Stabilization of Fuzzy Control Systems

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Abstract: - The paper presents the way to use Lyapunov techniques to analyse and to assure stability of control systems with fuzzy PI controllers. The stability is assured based on a compensation of the input-output characteristic of the fuzzy blocks used in PI fuzzy controller. The compensation is made on fuzzy control law by the action in parallel of a crisp block for non-linear parts of input saturations. Stability analysis of fuzzy control systems for general processes is framed in the field of non-linear systems. The paper presents the way to prove absolute global internal stability in general cases. The correction principle is theoretically substantiated and methodologically structured with application at the fuzzy control of the second order processes.

Key Words: - Fuzzy control systems, Lyapunov techniques, circle criterion, absolute internal stability, non-linear systems.

1 Introduction

1.1 Generalities

This paper presents some notions about the way in which we may analyse absolute internal stability of fuzzy control systems using circle criterion and how we can assure this stability based on an original correction method. Fuzzy blocks used in control systems have a non-linear character, for any variant of implementation. In consequence, for stability analysis methods from non-linear systems are used. In literature there are many criteria of stability analysis for non-linear systems, expressed for general cases [1, 4, 8]. Control systems based on fuzzy logic may have certain properties, which allow in particular cases using of stability criterion valuable large classes of non-linear systems. In this paper we show that fuzzy block have the sector property. This property allows using of Lyapunov techniques for stability analysis and framing the problem of stability assurance in circle criterion for multivariable systems [1, 3, 4]. Usage of circle criterion may done for the following reasons: -transfer characteristics of fuzzy blocks accomplished in principal the sector condition; -it is a rigorous method, which allows sector determination in which stability is granted; -it is based on frequency characteristics, which may be easy obtained based on a frequency analysis of controlled process. A disadvantage of this method is it does not allow obtaining of information related to stability quality, of transient phenomenon damping.

In literature a lot of papers, which are treating the problem of stability analysis of fuzzy control systems, appeared [2, 5, 6, 7, 8, 9, 10, 11, 12]. The methods based on Popov criterion are taking in consideration fuzzy blocks linearized around origin, considering the transfer characteristic invariant in time and they are reducing fuzzy block at an element non-linear with one variable at the input and one variable at the output. So, an approximation is done, which in many cases has errors. This method is valid only locally, if some conditions are accomplished in a first approximation. In general, stability analysis for fuzzy control systems is hard because the designers do not know a usable model of controlled process. If such a model exists a stability analysis may be initiate. Generally, the literature spares the stability analysis of the control systems with fuzzy controllers. Even if there are a number of main papers on this theme, in the most cases the remake of the results is very difficult. In the concrete cases the difficulties increase. This paper is based on a study made by the author [14, 16] with an application in control of electrical drives [13, 15, 17] and, in principal, it systematises the theoretical aspects for the problem of the fuzzy control. The term *quasi-fuzzy* is used for a hybrid fuzzy-crisp structure. The situation is reduced at a known case [16], for what a theoretical substantiation, based on the non-linear characteristic of the fuzzy block and circle's criterion is developed.

2 Fuzzy Systems

Let us consider a system with two inputs u_1 and u_2 and one output y, like in Fig. 1.

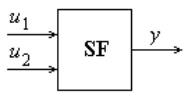

Fig. 1 A system with two input variables and one output variable

We may considered the physical variable of the systems have real values and they are defined on the following real sets, named universes of discourse [-u_{1M}, u_{1M}], [-u_{2M}, u_{2M}], [y_M, y_M]:

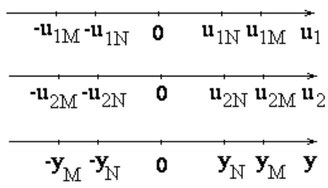

Fig. 2 The universes of discourse

where the base values of the variables are denoted with N and the maximum limit values are denoted with M. The variables are working in a symmetrical universe of discourse. The base values are the maximum values for a continuous working regime. The maximum limit values are the maximum values admitted in transient regimes.

For the real systems variables we may define fuzzy variables denoted with the same symbols u_1, u_2 and y. The fuzzy variables are taken fuzzy values, defined with membership functions on the universes of discourse. Let us considered three fuzzy values for the fuzzy variables: N- negative, ZE – zero and P - positive. We are chosen the following membership functions to define these fuzzy values:

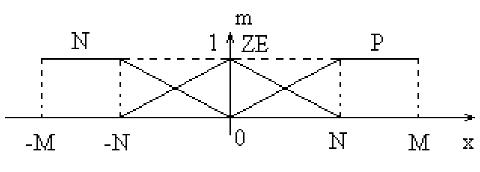

Fig. 3 Membership functions

The real set [-x_M, x_M] is called extended universe of discourse.

For the internal structure of the fuzzy system we are chosen the block diagram from the Fig. 4.

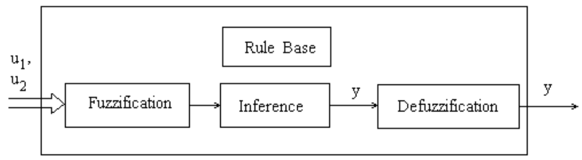

Fig. 4 The structure of fuzzy system

Different rule bases may be chosen related to the number of fuzzy values for the systems variables. We are chosen the primary rule base with only 9 rules, like in Fig. 5.

y		u_1		
		N	ZE	P
u_2	N	$N_{(2)}$	$N_{(4)}$	$ZE_{(6)}$
	ZE	$N_{(8)}$	$ZE_{(1)}$	$P_{(9)}$
	P	$ZE_{(7)}$	$P_{(5)}$	$P_{(3)}$

Fig. 5 The rule base

Different methods may be chosen for the inference. We are chosen, for example, the max-min method. The method of centre of gravity is chosen for defuzzification.

The fuzzy system with two inputs variables and one output variable, with different membership functions, inference and defuzzification methods, may be described by input-output transfer characteristics. We may obtain graphic representation for the following transfer characteristics:

1. The family of SISO transfer characteristics by the form $y=f_e(u_1; u_2)$, considering for the fuzzy block y as output variable and u_1 as a input variable and u_2 a parameter:

$$\begin{aligned} & y = f_e(u_1; u_2), \\ & u_1 \in [-a_1; a_1], u_2 \in \{u_{21}, u_{22}, ..., u_{2m}\}, \\ & u_{2k} \in [-1,1], k = 1,...,m \end{aligned} \qquad (1)$$

2. The MISO transfer characteristic, with the form of a three-dimensional surface:

$$\begin{aligned} & y = f_{BF}(u_1, u_2), \\ & u_1 \in [-a_1; a_1], u_2 \in [-a_2; a_2] \end{aligned} \qquad (2)$$

3. The family of translated transfer characteristics by the form $y=f(u_t; u_2)$, with u_2 parameter:

$$y = f(u_t; u_2),$$
$$u_t \in [-(a_1 + a_2); a_1 + a_2],$$
$$u_2 \in \{u_{21}, u_{22}, ..., u_{2m}\}, u_{2k} \in [-1,1], k = 1,..., m \quad (3)$$

where u_t is a new variable, compound, defined by the summation of those two input variables u_1 and u_2, like a state variable transformation:

$$u_t = u_1 + u_2,\ u_1 \in [-a_1; a_1],\ u_2 \in [-a_2; a_2] \quad (4)$$

The family of characteristics (3) are obtained by translation of the characteristics (1) in the following manner. The characteristics which are crossing the abscise axis at the left of the origin are translated from the left to he right. The characteristics which are crossing the abscise axis at the right of the origin are translated from the right to the left. By this translations the resultant characteristics are passing all through the origin.

4. The family of gain characteristics, defined by the function:

$$K_{SF}(u_t; u_2) = \frac{y}{u_t},$$
$$u_t \in (0; a_1 + a_2],\ u_2 \in \{u_{21}, u_{22}, ..., u_{2m}\},$$
$$u_{2k} \in [-1,1], k = 1,..., m \quad (5)$$

The utility of these characteristics is in the stability analysis and fuzzy control systems synthesis.

The function $K_{SF}(u_t;\ u_2)$ has a limit value in origin, which we may named gain in the origin, defined with the relation:

$$K_0 = \lim_{u_t \to 0} K_{BF}(u_t; 0) \quad (6)$$

The value K_0 may be appreciated from the characteristic $K_{SF}(u_t;\ u_2)$ obtained for u_2=0, for small values of u_t.

The families of characteristics defined above are presented in Fig. 6, 7, 8 and 9, as examples for a fuzzy block *33-mm-g*. The significance of notation *33-mm-g* is: the first digit is the number of the fuzzy values for the input variables of the fuzzy block, the second digit is the number of the fuzzy values for the output variable of fuzzy block, *mm* – min-max method for the inference and *g* – defuzzification with the method of centre of gravity.

