Advanced Algorithms for Coupled and Inverse Problems in Electrical Engineering

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Abstract: – This paper presents some theoretical and numerical problems that arise in the analysis of coupled electromagnetic-thermal problems, and inverse problems in electromagnetic devices.

The principal objective of the paper is to describe some computational aspects for coupled electromagnetic and thermal fields in the context of the finite element method, with emphasis on the reduction of the computing resources. We present coupled models for magnetic field and thermal field. The mathematical model for magnetic field is based on time-harmonic Maxwell equations in vector magnetic potential formulation for axisymmetric fields. The model for the heat transfer is the heat conduction equation.

We propose simplified numerical models for coupled fields in electromagnetic devices with target examples on the induction heating devices and high-voltage and large power cables. Domain decomposition is presented in the context of the coupled fields. The analysis domain is divided into two overlapping subdomains for the two coupled-fields considering physical significance of the pseudo-boundary of the two subdomains.

Key-Words: - Coupled fields; Inverse problems; Finite element method.

1 Introduction

The phenomena in the technical devices are not isolated but they were analysed independently because of some justified motivations. Ones of them are:

- **limited computational power** of the conventional computers
- the **complexity** of the **coupled problems**
- the **lack** of a **strong co-operation** between the engineers and mathematicians

There are **two standpoints**, which are not in contradiction, but they are linked. The former is the **mathematician's standpoint** that tries to prove that the problem has a solution and preferably a unique solution. The latter is the **engineer's standpoint** that wants the solution, and in practical cases an approximate solution. An engineer is concerned with large-scale physical achievement. We must not forget that each category is judged by different measures for their activities: a mathematician is judged for his publications in his area, and an engineer is judged by his physical achievements.

It is true that mathematics is with a step before the engineering, that is, sometimes, there are many years or decades between the mathematical researches and the application in the engineering. One of the motivations is limited technology for implementation of the mathematics results in practice. Now the time intervals are reduced. Are we clever? In my opinion, the answer is NO. We have more knowledge, we have a fast access to the information and we cooperate or we must cooperate in different disciplines. We dream more and have the tools to transform the dreams in reality.

Research engineers, that is the engineers devoted themselves to scientific research into engineering problems, use mathematics extensively. Mathematics enables the engineer to express his technical knowledge in clear and concise mathematical terms and arrange the components his knowledge in logical order. Engineering is a science so that an engineer without mathematics is a gardener without his special tools.

1.1.Motivations for advanced algorithms

It is well known that the nature is complex in its behaviour and the abstract models do not capture accurately the laws of the nature. So that we work with abstract models that try to describe the phenomena from nature and the technical devices. But it is a great mistake to think that we have perfect models of the natural phenomena. More, many numerical algorithms are not discovered so that, although we limit our discussion to our actual achievements in this area, we must dream and to seek permanently new and modern approaches for the actual problems in science, techniques and life.

Analytical solutions for the electrical engineering problems are limited to some simple applications and ignore some physical phenomena. For complex problems the accurate models are necessary and the numerical solutions are efficient approaches for an optimal design and operation.

With the advent of modern digital computers, many numerical models were developed and they become widely used in the scientific computing. We use the old algorithms and transform them for the new architectures but we must invent new algorithms having in our mind the computational power of the new computers.

The efficient design of the electromagnetic devices has resulted in more stringent specifications and a demand for optimal operation, which is very important in high-performance electrical power systems. More exacting specifications have demanded during the design stage the development of accurate methods of predicting the performance characteristics of these devices. Some of the performance indicators of concern in the design of the power devices are the electromagnetic forces, iron losses, the eddy-currents effects and the heat transfer between the component parts. Prediction of the flux densities and current densities can be used to compute forces and local heating, both of which are of a serious concern to the designer of the devices of high performance.

1.2.Motivations for coupled models

Many areas of electrical engineering require the solution of problem in which the electromagnetic field equations are coupled to other partial differential equations, such as those describing thermal field, fluid flow or stress behaviour. These phenomena are described by equations that are coupled [5]. The coupling between the fields is a natural phenomenon and only in a simplified approach the field analysis can be treated as independent problem.

In several cases, it is possible a decoupling and a cascade solution of the coupled equations. Another attractive and efficient approach of solving coupled differential equations is to consider the set as a single system. In this way a single linear algebraic system for the whole set of differential equations is obtained after discretization, and is solved to a single step. If one or more equations are non-linear, non-linear iterations of the whole system are required.

The equations of the electromagnetic fields and heat dissipation in electrical engineering are coupled because the most of the material properties are temperature dependent and the heat sources represent the effects of the electromagnetic field [5].

The thermal effects of the electromagnetic field are both desirable and undesirable phenomenon. Thus, in conducting parts of some electromagnetic devices (coils of the large-power transformers, current bars, cables conductors, conductors of the electric machines etc) the heating is an undesirable phenomenon. The heat is generated by ohmic losses of the driving currents and eddy currents induced in conducting materials. But in induction heating devices for welding the heating is a desirable phenomenon. The thermal effect of the electromagnetic field is the treatment base for many electric materials in industry [6].

1.3.Motivations for optimisation of coupled problems

In practical engineering synthesising the best engineering solution to a given design problem is of great interest. This requirement in engineering is called inverse problem and several methods have been developed for this purpose. Among them the deterministic method using design sensitivity analysis has proved to give a proper design in terms of computational efficiency.

Optimisation methods has been efficiently developed and applied to electromagnetic devices and mechanics. Unfortunately, the methods developed always deal with single systems. The reality is the coupled problems are complex because the critical design parameters are in both systems.

In the area in discussion, one of the principal criteria of performance is to control the distribution of the temperature in a device. In inverse problems the heat sources play the role of the control variable of the heat dissipation in an electromagnetic device [2].

