Pseudo-Equivalence of Fuzzy PID Controllers

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Abstract: - This paper presents a pseudo-equivalence of digital fuzzy PID controllers with linear PID controllers in the continuous time domain. Transfer functions and equivalence relations between controller's parameters are obtained for the common structures of the fuzzy PID controllers. The pseudo-equivalence is made using a graphic analytical analysis based on the input-output transfer characteristics of the fuzzy block, the linear characteristic of the fuzzy block around the origin and the usage of the gain in origin obtained as an origin limit of the variable gain of the fuzzy block. An algorithm of equivalence is presented. The paper presents a unitary theory, which may be applied to the most general fuzzy PID controllers, developed using all kind of membership functions, rule bases, inference methods and defuzzification methods. The term "pseudo-equivalence" is emphasized because there is no straight equivalence between the non-linear fuzzy controllers and the linear PID controllers. A case study of a control system using linear and fuzzy controllers is presented. The action mode of the fuzzy controller is analyzed. Based on the transient characteristics a comparison analyses is done. Better control quality criteria are demonstrated for control systems using fuzzy controllers.

Key Words: -Fuzzy logic, control systems, PID controllers.

1. Introduction

Some applications of fuzzy systems suddenly met in practice are so called PID fuzzy controllers. In the world scientific literature there were published many papers on this subject. Fuzzy controllers have a large field of applications and they may replace the linear PID controllers in process control. Some applications, from the most recent literature are, for example, the vector control of wind turbine [1] and the metal chamber temperature control [2].

The fuzzy PID controllers result after the introduction of a fuzzy block in the structure of the linear PID controller. The fuzzy block is developed using fuzzy logic and its basic theory is presented in many textbooks published in then last decades [3, 4]. In this paper we present a method and some synthesis relations for pseudo-equivalence of PID fuzzy controllers with PID linear controllers.

Many principles and methods were presented in literature for the equivalence (tuning, design) of the fuzzy PID controllers with the linear PID controllers [5, 6, 7, 8, 9] and a short survey of them is presented as follows. A related method is presented in [3]. That method makes an equivalence between the fuzzy PI controller and a linear control structure based on the state feedback. Relations for equivalence are derived. In the paper [5] the author proved that a fuzzy logic controller can be design to have an identical output to a given PI controller. Also, the reciprocal case is proven that a PI controller may be obtained with identical output to a given fuzzy logic controller with specified fuzzy logic operations. The paper [6] shows that it is possible to apply the empirical tool to predict the achievable performance of the conventional PID controllers to evaluate the performance of a fuzzy logic controller based on the equivalence between a fuzzy controller and a PI. The book [4] and other papers of the same author present a theory of fuzzy control, in which the fuzzy PID controllers are analyzed. Tuning fuzzy PID controller starting from a tuned linear PID controller, replacing it with a linear fuzzy controller, making the fuzzy controller nonlinear and then in the end making a fine tuning. In the paper [7] some mathematical models for the simplest fuzzy PID controllers and an approach to design fuzzy PID controllers are presented. The paper [8] analyses the analytical structure of a simple class of Takagi-Sugeno PI controller with respect to conventional control theory. An example shows an approach to Takagi-Sugeno fuzzy PI controllers tuning. But this type of controller is only a particular case in practice. In the paper [9] a tuning method based on gain and phase margins has been proposed to determine the weighting coefficients of the fuzzy PI controllers in the frame of the control o linear plant. Numerical simulations are presented.

In this paper we presents a method and some synthesis relations for pseudo-equivalence of the PID fuzzy controllers with the linear PID controllers, in the most general case, when the fuzzy block may be developed using all kind of: fuzzyfication methods, fuzzy values, membership functions, rule bases, inference methods and defuzzification methods. The equivalence is made based on the values taken from the graphical input-output transfer characteristics of the fuzzy block.

The linear PID controllers may be designed based on different methods, for example the modulus or symmetrical criterions, in Kessler's variant. The linear controller parameters may be used for an initial equivalence and after this other calculus must be applied, which will be presented in this paper.

The paper is treating the equivalence of the most common structures of fuzzy PID controllers.

Face to the above mentioned methods this paper is presenting a unitary theory, which is taking in consideration the input and output scaling coefficients, the analytic transfer characteristics of the fuzzy block, the nonlinear gain of the fuzzy block and the linear characteristic of the fuzzy block in the origin. The paper presents the calculus of the general gain of the fuzzy block and of the origin gain based on specific transfer characteristics obtained by digital computations.

For a good understanding of this method the readers may consult the paper [13] where the most important properties of the fuzzy systems used in this theory are presented.

The author used this equivalence theory in fuzzy control applications [11] and stability analysis [12].

2. Basic Elements

2.1. Fuzzy Controllers with Dynamics

The basic structure of the fuzzy controllers with dynamics is presented in Fig. 1.

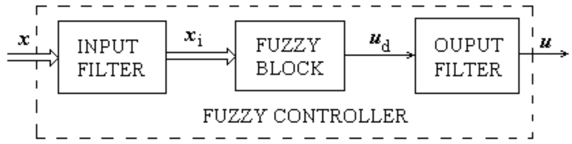

Fig. 1. The block diagram of a fuzzy controller with dynamics

So, the following fuzzy controllers, with dynamics, have as a central part a fuzzy block FB, an input filter and an output filter. The two filters give the dynamic character of the fuzzy controller.

The fuzzy block has the general structure from Fig. 2.

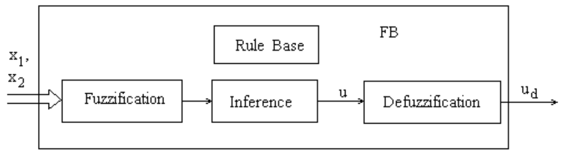

Fig. 2. The structure of the fuzzy block

The fuzzy block does not treat a well-defined mathematical relation (a control algorithm), as a linear controller is treating, but it is using inferences with many rules, based on linguistic variables. The inferences are treated with the operators of the fuzzy logic. The fuzzy block from Fig. 2 has in its structure three distinctive parts: fuzzyfication, inference and defuzzification. The fuzzy controller is an inertial system, but the fuzzy block is a non-inertial system.