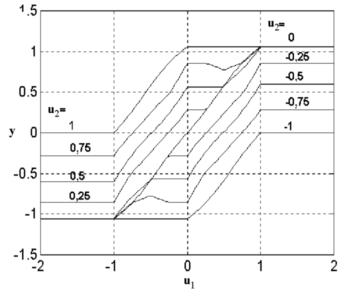

Fig. 6 SISO characteristics

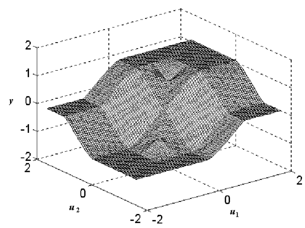

Fig. 7 MISO characteristic

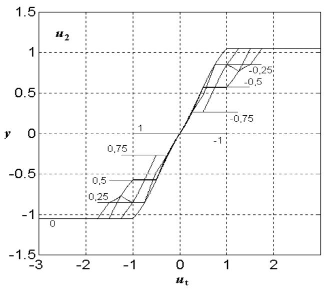

Fig. 8 Translated characteristics

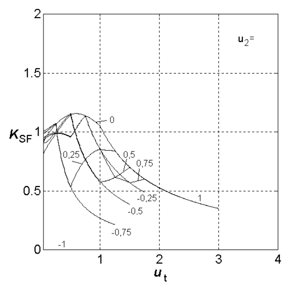

Fig. 9 Gain characteristics

Other characteristics like these may be represented for other fuzzy block, developed using other fuzzy values, membership functions, inference methods and so on. All of them have the same allures.

2 Control Structure

The structure of fuzzy control system is presented in Fig. 1.

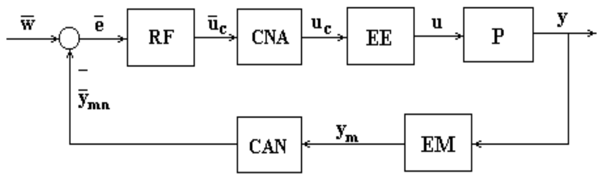

Fig.10 Control structure

In this structure the blocks have the following significance: RF - fuzzy controller, CAN – digital-to-analogue converter, EE – actuator, P – controlled process, EM – sensor, CAN – analogue-to-digital converter. The system variables are of two types: variables in continuous time: u_c – command variable, u – process control input variable, y – process controlled output variable, y_m – measured variable and variables in discrete time: $\bar{w}$ - reference input variable, $\bar{e}$ - control error, $\bar{u}_c$ - command input variable, $\bar{y}_{ma}$ - measured variable. This is a standard structure, in which the fuzzy controller is digital implemented.

For stability analysis we are working with a quasi-continual model of the fuzzy control system. This structure is presented in Fig. 11.

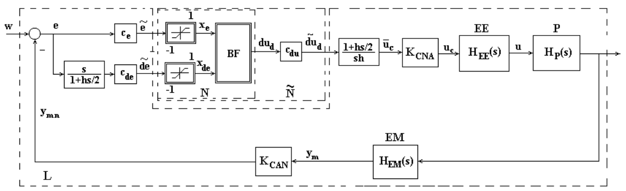

Fig. 11 The structure of fuzzy control system for stability analysis

The fuzzy controller is a fuzzy PI controller with integration at the output. The elements of derivation and integration, the sensor EM, the process P, the actuator EE are represented with operational transfer function in continuous time domain. All parts of the controlled process are considered linear systems. We suppose that the system parameters are known, or it is possible to obtain the frequency characteristics in open loop of the linear part, including the process. The fuzzy block BF together with the saturations elements form a non-linear part denoted N. This non-linear part together with a proportional block, which contains the incremental coefficient c_{du} form a global non-linear part denoted $\tilde{N}$. The rest of blocks from the control structure were grouped into a linear part denoted L. For the gain coefficients c_e and c_{de}, from the block inputs adequate values were be taken by design. The sampling period h is taken according to the digital control theory.

We are supposing that the process P, together with the actuator EE and the measurement element EM and the entire linear part have the input-state-output model in continuous time:

$$\begin{aligned} \dot{x}_{L1} &= A_{L1}x_{L1} + b_{L1}u_c \\ y_m &= c_{L1}^T x_{L1} \end{aligned} \quad (7)$$

For the filters from the input and the output of the fuzzy controller we may write the following equations:

$$\begin{aligned} e &= w - y_{mn} = w - K_{CAN}y_m \\ \tilde{e} &= c_e e \\ \tilde{de} &= c_{de}\frac{s}{1+sh/2}e \\ \bar{u}_c &= \frac{1+hs/2}{sh}\tilde{du}_d \end{aligned} \quad (8)$$

Two state variables, which appear in those two filters, are chosen, and they are given by the relations:

$$\begin{aligned} x_{a1} &= c_{de}e - \frac{h}{2}\tilde{de} \\ x_{a2} &= \bar{u}_c - \frac{1}{2}\tilde{du}_d \end{aligned} \quad (9)$$

With the new two auxiliary state variables the linear part L has the input-state-output model (10):

$$\begin{aligned} \dot{x}_{L1} &= A_{L1}x_{L1} + b_{L1}K_{CNA}x_{a2} + \frac{1}{2}b_{L1}K_{CNA}\tilde{du}_d \\ \dot{x}_{a1} &= -\frac{2c_{de}}{h}K_{CAN}c_{L1}^T x_{L1} - \frac{2}{h}x_{a1} + \frac{2c_{de}}{h}w \\ \dot{x}_{a2} &= \frac{1}{h}\tilde{du}_d \\ \tilde{e} &= -c_e K_{CAN}c_{L1}^T x_{L1} + c_e w \\ \tilde{de} &= -\frac{2c_{de}}{h}K_{CAN}c_{L1}^T x_{L1} - \frac{2}{h}x_{a1} + \frac{2c_{de}}{h}w \end{aligned} \quad (10)$$

The non-linear part $\tilde{N}$ is described by the equation:

$$\tilde{du}_d = c_{du} du_d = c_{du} f_{BF}(x_e, x_{de}) = c_{du} f_N(\tilde{e}, \tilde{de}) \qquad (11)$$

in which $f_{BF}(x_e, x_{de})$ represents the function of two variables associated to fuzzy block BF, and $f_N(\tilde{e}, \tilde{de})$, or with an equivalent notation: $f_N(\tilde{y})$, - represents the function of two variables associated to the nonlinearity N.

3 Control structure reconfiguration

The structure from Fig. 11 is reconfigured in a structure specific to the approach of absolute stability for non-linear systems, presented in Fig. 12.

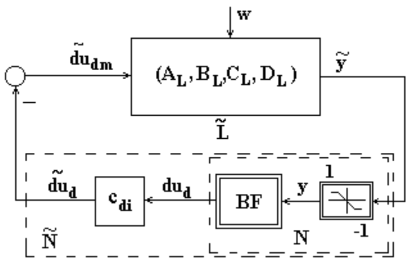

Fig. 12 Reconfigured structure of fuzzy control system

The linear part has a model in continuous time, with two input variables $\tilde{du}_d$ and w and two output variables $\tilde{e}$ and $\tilde{de}$. In Fig. 12 the linear part $\tilde{L}$ has the input variable:

$$\tilde{du}_{dm} = -\tilde{du}_d \qquad (12)$$

The input variable w of fuzzy control system is considered in the exterior of control structure. The block $\tilde{L}$ is different face to the block L by placement of an inverter at the input. The output variables of $\tilde{L}$ are grouped in the output variables vector $\tilde{y} = [\tilde{y}_1\ \tilde{y}_2]^T = [\tilde{e}\ \tilde{de}]^T$. With difference to Fig. 11 the non-linear part $\tilde{N}$ is placed in this case on the feedback channel. The inputs of fuzzy block are grouped in the vector $y = [y_1\ y_2]^T = [x_e\ x_{de}]^T$.