With the terminology of the system theory, we identify two kinds of the heat sources (and commands in an inverse problem):

- **Distributed sources** (electrical currents)
- **Boundary sources** (Dirichlet condition, Neumann condition, convection and radiation)

A control of the electromagnetic devices can be done by internal commands or/and boundary commands. For the first case the commands are the heat sources (position, amplitude).

In the heating of the electromagnetic devices, the **distributed commands** are the **internal heat sources** (position, amplitude) that are represented by:

- **Ohmic losses** from **driving (source) currents**
- **Ohmic losses** from **eddy currents** induced in conducting materials of the time variable magnetic field

- **Dielectric losses** due to friction in the molecular polarisation process in solid dielectrics that form the insulation of the high-voltage apparatus
- **Hysteresis loss** in magnetic problems. It is due to magnetic domain friction in ferromagnetic materials.

The **boundary commands** can be [3]:

- **Dirichlet command**, that is, an imposed temperature on the boundary of the spatial domain
- **Neumann command** that involves an imposed flux temperature on the boundary of the spatial domain
- **Convective command** (the temperature of the ambient medium or a cooling fluid, a parameter of the cooling fluid as the speed etc)
- **Radiation commands** (the temperature of the ambient medium or other parameters that are outside the spatial domain of the field problem and influences the temperature of a device by radiation phenomenon).

2 Mathematical modelling of the electromagnetic field

A complete physical description of electromagnetic field is given by Maxwell's equations in terms of five field vectors: the magnetic field **H**, the magnetic flux density **B**, the electric field **E**, the electric field density **D**, and the current density **J**. In low-frequency formulations, the quantities satisfy Maxwell's equations [5]:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \quad (1)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}} \quad (2)$$

$$div\ B = 0 \quad (3)$$

$$div\ D = \rho_C \quad (4)$$

with ρ_c the charge density, σ – the electric conductivity, and μ the magnetic permeability. For simplicity we give up to the bold notations for vectors.

The second set of relationships, called the constitutive relations, is for linear materials:

$$B = \mu H;\ D = \varepsilon E;\ J = \sigma E$$

The B-H relationship is often required to represent non-linear materials. The current density J in Eq. (1) must represent both currents impressed from external sources and the internally-generated eddy currents.

The formulation with vector and scalar potentials has the mathematical advantage that boundary conditions are more often easily formed in potentials than in the fields themselves. The magnetic vector potential is a vector A such that the flux density B is derivable from it by the *curl* ($\nabla \times$) *operation*

The complexity of the mathematical model for electromagnetic field was one of the main reasons to find and develop new computation methods. All methods can be included in one of the following classes [5]:

- Manipulation of the equations so that some unknowns are eliminated
- Definition of some potential functions from where the field unknowns can be obtained by simple processing
- Finding of some assumptions that simplifies the computation for practical problems

The potential formulations seem attractive because of their computational advantages. One of these consists in the fact the boundary conditions are easily framed in the potentials than in the field themselves.

2.1.The eddy-currents problem

The time-varying magnetic field within a conducting material causes circulating currents to flow within the material. These currents called eddy-currents can be unwanted or desirable phenomena. Thus, the eddy-currents in electrical machines give rise to unwanted power dissipation. On the other hand the induction heating is a wanted phenomenon in industry of the metal treatment.

Industrial equipment in which the eddy currents are essentially can be included in one of the following classes:

- **long structures**, in which the electric field and the current density posses only one component
- **complex structures** in which we use models 3D

In the *long structures*, the currents are generated by an electric field applied at the terminals of the conductor or by a time-varying magnetic field linking the loop formed by the conductors. These structures belong to electric transmission network or the distribution networks (bus bars, large-power cables etc). In these problems the applied voltage of the bar or cable is known and we seek to compute the current density distribution within the conductor in order to determine some electromagnetic quantities of interest (the electrodynamic forces, mutual inductances, local heating etc).

The *complex structures* generate difficulties in simulation and computation of their characteristics although these structures possess construction simplicity. One of these structures is the device for electric heating by electromagnetic induction. In these type the applications it is necessary to compute accurately the eddy currents. If the eddy-currents distribution is non-uniform, the resulting high-temperature gradients may crack the workpiece.

The problems are different in the two different types of applications but for any given application the presence of the saturable iron sheets introduces saturation phenomena and the problem becomes non-linear.

For each class we can apply general mathematical methods but it is more efficient to develop a particular algorithm for each kind of classes.

The **effects** of the eddy currents are:

- The time-varying magnetic **flux density** is **nonuniform** within the conductor. The alternating magnetic flux is concentrated toward the outside surface of the material (phenomenon known as the skin effect).
- **Power losses** are increased in the material

Eddy current computation appears in two types of problems:

- **Stationary** problems where the structures are fixed and source currents are time varying
- **Motion** problems where the field source is a coil in moving

Many practical engineering problems involve geometric shape and size invariant in one direction. Let z denote the Cartesian co-ordinate direction in which the structure is invariant in size and shape. This is the case of a **plane-parallel field** or **translational field** problem, where A has one component, namely A_z. It is independent of the z co-ordinate and the Coulomb Gauge is automatically imposed and V is independent of x and y. In such a case both the magnetic vector potential and the source current J_S reduce to a single component oriented entirely in the axial direction and vary only with the co-ordinates x and y. Consequently, the component A_z (for simplicity we give up the subscript z) satisfies the diffusion equation in fixed domains [5]:

$$\nabla(\nu\nabla A) - \sigma\frac{\partial A}{\partial t} = -J_S \qquad (5)$$

or in Cartesian co-ordinates:

$$\frac{\partial}{\partial x}(\upsilon\frac{\partial A}{\partial x}) + \frac{\partial}{\partial y}(\upsilon\frac{\partial A}{\partial y}) - \sigma\frac{\partial A}{\partial t} = -J_S \qquad (6)$$

The boundary conditions are set-up for the single component A and can be Dirichlet and/or Neumann's condition. The interface conditions between two materials with different properties are defined by the following equations:

$$A1 = A2; \quad v1\frac{\partial A1}{\partial n} = v2\frac{\partial A2}{\partial n}$$

2.2. Modelling of time-dependent fields

The time dependent electromagnetic field problems are usually solved using differential models of diffusion type. Many practical problems of great interest in electromagnetics involve time-harmonic fields and this case will be considered in this work.