The fuzzy controller has in the most common case two input variables x_1 and x_2 and one output variable u. The input variables are taken from the control system. The inference interface of the fuzzy block makes a treatment by linguistic variables of the input variables, obtained by the filtration of the controller input variables. For the linguistic treatment a definition with membership functions of the input variable is needed. In the interior of the fuzzy block the linguistic variables are linked by rules that are taking account of the static and dynamic behavior of the control system and also they are taken account of the limitations imposed to the controlled processed. In particular, the control system must be stable and it must assure a good amortization. After the inference fuzzy information for the output variable is obtained. Because in general the actuator that follows the controller must be commanded with a crisp value u_d, the defuzzification is used. The command variable u, furnished by the fuzzy controller, is obtained by filtering the defuzzified variable u_d. The output variable of the controller is the command input for the process. The fuzzification, the inference and the defuzzification bring a nonlinear behavior of the fuzzy block. The nonlinear behavior of the fuzzy block is transmitted also to the fuzzy controller. By an adequate choosing of the input and output filters we may develope different structures of the fuzzy controllers with imposed dynamics. So, we may obtain PI, PD and PID fuzzy controllers. The PI fuzzy controller may be with integration at the output or at the input.

2.2. Characteristics of the Fuzzy Block

The fuzzy block FB has a MISO transfer characteristic:

$$u_d = f_{FB}(x_e, x_{de}),\ x_e, x_{de} \in [-a, a] \tag{1}$$

From this transfer characteristic a SISO transfer characteristic may be obtained:

$$u_d = f_e(x_e; x_{de}),\ x_e \in [-a, a] \tag{2}$$

where x_{de} is a parameter.

A composed variable is introduced:

$$x_t = x_e + x_{de} \tag{3}$$

A family of translated characteristics may be obtained using this composed variable:

$$u_d = f_t(x_t; x_{de}),\ x \in [-2a, 2a] \tag{4}$$

with x_{de} as a parameter.

Using the above translated characteristics we may obtain the characteristic of the variable gain of the fuzzy block:

$$K_{FB}(x_t; x_{de}) = f_t(x_t; x_{de}) / x_t,\ x_t \neq 0 \tag{5}$$

Now, the MISO transfer characteristic of the fuzzy block may be seen as follows:

$$u_d = f_{FB}(x_e, x_{de}) = K_{FB}(x_e, x_{de}). \\ .(x_e + x_{de}) = K_{FB}(x_t; x_{de}).x_t \tag{6}$$

For the fuzzy block a linear characteristic may be used, around the point of the permanent regime x_e=0, x_{de}=0 and u_d=0:

$$u_d = K_0(x_e + x_{de}) \tag{7}$$

The value K_0 is the value at the limit, in origin of the characteristic $K_{BF}(x_t; x_{de})$:

$$K_0 = \lim_{xe \to 0} K_{FB}(x_t; x_{de}), x_{de} = 0 \tag{8}$$

This value may be determined with a good approximation, at the limit, from the gain characteristics.

Some examples of the above characteristics for the fuzzy block with max-min inference, defuzzification with center of gravity were the input variables and the output variables have 3 membership values from Fig. 3 and the 3x3 primary rule base from Tab. 1.

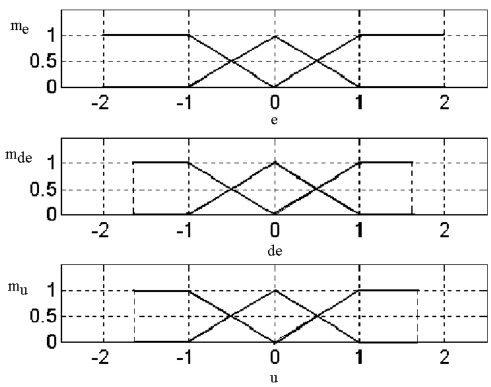

Fig. 3. Membership functions

Tab. 1. The 3x3 (primary) rule base

u		x_e		
		NB	ZE	PB
x_{de}	NB	NB	NB	ZE
	ZE	NB	ZE	PB
	PB	ZE	PB	PB

The MISO characteristic is presented in Fig. 4.

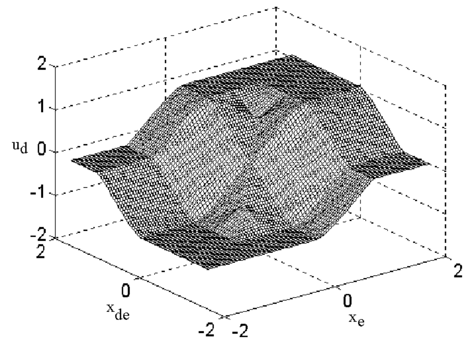

Fig. 4. MISO transfer characteristic

The SISO characteristics are presented in Fig. 5.

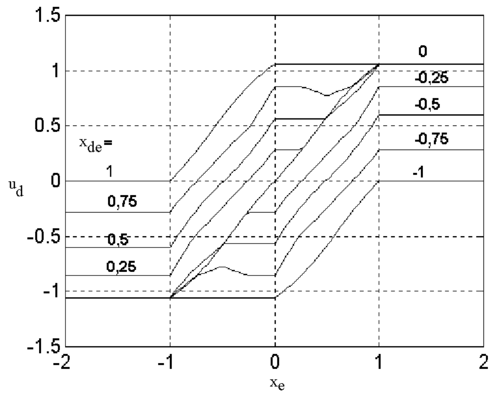

Fig. 5. SISO transfer characteristics

The translated characteristics are presented in Fig. 6.