In a concentrated way the equations (4) of the control system based on fuzzy controller may be expressed synthetically, after the intercalation of the inverter from the input, by the mathematical model:

$$\dot{x}_L = A_L x_L + B_L u_L$$
$$\tilde{y} = C_L x_L + D_L u_L \qquad (13)$$

where:

$$u_L = \begin{bmatrix} \tilde{du}_{dm} \\ w \end{bmatrix}$$

$$u_L = \begin{bmatrix} \tilde{du}_{dm} \\ w \end{bmatrix}$$

$$\tilde{y} = \begin{bmatrix} \tilde{e} \\ \tilde{de} \end{bmatrix}$$

$$A_L = \begin{bmatrix} A_{L1} & | & 0 & 0 \\ \hline -\frac{2c_{de}}{h} K_{CAN} c_{L1}^T & & 0 & b_{L1} K_{CNA} \\ 0 & & 0 & 0 \end{bmatrix}$$

$$B_L = \begin{bmatrix} -\frac{1}{2} b_{L1} K_{CNA} & | & 0 \\ \hline 0 & & \frac{2c_{de}}{h} \\ -\frac{1}{h} & & 0 \end{bmatrix} \qquad (14)$$

$$C_L = \begin{bmatrix} -c_e K_{CAN} c_{L1}^T & | & 0 & 0 \\ -\frac{2c_{de}}{h} K_{CAN} c_{L1}^T & | & -\frac{2}{h} & 0 \end{bmatrix}$$

$$D_L = \begin{bmatrix} 0 & c_e \\ 0 & \frac{2c_{de}}{h} \end{bmatrix}$$

$$\tilde{du}_{dm} = -f_{\tilde{N}}(\tilde{e}, \tilde{de}) = -c_{du}.du_d$$

$$du_d = f_N(\tilde{e}, \tilde{de})$$

and $f_N(\tilde{e}, \tilde{de})$ is the function associated to the non-linear part N.

The non-linear function with two inputs is associated to fuzzy block:

$$du_d = f_{BF}(x_e, x_{de}) \qquad (15)$$

with the universes of discourse U_e, U_{de} chosen for the input variables and U_{du} for the output variable.

With the compound variable x_t

$$x_t = [1 \quad 1]y = x_e + x_{de} \quad (16)$$

chosen by summation of two inputs of the fuzzy block and the function of variable x_t and parameter x_{de}:

$$K_{BF}(x_t; x_{de}) = \frac{f_{BF}(x_e, x_{de})}{x_t}, \; pt.\; x_t \neq 0 \quad (17)$$

The variable du_d is expressed by the form:

$$du_d = \begin{cases} K_{BF}(x_t; x_{de})x_t, & pt.\; x_t \neq 0 \\ 0, & pt.\; x_t = 0 \end{cases} \quad (18)$$

The families of characteristics $du_d=f(x_t;\ x_{de})$ of the fuzzy blocks presents the property they are situated only in the I and III quadrants.

The values of function $K_{BF}(x_t;\ x_{de})$ may be determinated from the family of characteristics for the fuzzy blocks. The last families of characteristics are inducing the necessity of relation:

$$0 \le K_{BF}(x_t; x_{de}) \le K_M \quad (19)$$

Based on the transformation given by the relations (18) the block scheme from Fig. 12 is detailed like in Fig. 13.

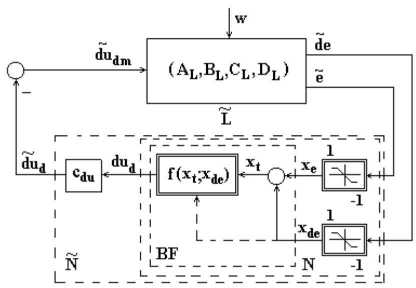

Fig. 13 First equivalent block diagram of fuzzy control system

Because in the control structure based on fuzzy controller the fuzzy block was chosen to work on the universes of discourse [-1, 1] of the input variables, we are deducing that $x_t \in [-2, 2]$.

The family of characteristics $du_d=f(x_t;\ x_{de})$ for the fuzzy block *33-mm-g* are presented in Fig. 14.

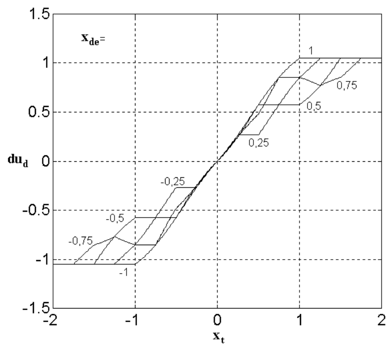

Fig. 14 Family of characteristics $du_d=f(x_t;x_{de})$

In the case in which the fuzzy block is working on the extended universe of discourse an "undetermination syndrome" is acting over the fuzzy block, due to the saturations of the membership functions of the input variables at the limits of the universes of discourse. So, du_d is taken the value 0 for every $x_e>1$ and $x_{de}<-1$ or very $x_e<-1$ and $x_{de}>1$. This aspect may be noticed on the input-output transfer characteristics.

In stability analysis of the fuzzy control system we must take in consideration the fact that the non-linear part N has in its structure also limitation elements. The non-linear block N has the non-linear characteristic:

$$du_d = f_N(\tilde{e}, \tilde{de}) \quad (20)$$

With the new compound variable:

$$\tilde{x}_t = [1 \quad 1]\tilde{y} = \tilde{e} + \tilde{de} \quad (21)$$

a new function of the compound variable $\tilde{x}_t$ and parameter $\tilde{de}$ may be introduced:

$$K_N(\tilde{x}_t; \tilde{de}) = \frac{f_N(\tilde{e}, \tilde{de})}{\tilde{x}_t}, \; pt.\; \tilde{x}_t \neq 0 \quad (22)$$

The variable du_d may be expressed under the form:

$$du_d = \begin{cases} K_N(\tilde{x}_t; \tilde{de})\tilde{x}_{t1}, & pt.\; \tilde{x}_t \neq 0 \\ 0, & pt.\; \tilde{x}_t = 0 \end{cases} \quad (23)$$

Also, the families of characteristics $du_d = \tilde{f}(\tilde{x}_t; \tilde{de})$ present the sector property to be placed only in the quadrants I and III. The family of characteristics $du_d = \tilde{f}(\tilde{x}_t;\tilde{de})$ is presented in Fig. 15, where $\tilde{x}_t, \tilde{de} \in R$.

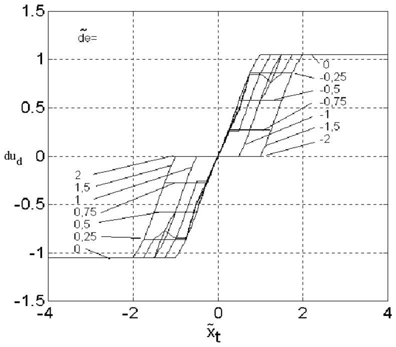

Fig. 15 Family of characteristics $du_d = f(\tilde{x}_t;\tilde{de})$

The family of characteristics $K_N(\tilde{x}_t, \tilde{de})$ are inducing the consideration of the relation:

$$0 \le K_N(\tilde{x}_t;\tilde{de}) \le K_M \tag{24}$$

In Fig. 16 we are presenting, like an example, the family of characteristics $K_N(\tilde{x}_t, \tilde{de})$, for $\tilde{x}_t \neq 0$.

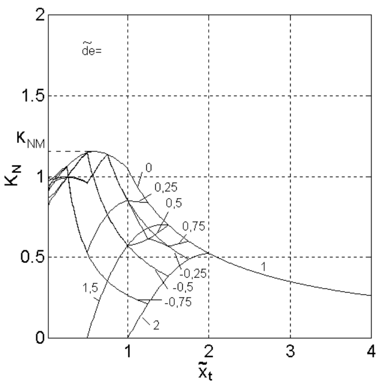

Fig. 16 Family of characteristics of the function K_N

From the family of characteristics $du_d = \tilde{f}(\tilde{x}_{t1};\tilde{de})$ we may deduce the spatial sector property associated to the non-linear part *N*, by the form:

$$f_N(\tilde{e}, \tilde{de})\{f_N(\tilde{e}, \tilde{de}) - K_{NM}[1\ 1][\tilde{e}\ \tilde{de}]^T\} \le 0, \quad \forall[\tilde{e}\ \tilde{de}] \in R^2 \tag{25}$$

The relation (25) describes a sector condition for a nonlinearity with two inputs and one output, which is framed in a general form suggested in literature for multivariable non-linear systems [4].

Based on the description of the non-linear part N with relations (21, 22, 23) a second equivalent block diagram of the non-linear control systems may be developed, like in Fig. 17.

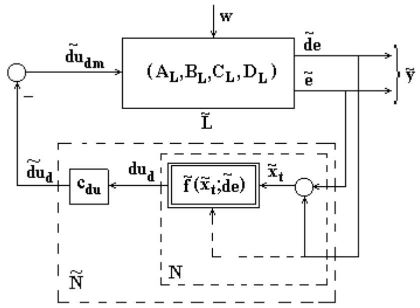

Fig. 17 The second equivalent block diagram

From the analysis of characteristics from Fig. 15 we notice again that parts of characteristics with null intervention of the non-linear part N appear, due to the limitations placed at the inputs of the fuzzy block: du_d take the value 0 for $\tilde{x}_t \in [-1, 1]$, for $|\tilde{de}| = 1; 1{,}5; 2$ and so on.