In general, computer software for time-varying problem can be classified into two classes [5]:

1. **time-domain** programs
2. **frequency-domain** programs

Time-domain programs generate a solution for a specified time interval at different time moments. Frequency-domain programs solve a problem at one or more fixed frequencies.

The first class has some disadvantages. One of these consists in the large amount of data that must be stored to recover the field behaviour. Although the second class has an essential advantage (a compact and a cheap program in terms of the computer resources), the area of problems that can be solved is limited. It is applicable only to linear problems (all phenomena are sinusoidal).

The usual mathematical model for time dependent electromagnetic field problems is with Maxwell's equations in their normal differential form. For low frequency the displacement current term in Maxwell's equations can be neglected. At a surface of a conducting material the normal component of current density J_n can be assumed to be zero.

In 2D problems, there are two limiting cases:

1. A formulation with H field
2. A formulation with magnetic vector potential

Both cases are PDEs of the diffusion type. More, the latter case is of greater practical interest because can be solved by numerical methods.

In general the time dependent problems after a spatial discretization can lead to a lumped-parameter model. For example, Maxwell's equations in differential form for low frequency in 2-D case, after spatial discretization, lead to a system of ordinary differential equations by the form [5]:

$$[S]\left\{\frac{\partial A}{\partial t}\right\} + [R]\{A\} + \{b\} = 0 \qquad (7)$$

where [R] and [S] are matrices and b is the vector of the free terms.

To simplify the computation, one approach is to separate the spatial domain of the problem in conducting and non-conducting parts, such that A_1 is the solution vector in conducting regions and A_2 is the solution vector in the non-conducting regions. By reordering the matrices, the system of equations is divided in two systems [5]:

$$[R_{11}]\{A_1\} + [R_{12}]\{A_2\} + [S]\left\{\frac{\partial A_1}{\partial t}\right\} + \{b_1\} = 0 \qquad (8)$$

$$[R_{21}]\{A_1\} + [R_{22}]\{A_2\} + \{b_2\} = 0 \qquad (9)$$

The system (9) is formed of algebraic equations; the system (8) is formed of differential equations. These systems are solved by an iterative procedure in time. The algorithm in pseudo-code has the following structure:

1. Choose the starting value for $\{A_1\}$ at t=0;
2. Compute $\{A_2\}$ from the system (9)
3. Compute $\{\partial A_1/\partial t\}$ from the system (8)
4. Compute the values of $\{A_1\}$ at $t=t+\delta t$;
5. Repeat the steps 2-4 for each time step until the final time is reached.

3 Mathematical modelling of the thermal field

The thermal field is described by the heat conduction equation [5]:

$$\frac{\partial}{\partial t}[(c\gamma)(T)\cdot T]+\nabla[-k(T)\cdot\nabla T]=q \qquad (10)$$

where: T (x, t) is the temperature in the spatial point x at the time t; point k is the tensor of thermal conductivity; γ is mass density; c is the specific heat that depends on T; q is the density of the heat sources that depends on T. In the coupled problems we use the formula:

$$q=\rho(T)\cdot J^2 \qquad (11)$$

with ρ the electrical resistivity of the material. Equation (10) is solved with boundary and initial conditions. The boundary conditions can be of different types: Dirichlet condition for a prescribed temperature on the boundary; convection condition; radiation condition, and mixed condition [5].

For many eddy-current problems the magnetic flux penetration into a conductor without internal sources of the magnetic field is confined mainly to surface layer. This is the skin effect. The skin depth δ depends on the material properties μ, ω and σ so that for the small depths all of the effects of the magnetic field is confined to a surface layer.

In steady-state low-frequency eddy current problems in magnetic materials, the mathematical model is the diffusion equation (6).

The skin effect can be exploited in two directions:

- To reduce the space domain in analysis with a fine mesh close to conductor surfaces
- To reduce the material volume since a significant proportion of the conductor is virtually unused

The penetration depth is given by the formula:

$$\delta=\sqrt{\frac{2}{\omega\sigma\mu}} \qquad (12)$$

For example, in a semi-infinite slab of conductor with an externally applied uniform alternating field, parallel to the slab, the amplitude of flux decays exponentially. In other words for problems with the skin depth very small all the effect of the field is confined to a surface layer. In a numerical model based on finite element method (FEM) this effect can be exploited by the use of a special boundary condition, known as the surface impedance condition. In this way we don't waste run-time of a program based on FEM.

Designer engineers use the formula (12) considering the permeability and the conductivity as numbers. In reality the two physical parameters change during heating. The changes in the value of δ affect the loss in the material and depend on the process (conduction or induction). For example, if the conductivity decreases by x, the depth depends on √x; that is, the current penetrates deeper into the metal. If the magnetic material heats, its resistivity (the inverse of the conductivity) rises but its relative permeability remains substantially constant up to the Curie point. In this point it drops suddenly to unit.

Another simplifying assumption for the designer engineers is based on that all heat enters at the surface of the conductor. In reality, this is only true if the frequency of the magnetic field source is very high and the depth of heating is small compared with the geometrical dimensions of the conductor.

