On the Robustness of the Slotine-Li and the FPT/SVD-based Adaptive Controllers

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Abstract: - A comparative study concerning the robustness of a novel, *Fixed Point Transformations/Singular Value Decomposition* (*FPT/SVD*)-based adaptive controller and the *Slotine-Li* (*S&L*) approach is given by numerical simulations using a three degree of freedom paradigm of typical Classical Mechanical systems, the cart + double pendulum. The effects of the imprecision of the available dynamical model, presence of dynamic friction at the axles of the drives, and the existence of external disturbance forces unknown and not modeled by the controller are considered. While the Slotine-Li approach tries to identify the parameters of the formally precise, available analytical model of the controlled system with the implicit assumption that the generalized forces are precisely known, the novel one makes do with a very rough, affine form and a formally more precise approximate model of that system, and uses temporal observations of its desired vs. realized responses. Furthermore, it does not assume the lack of unknown perturbations caused either by internal friction and/or external disturbances. Its another advantage is that it needs the execution of the SVD as a relatively time-consuming operation on a grid of a rough system-model only one time, before the commencement of the control cycle within which it works only with simple computations. The simulation examples exemplify the superiority of the FPT/SVD-based control that otherwise has the deficiency that it can get out of the region of its convergence. Therefore its design and use needs preliminary simulation investigations. However, the simulations also exemplify that its convergence can be guaranteed for various practical purposes.

Key-Words: - Fixed Point Transformation, Singular Value Decomposition, Slotine-Li Robot Control, Adaptive Control, Robustness Analysis, Complete Stability, Lyapunov Function, Sliding Mode/Variable Structure Controllers

1 Introduction

The adaptive robot control developed by Slotine and Li is a *classical adaptive solution in robot control* literature [1]. It utilizes *very subtle details* of the structurally and formally exact analytical model of the robot in each step of the control cycle in which *only the exact values of the parameters are unknown or known only with very rough approximation*. The application of this very sophisticated approach requires the precise calculation a lot of complicated analytical expressions within each control step that may require quite considerable computational time. Furthermore, this method has the drawback that it is apt to apply false compensation for the unknown perturbations that may originate either from internal friction between the relatively moving components of the robot or/and by external disturbances both unmodeled by the controller.

Due to the complications related to the Slotine-Li controller in robotics, and in other fields of nonlinear control, alternative approaches are sought even in these days. A popular model-based approach is the use of *Model Predictive Controllers (MPCs)* (e.g. [2]) applied within the frames of the *Receding Horizon Control (RHC).* Typical field of application is chemistry in the control of relatively slow processes in which satisfactory time is available for the computations (e.g. [3]). However, via restricting ourselves to the use of very special goal functions, model linearization along special phase trajectories at the present technological level MPC/RHC became applicable even in the control of *Classical Mechanical Systems* that normally require speedy control actions. It can be a proper approximation for modeling special human gaits as running or walking for which *Linear Time Invariant (LTI)* or *Linear Time Varying (LTV)* models can be derived that can be handled by some improved MATLAB functions based on static memory variables in order to evade the unpredictable time consumption of the "originally used", *Dynamically Linked Libraries (DLL)*. Such a system can be used for the control of powered lower limb prosthetic [4]. However, from the special restrictions functional relationships are established between the behavior (stability) of the solution and the horizon-length [5] that not always is advantageous. In general MPC/RHC may lead to hard computations needing exact solution to quite complex nonlinear tasks as the solution of non-convex optimization problems, in limited time. A suboptimal min-max MPC scheme was recently proposed for nonlinear discrete-time systems subjected to constraints and disturbances in general without considering a particular physical system in [6]. A possible alternative approach is the use of *Fuzzy Logic (FL)* in the control. For scalar input and output systems adaptive fuzzy logic was used to approximate the unknown dynamics in [7]. It was exploited that the linear structure of a Takagi-Sugeno fuzzy system with constant conclusion was applicable for the design of an indirect adaptive fuzzy controller. The problem class here considered also had strong restrictions and showed similarities with structures for which global linearization could be applied. In [8] the global linearization approach was extensively used by applying the Lie-derivatives in the *Adaptive Model Reference Fuzzy Controllers (AMRFC)*, too. In [9] stable direct and indirect decentralized adaptive fuzzy controllers were proposed for a class of large-scale nonlinear systems having strong internal coupling. The systems considered are of different order, and seem to have a structure for which global linearization is applicable. Normally, Classical Mechanical systems behave accordingly, but other systems as e.g. wings coupled by flowing air in a wind channel [10] may satisfy similar restrictions. In general it can be stated that the above mentioned problem tackling methods need complicated proofs and are not very lucid for practical applications. This fact motivated a search for even simpler and more lucid, geometrically interpretable approaches.