For determination of a Lyapunov function based on which to prove the absolute internal stability of fuzzy control system, the nonlinearity $\tilde{f}_{\tilde{N}}(\tilde{e};\tilde{de})$, which characterises the global non-linear part, or with the equivalent expression $\tilde{f}_{\tilde{N}}(\tilde{y})$, may be variable or time invariant, but it must be continuous on subintervals and it must accomplish locally the Lipschitz property related to $\tilde{y}$, in the way that it must exist a ν>0, finite, so:

$$\| \tilde{f}_{\tilde{N}}(\tilde{y}_1) - \tilde{f}_{\tilde{N}}(\tilde{y}_2) \| \le \nu \| \tilde{y}_1 - \tilde{y}_2 \|, \quad \forall\ \tilde{y}_1 = [\tilde{e}_1\ \ \tilde{de}_1]\ \text{şi}\ \tilde{y}_2 = [\tilde{e}_2\ \ \tilde{de}_2] \tag{26}$$

In the case of control systems based on fuzzy logic the fuzzy block is a no inertial element and the values of the nonlinearity output depend in every moment only of the values of two inputs $\tilde{e}$ and $\tilde{de}$.

The global nonlinearity $\tilde{N}$ satisfies the spatial sector condition:

$$\tilde{f}_{\tilde{N}}(\tilde{e},\tilde{de})[\tilde{f}_{\tilde{N}}(\tilde{e},\tilde{de}) - K_{\max}[1 \quad 1]\tilde{y}] \leq 0, \\ \forall t \geq 0,\ \forall \tilde{y} \in R^2 \tag{27}$$

and

$$K_{\max} = c_{du}K_M > 0 \tag{28}$$

4 Equilibrium point

For stability analysis we must determine the equilibrium points of the control system. In general, the linear control systems have as equilibrium points the origin state x_L=0. For the cases in which the fuzzy blocks are used we may show that the equilibrium state of the control systems with a linear process P is a unique one and it is the state for zero inputs. For this purpose we are considering the system with the equations (14), with zero input $w = 0$, and also we consider the system in a stationary regime, so the derivatives of the state variables are zero. We may notice immediately from (10) that $\tilde{de} = de = 0$. On the other hand, from (23) it results that $\tilde{du}_d = 0$. In these conditions from (14, 20) the next relation results:

$$\tilde{f}_{\tilde{N}}(\tilde{e},0) = 0 \tag{29}$$

Based on the families of characteristics of fuzzy blocks we may affirm that the dependence $du_d = \tilde{f}_{\tilde{N}}(\tilde{e},0)$ is bijectiv, so the equation (29) has a unique solution $\tilde{e} = 0$.

Coming back to this notice in (14) it results that absolute all the characteristic variables of the system take the zero value. So, the origin point is the equilibrium point. And because the equation (29) has $\tilde{e} = 0$ as unique solution, it is the unique equilibrium point.

Mutatis mutandis, to assure the desired conditions in the permanent regime it is necessary that when x_{de}=0 the fuzzy block to give at its output du_d=0 only when x_e=0. If we analyse on the SISO families of characteristics du_d=$f(x_e)$, with x_{de} as parameter, the characteristic for x_{de}=0, we may notice that the above condition is accomplished for all the fuzzy blocks which are using defuzzification with the method of the centre of gravity. The fuzzy blocks, which are using the mean of maxima defuzification method does not accomplishe this condition.

5 Linguistic state portrait

The link between the dynamic behaviour of the control system and the rule base of the fuzzy block may be analysed qualitatively in the vague state space. In Fig. 18 we present so called the state portrait in the case of the primary rule base.

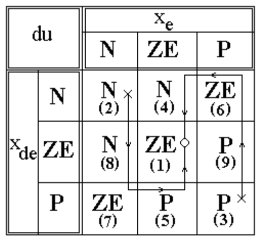

Fig. 18 The vague state portrait

So, if we suppose that at the initial moment the initial conditions are: x_{e0}=P and x_{de0}=P, according the rule number (3) from the primary rule base from Fig. 18, then the command given by the fuzzy block will be du_d=P. If the control system is considered stable and the input is on a controllable way the state trajectory may pas through the following rule: (9), (6), (4), touching the rule number (1) in the permanent regime. If the system is at the initial moment in one of the positions of rules (9), (6) or (4), the stable trajectory may also passing from this initial state to the rule number 1 of permanent regime. An other example for a possible trajectory, for an "aperiodical linguistic behaviour" is: (2), (8), (7), (5) and (1). The rule base is conceived as it contains the trajectories of vague states, which due to the integration character of the linear part of the control system lead the system, from any starting point of the rule base, to the stable permanent regime. We may notice that many rule bases may assure linguistic trajectories of the variables x_e, x_{de} and du_d resembling to this descried above.

6 Notice on the linear part

Considering the structures from Fig. 13 and 17, in situation that w=0, the linear part $\tilde{L}$ has a transfer matrix $H_0(s)$, and the expression of system output is:

$$\tilde{y}(s) = H_0(s)\tilde{du}_{dm}(s) \tag{30}$$

The characteristic polynomial of the linear part has a zero in origin, due to the integral character introduced in controller dynamic, so $H_0(s)$ is not a Hurwitz transfer matrix. Using (20, 22, 23) the neliniarity $\tilde{f}_{\tilde{N}}(\tilde{y})$, with two inputs and one output, may be assimilated with an equivalent SISO neliniarity $\tilde{f}_{\tilde{N}}(\tilde{x}_t;\tilde{de})$, and time variant, due to the influence of $\tilde{de}$. The transfer function, which links the input variable $\tilde{du}_{dm}$ to $\tilde{x}_t(s)$, is obtained with the relation:

$$H_{01}(s) = \frac{\tilde{x}_t(s)}{\tilde{du}_{dm}(s)} = [1 \quad 1]H_0(s) \tag{31}$$

and it has the expression:

$$H_{01}(s) = \frac{1+hs/2}{hs} H_{EE}(s)H_P(s)H_{EM}(s). \\ \cdot\left(c_e + c_{de}\frac{s}{1+hs/2}\right) \tag{32}$$

In the deduction of the above expression we took account on the fact that in some particular cases there is the equality $K_{CAN} = 1/K_{CNA}$.

In the analysis of the possibility of stabilization of the linear part the structure from Fig. 19 and the root locus for the equation $1+KH_{01}(s)$=0 are using.

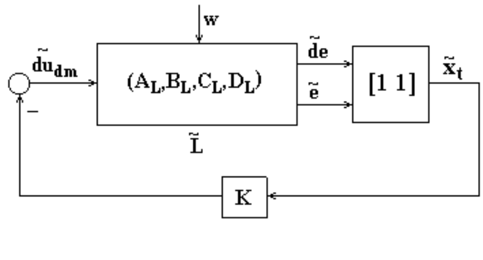

Fig. 19 The block diagram of the linear part, stabilized with a linear feedback

In Fig. 20 we present an example of root locus.

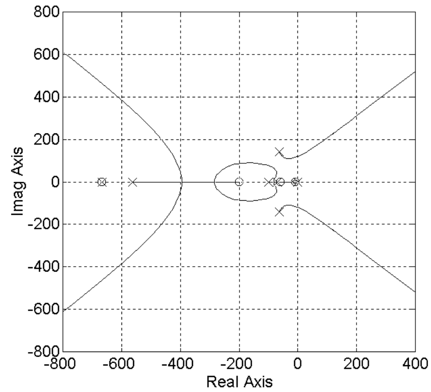

Fig. 20 Example of the root locus for the function $1+KH_{01}(s)$ [17]

The poles of the linear part are: -666,67; -100; -562,66; -63,25+140,40j; -63,25-140,40j; -69,64; -0,11; 0. The zeros of the linear part are: –8,51; -58,82; -666,67; -200.

In Fig. 21 we present a detail of this root locus, to put in evidence the roots closed to imaginary axis.

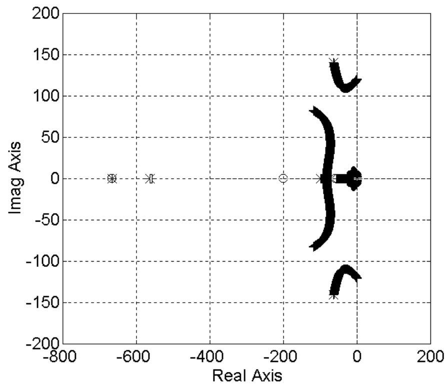

Fig. 21 Detail of the root locus for the function

The detail from Fig. 21 of the root locus was obtained for $K\in[0; 4795]$.

The root locus is crossing the imaginary axis for K=K_H – Hurwitz gain coefficient. For the root locus taken as an example $K_H \cong 4794$. So, the linear part may be stabilized by a linear feedback, like in Fig. 19, with a $K\in[\varepsilon; 4794)$, where $\varepsilon>0$, may be small as we wish. Such situation is called situation of ε-stability.