For an accurate computation of the penetration depth of the magnetic field we must consider two practical conditions:

- The heat is distributed in the conducting part
- There is an important heat lost by radiation at the conductor surface

Radiation can be regarded as a simple surface loss subtracting from the surface power input. The Stefan-Boltzmann law gives the radiation loss. If the body is radiating to a surface at absolute temperature T_∞ Kelvin, the radiation loss is defined by:

$$P_r=\varepsilon_r C_0(T^4-T_\infty^4)$$

where ε_r is the emissivity coefficient of the surface (dimensionless) and T is the absolute surface temperature in Kelvin (K). The constant C_0 is $5.67.10^{-8}$ W/m^2K^4. For low temperatures the radiation loss is negligible but in the induction-heating device it must be considered.

Consequently, it is convenient to use coupled models and accurate methods for computation of the heat penetration in the conductors, especially in the induction heating devices.

4 Iterative algorithms for coupled problem

A complete mathematical model for coupled fields involves Maxwell's equations and the heat conduction equation. Combining these equations yields a coupled system of non-linear equations. In a discrete form the unknowns are the nodal values of the temperature T and the magnetic vector potential A.

For electromagnetic field we consider the A-formulation, that is we define the magnetic vector potential A by B = *curl* A. More, the domain is the same for temperature and the electromagnetic field although in practice the interest is for different field domains.

The non-linear equations for T and A are straightforwardly obtained by a Galerkin's finite element method. For the 2D steady-state problems we do the approximations at the element level [8]:

$$T(x,y) = \sum_{j=1}^{r} N_j(x,y)T_j$$

$$A(x,y) = \sum_{j=1}^{r} N_j(x,y)A_j$$

where the interpolation functions N_j are basis functions in the mesh over Ω, and r is the number of nodes of an element.

The usual procedure for the FEM applications leads to a system of *2p* equations where p is the total number of the unknowns in each field problem. These non-linear equations can be solved by two different basic strategies [9]:

- Solving the equations for T_i and A_i simultaneously
- Solving the equations for the two fields in sequence with an outer iteration, technique known as operator-splitting technique (for example Newton-Raphson procedure)

In the area of the first strategy, Gauss-Seidel and Jacobi methods are well known. We present these methods in brief [2]. For this, let us define the two discrete equations derived from the electromagnetic field model and the thermal field model in the form:

$$f_A(A_1,...,A_p,T_1,...,T_p) = 0$$

$$f_T(A_1,...,A_p,T_1,...,T_p) = 0$$

where the subscript denotes the original problem (A – for the magnetic field in the magnetic vector potential formulation, and T – for the thermal field).

The Gauss-Seidel algorithm for coupled fields has the following pseudo-code [9]:

- *For m:=1 , 2, … until* **convergence** *DO*
- *Solve*

$$f_A(A_1^{(m)},...,A_p^{(m)};T_1^{(m-1)},...,T_p^{(m-1)}) = 0$$

with respect to $A_1^{(m)}, \dots A_p^{(m)}$

- *Solve*

$$f_T(A_1^{(m)},...,A_p^{(m)};T_1^{(m)},...,T_p^{(m)}) = 0$$

with respect to $T_1^{(m)}, \dots T_p^{(m)}$

In other words, the system is solved firstly with respect to A, using the values of T from the previous iteration. Afterwards, the equation derived from the thermal field model is solved using the computed values of A from the current iteration. The equations $f_A=0$ or/and $f_T=0$ are non-linear and must be solved by an iterative procedure (for example Newton-Raphson method).

The algorithm Jacobi-type is similar to Gauss-Seidel method, except that at the iteration m when we must solve the model for T, the values for A are from the previous iteration, that is $A^{(m-1)}$. The algorithm has the following pseudo-code:

- *For m:=1 , 2, … until* **convergence** *DO*
  - *Solve*

$$f_A(A_1^{(m)},...,A_p^{(m)};T_1^{(m-1)},...,T_p^{(m-1)}) = 0$$

with respect to $A_1^{(m)}, \dots A_p^{(m)}$

  - *Solve*

$$f_T(A_1^{(m-1)},...,A_p^{(m-1)};T_1^{(m)},...,T_p^{(m)}) = 0$$

with respect to $T_1^{(m)}, \dots T_p^{(m)}$

The domain decomposition method is the best among three possible decomposition strategies for the parallel solution of PDEs, namely, operator decomposition, function-space decomposition and domain decomposition [10]. This is one of the motivations to present the principles of the domain decomposition methods in this section.

The domain decomposition could be determined from mathematical properties of the problem (real boundaries or interfaces between subdomains), or from the geometry of the problem (pseudo-boundaries). For elliptic partial differential equations, there exists a mathematical approach based on the ideas given earlier in 1890 by Schwarz [1]. In Schwarz procedure there is an inherent parallelism with a data communication time for the passage of pseudo-boundary data between processors.

There is no general rule for the domain or/and operator decomposition. It is defined in a somewhat random fashion. The problems and questions that appear in the decomposition technique are:

- do *domain decomposition* or the *operator* decomposition
- Which approach is the best: *disjoint* or *overlapping* sub-domains?
- What *kinds of boundary conditions* are set up on the pseudo-boundaries of the sub-domains

- What kind of domain decomposition is useful for a particular problem: *static* or *dynamic* decomposition?

4.1 Decomposition techniques

The desire of the scientific community for faster processing on lager amounts of data has driven the computing field to a number of new approaches in this area [10]. The main trend in the last decades has been toward advanced computers that can execute operations simultaneously, called parallel computers. For these new architectures, new algorithms must be developed and the domain decomposition techniques are powerful iterative methods that are promising for parallel computation. Ideal numerical models are those that can be divided into independent tasks, each of which can be executed independently on a processor. Obviously, it is impossible to define totally independent tasks because the tasks are so inter-coupled that it is not known how to break them apart. However, algorithmic skeletons were developed in this direction that enables the problem to be decomposed among different processors. The mathematical relationship between the computed sub-domain solutions and the global solution is difficult to be defined in a general approach.

