Optimal Adaptive Fuzzy Control for a Class of Unknown Nonlinear Systems

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Abstract: - In this paper, we present an optimal adaptive fuzzy controller for a class of nonlinear systems with unknown nonlinear dynamics. The strategy of control is based on the fuzzy systems and linear matrix inequalities (LMI). The adaptive fuzzy logic system is used to approximate the unknown dynamics; we exploit the linear structure of a Takagi-Sugeno fuzzy system with constant conclusion to design an indirect adaptive fuzzy controller. The auxiliary compensation control is added to attenuate the influence of external disturbances and to remove fuzzy approximation error. The design of the auxiliary control relies on LMI and the Lyapunov technique for achieving stability and specified performance. The adaptation laws of the adjustable parameters are deduced from the stability analysis, in the sense of Lyapunov, to get a more accurate approximation level. Simulation results are given to illustrate the tracking performance.

Key-Words: - Nonlinear systems, Adaptive control, Fuzzy systems, Takagi-Sugeno Fuzzy Model, LMI, Lyapunov Approach.

1 Introduction

Traditionally, control system design has been tackled using mathematical models derived from physical laws. In fact, most of the parameters and structure of the system are unknown due to environment changes, modelling error and unmodelled dynamics. To overcome the above problems, several techniques, in particular, the intelligent technologies as neural networks, fuzzy logic, and genetics algorithm have been developed [2], [9], [12].

In the past few years, fuzzy control of nonlinear systems has been implemented successfully in many applications. In most of these applications, the so called Takagi-Sugeno (T-S) type fuzzy model is used to represent a nonlinear system [15]; then based on this model, a fuzzy controller is designed. Fuzzy logic, as one of the most useful approaches for utilizing expert knowledge, has been an active filed of research during the past decade [6], [11]. Fuzzy logic control has found promising applications for a wide variety of industrial systems specifically applicable to plants that are mathematically poorly modelled [10]. Based on the universal approximation capability, many effective adaptive fuzzy control schemes have been developed to incorporate with human expert knowledge information in a systematic way, which can also guarantee stability and performance criteria [7], [9], [12].

There have been many successful applications in fuzzy control in recent years. In spite of the success, there are still many basic issues that remain to be further addressed. Stability analysis and systematic design are certainly among the most important issues for fuzzy control systems [14],[16].

Some research has been focused on the Lyapunov synthesis approach to construct stable adaptive fuzzy controllers. The design procedure aims at rendering stable fuzzy controllers. More significantly, the stability analysis and control design problems and reduced to LMI problem [18], [14]. Numerically, the LMI problems can be solved very efficiently by means of some of the most powerful tools available to date in the mathematical programming literature. Therefore, recasting the stability analysis and control design problems as LMI problems is equivalent to finding solutions to original problems.

In this respect, several direct and indirect adaptive fuzzy control schemes have been introduced for controlling nonlinear systems [5], [9], [11]. In the direct scheme, the fuzzy system is used to approximate an unknown ideal controller. On the

other hand, the indirect scheme uses fuzzy systems to estimate the plant dynamics and then synthesizes a control law based on these estimates. In these adaptive fuzzy control schemes, the controllers are generally composed of two main components.

The ability of converting linguistic descriptions into automatic control strategy makes it a practical and promising alternative to the classical control scheme for achieving control of complex nonlinear systems. Recently, a great amount of the effort has been devoted to describing nonlinear system using a T-S fuzzy model [20].

Feedback linearization based on adaptive control is suitable for the control of nonlinear systems with accurate nominal models or linearly parametrical dynamical models. However, due to modelling errors, these controls may not be very effective without proper compensation to over came the modelling error effects [1], [4].

The basic idea of most of these works is that with the universal approximation ability of fuzzy systems, the systems uncertainties can be represented by linearly parameterized uncertainties so, that the standard parametric adaptive techniques can be utilized.

First, we approximate this class of uncertain nonlinear systems by T-S fuzzy model. Then base on an LMI approach, we develop a technique for designing robust H_∞ fuzzy state feedback and output feedback controllers such that gain of the mapping from the exogenous input noise to the regulated output is less than a presented value [19]. Thus, it is also very important to study the robust control against parameter uncertainties in the T-S fuzzy control systems.

The apparent similarities between sliding mode control and fuzzy control motivate considerable research efforts in combining the two approaches for achieving more superior performances such as overcoming some limitations of the traditional sliding mode control [3].

Many adaptive fuzzy sliding mode control schemes have been proposed and the chattering phenomena in the controlled system can be avoided by using the fuzzy sliding surface in the reaching condition of the sliding mode control [3].