In the case of the structure from Fig. 17 the non-linear block $\tilde{N}$ must stabilize the linear part $\tilde{L}$, which is unstable due to the pole from origin.

According [4] the nonlinearity $\tilde{f}_{\tilde{N}}(\tilde{y})$ must be framed in a sector under the form [K_{min}, K_{max}], which sector cannot overpass the Hurwitz sector [ε, K_H):

$$[K_{\min}, K_{\max}] \subset [\varepsilon, K_H) \tag{33}$$

So, K_{min} must be chosen so that the transfer function

$$H_1(s) = \frac{H_{01}(s)}{1 + K_{\min} H_{01}(s)} \tag{34}$$

associated to the system from Fig. 17 to be a Hurwitz function.

In the next paragraph we shall present the solution to assure stability.

7 Correction of the non-linear part

The relation, which characterises the fuzzy block, depends in the first by the rule base. To the fuzzy blocks we may attach a fuzzy relation of which characteristic is placed only in the quadrants I and III. From the relation $f_{BF}(x_e, x_{de})$, which is describing the fuzzy block, a source of neliniarity is done by the membership functions. If the block will work on the universe of discourse [-1, 1] its characteristic will be only in the sector [K_1, K_2], $0<K_1<K_2$, as it is shown in Fig. 14.

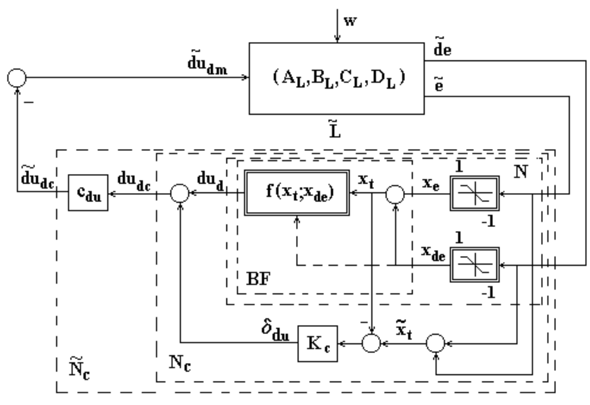

Fig. 22 The structure of the control system with the correction of the non-linear part N

By introducing the saturation elements with a role of limitation at the inputs of the fuzzy block the nonlinearity $\tilde{N}$ is placed in a sector [0, K].

To accomplish the sector condition (33) necessary to the stability assurance, in the following a correction will be used to the non-linear part. It consists in summation at the output du_d of the fuzzy block of the quantity δ_{du}:

$$\delta_{du} = K_c[(\tilde{e} - e) + (\tilde{de} - de)] = K_c(\tilde{x}_t - x_t) \tag{35}$$

It results:

$$du_{dc} = f_{\tilde{N}}(\tilde{e}, \tilde{de}) = di_d + \delta_{du} \tag{36}$$

The value $K_c>0$ will be choose so that the neliniarity $\tilde{N}_C$ characteristic to be framed in an adequate sector [K_{min}, K_{max}].

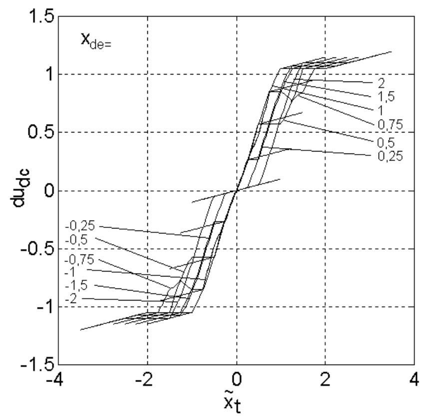

Fig. 23 The family of characteristics for K_c=0,1

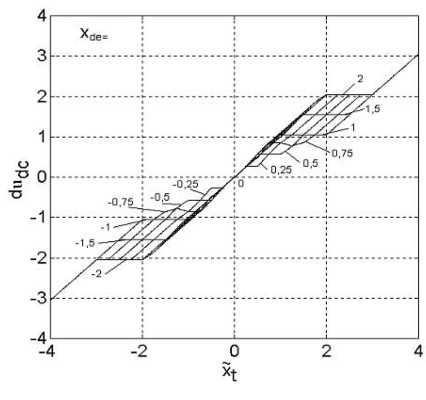

Fig. 24 The family of charactersitics for K_c=1

In Fig. 23 and 24 we present the families of characteristics $du_{dc}=f(\tilde{x}_t;\tilde{de})$, for K_c=0,1 and respectively for K_c=1.

We may notice the placement in quadrants I and III and the accomplished of the sector condition:

$$[f_{\tilde{N}}(\tilde{e},\tilde{de}) - K_m[1\quad 1]\tilde{y}][f_{\tilde{N}}(\tilde{e},\tilde{de}) - K_M[1\quad 1]\tilde{y})] \le 0,\quad y \in R^2 \tag{37}$$

This condition may be rewrite in the form:

$$(du_{dc} - K_m\tilde{x}_t)(du_{dc} - K_M\tilde{x}_t) \le 0,\ \forall \tilde{x}_t \in R \tag{38}$$

To determine K_m and K_M we consider the families of characteristics:

$$K_{Nc}(\tilde{x}_t;\tilde{de}) = \frac{du_{dc}}{\tilde{x}_t},\ \tilde{x}_t \ne 0 \tag{39}$$

from Fig. 25 and Fig. 26, associated to the characteristics from Fig. 23 and 24.

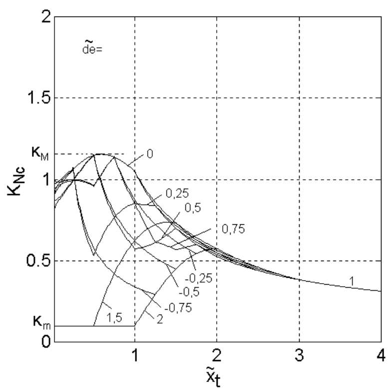

Fig. 25 The family of characteristics $K_{Nc}(\tilde{x}_{t1};\ \tilde{de})$, for K_c=0,1

We may notice the family of characteristics $K_{Nc}(\tilde{x}_t;\ \tilde{de})$ has the limits K_m and K_M depending of coefficient K_c.

In Fig. 27 we present the way in which the limits K_m and K_M vary function of K_c, on the domain [0, 1]. This variation is determined by interpolation between the points obtained for different values of the correction coefficient K_c. Analysing Fig. 27 we may see that the value of K_M is constant, being approximately at 1,17, for any value of K_c. So, the introducing of the correction does not modify the maximum value associated to the family of functions $K_{Nc}(\tilde{x}_t;\ \tilde{de})$.

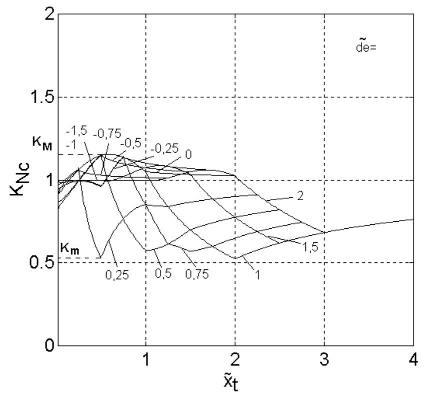

Fig. 26 The family of characteristics $K_{Nc}(\tilde{x}_{t1};\ \tilde{de})$, for K_c=1

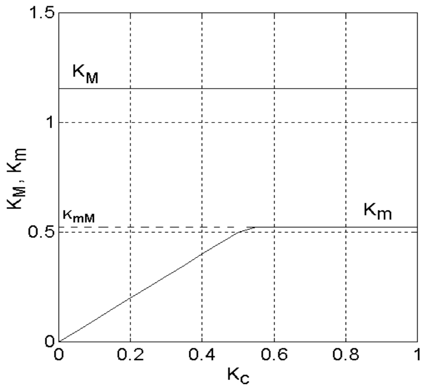

Fig. 27 The variation of sector limits function of K_c

The value of K_m vary linearly with K_c, from zero to a maximum value K_{mM} of approximate 0,52, for K_c=0,55. Then K_m remains constantly at the value of approximate 0,52 for values of K_c greater then 0,55. The rapport between K_m and K_M vary function of K_c between 0 and approximative 0,32.

Linking this results with the condition (33) it results that c_{du} and K_c must be chosen so:

$$[c_{du}K_m, c_{du}K_M] \subset [K_{\min}, K_{\max}] \subset [\varepsilon, K_H) \tag{40}$$

As we may notice from the equivalent block diagrams the incremental coefficient of the command variable c_{du} is included in the structure of the non-linear part. It affects in the same time the both limits of the sector in which the neliniarity $\tilde{N}_c$

is framed. The inferior limit of the sector may be corrected with the correction coefficient K_c.