The novel adaptive nonlinear control approaches recently developed at *Budapest Tech* for "*Multiple Input-Multiple Output (MIMO) Systems*" (e.g. [11, 12]) were based on simple geometric considerations taking into account the *positive definite nature of the inertia matrix of the robots* in the construction of convergent iterations obtained from *Fixed Point Transformations (FPT)*. For instance centralized and decentralized adaptive control of approximately and partly modeled coupled cart plus double pendulum systems in [13], and adaptive control of a polymerization process were considered on similar basis [14]. In a newer version of this approach the method of the *Singular Value Decomposition (SVD)*" (e.g. [15, 16]) was applied for dropping the requirement of positive definiteness of the controlled system [17]. This extension of the control possibilities is important in robotics in which "non positive definite behavior" can occur when certain axles [*Degrees Of Freedom (DOF)*] are driven by the drives of another ones via the dynamic coupling between them [18].

The main difference between the Slotine-Li and the FPT/SVD-based approaches is that while the proof of the asymptotic stability and convergence to an exact trajectory tracking of the Slotine- Li control is based on "Lyapunov's 2nd Method" [19, 20], in the new approach the control task is formulated as a "*Fixed Point Problem*" for the solution of which a *Contractive Mapping* is created that generates an *Iterative Cauchy Sequence*. Consequently it converges to the fixed point that is the solution of the control task. Besides its using very subtle analytical details, the main drawback of the Slotine-Li approach is that it assumes that the generalized forces acting on the controlled system are exactly known and are equal to that exerted by its drives. So unknown external perturbations can disturb the operation of this sophisticated method. In contrast to that, in the novel method the computationally relatively costly SVD operation on the formally almost exact model need not to be done within each control cycle. Depending on the variation of the inertia data of the system along/in the neighborhood of the nominal trajectory it has to be done either only one times in a more or less arbitrarily chosen point of the configurational space, or in a few "grid points" representing typical segments of this space, before the control action is initiated. Within the control cycle the inertia matrix is modeled only by a simple scalar. For obtaining the other parameters of the control the resulting matrices of the SVD in the

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case of using a single point, or simply their weighted linear combination in the case of using a grid, can be utilized. In the present paper this latter solution was chosen that practically corresponds to the approximation via *Radial Basis Function Networks (RBFN)* (more details about RBFN can be found in [21, 22]). As a geometric interpretation of the use of RBFNs in a 3D real space approximation of a 2D surface defined over a 2D plane by spanning a deformable tent-cloth over various masts of different heights and locations can be imagined. Though from mathematical point of view this approximation is "*smooth*" (i.e. it is everywhere differentiable), it approximates the surface only in a "*wavy*" manner that is quite satisfactory in the case of many practical situations in which neither very precise approximation is needed and nor the derivatives of the approximated functions are used. From this special point of view our approach completely corresponds to this latter case.

To illustrate the usability of the proposed method adaptive control of a Classical Mechanical paradigm, a cart plus double pendulum system is considered and discussed by the use of simulation results. It is assumed that the axles of this system suffer from friction unknown by the controllers. For modeling friction phenomena a dynamic approach (e.g. [24, 25]) is used in the simulations.

The paper is structured as follows: at first the comparison of the basic principles of the adaptive methods considered are given in general. Following that the dynamic model of the cart + double pendulum system is discussed. Then simulation results, and finally the conclusions are presented.