In order to improve the steady performance of the adaptive fuzzy sliding mode control, an adaptive fuzzy logic controller combining a proportional plus integral (PI) controller and the sliding mode control is considered in [3].

In this paper, an optimal adaptive fuzzy control is proposed for nonlinear systems. The controller is designed for a class of nonlinear systems with unknown nonlinear dynamics.

The adaptive fuzzy model type T-S is used to approximate the unknown dynamics systems [5], [7]. The adjustable fuzzy parameters are updated on line by the adaptive algorithm. The stability and convergence analysis is ensured from the Lyapunov approach. The auxiliary compensation control is designed to attenuate the influence of external disturbances and the fuzzy approximation error. The design of this signal depends on the well-known upper bounds of both the approximation error and the external disturbances, which is a restrictive assumption due to the fact that these bonds are generally unknown.

This paper is organized as follows: The problem formulation is presented in section 2. In section 3, the optimal fuzzy control design is proposed for nonlinear systems with unknown dynamics. In Section 4, simulation examples are shown to demonstrate the effectiveness of the proposed method.

2 Problem Formulation

For a class of nth order single input and single output (SISO) nonlinear systems in continuous time domain, the dynamics equation can be express as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(\underline{x},t) + bu(t) + d(t) \\ y = x_1 \end{cases} \tag{1}$$

Or equivalently

$$\begin{cases} x^{(n)} = f(\underline{x},t) + bu(t) + d(t) \\ y = x \end{cases} \tag{2}$$

The $f(\underline{x},t)$ is an unknown nonlinear continuous function, b is a positive constant and $d(t)$ is an unknown external disturbances.

$\underline{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^T = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the state vector of the systems which is assumed to be available for measurement.

$u(t) \in R$ and $y(t) \in R$ are, respectively, the scalar control input and the scalar output of the system.

The objective of the control is to determine a control law $u(t)$ to force the system output $y(t)$ to follow a given bounded reference signal $y_r(t)$, that is to minimise the tracking error $e = y(t) - y_r(t)$

and its forward shifted values, defined as $e^{(i)} = y(t)^{(i)} - y_r(t)^{(i)}$.

Assume that the given reference y_r is bounded and have up to $(n-1)$ bounded derivatives.

The reference vector is denoted as:

$$\underline{y}_r = \left[y_r, \dot{y}_r, \dots, y_r^{(n-1)} \right]^T.$$

Then the tracking error vector is given by:

$$\underline{e} = [e, \dot{e}, \dots, e^{(n-1)}]^T \in R^n.$$

It is desired that the output error of the system follow:

$$e^{(n)} + k_{n-1}e^{(n-1)} + \cdots + k_0 e = 0$$

To ensure a good tracking, the selection of $k_i, i = 0, 2, \dots, n-1$ must satisfy the following Hurwitz polynomial:

$$S = s^{(n)} + k_{n-1}s^{(n-1)} + \cdots + k_0$$

All roots of S are situated in the left-half-complex plane.

In general, the system is unknown and the design of the control law is confronted with many problems due to environment changes, modelling errors and unmodelled dynamics.

In this work, we propose a fuzzy system to approximate the unknown function and to use the LMI and Lyapunov approach for solving the problem of stability and tracking performance.

We remark that, when the system (1) is well known, free of the external disturbances, $b \neq 0$ and to guarantee that the state $\underline{x}$ of the closed-loop system will follow the desired state $\underline{y}_r$, in other words, the tracking error will converge to zero, the control should be designed to have the following idealized control law [3]:

$$u^* = \frac{1}{b}(-f(x,t) + y_r^{(n)} - \sum_{i=0}^{n-1} k_i e^{(i)}) \qquad (3)$$

However, the control law (3) can not be implemented for large-scale system with unknown dynamics. Thus, the adaptive fuzzy system will be used to approximate the unknown dynamics.

3 Optimal Adaptive Fuzzy Control Design

The adaptive optimal fuzzy control is designed for a class of nonlinear system with unknown nonlinear dynamics. The strategy of control is based on fuzzy system, LMI and Lyapunov approach to ensure stability, tracking and consistent performance.

At first part, the dynamics system $f(\underline{x},t)$ is estimated by the adaptive fuzzy model type T-S.

The T-S fuzzy model is widely accepted as a powerful tool for design and analysis of fuzzy control systems and applications of the T-S models to various kinds of nonlinear systems can be found. The T-S fuzzy model uses smooth aggregation of local linear mathematical models to represent dynamical systems, which are useful because they can provide description of a physical phenomenon or a process, and can be well suited to analysis, prediction and design of dynamic control systems.

The fuzzy parameters can be tuned on-line by adaptive law based on Lyapunov approach.