The accessible values of K_{min}, K_{max} and c_{du} for the fuzzy control system will be determined after a stability analysis, which will be presented as follows.

8 Internal Stability Analysis

To analyse the stability of control system based on fuzzy controller a fiction transformation of the structure from Fig. 22 in the structure from Fig. 28.

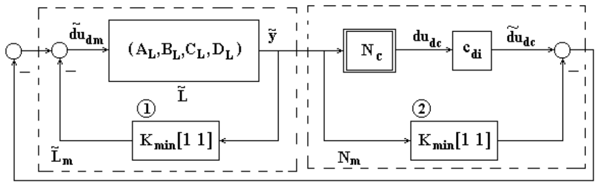

Fig. 28 The block diagram of the control loop transforation

It consists in the application of a negative feedback (the block 1) with the form $K_{min}[1\ 1]$ to the linear part $\tilde{L}$ and the total compensation of this (block 2). It results a linear modified block $\tilde{L}_m$ and a new non-linear block N_m. In this structure the linear part $\tilde{L}$ has a non Hurwitz state matrix A_L, and the corrected neliniarity $\tilde{f}_{\tilde{N}}(\tilde{y})$ satisfies the sector condition:

$$[\tilde{f}_{\tilde{N}}(\tilde{y})-K_m[1\quad 1]\tilde{y}][\tilde{f}_{\tilde{N}}(\tilde{y})-K_M[1\quad 1]\tilde{y})]\le 0,\quad \forall \tilde{y}\in R^2 \tag{41}$$

The linear part $\tilde{L}_m$ has the mathematical model:

$$\dot{x}_L=(A_L-B_LK_{\min}[1\quad 1]C)x_L+B_Lu_{L_s}$$
$$\tilde{y}=C_Lx_L+D_Lu_L \tag{42}$$

Let be K_{min} so the matrix of the system $\tilde{L}_m$

$$\tilde{A}_L=A_L-B_LK_{\min}[1\quad 1]C \tag{43}$$

to be a Hurwitz matrix.

Then, the new part N_m is described by the equation:

$$f_m(\tilde{y})=\tilde{f}_{\tilde{N}}(\tilde{y})-K_{\min}[1\quad 1]\tilde{y} \tag{44}$$

We may notice simply that from (41) it results:

$$f_m(\tilde{y})[f_m(\tilde{y})-K[1\quad 1]\tilde{y})]\le 0,\ \forall \tilde{y}\in R^2 \tag{45}$$

with K=K_{max}-K_{min}.

For internal stability analysis we are taken in consideration the free system, in which w=0. The mathematical model of the free system is

$$\dot{x}_L=\tilde{A}_L x_L+B_L\tilde{di}_{dm}$$
$$\tilde{y}=C_Lx_L \tag{46}$$
$$\tilde{du}_{dm}=-f_m(\tilde{y})$$

The system (46) has a non-linear part with two input variables and one output variable. In [4] the circle criterion is recommended for multivariable systems were the non-linear part has the same number of outputs as the input. We must notice that this is not the case in our problem.

Using the modification of the non-linear part done above from the form with two inputs at the form with only one input and a parameter, the circle criterion proposed in [4] may be applied in this case, at a control system with a SISO non-linear parts, but time variant.

In the following we shall show it is possible to give an enounce of the circle criterion also in this case in which the non-linear part has two inputs and only one output, and it has the property (45).

The demonstration has three stages.

First stage. In the first stage we make appeal at the following variant of the Lemma Kalman-Yakubovici-Popov:

Let it be:

$$F(s)=1+KH_1(s)=1+K[1\quad 1]C(sI-\tilde{A})^{-1}b_r \tag{47}$$

a transfer function, where $H_1(s)$ is given by the relation (31), $\tilde{A}$ is the matrix of the linear modified part $\tilde{L}_m$, which is a Hurwitz matrix, $(\tilde{A},b_r)$ is controllable and $([1\,1]C,\tilde{A})$ observable. Then, $F(s)$

is strictly positive real if and only if there is a matrix P symmetrically positive defined ($P=P^T>0$), a vector M and a constant ε>0 then:

$$P\tilde{A}+\tilde{A}^T P=-M^T M-\varepsilon P \qquad (48)$$
$$Pb_r=C^T K[1 \quad 1]^T-\sqrt{2}M^T$$

Taking account of relation (31, 43, 46) it results:

$$F(s)=1+(K_{\max}-K_{\min})\frac{H_{01}(s)}{1+K_{\min}H_{01}(s)}= \qquad (49)$$
$$=\frac{1+K_{\max}H_{01}(s)}{1+K_{\min}H_{01}(s)}=\frac{1+K_{\max}[1 \quad 1]H_0(s)}{1+K_{\min}[1 \quad 1]H_0(s)}$$

The second stage. We suppose that the conditions from Kalman-Yakubovici-Popov Lemma are accomplished and there are the matrix P and M. Then with the matrix P and M we construct a Lyapunov function $V(x)$ and we demonstrate that it derivative $\dot{V}(x)$ is negative then the sector condition is accomplished.

In this way a Lyapunov function is chosen:

$$V(x)=x^T Px \qquad (50)$$

where $x=x(t)$ is the vector of the state variables of the control system, given by the equations (8, 9, 10, 13).

In consequence:

$$\dot{V}(x)=x^T(P\tilde{A}+\tilde{A}^T P)x-2x^T Pb_r f_m(\tilde{y}) \qquad (50)$$

Increasing the right member of the relation (5.59) with $-2f_m(\tilde{y})(f_m(\tilde{y})-K[1 \quad 1]\tilde{y})\geq 0$, see (45), we obtain:

$$\dot{V}(x)\leq x^T(P\tilde{A}+\tilde{A}^T P)x-2x^T Pb_r f_m(\tilde{y})-$$
$$-2f_m(\tilde{y})\{f_m(\tilde{y})-K[1 \quad 1]\tilde{y}\}=x^T(P\tilde{A}+$$
$$+\tilde{A}^T P)x-2x^T Pb_r f_m(\tilde{y})-2f_m(\tilde{y})\{f_m(\tilde{y})- \qquad (52)$$
$$-K[1 \quad 1]Cx\}=x^T(P\tilde{A}+\tilde{A}^T P)x+$$
$$+2x^T(C^T K[1 \quad 1]^T-Pb_r)f_m(\tilde{y})-2f_m^2(\tilde{y})$$

Making changes in the right member of the equation (52) in the first and the second term based on the relations (44, 48), using the matrix P and M and the number ε, it results:

$$\dot{V}(x)\leq x^T(-M^T M-\varepsilon P)x+2\sqrt{2}x^T M^T f_m(\tilde{y})-$$
$$-2f_m^2(\tilde{y})=-\varepsilon x^T Px-x^T M^T Mx+ \qquad (53)$$
$$+2\sqrt{2}x^T M^T f_m(\tilde{y})-2\tilde{f}_m^2(\tilde{y})=-\varepsilon x^T Px-$$
$$-[Mx-\sqrt{2}f_m(\tilde{y})]^T[Mx-\sqrt{2}f_m(\tilde{y})]\leq-\varepsilon x^T Px$$

So, if there is P, M and ε which satisfies the relations (48) then $\dot{V}(x)$ is negative.

After the above demonstration the following consequence appears:

Consequence. Let us consider the system (46), where $\tilde{A}$ is a Hurwitz matrix, ($\tilde{A}$, b_r) is stabilized and $f_m(\tilde{y})$ satisfies the sector condition (45) on R^3. Then the system (46) is global absolute stable if the transfer function $F(s)$, given by (49), is strictly positive real.

The third stage. Taking account then we have started from a non-linear system with the linear part ε-stable and a transformation of the control loop is done from Fig. 28, 22, after the above demonstration we may affirm that the following enounce of the circle criterion is valuable, in the case in which the non-linear part has two inputs and one output:

Theorem (*Circle criterion*). The non-linear system with the linear part ε-stable, controllable and observable, with the transfer function $H_{01}(s)$ and the nonlinearity of the fuzzy block satisfying the sector condition (45) is considered. The system is global absolute stable if $H_1(s)$ is Hurwitz and $F(s)$ is strictly positive real.

In the expression (49) of function $F(s)$ we notice that the transfer matrix $H_0(s)$ of the linear part appears with one input variable and two output variables. Using the consequence from the second stage and the fact that $F(s)$ has the expression (49) we may say the non-linear fuzzy control system is absolute stable in the sector [$K_{\min}$, $K_{\max}$] if the hodograph $H_1(j\omega)$ is situated at the right of a vertical straight line which is crossing through the point of the abscise (-1/K, 0), with $K\geq K_{\max}-K_{\min}$.