In the area of the coupled fields we define two levels of decomposition, that is we define a hierarchy of the decompositions:

- One at the level of the problem
- The other at the level of the field

In other words, we decompose the coupled problem in two sub-problems: an electromagnetic problem and a thermal problem, each of them with disjoint or overlapping spatial domains. This is the first level of decomposition. At the next level, we decompose each field domain in two or more subdomains. The decomposition is guided both by the different physical properties of the materials, and the difference of the mathematical models. At this level of decomposition the Steklov-Poincaré operator can be associated with field problem [10]. This operator reduces the solution of the coupled subdomains to the solution of an equation involving only the interface values. One efficient and practical solution of elliptical partial differential equations is the dual Schur complement method [10].

5 Induction heating

As target example we consider a long cylindrical workpiece excited by a close-coupled axial coil (figure 1). The problem is an axisymmetric heating device. An axial section is presented in the figure 2 with 1- the workpiece, 2 – the air and 3 – the coil. The coil is assimilated with a massive conductor. In this case we can not ignore the eddy currents in the coil.

We consider a low-frequency current in the coil so that the penetration depth is large. We can decompose the whole domain of the field problem into *overlapped subdomains* for the two coupled-fields.

The domain for the magnetic field is the whole device bounded by a boundary at a finite distance from the device. For the thermal field we consider the workpiece as the analysis domain. The penetration depth of the magnetic field in the workpiece imposes the overlapping. The radiation plays an important role in induction heating at high temperature. Convection losses are small in through-heating, as the workpiece is contained in a shell which dos not permit air movement. In the case the workpieces are in the open air, the convection losses are very important.

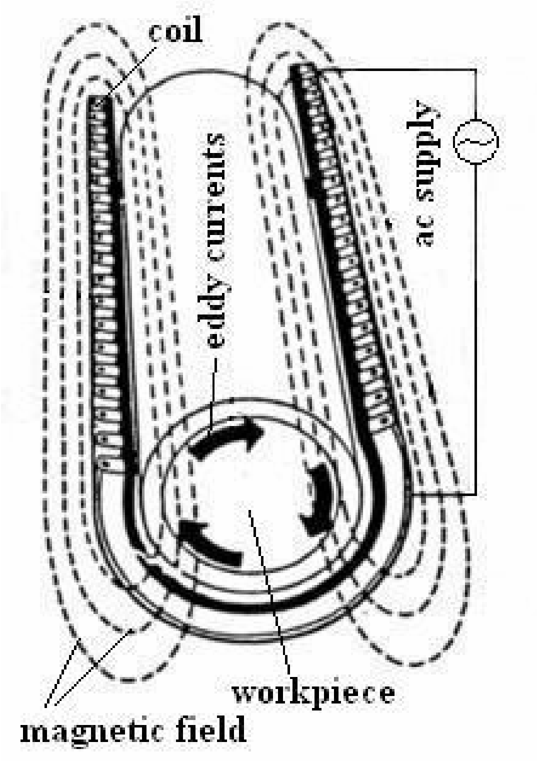

Fig.1 - *Device for induction heating*

In Fig.2 an axial section is presented. The coil is assimilated with a massive conductor. In this case we can not ignore the eddy currents in the coil. We consider a low-frequency current in the coil so that the penetration depth is large. In this case we can decompose the whole domain of the field problem into *overlapped subdomains* for the two coupled-fields.

The domain for the magnetic field is shown in the Fig. 3, that is a quarter of the device bounded by a boundary at a finite distance from the device. For the thermal field we consider the workpiece as the analysis domain. The penetration depth of the magnetic field in the workpiece imposes the overlapping domains for the two fields [7]. The numerical model is considered in a cylindrical co-ordinates with the vertical axis Or and the horizontal axis Oz.

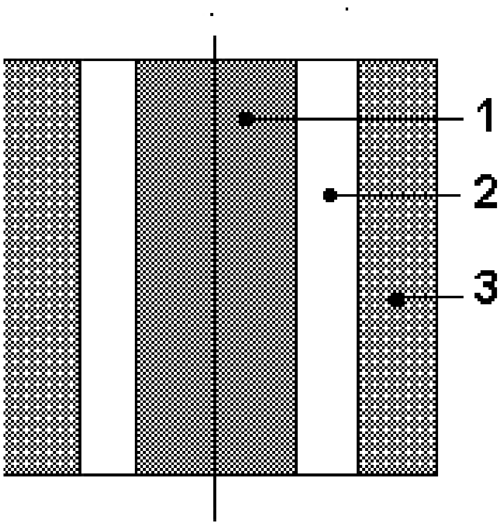

Fig. 2 – *Axial section*

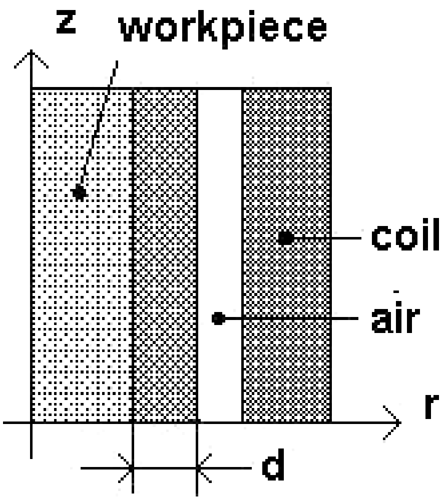

Fig. 3 – *The analysis domain*

A complete mathematical model for coupled fields involves Maxwell's equations and the heat conduction equation. Combining these equations yields a coupled system of non-linear equations. In a discrete form the unknowns are the nodal values of the temperature T and the magnetic vector potential A.