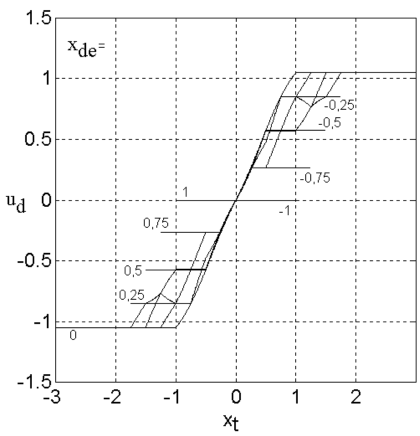

Fig. 6 The translated characateristics

And the characteristics of the variable gain are presented in Fig. 7.

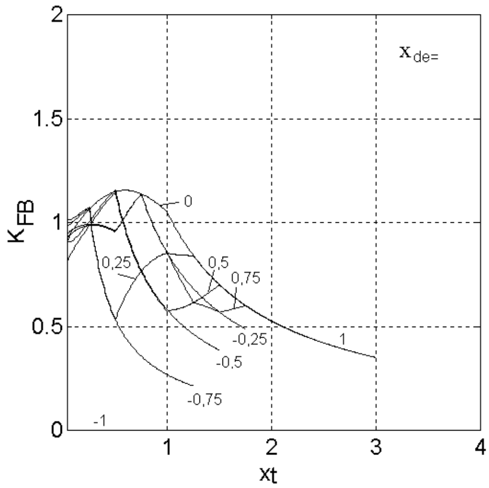

Fig. 7 The gain characateristics

From the Fig. 7 we may notice the value of the gain in origin is $K_0 \approx 1,2$.

Taking account of the correction made upon the fuzzy block with the scaling coefficient c_{du}, the characteristic of the fuzzy bloc around the origin is given by the relation:

$$\tilde{u}_d = c_{du} K_0 (x_e + x_{de}) \tag{9}$$

We use:

$$\tilde{c}_{du} = c_{du} K_0 \tag{10}$$

3 PI Fuzzy Controller with Integration at the Output

3.1 Basic Structure

The structure of a PI fuzzy controller with integration at its output (FC-PI-OI) is presented in Fig. 8.

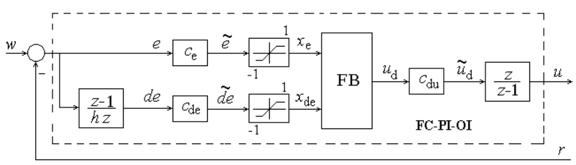

Fig. 8 The block diagram of the fuzzy PI controller

The controller is working after the error *e* between the reference input variable and the feedback variable *r*. In this structure we may notice that two filter were used. One of them is placed at the input of the fuzzy block FB and the other at the output of the fuzzy block. Their discrete time mathematical models present the filters. In the approach of the PID fuzzy controllers the concepts of integration and derivation are used for describing that these filters have discrete mathematical models obtained from continuous time mathematical models for integrator and derivative filters.

The filter from the controller input, placed on the low channel, realizes the operation of digital derivation at its output we obtain the derivative *de* of the error *e*:

$$de(t) = \frac{d}{dt}e(t) \quad \circ\!-\!\bullet \quad de(z) = \frac{z-1}{hz}e(z) \tag{11}$$

were *h* is the sampling period. In the domain of discrete time the derivative block has the input-output model:

$$de(t+h) = \frac{1}{h}e(t+h) - \frac{1}{h}e(t) \tag{12}$$

That show us that the digital derivation is made based on the information of error at the time moments $t=t_k=k.h$ and $t_{k+1}=t_k+h$:

$$\begin{aligned} e_k &= e(kh) \\ e_{k+1} &= e((k+1)h) \end{aligned} \tag{13}$$

So, the digital equipment is making in fact the subs traction of the two values.

The error *e* and its derivative *de* are scaled with two scaling coefficients c_e and c_{de}, as it follows:

$$\tilde{e}(t) = c_e e(t) \tag{14}$$

$$\tilde{de}(t) = c_{de} de(t) \tag{15}$$

The variables x_e and x_{de} from the inputs of the fuzzy block FB are obtained by a superior limitation to 1 and an inferior limitation to –1, of the scaled variables *e* and *de*. This limitation is introduced because in general case the numerical calculus of the inference is doing only on the scaled universe of discourse [-1, 1].

The fuzzy block offers the defuzzified value of the output variable u_d. This value is scaled with an output scaling coefficient c_{du}:

$$\tilde{u}_d = c_{du} u_d \tag{16}$$

In the case of the PI fuzzy controller with integration at the output the scaled variable $\tilde{u}_d$ is the derivative of the output variable *u* of the controller. The output variable is obtaining at the output of the second filter, which has an integrator character and it is placed at the output of the controller:

$$u(t) = \int_0^t \tilde{u}_d(\tau)d\tau \quad \circ\!-\!\bullet \quad u(z) = \frac{z}{z-1}\tilde{u}_d(z) \tag{17}$$

The input-output model in the discrete time of the output filter is:

$$u(t+1) = u(t) + \tilde{u}_d(t+1) \tag{18}$$

The above relation show that the output variable is computed based on the information from the time moments *t* and *t*+*h*:

$$\begin{aligned} u_{k+1} &= u((k+1)h) \\ u_k &= u(kh) \\ \tilde{u}_{dk+1} &= \tilde{u}_d((k+1)h) \end{aligned} \tag{19}$$

From the above relations we may notice that the “integration” is reduced in fact at a summation:

$$u_{k+1} = u_k + \tilde{u}_{dk+1} \tag{20}$$

That could be easily implemented in digital equipments.

Due to this operation of summation, the output scaling coefficient c_{du} is called also the increment coefficient.

Observation: The controller presented above could be called “fuzzy controller with summation at the output” and not with “integration at the output”.

3.2 Equivalence

For the fuzzy controller FC-PI-OI with the fuzzy block BF with a linear characteristic around the origin we may write the following input-output relation in the *z*-domain:

$$u(z)=\frac{z}{z-1}\tilde{c}_{du}(e(z)+de(z))= \\ =\frac{z}{z-1}\tilde{c}_{du}\left[c_e+c_{de}\frac{z-1}{hz}\right]e(z) \tag{21}$$

The transfer function of the PI fuzzy controller with integration at the output become:

$$H_{RF}(z)=\frac{u(z)}{e(z)}=\frac{z}{z-1}\tilde{c}_{du}\left(c_e+c_{de}\frac{z-1}{hz}\right) \tag{22}$$

A pseudo-equivalence may be done for the fuzzy controller with a linear PI controller in the continuous time, used in common applications. The equivalence is a false one, because the fuzzy controller is not linear, so we use the word “pseudo”.

