# Interactive Pareto Optimal with Advantage and Application in Distribution Planning

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*Abstract:* - An interactive pareto optimal with advantage (IPOA) approach for bi-objective programming is proposed and applied on capacitor placement and sizing problem. Two main contradictory concerned which including cost and quality properties are considered as bi-objective programming formulation. The IPOA approach can provide a valuable trade-off pareto-optimal solution by following the intention of decision makers (DMs). Many nonlinear characteristics of distribution feeders, and their load, operating and expansion constraints could be all considered for practical operation. Also, both fixed and switched types of capacitors are included. The effectiveness and feasibility of the proposed approach were demonstrated by an actual feeder study systematically with particle swarm optimization method. The experiment showed encouraging results, suggesting that the proposed approach was capable of efficiently determining higher quality solutions about distribution planning problem.

*Key-Words:* - Interactive pareto optimal with advantage, Capacitor placement and sizing, Particle swarm optimization, Distribution planning, Bi-objective programming.

## 1 Introduction

Capacitors have been widely employed in radial distribution systems for reactive power compensation. The benefits greatly depend on how the capacitors are installed and dispatched in the system. This kind of problem is termed the general capacitor planning problem. It consists of determining the locations, types (fixed or switched) and sizes of capacitors to be installed in the system such that the cost profits and quality conditions of the system are improved considering the load, operating, and expansion constraints. Most previous studies [1-7] formulated the problem with a single objective. Generally, cost is employed as the objective function and the other possible objectives, such as voltage deviation and system capacity, are treated as constraints. However, power quality plays an important role for the loyalty of customers after the deregulation of power system. It is necessary for utilities to take not only the cost but also the quality into consideration.

In the past decade, some evolutionary computational techniques, such as ant colony [2], genetic algorithms (GA) [4, 8-9], simulated annealing (SA) [5, 10] and tabu search (TS) [11] have been widely used to solve power optimization problems. These algorithms are in the form of probabilistic heuristics, with global search properties. Though GA methods have been employed successfully to solve complex optimization problems, recent research has identified deficiencies in GA performance. This degradation in efficiency is apparent in applications with highly epistatic objective functions (i.e., where the parameters being optimized are highly correlated) [the crossover and mutation operations can not ensure improved fitness of offspring because chromosomes in the population have similar structures and their average fitness is high toward the end of the evolutionary process][12]. Moreover, the premature convergence of GA degrades its performance and reduces its search capability, which leads to a higher probability for obtaining a local optimum [13].

Recently, the use of a global optimization technique called particle swarm optimization (PSO) [14]used to solve real world problems have aroused researchers' interest due to its flexibility and efficiency. Limitations regarding the form of the fitness function employed and the continuity of variables used for the classical greedy search technique can be completely eliminated. The PSO, first introduced by Kennedy and Eberhart [15], is one of the modern heuristic algorithms. It was developed through the simulation of a simplified social system and has been found to be robust in solving continuous nonlinear optimization problems.

Therefore, this paper aims to develop a unified approach to solve general capacitor planning problem. A bi-objective formulation combined with PSO for the above problems is presented. In bi-objective problems, the objectives are usually non-commensurable and conflict with each other. Hence, any improvement of one objective may be reached only by the reduction of another. The interactive pareto optimal with advantage (IPOA) method proposed in this paper is a powerful tool, which can provide a flexible best-advantage solution for capacitor planning problem by following the intention of decision makers. Two important objectives are included, one is the cost operation, and the other is the maximum voltage deviation of the system. The load, operating and expansion constraints of the system are considered. Also, the fixed and switched types of capacitors are included for increased realism.

## 2 Problem Formulation

In this section, a bi-objective formulation of the capacitor planning problem is proposed. It aims to simultaneously optimize each objective, while satisfying the equality and inequality constraints given below:

### 2.1 Operating constraints

The voltage magnitude at each bus of each load period has to lie in a permissible range. The current on each branch must stay within its capacity limits for security reasons. Also, the number of capacitors mounted on the buses should be below the total number of installed capacitors. These constraints are expressed as follows.

$$V_i^{\min} \le V_i^k \le V_i^{\max} \quad , i = 1 \sim N_b \quad , k = 1 \sim N_p \tag{1}$$

$$I_i^k \le I_i^{\max} \quad , i = 1 \sim N_l \quad , k = 1 \sim N_p \tag{2}$$

$$N_{c,i}^k \le [N_{f,i} + N_{s,i}] \quad , i = 1 \sim N_b \quad , k = 1 \sim N_p \tag{3}$$

where:

$V_i^k$ voltage at bus i of period k,

$V_i^{\max}$ maximum allowable voltage of bus i,

$V_i^{\min}$ minimum allowable voltage of bus i,

$N_b$ total number of buses,

$N_p$ total number of different load periods for yearly load duration curve,

$N_l$ total number of branches,

$I_i^k$ current at bus i of period k,

$I_i^{\max}$ maximum allowable current of feeder section i,

$N_{c,i}^k$ number of capacitor banks mounted on bus i of period k,

$N_{f,i}$ number of fixed type capacitors installed at bus i,

$N_{s,i}$ number of switched type capacitors installed at bus i.