In the second part, the auxiliary part of control is used to suppress the external disturbances and to remove fuzzy approximation error. The design of this control law is derived from LMI and the Lyapunov approach.

In the proposed design procedure, we represent a given nonlinear system by the so-called T-S fuzzy model. This fuzzy modelling method is simple and natural. The system dynamics is captured by a set of fuzzy implication which characterizes local relations in the stable state space. The main feature of T-S fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy blending of the linear system models.

Specifically, the T-S fuzzy system is described by fuzzy if-then rules, which locally represent linear input-output relations of a system. The fuzzy system is of the following form:

$$R^j : \; if \; x_1 \; is \; F_1^j \; and \, x_2 \; is \; F_2^j \dots$$

$$and \; x_n \; is \; F_n^j \; then \; \hat{f} \; is \; \theta_j \qquad (4)$$

$$j = 1, \dots, M$$

where $R^j (j = 1, \dots, M)$ denotes the j th implication, $F_i^{\,j}, i = 1, \dots, n$ are fuzzy variables characterized by membership functions $\mu_{F_i^j}(x_i)$ and θ_j is the corresponding value of the output fuzzy singleton.

The output of the fuzzy system with singleton fuzzification, product inference and centre average defuzzification can be expressed as:

$$\hat{f}(\underline{x}, \underline{\theta}) = \frac{\sum_{j=1}^{M} \theta_j (\prod_{i=1}^{n} \mu_{F_i^j}(x_i))}{\sum_{j=1}^{M} (\prod_{i=1}^{n} \mu_{F_i^j}(x_i))} \qquad (5)$$

$$= \underline{\theta}^T \xi(\underline{x})$$

where M is the total number of the fuzzy rules, $\underline{\theta} = [\theta_1, \theta_2, \dots, \theta_M]^T$ is the adjustable parameter

vector grouping all consequent parameters and $\xi(\underline{x}) = [\xi_1(x), \xi_2(x), \ldots, \xi_M(x)]^T$ is the vector of the fuzzy basis functions.

$$\xi_j(x) = (\prod_{i=1}^{n} \mu_{F_i^j}(x_i)) / \sum_{j=1}^{M} (\prod_{i=1}^{n} \mu_{F_i^j}(x_i)) \quad (6)$$

The fuzzy system (5) is assumed to be well defined so that $\sum_{j=1}^{M} (\prod_{i=1}^{n} \mu_{F_i^j}(x_i))$ for all $\underline{x} \in R^n$.

The fuzzy system (5) is a universal approximator of continuous functions over a compact set if its parameters are suitably selected.

We define the optimal parameters vectors and fuzzy approximation error as:

$$\underline{\theta}^* = \arg \min_{\theta \in \Omega_f} \left(\sup_{x \in R^n} \left| f(\underline{x}, t) - \hat{f}(\underline{x}, \underline{\theta}) \right| \right) \quad (7)$$

$$w = f(\underline{x}, t) - \hat{f}(\underline{x}, \underline{\theta}^*)$$

Ω_f is the convex compact sets, which contain feasible parameter sets for θ,

$\Omega_f = \{ \underline{\theta} \in R^n \ \|\underline{\theta}\| \le M_f \}$,

M_f is given constant.

Sufficient conditions for the stability of T-S systems were first proposed [17]. These sufficient conditions required the existence of a positive definite matrix P.

This would satisfy a set of Lyapunov inequalities.

This problem is transformed into a minimization problem subject to a LMI, which can be solved efficiently by using the existing convex optimization algorithms [13].

The controller design is based on the fuzzy system and the LMI optimization techniques.

If there exist a matrix $P > 0$, satisfying the following LMI:

$$A^T P + PA < 0 \quad (8)$$

where A is given by:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \vdots \\ \vdots & & \ddots & & \\ \vdots & & & & 1 \\ -k_0 & -k_1 & \cdots & \cdots & -k_{n-1} \end{bmatrix}$$

We can use LMI convex programming techniques to solve this stability analysis problem.

In this paper, we will make the following assumptions regarding the system (1)

Assumptions:

1) There exist a function f^u positive such that $|f(\underline{x}, t)| \le f^u$.

2) The unknown external disturbance is bounded and satisfies the condition as follows $|d(t)| \le d_{max}$ where d_{max} is upper bound.

We present a design methodology for control of a class of unknown nonlinear systems. First, we present a nonlinear plant with T-S fuzzy model. Whereas, the auxiliary control action is added to attenuate the influence of external disturbances and to remove fuzzy approximation error.