The PI controller has the general transfer function:

$$H_{RG}(s)=\frac{u(s)}{e(s)}=K_{RG}\left(1+\frac{1}{sT_{RG}}\right) \tag{23}$$

We use the quasi-continual form of the transfer function, obtained by the conversion from the discrete time in the continuous time with the transformation [1]:

$$z=\frac{1+sh/2}{1-sh/2} \tag{24}$$

were h is the sampling period for the conversion of the transfer function:

$$H_{RF}(s)=\frac{u(s)}{e(s)}=H_{RF}(z)\Big|_{z=\frac{1+sh/2}{1-sh/2}}= \\ =\frac{\tilde{c}_{du}}{h}\left(c_{de}+\frac{h}{2}c_e\right)\left[1+\frac{c_e}{(c_{de}+c_e h/2)s}\right] \tag{25}$$

We notice that the above transfer function matches the general transfer function of the linear PI controller (23).

From the identification of the coefficients of the two transfer functions the following relations result:

$$K_{RG}=\frac{\tilde{c}_{du}}{h}\left(c_{de}+\frac{h}{2}c_e\right) \tag{26}$$

$$T_{RG}=\frac{c_{de}+\frac{h}{2}c_e}{c_e} \tag{27}$$

From relation (26) we may notice that the value of the gain coefficient K_{RG} of the PI fuzzy controller depends on the all three scaling coefficients, and what it is the most important, it depends on the gain in the origin of the fuzzy block.

And from the relation (27) we may notice that the time constant T_{RG} depends only on the scaling coefficients c_e and c_{de} from the inputs of the fuzzy block. At the limit, for $h\rightarrow 0$, the gain coefficient of the fuzzy controller has the value

$$K_{RG}=c_{de}K_0c_{du}/h \tag{28}$$

and the time constant of the fuzzy controller has the value

$$T_{RG}=c_{de}/c_e \tag{29}$$

Observations: A great value of c_e assures a small value of time constant of the fuzzy controller based on the relation (27). The value $c_e=1/e_M$, were e_M is the superior limit of the universe of discourse of the variable e, assures a dispersion of the values from the input e of the fuzzy block on the entire universe of discourse, without limitation for large variations of the error e.

A great value of c_{de} makes a great value of the time constant of the controller. A small value of c_{de} makes smalls values for the time constant and also for the gain.

But increasing c_{du} we may compensate the decreasing of the gain due to the decreasing of c_{de}.

Chosen of other fuzzy block with other membership functions and inference method is equivalent to the chosen of other K_0, greater or smaller.

From these relations we obtain the relation for designing the scaling coefficients based on the parameters of the linear PI controller:

$$c_e = \frac{hK_{RG}}{c_{du}K_0T_{RG}} \tag{30}$$

$$c_{de} = c_e(T_{RG} - h/2) \tag{31}$$

We may notice the influence of the gain in origin on c_e and also c_{de}.

The linear PI controller may be designed with different methods taken from the linear control theory.

Because the gain in origin is the main issue in this equivalence we present the algorithm of computation of the gain in origin is:

1. Obtaining the MIMO transfer characteristic of the fuzzy block.
2. Obtaining the family of SISO transfer characteristics from the MIMO characteristic, using one of the input variables as a parameter.
3. Obtaining the family of translated characteristic from the SISO characteristic, using a compound variable as summation of the two input variables.
4. Obtaining the gain characteristic by dividing the translated characteristic to the compound variable.
5. Obtaining the gain in origin by computing the limit in origin of the families of gain characteristics.

3.3. Anti-wind-up circuit

As in the case of the analogue linear PI controllers for the digital fuzzy controllers with integration an anti-wind-up circuit is necessary. For the PI controller with integration at the output an equivalent anti-wind-up circuit may be implemented as it is shown in Fig. 9.

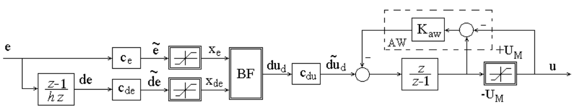

Fig. 9 The structure of the fuzzy PI controller with an anti-wind-up circuit

This structure is different from the structure from Fig. 8. Because the integration block a feedback is made with the anti-wind-up circuit AW. The circuit is necessary because the output of the controller is limited at maximum and minimum values +/-U_M imposed by the maximum command u of the process.

3.4 Correction of Fuzzy Controller

To assure stability to control systems using fuzzy PI controllers a correction is needed to modify the input-output transfer characteristic and a quasi-fuzzy controller results, with the structure from Fig. .

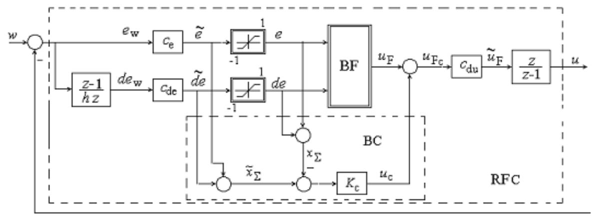

Fig. 10 Fuzzy controller with stability correction

With the correction from Fig. 10 the command is given by relation:

$$u_{Fc} = u_F + K_c(\tilde{e} + \tilde{de} - e - de) \tag{32}$$

and the characteristic of the nonlinear part of the control system is placed only in the I-st and III-rd quadrants [12].

In the quasi-fuzzy structure in parallel with the fuzzy block BF a linear structure is introduced. The correction is nonlinear.

4 PI Fuzzy Controller with Integration at the Input

The PI Fuzzy Controller with integration at the Input has the block diagram from the fig. 11.