For circle criterion using the hodograph $H_{01}(j\omega)$ of the linear part is plotting. In Fig. 29 an example of such hodograph is presented [17].

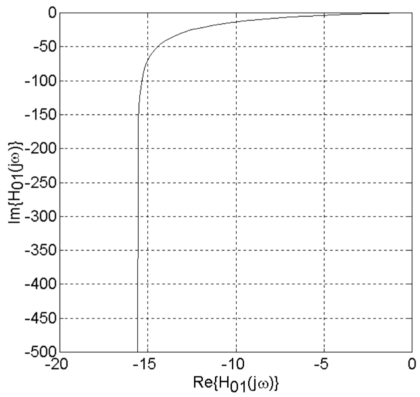

Fig. 29 Example of hodograph for the function $1+KH_{01}(s)$

A detail of this hodograph is presented in Fig. 30.

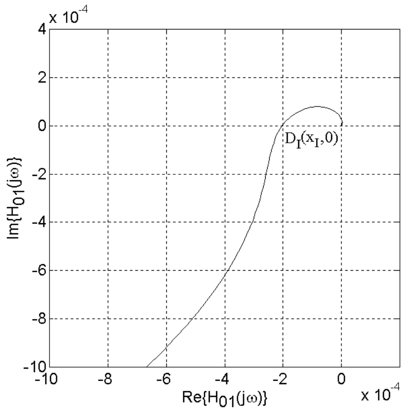

Fig. 30 Detail of the hodograph $H_{01}(j\omega)$

For plotting the hodograph of the linear part a Simulink block diagram of the linear part may be used.

Between the value of K_{max} and K_{min} there is dependence due to the allure of the hodograph of the linear part $H_{01}(j\omega)$. The circles which may be plotted in the plane of the hodograph $H_{01}(j\omega)$ must be placed at the left of the hodograph and they can be at least tangential to the hodograph.

If the correction is not done to move the nonlinearity in the sector [K_{min}, K_{max}] and the nonlinearity is let in the sector [0, K] about the equilibrium point x=0 of the control system based on fuzzy logic we can not say, based on the chosen Lyapunov function, if it is stable or unstable. In the situation when the non-linear part is not corrected there are states of the control systems for which $V(x)>0$ and $\dot{V}(x) \geq 0$, but we cannot say if the system is passing or not from these states in which $V(x)>0$ and $\dot{V}(x) < 0$.

9 Limits of stability sector

In the determination of the domain in which a stability sector may be chosen it must be taken account of the fact that both the linear part and the nonlinear part are imposing constraints to this domain.

An example of the way in which the maximum value of K_{max} and the minimum value of K_{min} depend on the abscise of the tangential point of the circle at the hodograph $Re\{H_{01}(j\omega)\}$ is presented in Fig. 31.

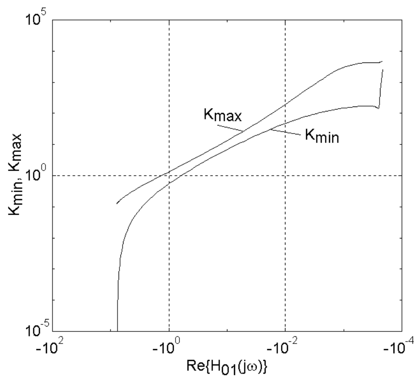

Fig. 31 The extreme values of K_{max} and K_{min} function of the position on the hodograph

Elements of analytical geometry help the determination of the stability sector limits imposed by the hodograph of the linear part.

We may notice that for the points from the hodograph placed closed to the point D_I, characterised by the value of the limit of the Hurwitz sector K_H=2500, the maximum limit of the stability sector is increasing to K_H. The rapport K_{max}/K_{min} between the extreme values has a maximum value of approximate 12, on the part from the crossing point of the hodograph with the real axis. The value of the rapport is decreasing to the central zone of the hodograph. The rapport is increasing at the left of the asymptotic line to the hodograph. The constraint imposed by the non-linear part depends on the value of the correction coefficient K_c and the value of the command coefficient c_{du}.

For the coefficient of the command increment c_{duM} a maximum value may be determined. The maximum value c_{duM} is limited by the control system capacity to furnish the command value to the process. It is determined by the maximum value of the command variable. The value of the product $c_{du}K_M$, for a given value of K_M, specific of a nonlinearity containing a certain fuzzy block give the value of K_{max}. From Fig. 31 it results that the maximum value of K_M is constant function of the correction coefficient K_c. Also from this figure it resulted that the value of K_{mM} is varying with K_c.

To determine a maximum domain in which a stability sector may be chosen a diagram as the example from Fig. 32 may be used.

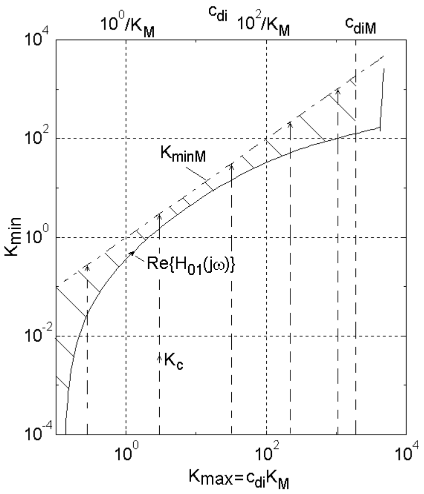

Fig. 32 The domain of the stability sector

In Fig. 32 the characteristic $K_{min}=f(K_{max})$ is presented in logarithmic coordinates, to show in a better image different parts of this characteristics. Also, on the abscise axis we may represent the values of c_{du}, see the superior part, because:

$$c_{du} = K_{\max} / K_M \tag{54}$$

Also, in Fig. 32 the characteristic

$$K_{\min M} = c_{du} K_{mM} \tag{55}$$

is represented, in which K_{mM} depends on K_M. The value of K_M is rising in the same time with K_M. In the presented example the stability sector may be chosen only in the represented domain because only this domain is placed between the characteristic $K_{min}=f(K_{max})$ and the characteristic $K_{min}=f(K_{max})$. The domain is superior limited by the value

$$K_{\max M} = c_{duM} K_M \tag{56}$$

The verification of these observations related to the stability domains may be done in practice by a transient regime analysis for the control system in a free regime (w=0).

10 Design recommendations

For determination of an acceptable dynamic behaviour of the control system in closed loop the values of scaling coefficients c_e, c_{de} and c_{du} may be determined. In the same time, for the correction of the nonlinearity characteristic the value of coefficient K_c may be modify. In the following some indications to chose the parameter c_{du} and K_c, considering the values of the parameters c_{du} and K_c chosen before.

#1. For a certain fuzzy block type the minimum value of K_m and the maximum value of K_M are chosen from the curves families $K_{Nc}=f(\tilde{x}_t)$, or $du_{dc}=f(\tilde{x}_t)$, with $\tilde{de}$ as parameter.

#2. The value of incremental coefficient of the command variable is limited by the capacity of control system to furnish the command variable to the process. The difference of the command variable referred at a sample period h is

$$\Delta u^*_{nk+1} = u^*_{nk+1} - u^*_{nk} \tag{57}$$

The incremental coefficient of the command variable may be determined with the relation that is describing the digital integration (incrementation):

$$c_{du} = \frac{\Delta u^*_{nk+1}}{du_{dk+1}} \tag{58}$$

The maximum value of the command variable cannot overpass:

$$U^*_M = K_{CAN} U_{aM} K_{EM} \tag{59}$$

This value is adimensional. It appears only in theoretical computations.

The maximum value from the output of the fuzzy block may be approximated from the transfer characteristics. As an example, for a *33-mm-g* fuzzy block this value is $du_{dM} = 1{,}4$.

So, at an incremental step, on a sampling period h, for the incremental of the command variable a value greater then

$$c_{duM} = \frac{U^*_M}{du_{dM}} \tag{60}$$

is not recommended. For this maximum value of the incremental coefficient of $c_{duM}.K_M$ may be chosen.

#3. The values of coefficients c_{du} and K_c may be chosen to assure

$$K_{\min} \le c_{du} K_m < c_{du} K_M \le K_{\max} < K_H \quad (61)$$

In practice this option may be done after repeated transient regime analysis, and a value of c_{du} must be chosen to assure an adequate dynamical behaviour.

#4. In the choosing of c_{du} we must take account to the maximum values of K_M of the superior limit of the nonlinearity of the fuzzy block. As an example, for a *33-mm-g* fuzzy block a value of K_M=1,17 results:

$$c_{du} = K_{\max} / K_M \quad (62)$$

This value must be less then the maximum justified value c_{duM}.