The proposed fuzzy control law is as follows:

$$u = u_c + u_r \quad (9)$$

In real system, $f(\underline{x}, t)$ is unknown. Thus, it is impossible to generate the control law (3). To overcome these difficulties, we use fuzzy system $\hat{f}(\underline{x}, \underline{\theta})$ to approximate $f(\underline{x}, t)$, we introduce a certain type of fuzzy system that is based on T-S fuzzy model to approximate nonlinear system. Then the idealized control law (3) can be approximated as follows:

$$u_c = \frac{1}{b}[-\hat{f}(\underline{x}, \underline{\theta}) + y_r^{(n)} - \sum_{i=0}^{n-1} k_i e^{(i)}] \quad (10)$$

In order to obtain a good results and tracking performance, we can use the auxiliary action control. The synthesis of this control relies on LMI and Lyapunov approach.

The auxiliary control part is given as:

$$u_r = -\frac{1}{b} sat(e^T PB) \left[|f^u| + |\hat{f}^*| + d_{max} \right] \quad (11)$$

The function $sat(\underline{e}^T PB)$ may be written as:

$$sat(\underline{e}^T PB) = \begin{cases} \text{sgn}(\underline{e}^T PB) & |\underline{e}^T PB| > \varepsilon \\ \dfrac{\underline{e}^T PB}{\varepsilon} & |\underline{e}^T PB| \le \varepsilon \end{cases} \quad (12)$$

ε is a small positive constant.

The adjustable fuzzy parameters can be tuned on-line by the adaptive law based on the Lyapunov technique. In order to guarantee that the adaptive parameters are bounded, we introduce the projection operator [10], to restrict them in the closed set Ω_f.

$$\dot{\underline{\theta}} = \begin{cases} \gamma \underline{e}^T PB\xi(\underline{x}) \ \text{if} \\ (\ \|\underline{\theta}\| < M_f \ \text{or} \ \|\underline{\theta}\| = M_f \ \text{and} \ \underline{e}^T PB\underline{\theta}^T \xi(\underline{x}) > 0) \\ \text{Pr}oj(\gamma \underline{e}^T PB\xi(\underline{x})) \qquad \text{otherwise} \end{cases} \quad (13)$$

The projection operator is given by:

$$\mathrm{Pro}j(\gamma \underline{e}^T PB\xi(\underline{x})) = \gamma \underline{e}^T PB\xi(\underline{x}) - \gamma \underline{e}^T PB\frac{\underline{\theta}\underline{\theta}^T \xi(\underline{x})}{\|\underline{\theta}\|^2}$$

where γ is fixed adaptive gain.

Theorem:

Consider the nonlinear system (1), satisfying the assumptions (1-2). If there is a matrix $P > 0$ satisfying the LMI: $A^T P + PA < 0$, the optimal fuzzy controller is chosen as (9) with parameter adaptation law (13), then the proposed fuzzy control scheme can guarantee that:

    i)   all the variable of the closed-loop system are bounded

    ii)   the tracking performance is achieved.

Proof:

Consider the system (2)

$$x^{(n)} = f(\underline{x},t) + bu(t) + d(t)$$

Substituting (9) and (10) into (2), the output error dynamics can be expressed as:

$$x^{(n)} - y_r^{(n)} + \sum_{i=0}^{n-1} k_i e^{(i)} = f(\underline{x},t) - \hat{f}(\underline{x},\underline{\theta}) + bu_r + d(t) \tag{14}$$

Then

$$e^{(n)} = -\sum_{i=0}^{n-1} k_i e^{(i)} + f(\underline{x},t) - \hat{f}(\underline{x},\underline{\theta}) + bu_r + d(t)$$

After some manipulation, the error dynamic can be represented by:

$$\dot{\underline{e}} = A\underline{e} + \underline{B}[f(\underline{x},t) - \hat{f}(\underline{x},\underline{\theta}) + bu_r + d(t)] \tag{15}$$

with $\underline{B} \in R^n$, defined by:

$$\underline{B} = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \end{bmatrix}^T$$

Consider the following Lyapunov function:

$$V = \frac{1}{2}\underline{e}^T P\underline{e} + \frac{1}{2\gamma}\Phi^T \Phi \tag{16}$$

where $\Phi = \underline{\theta}^* - \underline{\theta}$ and γ is a positive constant specified by the designer.