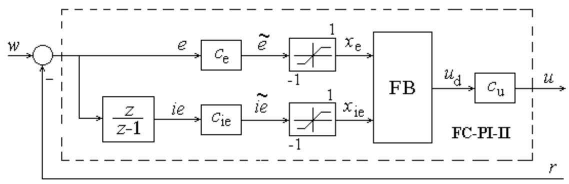

Fig. 11 The block diagram of the PI fuzzy controller with input integration

In the case of this structure the integration is made at the input of the fuzzy block on the error e.

In this case the transfer function of the fuzzy controller may be obtain as follow. The fuzzy block is described with a linear around the origin, for the permanent regime x_e=0, x_{ie}=0 and u_d=0:

$$u_d = K_0(x_e + x_{ie}) \tag{33}$$

The value of K_0 is the value at the limit in origin of the gain characteristic $K_{BF}(x_t; x_{ie})$, were:

$$x_t = x_e + x_{ie} \tag{34}$$

Taken account of the correction made upon the fuzzy block with the increment c_u, the characteristic of the fuzzy is:

$$u = c_u K_0(x_e + x_{ie}) \tag{35}$$

with

$$u = c_u K_0(x_e + x_{ie}) \tag{36}$$

For the fuzzy controller FC-PI-II with the linearized fuzzy block we may write the following input-output relation in the z-domain:

$$\begin{aligned} u(z) &= \tilde{c}_u[x_e(z) + x_{ie}(z)] = \\ &= \tilde{c}_{du}\left(c_e + c_{ie}\frac{z}{z-1}\right)e(z) \end{aligned} \tag{37}$$

With these observations the transfer functions of the fuzzy controller become:

$$H_{RF}(z) = \frac{u(z)}{e(z)} = \tilde{c}_{du}\left(c_e + c_{ie}\frac{z}{z-1}\right) \tag{38}$$

A pseudo-equivalence may be done with a PI linear controller described with the relation (23). For this reason the following quasi-continuous form is used:

$$\begin{aligned} H_{RF}(s) &= \frac{u(s)}{e(s)} = H_{RF}(z)\Big|_{z=\frac{1+sh/2}{1-sh/2}} = \\ &= \frac{\tilde{c}_{du}}{h}\left(c_{de} + \frac{h}{2}c_e\right)\left[1 + \frac{c_e}{(c_{de} + c_e h/2)s}\right] \end{aligned} \tag{39}$$

We may notice that the above transfer function matches the transfer function of the common PI linear controller.

From the identification of the coefficients of the two transfer functions from the relations (38) and (23) the following equalities result:

$$K_{RG} = \frac{\tilde{c}_{du}}{h}\left(c_{de} + \frac{h}{2}c_e\right) \tag{40}$$

$$T_{RG} = \frac{c_{de} + \frac{h}{2}c_e}{c_e} \tag{41}$$

From relation (40) we may notice that the value of the gain coefficient K_{RG} of the PI controller depends on the all three scaling coefficients.

From the relation (41) the time constant depends only the scaling coefficients c_e and c_{de} from the inputs of the fuzzy block. At the limit, for $h \to 0$, the gain coefficient of the fuzzy controller has the value $K_{RG} = c_{de}K_0 c_{du}/h$, and the time constant has the value $T_{RG} = c_{de}/c_e$.

Observation. A great value of c_e assures a small value of time constant. The value $c_e = 1/e_M$ were e_M is the maximum error on the universe of discourse, assure a repartition of the values from the input e on the all universe of discourse. A great value c_{de} leads to a grea time constant. Increasing the value of c_{du} we may compensate the decreasing of the gain by the decreasing of c_{de}.

From the above equalities we may obtain the relations for the scaling coefficients based on the parameters of the linear PI controller:

$$c_e = \frac{hK_{RG}}{c_{du}K_0 T_{RG}} \tag{42}$$

$$c_{de} = c_e(T_{RG} - h/2) \tag{43}$$

The linear controller may be designed using different methods from the linear control theory.

5 PD Fuzzy Controller

The structure of the PD fuzzy controller is presented in fig. 12.

In this case the derivation is made at the input of the fuzzy block, on the error e.

The fuzzy block is described with a linear characteristic around the origin, for the permanent regime $x_e=0$, $x_{de}=0$ and $u_d=0$:

$$u_d = K_0(x_e + x_{de}) \tag{44}$$

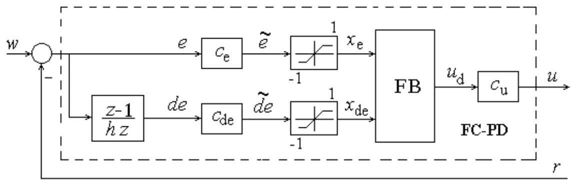

Fig. 12 The block diagram of the PD fuzzy controller

The value of K_0 is obtained from the characteristic $K_{BF}(x_t; x_{de})$, were:

$$x_t = x_e + x_{ie} \tag{45}$$

Taking account of the correction with the increment c_u, the characteristic of the fuzzy block is:

$$u = c_u K_0 (x_e + x_{de}) \tag{46}$$

with

$$\tilde{c}_u = c_u K_0 \tag{47}$$

For the fuzzy controller FC-PD the following relation in the z-domain is obtained:

$$u(z) = \tilde{c}_u [x_e(z) + x_{de}(z)] = \\ = \tilde{c}_u \left[c_e + c_{de} \frac{z-1}{hz} \right] e(z) \tag{48}$$

With this relation the transfer function results:

$$H_{RF}(z) = \frac{u(z)}{e(z)} = \tilde{c}_u \left(c_e + c_{de} \frac{z-1}{hz} \right) \tag{49}$$

The PD linear controller has the transfer function:

$$H_{RG}(s) = K_{RG}(1 + T_D s) \tag{50}$$

Pseudo-equivalence may be done, using the quasi-continual form:

$$H_{RF}(s) = \frac{u(s)}{e(s)} = \tilde{c}_u (c_e + c_{de} s) = \\ = \frac{\tilde{c}_u}{c_e} \left(1 + \frac{c_{de}}{c_e} s \right) \tag{51}$$

From the identification of the coefficients from the two relations (50) and (49) result the relations:

$$K_{RG} = \frac{\tilde{c}_u}{c_e} \tag{52}$$

$$T_{RG} = \frac{c_{de}}{c_e} \tag{53}$$

From relation (52) we may notice that the value of the gain K_{RG} of the controller depends on two scaling coefficients. And the time constant T_{RG} depends only on the scaling coefficients from the input of the fuzzy block c_e and c_{de}.