#5. The chosen of K_c is done taken account to the rapport $r_k=K_{min}/K_{max}$:

$$K_c = r_k K_M \quad (63)$$

If for K_c it results relative small values the nonlinearity of the fuzzy block do not have great modifications.

We recommend the characteristic family $f_{Nc}(\tilde{x}_t)$ to be in the stability sector so at variations of the controlled process parameters it does not get off the stability sector.

In Fig. 33 an example of the way in which the circle is placed in the following coordinates: x_m=-0,0048 and x_M=-6,416.10^{-4}.

For this circle the stability sector has the limits K_{min}=206,26 and respectively K_{max}=1558,4. The circle centre has the abscise x_c=-0,0027. The distance from the centre to the hodograph has the value d=0,0021. The tangential point of the circle with the hodograph has the coordinates (-0,0016; -0,0017). The rapport between the extreme values of the sector limits has the value r_K=7,5553. For this circle e may approximate c_{du}≅1332 and K_c≅0,1 [17].

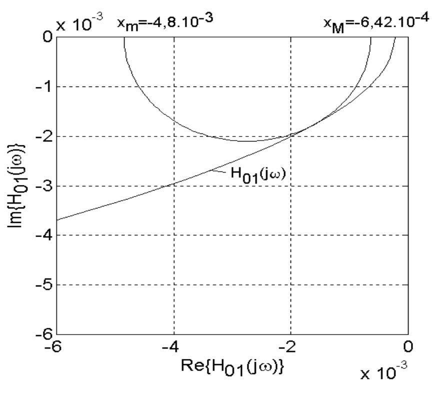

Fig. 33 Example of a circle placed close to $-1/K_H$

In Fig. 34 we present an example of placing of a family of curves $du_{dc}(\tilde{x}_{t1};\ \tilde{de})$ of the corrected nonlinearity N_c in the first and third quadrants, in sector [K_m, K_M], admitted in the procedure of stabilization of the control system bas on fuzzy logic.

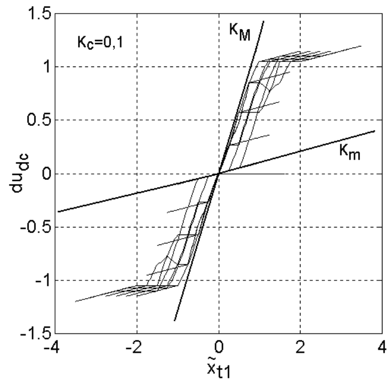

Fig. 34 Framing of characteristics of the nonlinearity N_c in the stability sector

11 Application

In the following paragraph an application of a fuzzy control system for a second order process is presented. The structure of this control system is presented in Fig. 36.

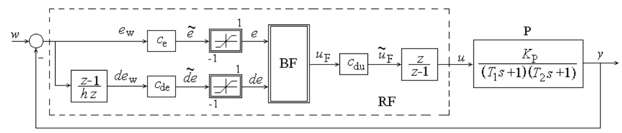

Fig. 35 The diagram of a conventional fuzzy control system of a second order process

A PI fuzzy controller with output integration is used. This incremental fuzzy controller RF is characterised by the utilisation of a fuzzy block BF, which has as input variables the control error e and its derivative de, and as output variable the increment u_F of the control variable u. The fuzzy block *BF* has the classical structure with fuzzyfication, inference on a rule base and defuzzyfication, from Fig. 4. Different rule bases, inference methods and defuzzyfication may be used. The controller commands a process P, and w represents the reference input of the control system and y the output.

The fuzzy block BF is a non-linear element, non-inertial, with two inputs and one output. The input-output characteristics (1, 2, 3, 4, 5) may be used to describe the fuzzy block.

To achieve system stabilisation the non-linear characteristic of the fuzzy block, transformed in a convenient manner, must fulfilled the sector condition. Generally, the fuzzy block BF does not satisfy the sector conditions.

To apply circle's criterion the control system structure from Fig. 35 is transformed in a quasi-continual one, shown in Fig. 36, using the Pade transformation.

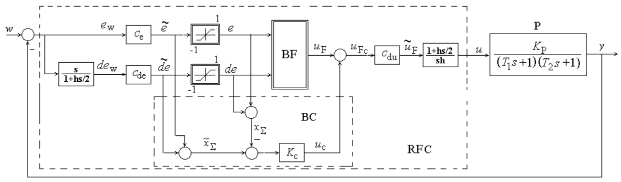

Fig. 36 The quasi-continual control structure

The quasi-continual diagram is then transformed in an adequate one, which matches the standard structure for stability analysis of non-linear systems.

The hodograph of the linear part and an example of circle is presented in Fig. 37.

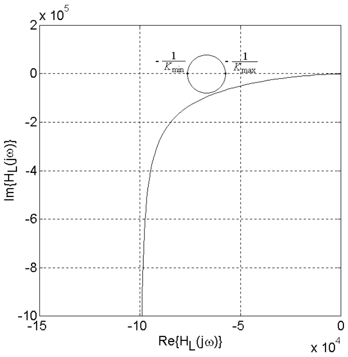

Fig. 37 The hodograph of the linear part and the circle

12 Conclusions

In this paper a method to assure and to analyse internal stability of fuzzy control system, which are using fuzzy PI incremental controllers, is presented.

The stability analysis of a control system with an incremental PI fuzzy controller leads to the conclusion of the theoretical necessity of the foreseen of a parallel correction block. A structure of a quasi fuzzy controller is developed. With the help of this structure absolute internal stability of control system may be demonstrated. It is demonstrating that the structure is absolute stable if the resultant quasi-fuzzy controller with the help of the correction block fulfils some sector conditions. Based on the observations done in stability analysis some indications to calculate controller parameters, to guarantee absolute stability of control system with the quasi fuzzy controller are given.

The fuzzy control system was treated as a nonlinear system, which has a linear subsystem and a nonlinear subsystem. The nonlinear subsystem has the fuzzy block as the main part. The nonlinearity may be seen as placed in a negative feedback, with the role of stabilization of the closed loop fuzzy control system. The linear part is ε-stable.

To assure the stability of the fuzzy control system, which has as a controller a fuzzy PI controller, a first condition may be assured that by introducing of the nonlinearity due to the fuzzy block, the linear part to be stabilized. This condition must be assured for all fuzzy block type used in the controller structure.

The rule bases of the fuzzy controllers may be chosen to assure the linguistic stabilization of the fuzzy control system. The nonlinearity from the control structure, which is containing the fuzzy block, which can be at its turn from a larger class of fuzzy blocks, developed using different fuzzy values, membership functions, rule bases, inference methods and defuzzification methods is framed in a sector by the form $[0, K]$. This property is due to the following causes: 1. Using of membership functions which are introducing saturations at the limits of universe of discourse. 2. Using of saturation elements at the fuzzy block inputs.

We notice that to assure stability the nonlinearity characteristic must be corrected in such a manner that it must be situated in a sector by the form $[K_{min}, K_{max}]$.

The correction of the nonlinear part is made adding on the direct way of the nonlinearity the summation of differences between the nonlimited inputs and the limited inputs, multiplied with a correction coefficient. This correction is a nonlinear one. A second correction, of the control system gain in the open loop, is made with the help of an incremental coefficient of the controller command output.

The paper shows that the absolute internal stability analysis of the fuzzy control system may be framed in the circle criterion for multivariable systems, if the quasi-fuzzy block accomplishes the specific sector condition.

The demonstration is made starting from Lyapunov's absolute stability theory. The way to use the circle's criterion is presented. The stability sector is imposed by the hodograph of the linear part of the fuzzy control system and also by the characteristic of the non-linear part, which includes the fuzzy block.

The sector may be corrected with the help of two correction coefficients. The values of the two correction coefficients may be chosen according to the fuzzy block type, in a way that the nonlinearity characteristic to be placed in the sector [K_{min}, K_{max}], which assures system's absolute global stability. In this paper some indications to choose these coefficients are given. Some indications to choose the limits of the stability sector are given.

An application at the second order process is presented.

Appreciations for internal stability cannot be done for the fuzzy blocks, which are using defuzzification with the mean of maxima method, due to the fact that the nonlinearity of these blocks is not continuous. And their nonlinearity is placed in the sector [0, +∞]. And this aspect make the fuzzy control system unstable due to the integral character of te linear part.

If, for the fuzzy control system the internal stability may be assured, then also the input-output stability may be assured, based to the concept of BIBO stability applied to the multivariable systems.

The fuzzy blocks used in practice present the sector property on their universe of discourse and this method of stability assurance may be used on a large scale.

The correction to the fuzzy blocks may be applied on a large scale also at other control systems based on fuzzy PI controllers. It assures stabilization in the case of a large class of control systems based on fuzzy logic.

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