The time derivative of V along the trajectories of (15) equals

$$\dot{V} = \frac{1}{2}\underline{e}^T(A^T P + PA)\underline{e} + \underline{e}^T PB[f(\underline{x},t) - \hat{f}(\underline{x},\underline{\theta}) + bu_r + d(t)] + \frac{1}{\gamma}\Phi^T \dot{\Phi} \tag{17}$$

$$\dot{V} = \frac{1}{2}\underline{e}^T(A^T P + PA)\underline{e} + \underline{e}^T PB[f(\underline{x},t) - \hat{f}(\underline{x},\underline{\theta}^*) + \hat{f}(\underline{x},\underline{\theta}^*) - \hat{f}(\underline{x},\underline{\theta}) + bu_r + d(t)] + \frac{1}{\gamma}\Phi^T \dot{\Phi}$$

$$\dot{V} = \frac{1}{2}\underline{e}^T(A^T P + PA)\underline{e} + \underline{e}^T PB[f(\underline{x},t) - \hat{f}(\underline{x},\underline{\theta}^*) + \Phi^T \xi(\underline{x}) + bu_r + d(t)] + \frac{1}{\gamma}\Phi^T \dot{\Phi}$$

$$\dot{V} = \frac{1}{2}\underline{e}^T(A^T P + PA)\underline{e} + \underline{e}^T PB[f(\underline{x},t) - \hat{f}(\underline{x},\underline{\theta}^*) + bu_r + d(t)] + \tag{18}$$

$$\frac{1}{\gamma}\Phi^T(\dot{\Phi} + \gamma \underline{e}^T PB\xi(\underline{x}))$$

By consideration of the update law (13) and satisfying assumptions, $\dot{V}$ can be written as:

$$\dot{V} \le \frac{1}{2}\underline{e}^T(A^T P + PA)\underline{e} + \left|\underline{e}^T PB\right|\left[\left|f^u\right| + \left|\hat{f}(\underline{x},\underline{\theta}^*)\right| + d_{max}\right] + \tag{19}$$

$$\underline{e}^T PB\, bu_r$$

with P verifying the LMI form:

$$A^T P + PA < 0$$

Then

$$\dot{V} \le \left|\underline{e}^T PB\right|\left[\left|f^u\right| + \left|\hat{f}(\underline{x},\underline{\theta}^*)\right| + d_{max}\right] + \tag{20}$$

$$\underline{e}^T PB\, bu_r$$

if $\left|e^T PB\right| > \varepsilon$, $sat(e^T PB) = \mathrm{sgn}(e^T PB)$

Then

$$u_r = -\frac{1}{b}\mathrm{sgn}(e^T PB)\left[\left|f^u\right| + \left|\hat{f}(\underline{x},\underline{\theta}^*)\right| + d_{max}\right] \tag{21}$$

and

$$\dot{V} \le \left|e^T PB\right|\left[\left|f^u\right| + \left|\hat{f}(\underline{x},\underline{\theta}^*)\right| + d_{max}\right] - \left|e^T PB\right|\left[\left|f^u\right| + \left|\hat{f}(\underline{x},\underline{\theta}^*)\right| + d_{max}\right]$$

$$\dot{V} \le 0$$

if $\left|e^T PB\right| \le \varepsilon$, $sat(e^T PB) = \frac{(e^T PB)}{\varepsilon}$

Then

$$u_r = -\frac{1}{b}\frac{(e^T PB)}{\varepsilon}\left[\left|f^u\right| + \left|\hat{f}(\underline{x},\underline{\theta}^*)\right| + d_{\max}\right]$$

and

$$\dot{V} \le \left|e^T PB\right|\left[\left|f^u\right| + \left|\hat{f}(\underline{x},\underline{\theta}^*)\right| + d_{\max}\right] -$$

$$\frac{(e^T PB)^2}{\varepsilon}\left[\left|f^u\right| + \left|\hat{f}(\underline{x},\underline{\theta}^*)\right| + d_{\max}\right] \qquad (22)$$

$$\dot{V} \le 0$$

This completes the proof of the theorem.

4 Simulation Results

To illustrate the effectiveness of the proposed adaptive controller, we test our proposed controller on two nonlinear systems. The first example is a regulation problem of a nonlinear servomechanism [8]. The second example is to let the doffing forced-oscillation system to track a desired trajectory [10].

Example 1:

The nonlinear servomechanism is modelled by the following second order differential equation:

$$m\ddot{q} + l\dot{q} + \Delta f(q) = \tau + d \qquad (23)$$

$\dot{q}$: Velocity

q : Position

$\Delta f(q)$: Nonlinear term depending on q .

m, ℓ : Mass and damping,

τ : Torque

d : Disturbance included in order to test the robustness of the adaptive controller against external disturbances.

We suppose that the position $x_1 = q$ and the velocity $x_2 = \dot{q}$ are available from measurements.

