen Mach number will be used in command relation previously established by relations (57).

## 7.3 Results for formation fly

### 7.3.1 Calculus algorithm

In order to solve the problem of formation flight, we most solve ordinary differential equation systems previously described by the simplified model (3 DOF) and for complex model (6 DOF). The calculus algorithm consists in a multi-step method Adams' predictor-corrector with variable step integration method: [6] [19]. Absolute numerical error was 1.e-8, and relative error was 1.e-6.

### 7.3.2 Results for simplified model (3 DOF)

Relations (5), (26), (31), (33), (35), (36), (37), describe the simplified model (3 DOF). First, we present a few results for individual flight of the UAV. As test case, we choose an ascension flight with constant climb angle simultaneously with turning flight, as we can see in figure 9. For this evolution, in figure 7 we showed the velocity diagram, and in figure 8 the velocity angles.

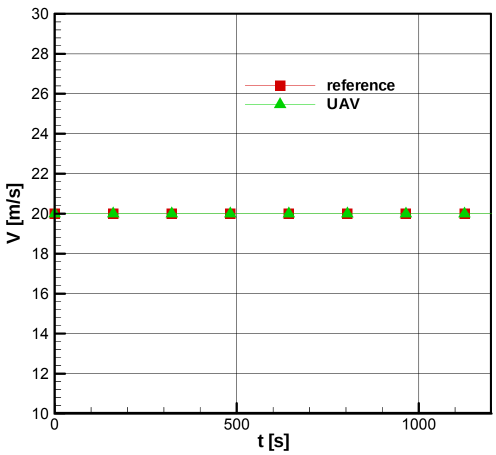



Fig. 7 Velocity Diagram

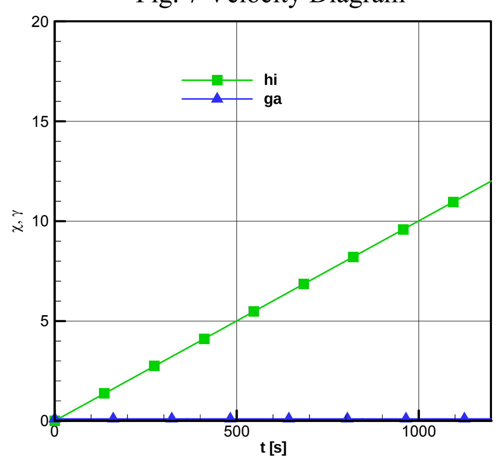



Fig. 8 Velocity angles

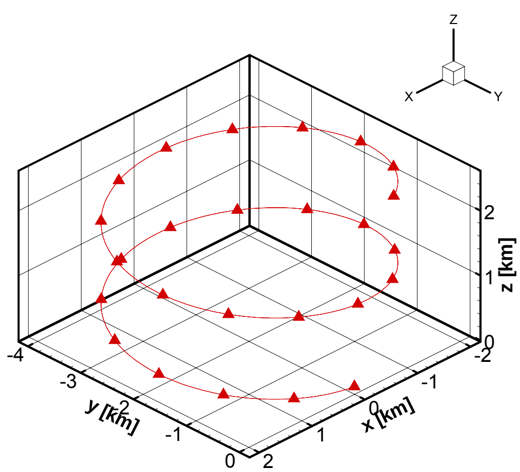



Fig. 9 Ascension flight, 1 aircraft, simplified model (3 DOF)

Next, for the same test case we present in figure 10 evolution of a UAV formation containing 3 aircrafts.

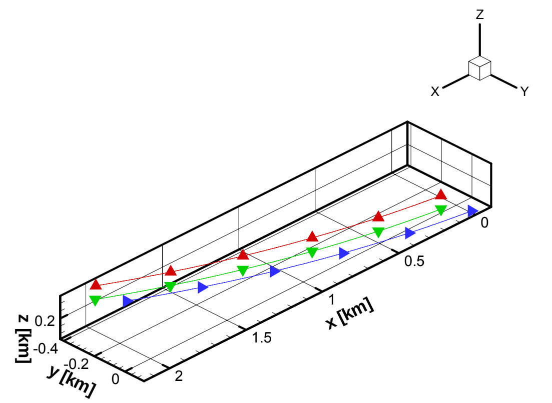



Fig. 10 Climbing flight, 3 aircrafts formation, simplified model (3 DOF)

It can be seen that with increasing distance between UAV the ability to realize the turn's flight decreases. This site because the airplane from the center of curvature has a short path, while being obliged to maintain reference speed. By increasing the distance between airplanes, and decreasing radius of curvature formation instability can be reached. This major shortcoming of the guidance method may be offset by switching the turning with short straight portions of the flight, ensuring relaxation of the formation.

### 7.3.2 Results for complex model (6 DOF)

Relations (40), (42), (44), (45), (50), (55), (56), (57) , describe the complex model (6 DOF). First, we present a few results for individual fly of the UAV. As test case, we choose an ascension flight with constant climb angle simultaneously with turning flight, as we can see in figure 14, which is quite similarly with the simplified model showed in figure 9.