Observation. A great value of $\tilde{c}_u$ assures a great value of time constant, according to relation (52). A great value of c_{de} leads to a great time constant.
From the above relations we obtain the relations of the scaling coefficients, based on the parameters of the linear controller:

$$c_e = \frac{\tilde{c}_u}{K_{RG}} \tag{54}$$

$$c_{de} = \frac{T_{RG}\, \tilde{c}_u}{K_{RG}} \tag{55}$$

We may notice that the values of the input scaling coefficients do not depend on *h*.

6. PID Fuzzy Controller

The structure of the PID fuzzy controller is presented in fig. 13.

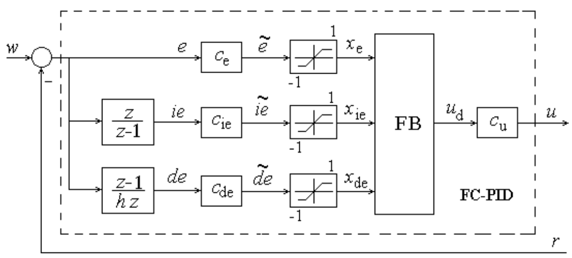

Fig. 13 The block diagram of PID fuzzy controller

In this case the PID characters are made at the input of the fuzzy block. The integration and the derivation are made at the input of the fuzzy block on the error *e*. The fuzzy block has three input variables x_e, x_{ie} and x_{de}.

For the PID fuzzy controllers introduced in this paragraph pseudo-equivalence with a linear PID controller in the continuous time domain may be done. This pseudo-equivalence will be proved as follows.

In the case of the PID fuzzy controller we extend the method of linear characteristic around the origin, for the permanent regime, for x_e=0, x_{ie}=0, x_{de}=0 and u_d=0 with the relation:

$$u_d = K_0(x_e + x_{ie} + x_{de}) \tag{56}$$

The fuzzy block as three input variables and it may be describe with the function:

$$K_{FB}(x_t; x_{de}, x_{ie} = 0) = \frac{u_d}{x_t},\ x_t \neq 0 \tag{57}$$

where

$$x_t = x_e + x_{ie} + x_{de} \tag{58}$$

The value of K_0 is the value at the limit in origin of the above function:

$$K_0 = \lim_{x_t \to 0} K_{BF}(x_t; x_{de}, x_{ie} = 0) \tag{59}$$

Taking account of the correction with the increment c_u, the characteristic of the fuzzy block is:

$$u = c_u K_0 (x_e + x_{ie} + x_{de}) \tag{60}$$

with

$$\tilde{c}_u = c_u K_0 \tag{61}$$

For the fuzzy controller the following relation results in the *z*-domain:

$$u(z) = \tilde{c}_u [x_e(z) + x_{ie}(z) + x_{de}(z)] = \\ = \tilde{c}_u \left[c_e + c_{ie} \frac{z}{z-1} + c_{de} \frac{z-1}{hz} \right] e(z) \tag*{62()}$$

and the transfer function is:

$$H_{RF}(z) = \frac{u(z)}{e(z)} = \\ = \tilde{c}_u \left(c_e + c_{ie} \frac{z}{z-1} + c_{de} \frac{z-1}{hz} \right) \tag{63}$$

The pseudo-equivalence is made with the linear PID controller described by the transfer function:

$$H_{RG}(s) = K_{RG} \left(1 + T_D s + \frac{1}{T_I s} \right) \tag{64}$$

The quasi-continual form used is:

$$H_{RF}(s) = \frac{u(s)}{e(s)} = H_{RF}(z)\Big|_{z=\frac{1+sh/2}{1-sh/2}} = \\ = \tilde{c}_u (c_e + c_{ie}/2). \\ .\left[1 + \frac{c_{ie}}{h(c_e + c_{ie}/2)s} + \frac{c_{de}}{c_e + c_{ie}/2} s \right] \tag{65}$$

In the calculus we considered h^2=0.

We may notice the match between the transfer functions.

After the identification of the coefficients from the relations (64) and (65) the designing relations result:

$$K_{RG} = \tilde{c}_u (c_e + c_{ie}/2) \tag{66}$$

$$T_I = \frac{h(c_e + c_{ie}/2)}{c_{ie}} \tag{67}$$

$$T_D = \frac{c_{de}}{c_e + c_{ie}/2} \tag{68}$$

Observations. A great value of $\tilde{c}_u$ assures a great value of the controller gain. A great value of scaling coefficient c_{ie} leads to a small value of the time constant of the integrator. A great value of c_{de} leads to a great derivation time constant.

From the above equalities we obtain the relations for the scaling coeffients based on the parameters of the linear controller:

$$c_e = \left(\frac{T_I}{h} - \frac{1}{2} \right) \frac{h K_{RG}}{\tilde{c}_u T_I} \tag{69}$$

$$c_{ie} = \frac{hK_{RG}}{\tilde{c}_u T_I} \tag{70}$$

$$c_{de} = \frac{K_{RG}}{\tilde{c}_u} T_D \tag{71}$$

7 Case Study

To demonstrate the advantages of the above equivalence method a fuzzy control system for an electric drive is analyzed [11].

The block diagram of the fuzzy control system of a d. c. drive is presented in Fig. 14.

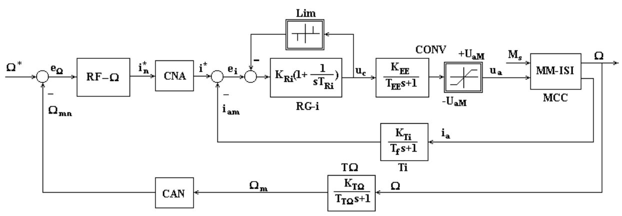

Fig. 14 The block diagram of the fuzzy control system

A fuzzy PI digital controller RF-Ω with the structure from Fig. 15 is used.