2 Comparison of the Adaptive Approaches Investigated

In this section the fundamental characteristics of the Slotine--Li adaptive control and that of the novel approach are compared to each other. The adaptive version of the Slotine-Li control [1] strongly utilizes that the Lagrangian of a robot of open kinematic chain has the form as follows:

$$L=\frac{1}{2}\sum_{i,j}H_{ij}(\mathbf{q})\dot{q}_i\dot{q}_j-V(\mathbf{q}) \qquad (1)$$

where $\mathbf{q}\in\Re^n$ denotes the *generalized coordinates*, $H_{ij}=H_{ji}$ is the symmetric inertia matrix, and $V(\mathbf{q})$ is the potential energy of the robot. Via analyzing the symmetries in the terms obtained in the Euler-Lagrange equations by substituting the above expression into the appropriate operations they arrived at the conclusion that the equations of motion have the general form as

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})=\mathbf{Q} \qquad (2)$$

in which the ingenious idea is also incorporated that though the originally obtained expressions are quite symmetric in the positions of the components that are quadratic in $\dot{q}_i$, they can be treated in an "asymmetric" manner by including them partly in the matrix $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$, too. This decomposition is unambiguous since $\mathbf{C}$ must be linear in the $\dot{q}_i$ components. The term $\mathbf{g}(\mathbf{q})$ originates from the gravitation. The second great idea in this approach is that the available, formally exact but numerically inexact model marked by the caret (^) symbol can be used for asymmetrically calculating a feedback force that contains PD-type terms plus an additional one that is similar to the "*Error Metrics*" $\mathbf{S}=\dot{\mathbf{e}}+\mathbf{\Lambda e}$ (the tracking error is $\mathbf{q}$-$\mathbf{q}^N$, $\mathbf{q}$ and $\mathbf{q}^N$ denote the *actual* and the *nominal* coordinates in the given time instant, respectively) normally used in the robust "*Variable Structure / Sliding Mode (VS/SM)*" controllers (e.g. [26]). In the case of higher order systems $\mathbf{S}$ can be defined with constant $\mathbf{\Lambda}$ as $\mathbf{S}:=(d/dt+\mathbf{\Lambda})^{m-1}\mathbf{e}$ where the positive integer m is the order of the system. On the basis of the available rough model normally strong torque/force overestimation is applied to drive $\mathbf{S}$ to the vicinity of $\mathbf{0}$ during finite time by approximating some simple differential equation prescribed for $d\mathbf{S}/dt$ necessarily containing $d^m\mathbf{e}/dt^m$ that normally can physically be manipulated in the case of an m^{th} order system. The precise realization of this differential equation has no practical significance: as soon as $\mathbf{S}$ approximates $\mathbf{0}$, due to its definition its various, decreasing order derivatives have to converge to zero, too, so finally $\mathbf{e}$ itself must converge to $\mathbf{0}$ with characteristic exponents determined by $\mathbf{\Lambda}$. (The torque/force overestimations normally cause some chattering.) Essentially the same philosophy is applied when using Lyapunov functions in control technology. Normally it is satisfactory to guarantee only that the time-derivative of the Lyapunov function is zero or negative. It is not necessary to precisely prescribe this derivative, therefore some approximate model/information on the controlled system can suitably work. Therefore Lyapunov's 2[nd] method is extensively used in robust control (e.g. [27]). On this reason the robust VS/SM controllers are used in solving various problems in control and optimization fields [28]. In the Slotine-Li control $\mathbf{S}$ is used in the following manner

$$\begin{aligned}\mathbf{Q}&=\hat{\mathbf{H}}(\mathbf{q})\dot{\mathbf{v}}+\hat{\mathbf{C}}\mathbf{v}+\hat{\mathbf{g}}-\mathbf{K}_D[\dot{\mathbf{e}}+\mathbf{\Lambda e}]\equiv\\&\equiv\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\mathbf{v},\dot{\mathbf{v}})\hat{\mathbf{p}}-\mathbf{K}_D[\dot{\mathbf{e}}+\mathbf{\Lambda e}]\end{aligned} \qquad (3)$$

that corresponds to a 2nd order system. In (3) the term $\mathbf{v} = \dot{\mathbf{q}}^N - \mathbf{\Lambda e}$ is used for tracking correction, $\mathbf{K}_D$ is a positive definite symmetric matrix. The array $\mathbf{Y}$ is precisely known if the precise kinematical data and model of the robot are available, the *vector of the estimated dynamic parameters* is denoted by $\hat{\mathbf{p}}$. Since it is assumed that the so calculated $\mathbf{Q}$ is the only contribution to the generalized forces and no additional external perturbations are present, it also is related to the actual state propagation of the system as given in (2) that is a kind of "weak point" of this sophisticated approach leading to the equation