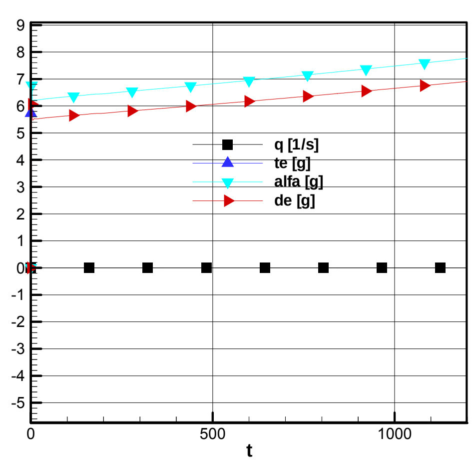



Fig. 11 Longitudinal fly parameters

For this evolution, in figure 11 we can see the longitudinal flight parameters, in figure 12 the lateral flight parameters and in figure 13 roll flight parameters.

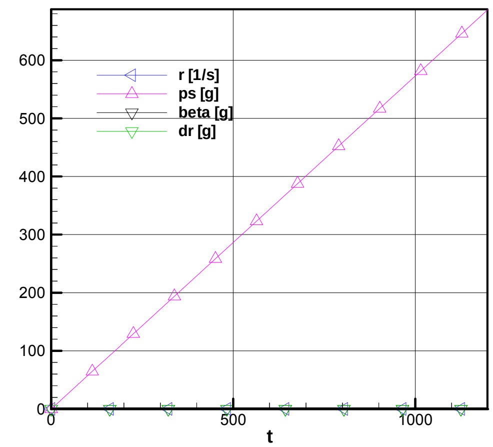



Fig. 12 Lateral flight parameters

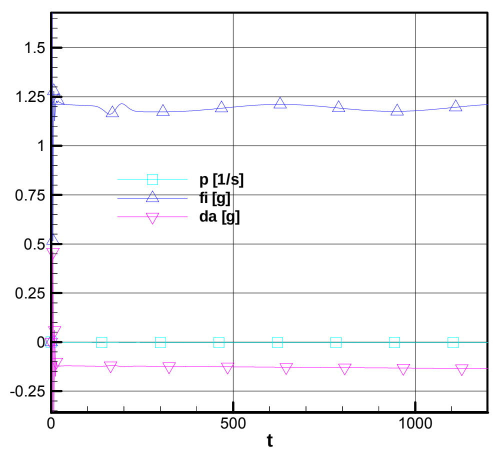



Fig. 13 Roll flight parameters

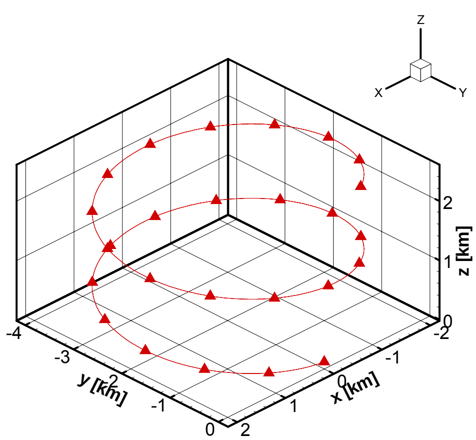



Fig. 14 Climbing flight, 1 aircraft, complex model (6 DOF)

Next, for the same test case we present in figure 15 evolution of a UAV formation containing 2 aircrafts.

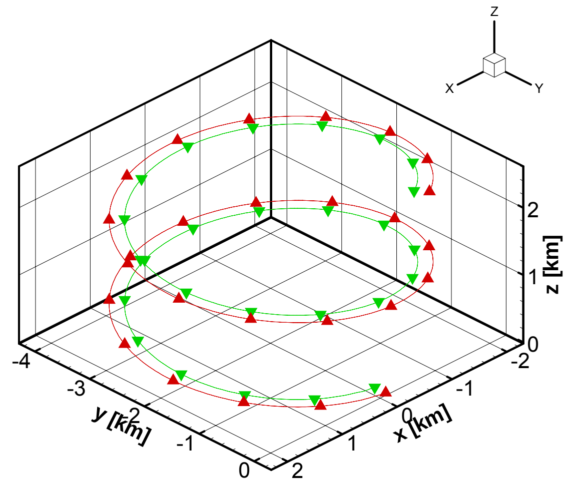



Fig. 15 Climbing flight, 2 aircrafts formation, complex model (6 DOF)

Unlike the case of simplified model for the complex model, we can obtain greater distances between airplanes at the same radius of curvature, due to the flexibility of command low.

## 8 Conclusions

For solving the UAV's formation control problem there have been successively developed two models. The first one is a simplified model with 3 degrees of freedom, accordingly to the methodology indicated in [1]. The second model , which is the contribution of this paper has a complex form, with 6 degrees of freedom, which allows the definition of the real commands needed for controlling the formation in order to do and experimental test of the model.

The question is which of the models are preferred, which is the better solution.

The first, the simplified one can be easy to use but the pseudo-commands are unhelpful as technical result. That because nobody applies directly the load factor, or the velocity bank angle without using the aerodynamic deflection.

The second one, the complex model involves many supplementary information, like real aerodynamic angles of each UAV, the balance deflection angle for the elevator, the aerodynamic coefficients of each UAV. Many of this data must be obtained through direct measurements, during the formation flight, or by estimator model, and finally can conduct to a sophisticate system.

In the next step of the work, we will try to build a linear model in order to analyze the stability of the system and to synthesis the command.

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