### 2.2 Expansion constraints

The number of capacitors installed at each bus should be limited due to some practical concerns. For example, it is impossible to install more capacitors if there is not enough space in the buses. These constraints are stated below.

$$[N_{f,i} + N_{s,i}] \le N_{c,i}^{\max} \quad , i = 1 \sim N_b \tag{4}$$

where:

$N_{c,i}^{\max}$ upper limit of installed capacitors at bus i.

### 2.3 Objective functions

The objective functions considered in the study are:

*1). Cost Objective Function:* The cost objective function employed is:

$$C(\overline{S}) = \sum_{i=1}^{N_b} [C_{I,i} \times u(N_{f,i} + N_{s,i}) + C_f N_{f,i} + C_s N_{s,i}] / Y + C_e \sum_{k=1}^{N_p} [T_i P_{loss}^k(\overline{S})] \tag{5}$$

where:

$E(\overline{S})$ annual cost of system under $\overline{S}$ configuration,

$C_e$ energy cost per kWh,

$C_{I,i}$ installation cost at bus i,

$C_f$ fixed type capacitor cost per bank,

$C_s$ switched type capacitor cost per bank,

$T_i$ time duration of the ith load period,

$P_{loss}^k$ total power loss of load period k,

$Y$ average lifetime of capacitors,

$u(.)$ unit step function.

In the right hand side of (5), the first term represents the annual cost of capacitor allocation, with two components: fixed installment cost and purchase cost. Generally, fixed type capacitors serve as the base compensation and they are cheaper than switched type capacitors that are used for additional compensation in different load periods. The second term represents the total annual cost of energy loss, where the energy loss is obtained by summing up the power losses for each load period multiplied by the duration of the load period. In fact, capacitors are grouped into banks of standard discrete capacities. Therefore, capacitor sizes are represented as discrete variables to meet the real situation.

*2). Quality Objective Function:* This objective is concerned with the voltage deviation of the system. Voltage deviation at bus i of period k is defined as:

$$VD_i^k = \left|V_i^{ideal} - V_i^k\right| \quad , i = 1 \sim N_b \, , k = 1 \sim N_p \qquad (6)$$

where:

$V_i^{ideal}$ ideal specific voltage at bus i.

Voltage deviation is important for both the utilities and customers. The more voltage deviation a system has, the shorter the lifetime and the less efficient the operation of any equipment mounted onto the system. Moreover, voltage collapse may arise due to the voltage deviation of some fixed power equipment such as synchronous machines. Hence, voltage deviation in a system represents the quality of the power that the utilities supply to their customers. Electricity quality and cost conditions of its supply are somewhat non-commensurable. To avoid customers' dissatisfaction and to maintain the stability of systems, it is beneficial to tackle the voltage deviation problem as an objective function instead of a constraint. In this paper, we attempt to minimize the maximum term of the voltage deviation of all buses and periods as shown below:

$$Q(\overline{S}) = \max_{i \in N_b \, , k \in N_p} VD_i^k = \max_{i \in N_b \, , k \in N_p} \left|V_i^{ideal} - V_i^k\right| \qquad (7)$$

## 3. The IPOA Approach

Complex real-world decision-making problems are multifarious and their multiple objectives are usually non-commensurable and often in conflict. The ultimate goal in multi-objective optimization is to seek the most preferred solution from the set of pareto optimal solutions. Thus, in this paper, our goal was to develop an efficient interactive solution methodology for generating preferred solutions.

Some literature translates the bi-objective characteristic into a single objective optimization model and, generally, in previous research, two methods have been commonly applied. In the “single objective with constraints” method, the most important item is selected as the main objective function, which is expressed by single objective programming. The other optimization objectives are treated with constraints. Although this method reduces solving difficulty, it does not provide complete programming for problems; thus, solved solutions may not conform to the principle of optimal benefit, especially when objective functions are in conflict with each other. To optimize the main objective function generally leads to the other objective function values being very close to the set constrained value. In other words, solved solutions can only meet the basic requirement of the objective function that is treated as a constraint. For example, suppose that one optimization problem seeks solution $x$ , which can minimize the value of objective functions $O_Q(x)$ and $O_E(x)$ that are conflicting with each other. To apply the single objective with constraints approach, the problem is expressed as follows, where $O_Q(x)$ is selected as the main objective:

Minimize $O_Q(x)$ such that $O_E(x) \le C$ is satisfied.