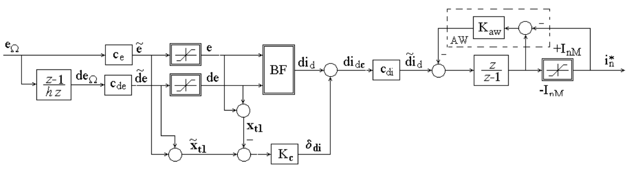

Fig. 15 The structure of the quasi fuzzy controller

This fuzzy controller has a correction which assures stability and the transfer characteristic of the non-linear part from Fig. 16, place only in the I-st and III-rd quadrants [13].

The controller has also an anti wind-up circuit.

The scaling coefficients were chosen after some iterative steps, using the quality criteria of the transient characteristics of the speed fuzzy control system at a step speed reference. The speed scaling coefficient c_e had the same value $c_e=1/e_M$. The first value of the derivative scaling coefficient was $c_{de}=1/de_M$.

The coefficients c_e and c_{de} from the inputs of the fuzzy block are chosen after some iterative testing with the fuzzy control system, to assure good control quality criteria. The incremental coefficient c_{di} from the output of the fuzzy block was chosen based on a stability analysis using circle criterion [12].

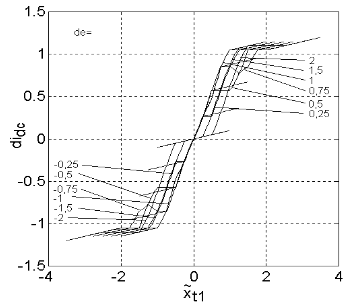

Fig. 16 The corrected transfer characteristic

The adopted solution contains the values of the scaling coefficients from the sixth step. The transient characteristics obtained in the process of choosing the scaling coefficients are presented in the following figures. Fig. 17.

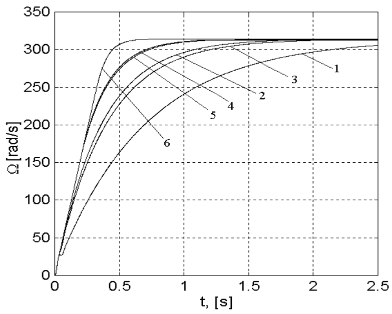

Fig. 17 The transient characteristics for scaling coefficients determination

We have started the selection from the initial values of the linear PI speed controller parameters obtain for the linear control system, tuned with the symmetrical criterion in Kessler's variant. The value of c_{de} was decreased to the final value from the sixth step. Decreasing more this scaling coefficient the fuzzy control system becomes unstable.

Using the above equivalence relation a transfer function for the equivalent controller resulted: $H_{RF0}(s)=11{,}46(1+1/0{,}12s)$.

The Simulink block diagram of the speed fuzzy control system of a d.c. motor is presented in Fig. 18.

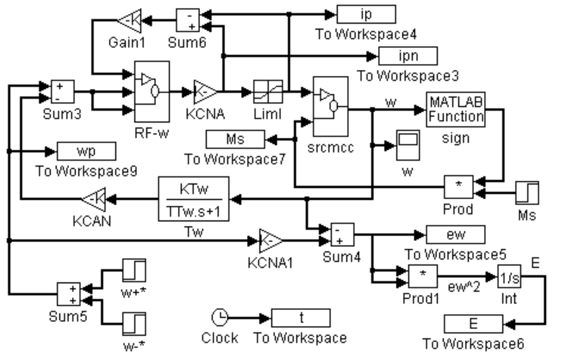

Fig. 18 Simulink block diagram of the speed fuzzy control system

The Simulink block diagram of the fuzzy controller is presented in Fig. 19.

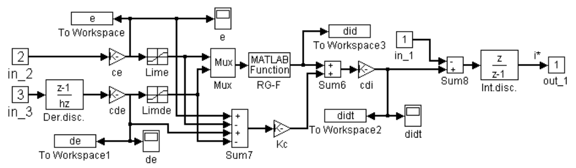

Fig.19 Simulink block diagram of the fuzzy controller

The transient characteristics for the main variables of the fuzzy control systems are preened as follows.

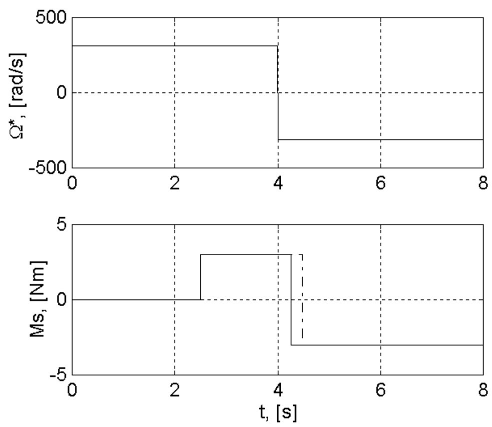

Fig. 20 Rotational speed reference Ω^* and the load torque M_s

Simulations are made for both control systems, conventional and fuzzy, with tuned parameters: J and k_f and detuned parameters: J^d=2J and k_f^d=2k_f. In the second case an error at the parameter identification is assumed.