The dynamic equations of the servomechanism can be described in space state as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(\underline{x},t) + u + d(t) \\ y = x_1 \end{cases} \qquad (24)$$

$$f(\underline{x},t) = -lx_2 - \Delta f(x_1)$$

$$u = \tau$$

The control objective is to maintain the system to track the desired angle trajectory:

$$y_r = 2*\sin(0.015t)$$

The parameters are given as:

$$m = 1kg, \ l = 1$$

$$d = 0.01*\cos(0.0015t)$$

Choose the initial condition:

$$x_1(0) = 3 \text{ and } x_2(0) = 0$$

The adaptive gain $\gamma = 0.5$ with $k_1 = 10$, $k_2 = 1$ and $\varepsilon = 0.1$.

The choice of the number of fuzzy set and the constant M_f is related to knowledge of expert on the system.

For simplicity, we consider $M_f = 1.5$ and the fuzzy membership functions are chosen as in Fig.1.

Then there are 9 rules to approximate the primary control law $\hat{f}(\underline{x},\underline{\theta})$.

P is given by solving the LMI form (8), using the Matlab Toolbox:

$$P = \begin{bmatrix} 1.7052 & 0.0614 \\ 0.0614 & 0.1815 \end{bmatrix}.$$

The function f^u as defined by:

$$f^u = \left|x_2\right|$$

From Fig.2, it can be seen that, the tracking performance is obtained with unknown nonlinear dynamics in presence of disturbances, and it is proven by Fig. 5, where the tracking error is illustrated.

The corresponding fuzzy control signal is shown in Fig.3. The tracking performance of the velocity is shown in Fig. 4.
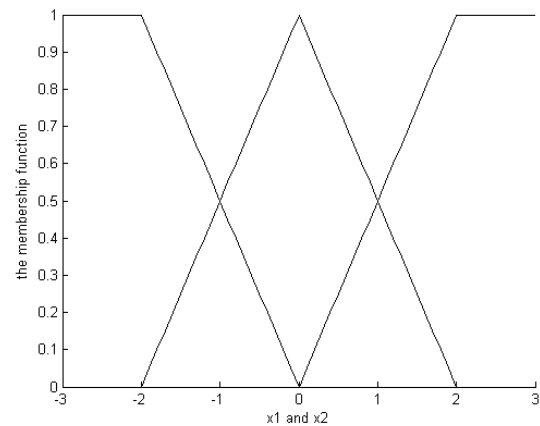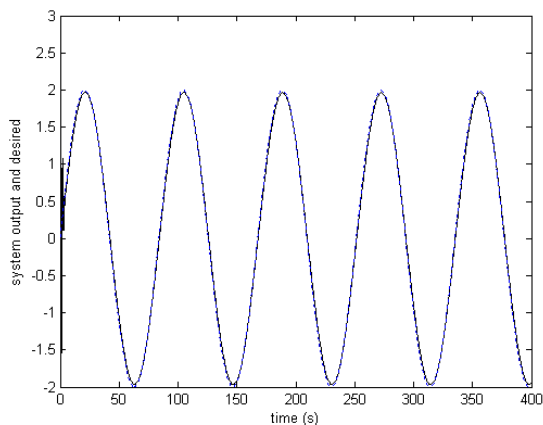