$$\mathbf{H\dot{S}} + \mathbf{CS} + \mathbf{K}_D\mathbf{S} = \tilde{\mathbf{H}}\dot{\mathbf{v}} + \tilde{\mathbf{C}}\mathbf{v} + \tilde{\mathbf{g}} = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{v}, \dot{\mathbf{v}})\tilde{\mathbf{p}} \quad (4)$$

in which $\mathbf{Y}$ is well known from the formally exact system model, and $\tilde{\mathbf{p}}$ denotes the error in the parameter vector. The third excellent trick applied in the Slotine-Li approach is that instead of introducing a more or less "arbitrary" positive definite matrix for constructing the Lyapunov function, the exact symmetric positive definite matrix $\mathbf{H}$ is used for constructing it. *It is very important to realize that the unknown* $\mathbf{H}$ *itself is not used in the calculation the control signal. Only the fact of its existence and known properties are used in the Lyapunov function* $U := \frac{1}{2}\mathbf{S}^T\mathbf{HS} + \frac{1}{2}\tilde{\mathbf{p}}^T\mathbf{G}\tilde{\mathbf{p}}$ with some symmetric positive definite matrix $\mathbf{G}$. This immediately yields the derivative $\dot{U} := \mathbf{S}^T\left[\frac{1}{2}\dot{\mathbf{H}}\mathbf{S} + \mathbf{H}\dot{\mathbf{S}}\right] + \tilde{\mathbf{p}}^T\mathbf{G}\dot{\tilde{\mathbf{p}}}$. From (4) it is obtained that $\mathbf{H\dot{S}} = \mathbf{Y}\tilde{\mathbf{p}} - \mathbf{CS} - \mathbf{K}_D\mathbf{S}$ that can be substituted into the expression of $\dot{U}$ as $\dot{U} = \mathbf{S}^T\left[\frac{1}{2}\dot{\mathbf{H}}\mathbf{S} - \mathbf{CS} - \mathbf{K}_D\mathbf{S} + \mathbf{Y}\tilde{\mathbf{p}}\right] + \tilde{\mathbf{p}}^T\mathbf{G}\dot{\tilde{\mathbf{p}}}$. The fourth great idea is the realization of the fact that the matrix $\left(\frac{1}{2}\dot{\mathbf{H}} - \mathbf{C}\right)$ is skew symmetric therefore yields zero contribution in $\dot{U}$. So the requirement of $\dot{U} < 0$ can be met by the parameter updating rule $\mathbf{S}^T\mathbf{Y}\tilde{\mathbf{p}} + \tilde{\mathbf{p}}^T\mathbf{G}\dot{\tilde{\mathbf{p}}} = \mathbf{0}$ since $\mathbf{K}_D$ is negative definite. *This condition yields the parameter updating rule in the Slotine-Li approach* $\tilde{\mathbf{p}}^T\left(\mathbf{Y}^T\mathbf{S} + \mathbf{G}\dot{\tilde{\mathbf{p}}}\right) = 0$, that is $\dot{\hat{\mathbf{p}}} = -\mathbf{G}^{-1}\mathbf{Y}^T\mathbf{S}$ since the precise inertia data of the system are constant. So, besides the initial estimations of the inertia values this approach has a lot of parameters concerning the quality of the control: the elements of $\mathbf{K}_D$, $\mathbf{\Lambda}$, and $\mathbf{G}$. While $\mathbf{\Lambda}$, and $\mathbf{G}$ directly concerns the speed of the parameter tuning process, $\mathbf{K}_D$ and $\mathbf{\Lambda}$ determines feedback term in (3), therefore the strength of the actual error compensation. This coupling in the effect of the control parameters is quite complicated due to the nonlinear functions in the matrix elements of $\mathbf{Y}$, and makes the method's behaviour "deceptive": for large eigenvalues of $\mathbf{K}_D$ strong temporal feedback can well work without considerable tuning speed that may not result in problems in the lack of external perturbations. The greater the eigenvalues of $\mathbf{G}^{-1}$ are, the faster the learning process becomes. When improper perturbations "fob" the tuning procedure, through a too large $\mathbf{K}_D$ this may result in the corruption of the tracking quality. It is worth noting, too, that for tuning the parameters the necessary information is present in the error metrics $\mathbf{S}$. In various sections of the trajectory of the robot various information contents may be available for this tuning that generally is not a monotone process. That will be exemplified by the simulation results presented in this paper, too.