Because $O_Q(x)$ and $O_E(x)$ conflict with each other, to ensure the minimal value for objective function $O_Q(x)$, the solved solutions may make the value of constraint $O_E(x)$ very close to constraint value $C$, in which case the optimization of $O_E(x)$ may not process properly. Moreover, it is also very difficult to choose ideal value $C$. Suppose that Fig. 1 shows the functions of $O_Q(x)$ and $O_E(x)$ corresponding to $x$. As shown, constraint value $C_1$ is very close to $C_2$. The selection of the constraint value of $O_E(x)$ between $C_1$ and $C_2$ exerts insignificant influence on $O_E(x)$ , but considerable effect on $O_Q(x)$ ; if $C_1$ is selected as the constraint value of $O_E(x)$ , $O_1$ can only be obtained for $O_Q(x)$ even in the best situation. On the other hand, if the constraint value of $O_E(x)$ is $C_2$, the optimal value $O_2$ can be obtained for $O_Q(x)$. It is obvious that there is large difference between $O_1$ and $O_2$ ; thus, constraint value $C_2$ should be much better than $C_1$. However, for ordinary optimization problems, a curve similar to Fig. 1 cannot be obtained in advance, thus it is difficult to select an adequate constraint value $C$.

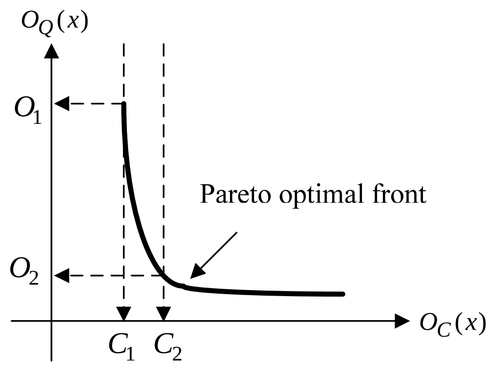


Fig. 1 Relational diagram of bi-objective functions

The other commonly used method for solving bi-objective programming problems is called the "weighting method". It multiplies all objective functions by weighted value and then adds up all the functions, so as to transform bi-objective programming problems into single objective programming problems, as expressed below:

$$T = W_1 \times O_Q(x) + W_2 \times O_E(x) \quad (8)$$

Whether $O_Q(x)$ or $O_E(x)$ is selected, they are regarded as objective functions and are important to users. However, because the units of objective function $O_Q(x)$ and $O_E(x)$ may be different, and there is no direct corresponding transformation relationship, weighted value cannot be used merely to represent the importance of different objective functions. Hence, it is difficult to determine the weighted value ($W_1$, $W_2$) intuitionally.

Based on the above discussion, this paper proposes the IBVT approach which uses a simple interactive method to satisfy users' preferences and obtains the most valuable trade-off solution for the bi-objective function. In the solving process, users do not need to input a weighted value, but only need to make a choice of favorite objective function and the bi-objective problem can be solved smoothly. The main advantage of this method is in providing a larger programming space in the modeling process which is not limited to a single objective programming model. The problems of bi-objective programming can be easily solved so as to provide users with more favorable solutions and a more convenient operating environment.

This paper first applied mathematical theoretical deduction to explain mathematical meaning of the most valuable advantage solution in bi-objective optimization, further construct a complete solving process of suggested IPOA method, and then assess its application value.

## 3.1 Mathematical deduction and explanation

Suppose that bi-objective function problem to be solved is as follows:

$$\text{Minimize } \begin{cases} C(S) \\ Q(S) \end{cases} \text{ and satisfy all constraints.} \quad (9)$$

Generally speaking, typical Pareto optimal front of bi-objective optimization function is shown as the thick solid line in Fig. 2. And $S_C$ and $S_Q$ are two extreme solutions with mere consideration of single objective optimization. $S_C$ is the single objective optimal solution with mere consideration of the objective function $C(S)$, thus corresponding $C_{ideal}$ should be the optimal value of $C(S)$. Because the effect of $Q(S)$ is not considered, and based on reciprocal effect among objective functions, corresponding $Q_{nonideal}$ should be the non-ideal value of $Q(S)$. Contrarily, $S_Q$ is the single objective optimal solution with mere consideration of $Q(S)$ objective function, thus corresponding $Q_{ideal}$ is the optimal value of $Q(S)$. Because the effect of $C(S)$ is not considered, and based on reciprocal effect among objective functions, corresponding $C_{nonideal}$ should be the non-ideal value of $C(S)$.

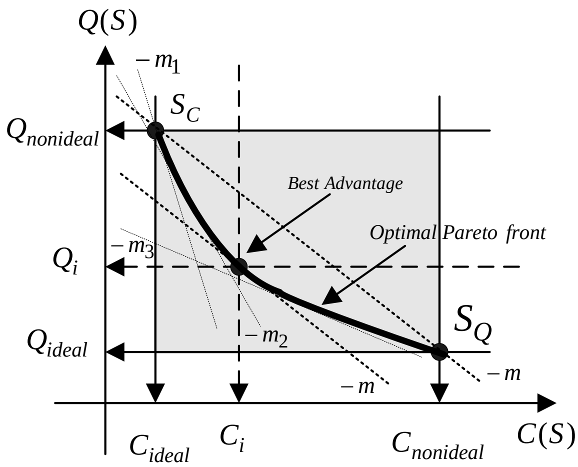


Fig. 2 Typical pareto optimal front of bi-objective optimization problem

The slope of the straight line connecting $S_C$ and $S_Q$ is: $-m = -\dfrac{Q_{nonideal} - Q_{ideal}}{E_{nonideal} - E_{ideal}}$. Here, $m$ represents maximum scope ratio which can be reached in optimization process of two objective functions. When the unit and value ranges of objective functions are different, m also represents the relationship of improvable ratio of two objective functions.