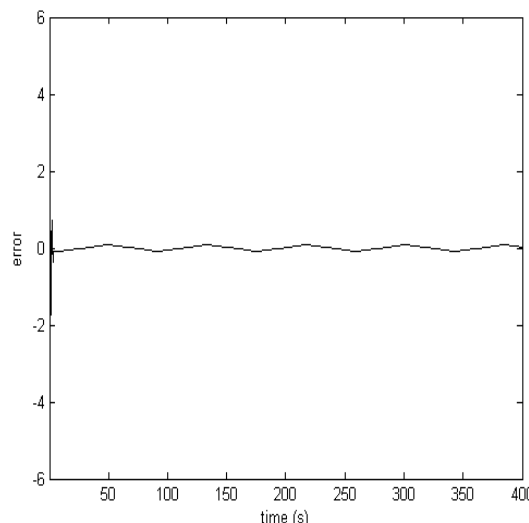Fig.1 Membership functions

Fig.2 Reponses of the $y(t)$ and $y_r(t)$

Fig.5 The tracking error
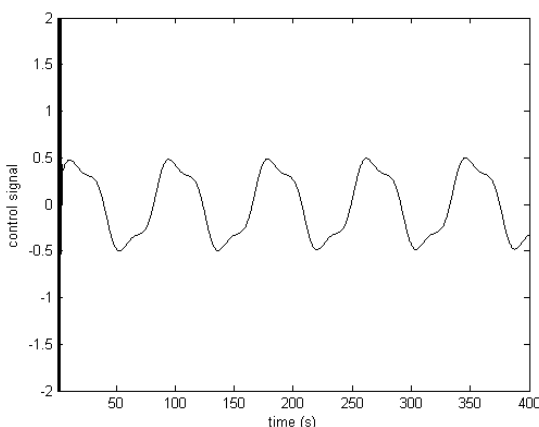
Fig.3 The control signal $u(t)$
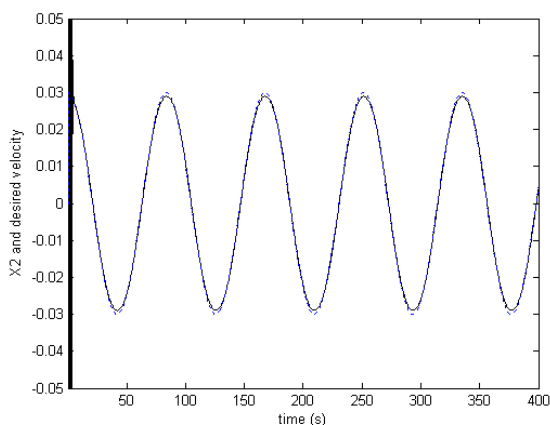
Example 2:

In this example, we consider the Duffing forced oscillation system. The equation which describes the motion of the system is defined by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.1x_2 - x_1^3 + bu(t) + d(t) \\ y = x_1 \end{cases} \qquad (25)$$

The desired trajectory is chosen as follows:

$$y_r(t) = 0.6 * \sin(0.01t)$$

Let $c_1 = 1$ and $c_2 = 7$.

The initial values of $x_1(0) = 0.4$ and $x_2(0) = 1$.

Let the adaptive gain $\gamma = 0.5$, $M_f = 1$ and $\varepsilon = 0.1$.

The external disturbance is represented by $d(t) = 12 * \cos(t)$ included in order to test the robustness of the adaptive controller against external disturbances.

The solution to LMI (8) using the Matlab Toolbox is:

$$P = \begin{bmatrix} 1.2580 & 0.1769 \\ 0.1769 & 0.1068 \end{bmatrix}.$$

The upper function f^u is presented as follows:

$$f^u = \left| x_1^3 \right| + \left| x_2 \right|.$$

To construct the fuzzy approximators $\hat{f}(\underline{x}, \underline{\theta})$, we define fuzzy sets for component of each x_1 and x_2 with the memberships function for

Fig.4 Reponses of the x_2 and $\dot{y}_r(t)$

system state is represented in fig.6. Hence, there are 14 rules to approximate $\hat{f}(\underline{x},\underline{\theta})$.

Fig.7, demonstrates the tracking performance in presence of disturbances. The corresponding fuzzy control signal is given in fig.8.
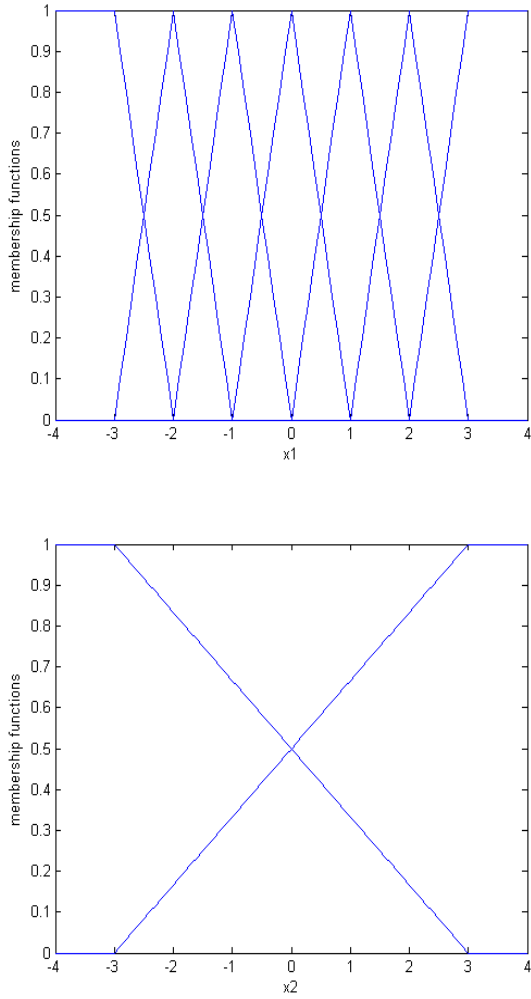