For electromagnetic field we considered the A-formulation, that is we defined the magnetic vector potential A by B = *curl* A. More, the domain was the same for temperature and the electromagnetic field although in practice the interest is for different field domains.

5.1. Numerical results

We can distinguish two practical cases: low frequency and high frequency. At high frequency the domain for the magnetic field can be reduced: a part of the workpiece is not penetrated by the magnetic field. The domain is the coil, the air and a layer of the thick. For the thermal field the analysis domain is the workpiece.

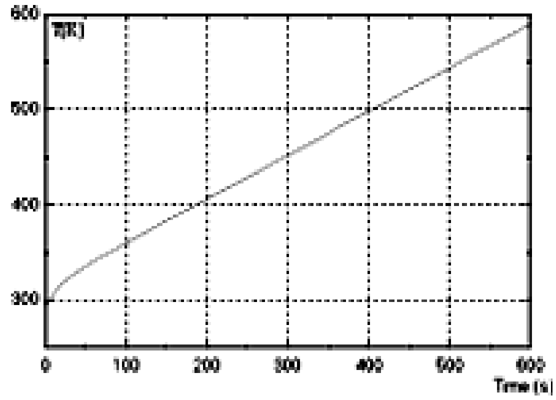

Fig. 4 – *Temperature vs. time in coil*

The radiation plays an important role in induction heating at high temperature. Convection losses are small in through-heating, as the workpiece is contained in a shell that does not permit air movement. In the case the workpiece is in the open air, the convection losses are very important.

The finite element method was used for the numerical results [8]. In Fig. 4 the temperature versus time in a point on the internal surface of the coil is plotted using the program Quickfield [11]. The initial temperature was 20^0C (293.15 K). The workpiece is a steel cylinder and the coil material is copper. The current intensity is 60000 [A] and the time duration is 600 [s]. We considered both forced convection and radiation conditions. The convection coefficient is 50 for the forced convection and radiation coefficient is 0.8.

In a numerical solution of the mathematical models for coupled problems, we determine an approximate solution for the unknown function at a finite number of discrete points in the domain. The finite element method (FEM) is presented in a large professional literature so that we do not discuss it.

6 Inverse problems

In this section we present some computational aspects for optimal control of the heat transfer in solids, both for single system and coupled systems. For the single system we consider the case of the conduction heat transfer using as mathematical model the heat equation in space 2D. The functional cost (objective function) is a quadratic form [4].

6.1.Optimal control by distributed-commands

The general class of the problems dealt with this paper is governed by the following differential equation [5]:

$$\frac{\partial}{\partial x}(k_x\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(k_y\frac{\partial u}{\partial y})+h+f=0 \qquad (13)$$

with boundary conditions:

$$u(x,y)|_{C1}=u0(x,y) \qquad (14)$$

$$[\frac{\partial u}{\partial n}+q]\,|\,C2=0 \qquad (15)$$

$$[\frac{\partial u}{\partial n}+\alpha(u-v)]\,|\,C3=0 \qquad (16)$$

$$[\frac{\partial u}{\partial n}+g(u,v)]\,|\,C4=0 \qquad (17)$$

where: u (x,y) is the temperature in the domain Ω C R^2; C =C_1 U C_2 U C_3 U C_4 is the boundary of the domain; h is a known function representing internal heat generation and f is the command (an unknown function). In (14) u_0 is a known function (Dirichlet condition) and in (15) we have a Neumann condition with q -the flux on the boundary. On the boundary C_3 we have a convective condition (16) with α the convection coefficient and v the ambient temperature. On the boundary C_4 we have a mixed-condition (as for example a convection and radiation condition), with g a known function. In (13) k_x , k_y, are the thermal conductivities in the directions of the axes of the co-ordinates system Oxy. In conditions (15)-(17), ∂/∂n is the directional derivative normal to the boundary C.

We consider a functional cost by the form [3]

$$J(w)=c_0\int_\Omega (u-u_D)^2\,dx\,dy \qquad (18)$$

with c_0 - a given positive coefficient and u_D - an imposed internal temperature distribution. This functional penalises the deviation of the temperature from an imposed (desired) distribution.

The problem of the optimal control consists in the minimisation of the functional (18), that is we seek a command f^* in F (an admissible set) such that [7]:

$$J(f^*)\le J(f);\quad \forall f\in F$$

in the conditions (13)-(17). Frequently, the set of admissible commands is by the form:

$$F=\{f\in L_2(\Omega):f_{\min}\le f\le f_{\max}\}$$

Two practical cases appear:

1.the positions (x_i, y_i) of the distributed sources are known and the intensities f_i of these sources are required, that is the command function has the form:

$$f_i=\sum_{i=1}^{n} f_i\delta(x-x_i)\delta(y-y_i)$$

with δ the Dirac's function.

2.the intensities f_i are known and the positions are required.

The first case is simpler than the second case because it doesn't involve geometrical parameters in the design of the device.