The slope of Pareto optimal close to $S_C$ is $-m_1$, as shown in Fig. 2, and $m_1 > m$. This shows that when solutions are close to $S_C$, and if inferior Pareto optimal solution of objective function $C$ is sought, less ratio concession of objective function $C$ can improve $Q$ significantly because the slope near $S_C$ is $m_1 > m$. The phenomenon still remains when the curve of Pareto optimal front deviates from $S_C$. For example, for the point where curve slope is $-m_2$, as shown in Fig. 2, $m_2 > m$, thus in solving bi-objective functions, continuing to seek more adequate Pareto optimal solutions along the direction of $S_Q$ must be rational and advantageous. When Pareto optimal reaches $S_i$, the slope of the point is $-m$. After Pareto optimal solution passes the point, the slope becomes less than $-m$. For example, the Pareto optimal solution shown in Fig. 2 has slope of $-m_3$, and because $m_3 < m$, the descending Pareto optimal solution will possess less advantage. In other words, more ratio concession of objective function $C$ is needed to make some improvement of $Q$. Therefore, this method attempts to seek the solution $S_i$ with best advantage among many Pareto optimal solutions as reference for users. Among these solutions, the curve slope at $S_i$ is $-m_1$.

Besides, to help users seeking adequate solutions with best advantage according to their own strategies, this method also can apply simple interactive method to provide solutions with best advantage suitable for users, in accordance with users' requirements.

How to find the pareto optimal solution $S_i$? In here we proposed a mathematical minimization programming below :

$$\min\ T(\bar{S}) = \frac{C(\bar{S}) - C_{ideal}}{C_{nonideal} - C_{ideal}} + \frac{Q(\bar{S}) - Q_{ideal}}{Q_{nonideal} - Q_{ideal}} \tag{10}$$

Find $S_i$ such that (10) can be minimized. Assume the pareto optimal front can be represented as a function $Q(C)$, then rewrite (10) as :

$$T(C) = \frac{C - C_{ideal}}{C_{nonideal} - C_{ideal}} + \frac{Q(C) - Q_{ideal}}{Q_{nonideal} - Q_{ideal}}$$

where $E \in E(S_{Pareto})$, (11)

Derivate the both sides of equation (11) and set $T'(C)$ to zero to get the minimized valve;

$$T'(C) = \frac{1}{C_{nonideal} - C_{ideal}} + \frac{Q'(C)}{Q_{nonideal} - Q_{ideal}} = 0 \tag{12}$$

Rearrange (12), then we can get:

$$Q'(C) = -\frac{Q_{nonideal} - Q_{ideal}}{C_{nonideal} - C_{ideal}} = -m \tag{13}$$

Therefore, the solution of (10) represents the best advantage point shown $S_i$ in Fig. 2.

## 3.2 Solution procedure of IPOA

According to the principle and discussion above, this section will explain the solving process of suggested IPOA method, and divide the process into 3 steps as follows:

*Step 1*

First, the optimal solution of single objective function is obtained one by one. Therefore, the $S_C$ and $S_Q$ can both be got and the relative values $C_{ideal}$, $C_{nonideal}$, $Q_{ideal}$ and $Q_{nonideal}$ are found.

*Step 2*

Find the optimal solution of (10) subject to following constraints:

$$\begin{cases} Eqs.(1) \sim (4) \\ C_{ideal} \le C(\bar{S}) \le C_{nonideal} \\ Q_{ideal} \le Q(\bar{S}) \le Q_{nonideal} \end{cases}$$

Suppose that $S_i$, solved in Eq.(10) and $S_i$, $C_i$ and $Q_i$ are shown in Fig. 3.

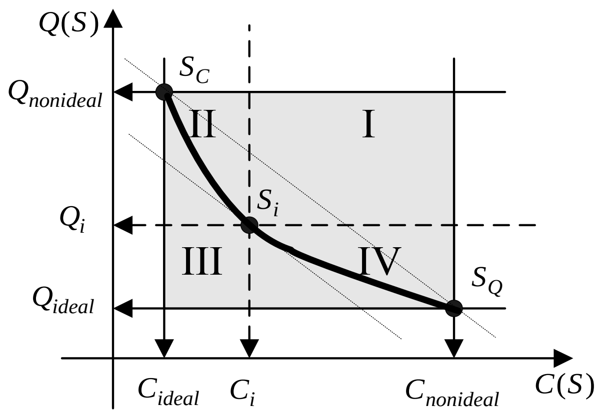


Fig. 3 Solution space of IPOA after $S_i$ is solved.