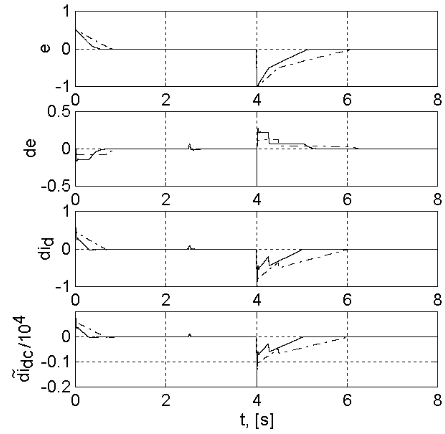

Fig. 21 The input and output variables of the fuzzy block $\tilde{e}, \tilde{de}$,

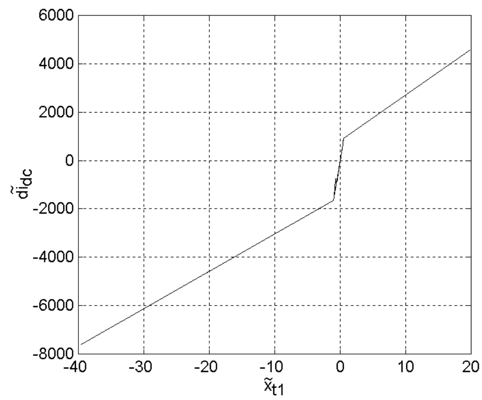

Fig. 22 The characteristic of the non-linear part, corrected

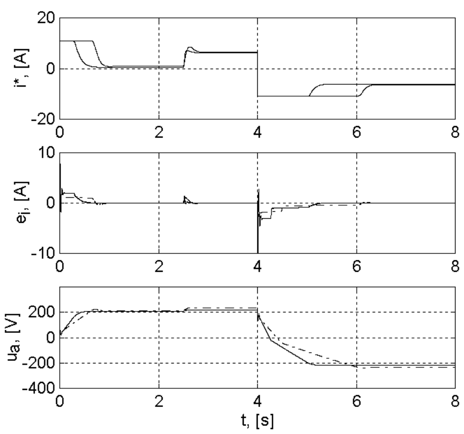

Fig. 23 The reference current i^*, the current error e_i the armature voltage u_a

The transient characteristics for the current and speed, for tuned and detuned parameters are presented in Fig. 24.

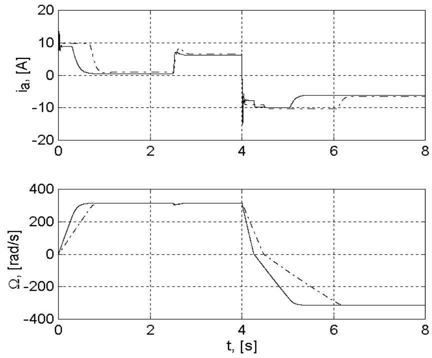

Fig. 24 Armature current i_a and the rotational speed Ω

With continuous line are represented the characteristics for tuned parameters and with dash-dot line are represented the characteristics for detuned parameters.

The regime consists in starting the process unloaded at t=0 with a constant the speed reference Ω*=314 rad/s. At the time 2,5 s a constant load torque M_s of M_N=3 Nm, in the range of the rated process torque, is introduced. At the time 4 s the speed is reversed, at Ω*=-314 rad/s, maintaining the constant load torque: M_s=$M_N$sign(Ω).

The comparative characteristics for fuzzy and linear control are presented in Fig. 25.

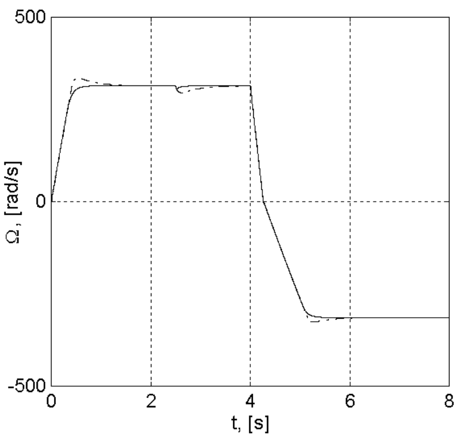

Fig. 25 Transient characteristics for current and speed

Based on a comparative analysis of the speed performance criteria the following may be presented.

The fuzzy controllers assure better control quality criteria: -overshoot for reference is zeros at start and at the reversing; the settling time for speed reference at start and reversing of the fuzzy control system is smaller; -the deviation of speed for perturbation of the fuzzy control system is smaller then in the conventional case; the performance criteria of the fuzzy control in the case of detuned parameters are sensitive better then the performance criteria for conventional control and the fuzzy control system is more robust at the identification errors then the convention control system.

8 Conclusion

In this paper some common controllers based on fuzzy blocks with the general structure and PID dynamics was analyzed. Pseudo-equivalences of them with linear PID controllers were made. The transfer characteristics of the fuzzy blocks were used in the design of the fuzzy controllers. The design of the fuzzy controller is based on: the input-output transfer characteristics of the fuzzy block, obtained by digital computer calculation and the linear characteristic of the fuzzy block around the origin, for the permanent regime. The gain in origin is obtained as a limit in origin of the function obtained from the translated SISO transfer characteristics.

Design relations were developed for all the fuzzy controllers: PI with output and input integration, PD and PID fuzzy controllers.

Analysis of the design relation was made.

Some observations related to the influence of the scaling coefficients were presented.

The results presented in this paper are important in the design of the control systems based on PID fuzzy controllers.

A pseudo-equivalence of them with linear PID controllers was made.

The method is developed for the most general structure of the fuzzy systems. This method for equivalence is valid for all kind of fuzzyfication and defuzzification methods, all types of membership functions, all inference methods, because it is based on analytic transfer characteristic, which may be obtained using computer calculations.

If there is a designed linear PID controller for a process control we may use the equivalent fuzzy PID controller in its place to control the process with better control quality criteria.

Based on the above notice the method may be used also for tuning the fuzzy PID controllers in a control system.

The paper uses the term of "pseudo-equivalence", because there is no direct equivalence between the nonlinear digital fuzzy PID controllers, with linear gain only in the origin, and a linear analogue PI controller.

Using a correction of the fuzzy block a quasi-fuzzy controller results, which assures stability of fuzzy control system [12].

Anti-wind-up circuit may be used for the integrator component I the same way as at the linear PID controllers.

The theory presented in this paper is used and proved by the author in practical control applications [11] and stability analysis of these fuzzy control systems [12].

Using this equivalence method we may obtain fuzzy controllers which used in control systems assure better quality criteria and a higher robustness of the control systems [11].

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