6.1.1. Necessary conditions for optimality

We transform the constrained optimal control problem into an unconstrained problem through the introduction of adjoint function Φ. We define the augmented cost-functional by [3]:

$$L=J(f)+\int_\Omega \Phi(x,y)[\frac{\partial}{\partial x}(k_x\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(k_y\frac{\partial u}{\partial y}) \qquad (19)$$
$$+h+f]dx\,dy$$

Necessary conditions for optimality are derived by a variational approach. It is considered a variation δf in the command f that introduces a variation δL. From the first variation of L, results the adjoint equation [2]:

$$\frac{\partial}{\partial x}(k_x\frac{\partial \Phi}{\partial x})+\frac{\partial}{\partial y}(k_y\frac{\partial \Phi}{\partial y})+2c0(u-u_D)=0 \qquad (20)$$

$$J'(f)=\phi \qquad (21)$$

with boundary conditions:

$$[\frac{\partial \Phi}{\partial n}+\Phi\frac{\partial g}{\partial u}]|_{C4}=0 \qquad (22)$$

$$[\frac{\partial \Phi}{\partial n}]|_{C2}=0 \qquad (23)$$

$$[\frac{\partial \Phi}{\partial n}+\alpha\Phi]|_{C3}=0 \qquad (24)$$

To obtain the optimal command f^* (practically, the method of gradient projection), the algorithm proceeds as follows:

1. make an initial guess of the command f_0 and set the iterations counter n to zero;
2. solve the state equation (13) with conditions (14)-(17);
3. solve the adjoint equation (20) with conditions (21)-(24);
4. compute the new command:
   - $f_{n+1}=f_n-s.J'(f_n)$ (25)

   with s the length of the step in the antigradient direction.
5. repeat the steps 2^0-4^0 until subsequent changes in J are less than a pre-set criterion.

The length of the step s is determined by a one-dimensional search technique [7] using the value of the cost functional.

6.2. Optimal control by boundary commands

The general class of the problems dealt with this paper is governed by the following differential equation:

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial u}{\partial y}\right)+f=0 \qquad (26)$$

with specified boundary conditions. In (26) f is a known function that represents internal heat sources-the Joule Lenz effect and eddy-current losses.

The boundary conditions are:

$$u(x,y)|_{C1}=u0(x,y) \qquad (27)$$

$$\left(\frac{\partial u}{\partial n}+q\right)|_{C2}=0 \qquad (28)$$

$$\left[\frac{\partial u}{\partial n}+\alpha(u-w)\right]|_{C3}=0 \qquad (29)$$

$$\left[\frac{\partial u}{\partial n}+g(u,w)\right]|_{C4}=0 \qquad (30)$$

where: u (x, y) is the temperature in the domain Ω C R^2 and C =C_1 U C_2 U C_3 U C_4 is the boundary of the domain. In (27) u_0 is a known function (Dirichlet condition) and in (28) we have a Neumann condition with q -the flux on the boundary. On the boundary C_3 we have a convective condition (29) with α the convection coefficient and w the ambient temperature. On the boundary C_4 we have a mixed-condition (as for example a convection and radiation condition), with g a known function. In (26) k_x and k_y are the thermal conductivities in the directions of the axes of the co-ordinates system Oxy. In conditions (28)-(30), ∂/∂n is the directional derivative normal to the boundary C.

The mathematical model of the heat equation in space 2D, also is met in axisymmetric field, where the equation (26) becomes:

$$\frac{\partial}{\partial r}\left(k_r r\frac{\partial u}{\partial r}\right)+\frac{\partial}{\partial z}\left(k_z r\frac{\partial u}{\partial z}\right)+rf=0 \qquad (31)$$

In a convective control, w can be chosen as a command variable. We consider a functional cost by the form:

$$J(w)=c_0\int_\Omega (u-u_D)dxdy \qquad (32)$$

with: c_0 - a given positive coefficient; u_D-an imposed internal temperature distribution.

The functional cost has a practical significance: it penalises the deviations of the temperature in the domain from the imposed standard (u_D). On the boundary C_3 U C_4 we apply a command w ε L^2 (C) - the space of the integrable-squared functions, with g a known function. The boundary command w can be the temperature of the cooling medium that is we have a convective control like in (29) where the coefficient α is supposed constant or depends by the boundary temperature. In another practical case, the command w is the speed of the cooling medium (like in the oil-immersed transformer), and g has the form g (u, w)=α(w)(u-u_∞), where u_∞ is the temperature of the cooling medium (supposed a constant). The dependence of α by w must be known but unfortunately this is a difficult task. It is determined from experimental data and is expressed using nondimensional parameters as Nusselt and Reynolds numbers.

The problem of the optimal control consists in the minimisation of the functional (32), that is we seek a command w^* ε W (an admissible set) such that:

$$J(w^*)\le J(w) \quad \forall w\in W \qquad (33)$$

in the condition (26), with specified boundary conditions (27)-(30).

Frequently, the set of admissible commands is by the form [7]:

$$W=\{w\in L_2(\Omega): w_{\min}\le w\le w_{\max}\} \qquad (34)$$

6.2.1.Necessary conditions for optimality

We transform the constrained optimal control problem into an unconstrained problem through the introduction of adjoint function Φ. We define the augmented cost-functional by [3]:

$$L=J(w)+\int_\Omega \Phi(x,y)\left[\frac{\partial}{\partial x}\left(k_x\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial u}{\partial y}\right)+f\right]dxdy \qquad (35)$$

Necessary conditions for optimality are derived by a variational approach. It is considered a variation δw in the command w that introduces a variation δL. From the first variation of L, results the adjoint equation [5]:

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial \Phi}{\partial y}\right)+2c_0(u-u_D)=0 \qquad (36)$$

with boundary conditions:

$$\frac{\partial\Phi}{\partial n}|_{C2}=0$$
$$\left[\frac{\partial\Phi}{\partial n}+\alpha\Phi\right]|_{C3}=0 \qquad (37)$$
$$\left[\frac{\partial\Phi}{\partial n}+\Phi\frac{\partial g}{\partial u}\right]|_{C4}=0$$

The gradient of the cost-functional is:

$$J'(w) = -\Phi \frac{\partial g}{\partial w}\Big|_{C4} \quad (38)$$

The gradient method [4] can be employed to obtain the optimal command w^* (or the method of gradient projection for the constrained problem).