$S_i$ can divide the shadow part in Fig. 3 into areas I, II, III and IV. If $S_i$ is the optimal value of (10), no more other solutions S can be obtained to generate combinations of $C(S)$ and $Q(S)$ in area III. Therefore, the area can be called "unreachable solution space". Contrarily, solutions of $C(S)$ or $Q(S)$ in area 1 are all inferior to $S_i$, thus there is no need to consider this area. Compared with the value T of objective function of (10), though the values of area II are inferior to $S_i$. The area possesses one characteristic, namely, $C(S)$ is superior to $C_i$ obtained by $S_i$. But $Q(S)$ is always inferior to $Q_i$. Similar situations also occur in area IV; and the characteristic can help users searching the solutions they want. This part is detailed in Step 3.

*Step 3*
To reach interactive relation with users, if system decision-maker is not satisfied with $S_i$ in Step 2, one may sacrifice one objective function to improve another objective function as follows:
If it is intended to further improve $C(S)$, $C_i$ obtained in Step 2 is regarded as new $C_{nonideal}$, and $Q_i$ is considered as new $Q_{ideal}$, as shown in (14). Thus at the time, the corresponding solution space is shown in Fig. 4:

$$\begin{cases} C_{nonideal} = C_i \\ Q_{ideal} = Q_i \end{cases} \quad (14)$$

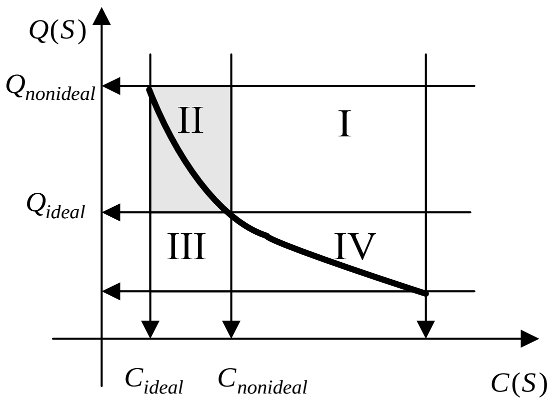


Fig. 4 Solution space of IPOA method after (14).

Contrarily, if it is intended to further improve $Q(S)$, $Q_i$ obtained in Step 2 is regarded as new $Q_{nonideal}$, and $C_i$ is considered as new $C_{ideal}$. Thus the corresponding solution space is within area IV.
If users decide to further improve $C(S)$, the new solved solution with best advantage is $S_i$ shown in Fig. 5.

As shown from the above discussion, solving direction conforms to users' requirements. Users only need to input objective function intended to be improved, and can find out an optimal comprised solution meeting requirement, without considering weighted value. As shown in Fig. 3-5, the whole space of feasible solution can automatically reduce its range according to the requirement of system decision-maker when search is done each time, thus the space turns to the direction appointed by system decision-maker gradually. Because each search may reduce the space of feasible solutions, $Dis\_C$ and $Dis\_Q$ provided in the research are shown in (15), as reference to system decision-maker on determining the next search.

$$\begin{cases} Dis\_C = C_{nonideal} - C_{ideal} \\ Dis\_Q = Q_{nonideal} - Q_{ideal} \end{cases} \quad (15)$$

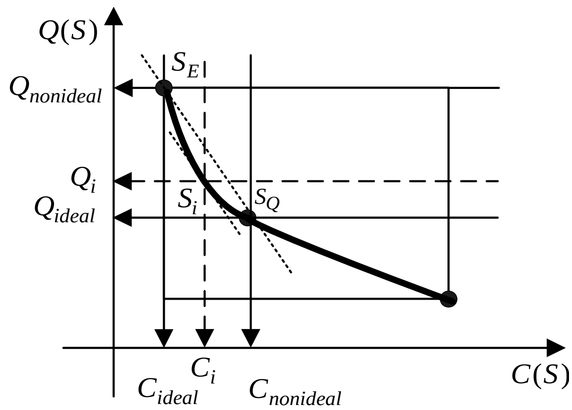


Fig. 5 Solution of IPOA after users decide to further improve $C(S)$

As shown in Fig. 3-5, $Dis\_C$ and $Dis\_Q$ in Eq.(15) represent the ranges of $E(S)$ and $Q(S)$ in space of feasible solution, respectively. If $Dis\_E$ and $Dis\_Q$ are too small, there will be no big change in the next search, thus it is unnecessary to do next search. Contrarily, if $Dis\_E$ and $Dis\_Q$ are too big, this reveals that there will be big change in the next search, thus it is necessary to do the next search.
These 3 steps stated above constitute the IPOA method. It is obvious that in the whole process, users only need to decide the subsequent solution direction based on optimal comprised solutions provided by the solution method, and it is not necessary to consider the selection of weighted value and unit difference of objective functions. Thus, the work load of system decision-makers would be reduced, human errors can be avoided, and solution efficiency can be effectively increased. Fig. 6 is the complete flow chart of IPOA method in bi-objective programming.

# 4. Implementation
In this section, the PSO is used as the tool to find the IPOA formulation of capacitor planning problem as stated detail below:

## 4.1 Representation of individual string
Implementation of a problem in the PSO framework starts from the parameter encoding, i.e., the representation of the problem. In this study, integer representation is chosen for each particle. The individual string structure is represented in Fig. 7.