Fig.7 Reponses of the $y(t)$ and $y_r(t)$
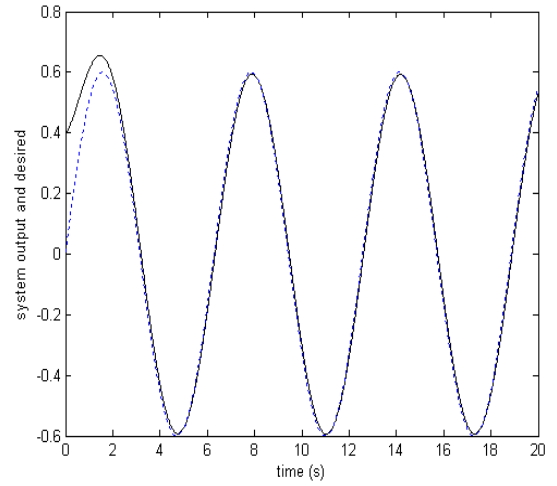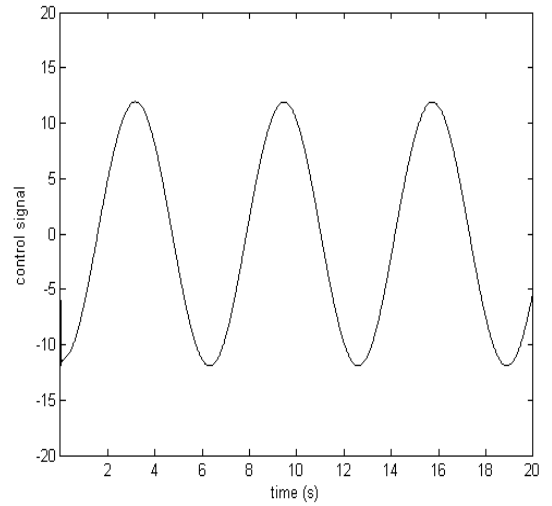

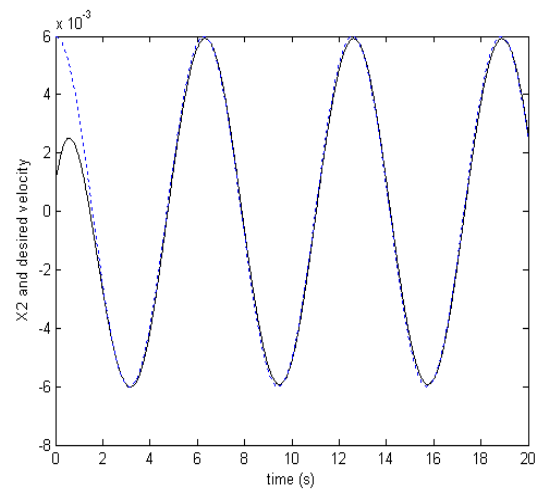Fig. 6 Membership functions

Fig. 8 The control signal $u(t)$
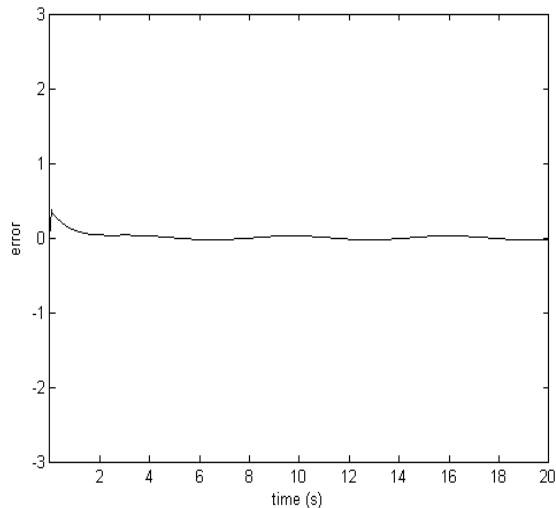

Fig. 9 Reponses of the x_2 and $\dot{y}_r(t)$

Fig.10 The tracking error

Fig. 9 shows the response of the x_2 and $\dot{y}_r(t)$. Fig. 10 illustrates the tracking error, which converges to zero. These results demonstrate the good performance of our proposed approach.

5 Conclusion

In this paper, we have presented an optimal adaptive fuzzy controller for a class of nonlinear systems with unknown dynamics. We have introduced the fuzzy controller combining LMI and Lyapunov approach to ensure the stability, robustness and tracking performance.

The adaptive fuzzy logic system type T-S is used to approximate the unknown nonlinear dynamics. The auxiliary part of control is implemented to improve the system performance by suppressing the influence of external disturbance and removing the fuzzy approximation error.

The adaptation laws of the adjustable parameters are deduced from the stability analysis, in the sense of Lyapunov, to get a more accurate approximation level.

The simulation results have shown that the proposed control strategy can guarantee the system stability as well as maintain a good tracking performance. In our future works, we will develop the new structure of control method of the complex and nonlinear system using the genetics algorithm combined with the fuzzy system.

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