6.2.2. A numerical model

For obtaining the optimal command w^*, the gradient method can be used with good results, especially for the unconstrained commands. For this case the gradient method proceeds as follows [4]:

- *make an initial guess of the command w_0, and set the iterations counter to zero;*
- *solve the state equation (26) with conditions (27)-(30);*
- *solve the adjoint equation (36) with the conditions (37);*
- *compute the new command:*

$$w_{n+1} = w_n - s.J'(w_n) \quad (39)$$

- *repeat the steps 2^0-4^0 until subsequent changes in J are less than a preset criterion.*

The length of the step s is determined by a one-dimensional search technique. Recent developments allow replacing the step length rule by a trust region method. In the application program developed by the authors, it was used the following rule: an initial value for s is chosen and the functional-cost is calculated and if its value isn't less than the old value, the length of the step is divided to two and this procedure continues until the monotony of the functional is satisfied. The disadvantage of this rule is that it requires an iterative method to determine s at each iteration. The steps 2^0 and 3^0 of the algorithm imply the solution of the state and adjoint equations. The finite element method was used to obtain approximate solutions in finite dimensional subspace.

Finally, by assembling the element equations, results an algebraic equations system. The adjoint equation (36) and cost-functional are discretized in the same manner.

6.3. Optimal control of the heat in electrical cables by boundary commands

As target examples we consider an infinitely long coaxial cable with a stranded inner conductor carrying the direct current. This problem can be treated as a two-dimensional problem. The current density is a constant and this assumption is valid in the analysis and synthesis of electrical devices where the current density J is a specified constant in conductors and zero elsewhere. This inherent approximation becomes more and more valid as we use smaller and smaller triangles. In the alternating current, the skin effect appears but in the most practical systems the conductor is stranded (that is made up several tightly wound strands of conductor insulated from each other) so as to force the currents to flow through the entire cross section of the conductor. In this way we utilise the material better. Hence the validity of assuming uniform density as in direct current systems can simplify the computation. This assumption can lead at some practical applications. For such a system it has seen that the governing equation is (26). With the origin of the co-ordinates system in the centre of the cable, only a part of the entire domain is used. The convective command w is applied on the shield of the cable. The functional cost is by the form (32). We considered an averaged value of the gradient so that we can obtain a sub-optimal command.

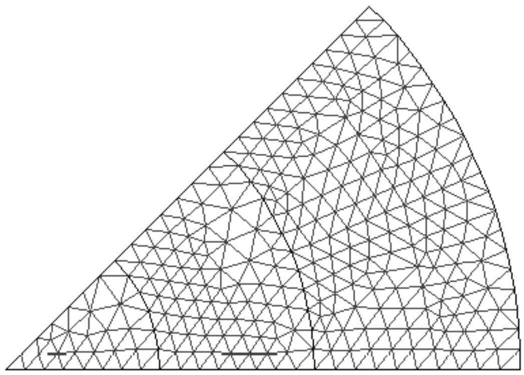

Fig. 5 – *Analysis domain and mesh*

In this target example we consider a coaxial cable with a nonuniform current density and two insulation layers. In the figure 1 the analysis domain is presented. The geometrical dimensions are: conductor radius is 15 mm, the outer radius of the first layer is 30 mm and the outer radius of the second layer is 50 mm. The resistivity of the copper was considered at the temperature 75 ^{0}C and equal to 1.78 .10^{-8} Ω/m. The physical properties are the following: thermal conductivities k_x=k_y=385 W/m.^{0}C in the copper and equal to k1=0.14 W/m.^{0}C and k2=0.175 W/m.^{0}C in the insulation layers; α=12 W/m^2.^{0}C .The current density is 5.0.10^{-5} A/mm^2. The minimum value of J (w) was found to be equal to 5.604 for c_0=0.0001. The number of iterations was 181 with the initial value of the command equal to 40^{0}C. The optimal command is 63.95 ^{0}C for u_D=75 ^{0}C.

In numerical simulation it is considered a medium value of the gradient on the boundary, that is in the formula (39) the command w, at each iteration step, is a constant (a frequent case in industry where we consider an average value of the command variable).

Any case may be treated in the same manner (for example, a piecewise command or a local command). It is a sub-optimal control but the accuracy of the solution is acceptable from the engineer's viewpoint.

7 Conclusions

The problem of coupled fields and inverse problems in electrical engineering is a complex problem in terms of computing resources. In practice the coupled fields are treated independently in some simplified assumptions. The accuracy of the numerical computation is poor. With the new architectures, a multidisciplinary research is possible. Some iterative procedures were presented with emphasis on the coupled problems and inverse problems.

In inverse problems we used a simple gradient technique. In the optimisation problems with restrictions we can use other techniques from the automatic control theory. In some previous works we used the method of the gradient projection. More, we can use a mixed control, that is, the boundary and distributed commands can be used.

In many practical applications the optimisation can be done with respect a parameter. This parameter can be a physical or geometrical parameter. Usually the physical parameter is a material property as the conductivity, permitivitty etc. The physical parameter is the device shape. The latter is difficult because it involves the mesh reconstruction at each step of the optimisation algorithm. In our future research we develop software for shape optimisation of the electromagnetic devices.

Domain decomposition offers an efficient approach for large-scale problems or complex geometrical configurations ([1],[10]). This method in the context of the finite element programs leads to a substantial reduction of the computing resources as the time of the processor.

In coupled problems a hierarchy of decomposition can be defined with a substantial reduction of the computation complexity.

Inverse problems in electrical engineering are complex problems that involve large resources in terms of computing. We presented some computational aspects with emphasis on simple gradient techniques. In our future research we shall develop algorithms based on gradient projection [5] and the second-order derivative of the functional cost. It is obviously that the algorithm complexity is increased by the accuracy of the optimisation solution is improved.

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