The parameter $N_{c,i}^{k}$ describes the number of capacitor banks mounted on bus i of period k, as defined previously. The value of each chromosomes' position should be limited so that they are not violating the expansion constraints $N_{c,i}^{\max}$. The value of each particle should be limited to 6 so that they are feasible solution. In the initial process, a random number from 1 to 6 will be generated to create the first positions of each individual.

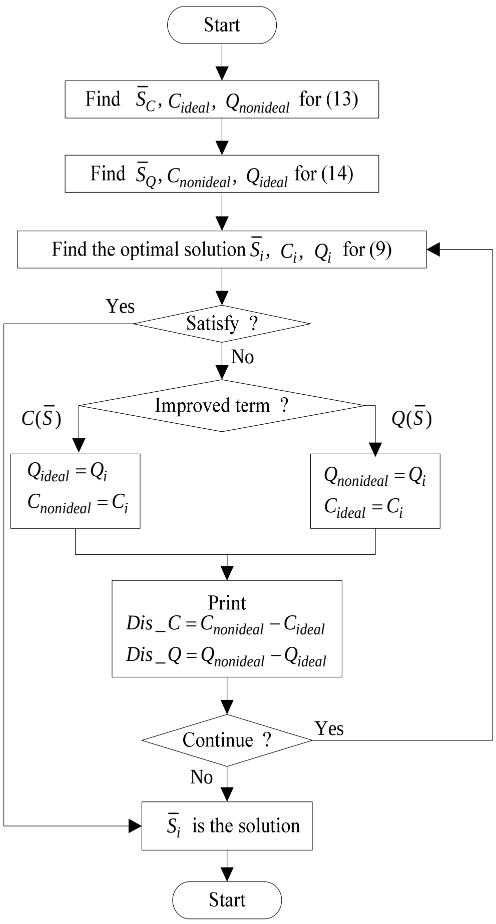


Fig. 6 The flowchart of IPOA method.

| | | | | |
|---|---|---|---|---|
| $N_{c,1}^{1}$ | $N_{c,2}^{1}$ | ............ | $N_{c,N_b-1}^{1}$ | $N_{c,N_b}^{1}$ |
| . | . | .............. | . | . |
| . | . | .............. | . | . |
| $N_{c,1}^{N_p}$ | $N_{c,2}^{N_p}$ | .............. | $N_{c,N_b-1}^{N_p}$ | $N_{c,N_b}^{N_p}$ |

Fig. 7 The individual string structure.

### 4.2 Evaluation function

Implementation of an optimization problem in PSO is realized within the evolutionary process of an evaluation function. The function adopted is given below.

$$Fitness\,(\bar{S}) = Const - \{\, Obj(\bar{S}) + Penalty \,\} \qquad (16)$$

where:

$Const$ a constant to convert minimized problem to maximum one,

$Obj()$ the objective function,

$Penalty$ a penalty term. If any constraint is violated. then the penalty will be set to 1.5, otherwise 1 is instead.

### 4.3 Parameter selection and convergence criterion

If one of the following conditions is met, the PSO process is considered converged.

(i). After 50 consecutive iterations, the best solution does not change.

(ii). The total iterations exceed the upper limit of 10000.

## 5. Test Study

To illustrate the performance of the proposed solution methodology, consider a practical 12-bus, 11.4 kV distribution feeder, as shown in Fig. 8, that is a portion of the Taiwan Power Company's distribution system. The line and load date of the test system are shown in Table 1. Also, the load duration data for the test system is shown in Table 2. The parameters that are the average values according to the real conditions in Taiwan are shown in Table 3 and each bank of capacitors is 300 kVar. The satisfaction rates for each objective are defined in (17). It represents the level of satisfaction within the attainable search region for each objective.

$$SA\begin{Bmatrix} C \\ Q \end{Bmatrix} = \begin{Bmatrix} SA_C \\ SA_Q \end{Bmatrix} = \frac{\max\begin{Bmatrix} C \\ Q \end{Bmatrix} - \begin{Bmatrix} C \\ Q \end{Bmatrix}}{\max\begin{Bmatrix} C \\ Q \end{Bmatrix} - \min\begin{Bmatrix} C \\ Q \end{Bmatrix}} \qquad (17)$$

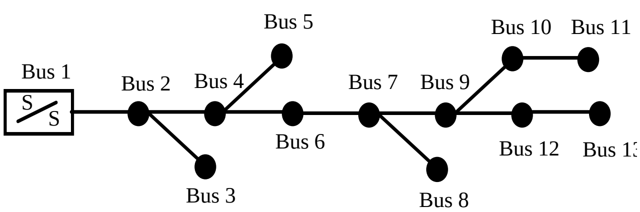


Fig. 8 The 12-bus distribution test system.

Table 1 Line and load date of the 12-bus Tai-power distribution feeder.

| Br. No. | Sd. Bus | Rv. Bus | Br. Parameter r(Ω) | Br. Parameter x(Ω) | Rv. Bus Load P(kW) | Rv. Bus Load Q(kvar) |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 0.2784 | 0.4437 | 124.8 | 127.2 |
| 2 | 2 | 3 | 0.0753 | 0.1200 | 184.8 | 163.2 |
| 3 | 2 | 4 | 0.4440 | 0.7074 | 302.4 | 308.4 |
| 4 | 4 | 5 | 0.1131 | 0.1800 | 99.6 | 87.6 |
| 5 | 4 | 6 | 0.1131 | 0.1800 | 319.2 | 326.4 |
| 6 | 6 | 7 | 0.2259 | 0.3597 | 216.0 | 190.8 |
| 7 | 7 | 8 | 0.2709 | 0.4317 | 1020.0 | 632.4 |
| 8 | 7 | 9 | 0.1506 | 0.2400 | 252.0 | 258.0 |
| 9 | 9 | 10 | 0.3630 | 0.5757 | 562.8 | 574.8 |
| 10 | 10 | 11 | 0.4818 | 0.7674 | 1214.4 | 753.6 |
| 11 | 9 | 12 | 0.5268 | 0.8394 | 886.8 | 783.6 |
| 12 | 12 | 13 | 0.2634 | 0.4197 | 672.0 | 686.4 |

Table 2 Load duration data for the Tai-power distribution feeder.

| Load levels (p.u.) | | | Time intervals (h) | | |
|---|---|---|---|---|---|
| Heavy | Normal | Light | Heavy | Normal | Light |
| 1.2 | 0.8 | 0.6 | 1200 | 6560 | 1000 |

The test results are summarized in Table 4, where the symbols F (fixed), or S (switched), represent the capacitor type, and H (heavy), N (normal), L (light), indicate the various load levels. The digits before the capacitor type and load level indicate the number of capacitors installed and the number mounted during different load levels, respectively. The second column represents the performance of the system before the capacitors were installed. Obviously, the voltage constraint is violated and the compensating capacitors are needed. The third and fourth columns each correspond to a single objective programming that minimizes voltage deviation and cost respectively. Columns five to seven show the results of the proposed bi-objective solution procedures that consider both cost and voltage deviation. In fourth column, to achieve the minimum cost, the minimum voltage is only 0.920, which is almost on the feasible margin (0.92~1.05). The fifth column represents the first result of the proposed algorithm. Comparing the fifth column and the fourth column, it is obvious that $SA_E$ has degraded slightly from 100% to 82.95% but $SA_Q$ has greatly improved from 0% to 75.11%. Generally, it is beneficial to perform such an investment.

The suitability of result 1 should be judged by the DMs of electricity utilities. If the DMs think that result 1 is not suitable for the policy of the utilities, then further compromise can be made according to the directions dictated by DMs. Unlike other approaches that indicate many unknown parameters such as weight values for further search, the DMs only have to choose one of the objectives (cost or voltage deviation) as the compromised term and then the proposed method can find a best-compromise and desirable solution for the bi-objective problem. Assume the DMs think that the power quality of result 1 should be further improved and decide to spend more money to reduce the voltage deviation of the system. The parameter of $Q_{nonideal}$ in result 1 is then changed, because further improvement of voltage deviation is needed. Similarly, the $E_{ideal}$ is also changed, because further compromise will be made on the cost. Note that the decision region is changed simultaneously with the ideal and nonideal values of both $E$ and $Q$ such that it shifts toward the region of interest as indicated by DMs. The values $Dis\_E$ and $Dis\_Q$ can help the DMs to understand the maximum improvement that a further step can achieve. If the DMs think that the maximum improvement in the desired term is too small to make further searching worthwhile, then they can stop the process. Assuming that the further step is allowed by the DMs, result 2 shows the consecutive result. Again, it is a flexible best-compromise solution within the decision region. However, if the DMs thought that the cost should be more reduced, similar procedure can be easy applied and the results shown in Result 1' and 2'.

The same procedure can be repeated again as shown in result 3. Gradually, the decision region will become smaller and focused on the intention of the DMs.

## 6. Conclusion

A bi-objective formulation for the general capacitor planning problem has been successfully applied. The objectives include both concerns of cost and quality. To get a more realistic solution, the load, operating, and expansion constraints of the system, together with the fixed and switched types of capacitors are considered. The IPOA method for solving general bi-objective optimization problems has been presented and tested on a real system. The results show that the proposed methodology can find a flexible best-compromise solution to be dictated by the DMs of the utilities.

## 7. Acknowledgment

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Table 3. Parameters for the study system.

| $V_i^{ideal}$ | $C_e$ | $C_{I,i}$ | $C_f$ | $C_s$ | $V_i^{\min}$ | $V_i^{\max}$ | $N_{c,i}^{\max}$ | $Y$ |
|---|---|---|---|---|---|---|---|---|
| 1 p.u. | 1.8NT/kWh | 387278 NT | 43362 NT/bank | 66241 NT/bank | 0.92 p.u. | 1.05 p.u. | 6 | 7 years |

Table 4 : Numerical results.

| | Original | Single objective | | Bi-objective programming | | | | |
|---|---|---|---|---|---|---|---|---|
| | system | Min $Q$ | Min $E$ | Result 1 | Result 2 | Result 3 | Result 1' | Result 2' |
| $E(\overline{S})$ (NT/year) | 6209146 | 6284417 | 3934274 | 4335098 | 5137890 | 4595468 | 4335098 | 4148589 |
| $Q(\overline{S})$ (V) | 1520 | 13 | 913 | 237 | 67 | 138 | 237 | 367 |
| $\min V_i^k$ (p.u.) | 0.867 | 0.999 | 0.920 | 0.979 | 0.994 | 0.988 | 0.979 | 0.968 |
| $SA_E$ (%) | ------------ | 0 | 100 | 82.95 | 48.79 | 71.87 | 82.95 | 90.88 |
| $SA_Q$ (%) | ------------ | 100 | 0 | 75.11 | 94.00 | 86.11 | 75.11 | 60.67 |
| Satisfy ? | ------------ | ------------ | No | No | No | Yes | No | Yes |
| Compromised term ? | ------------ | ------------ | $E(\overline{S})$ | $E(\overline{S})$ | $Q(\overline{S})$ | ------------ | $Q(\overline{S})$ | ------------ |
| $E_{ideal}$ | ------------ | ------------ | 3934274 | 4335098 | 4335098 | ------------ | 3934274 | ------------ |
| $E_{nonideal}$ | ------------ | ------------ | 6284417 | 6284417 | 5137890 | ------------ | 4335098 | ------------ |
| $Q_{ideal}$ | ------------ | ------------ | 13 | 13 | 67 | ------------ | 237 | ------------ |
| $Q_{nonideal}$ | ------------ | ------------ | 913 | 237 | 237 | ------------ | 913 | ------------ |
| $Dis\_E$ | ------------ | ------------ | 2350143 | 1949319 | 802792 | ------------ | 400824 | ------------ |
| $Dis\_Q$ | ------------ | ------------ | 900 | 224 | 170 | ------------ | 676 | ------------ |
| Continue ? | ------------ | ------------ | Yes | Yes | Yes | ------------ | Yes | ------------ |
| Bus 2 | None | 2F, 2S<br>4H, 3N, 2L | None | None | None | None | None | None |
| Bus 3 | None | 1F, 2S<br>3H, 1N, 1L | None | None | None | None | None | None |
| Bus 4 | None | 1F, 0S<br>1H, 1N, 1L | None | None | None | None | None | None |
| Bus 5 | None | 1F, 0S<br>1H, 1N, 1L | None | None | None | None | None | None |
| Bus 6 | None | 1F, 0S<br>1H, 1N, 1L | None | 0F, 5S<br>5H, 0N, 0L | 3F, 0S<br>3H, 3N, 3L | None | 0F, 5S<br>5H, 0N, 0L | 0F, 5S<br>5H, 0N, 0L |
| Bus 7 | None | 1F, 4S<br>5H, 1N, 1L | 3F, 1S<br>4H, 3N, 3L | None | 3F, 0S<br>2H, 0N, 3L | None | None | None |
| Bus 8 | None | 2F, 4S<br>6H, 2N, 3L | None | 3F, 3S<br>6H, 4N, 3L | 0F, 5S<br>4H, 5N, 0L | 0F, 6S<br>6H, 4N, 0L | 3F, 3S<br>6H, 4N, 3L | None |
| Bus 9 | None | 1F, 2S<br>1H, 3N, 1L | None | None | 0F, 4S<br>4H, 0N, 0L | None | None | 3F, 2S<br>5H, 4N, 3L |
| Bus 10 | None | 2F, 4S<br>2H, 6N, 4L | 0F, 5S<br>5H, 4N, 0L | 0F, 4S<br>4H, 1N, 0L | 0F, 5S<br>5H, 1N, 0L | 3F, 2S<br>5H, 3N, 3L | 0F, 4S<br>4H, 1N, 0L | None |
| Bus 11 | None | 1F, 5S<br>6H, 1N, 1L | None | 3F, 3S<br>6H, 3N, 3L | 4F, 0S<br>4H, 4N, 4L | 2F, 4S<br>6H, 2N, 3L | 3F, 3S<br>6H, 3N, 3L | 4F, 2S<br>6H, 3N, 2L |
| Bus 12 | None | 2F, 3S<br>5H, 3N, 2L | 5F, 0S<br>5H, 5N, 5L | None | 0F, 4S<br>4H, 0N, 1L | 0F, 6S<br>6H, 1N, 0L | None | None |
| Bus 13 | None | 2F, 3S<br>5H, 3N, 2L | None | 5F, 1S<br>6H, 6N, 5L | 5F, 1S<br>5H, 6N, 5L | 5F, 1S<br>5H, 6N, 5L | 5F, 1S<br>6H, 6N, 5L | 4F, 2S<br>6H, 5N, 4L |

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