The control based on the SVD of an approximate system model considers the control task as a "*known excitation – observable response*" scheme as follows. Let the "excitation" of the controlled system be $\mathbf{Q}$ to which it is expected to respond by some prescribed or "*desired response*" $\mathbf{r}^d$. The appropriate excitation can be computed by the use of some approximate and incomplete inverse dynamic model as $\mathbf{Q}=\mathbf{F}(\mathbf{r}^d)$. The actual response determined by the system's dynamics, $\mathbf{Z}$, results in a *realized response* $\mathbf{r}^r$ that normally differs from the desired one: $\mathbf{r}^r=\mathbf{Z}(\mathbf{F}(\mathbf{r}^d))\equiv\mathbf{f}(\mathbf{r}^d)\neq\mathbf{r}^d$. It is worth noting that the functions $\mathbf{F}()$ and $\mathbf{Z}()$ may contain various hidden parameters that partly correspond to the dynamic model of the system, and partly pertain to unknown external dynamic forces acting on it. Due to phenomenological reasons the controller can manipulate or "*deform*" the input value from $\mathbf{r}^d$ so that $\mathbf{r}^r=\mathbf{r}^d=\mathbf{f}(\mathbf{r}^d{}_*)$. Now it will be shown that in combination with the geometric interpretation of the SVD this idea can be used in the adaptive control of MIMO systems.

Consider the following task: it is given an initial $\mathbf{x}_0$ value, a smooth $\mathbf{f}:\Re^n\rightarrow\Re^n$ function, an $\mathbf{x}^d$ "desired" value, and the appropriate solution $\mathbf{x}_*$ is sought for which $\mathbf{x}^d=\mathbf{f}(\mathbf{x}_*)$. We should like to achieve a first order correction in the value of $\mathbf{f}(\mathbf{x})$ that moves $\mathbf{f}$ in the direction of $\mathbf{x}^d$, that is a small positive number $\alpha>0$ can be introduced for which

$$\Delta\mathbf{f} = \frac{\partial\mathbf{f}}{\partial\mathbf{x}}\Delta\mathbf{x} = \alpha\left(\mathbf{x}^d - \mathbf{f}(\mathbf{x})\right) \quad (5)$$

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If the Jacobian of f can be inverted then the following iterative sequence of points can be generated by (5)

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \alpha \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right]^{-1} \left(\mathbf{x}^d - \mathbf{f}(\mathbf{x}_n) \right) \tag{6}$$

To estimate the approximation error belonging to $\mathbf{x}_{n+1}$ the first order Taylor series expansion of $\mathbf{f}$ can be used as

$$\begin{aligned} &\mathbf{x}^d - \mathbf{f}(\mathbf{x}_{n+1}) \approx \\ &\approx \mathbf{x}^d - \mathbf{f}\left[\mathbf{x}_n + \alpha \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right]^{-1} \left(\mathbf{x}^d - \mathbf{f}(\mathbf{x}_n) \right) \right] \approx \\ &\approx \mathbf{x}^d - \mathbf{f}(\mathbf{x}_n) - \alpha \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right] \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right]^{-1} \left[\mathbf{x}^d - \mathbf{f}(\mathbf{x}_n) \right] \approx \\ &\approx (1-\alpha)\left[\mathbf{x}^d - \mathbf{f}(\mathbf{x}_n) \right] \end{aligned} \tag{7}$$

This error in absolute value evidently can be decreased if approximately $0<\alpha<2$. Normally (7) cannot exactly be realized since $\partial\mathbf{f}/\partial\mathbf{x}$ is not precisely known. To have better idea on the possibilities of reducing the approximation error imagine the application of the SVD for it as $\partial\mathbf{f}/\partial\mathbf{x}=\mathbf{UDV}^T$, $[\partial\mathbf{f}/\partial\mathbf{x}]^{-1}=\mathbf{VD}^{-1}\mathbf{U}^T$ that, in the above outlined iteration, leads to the step $\Delta\mathbf{x}=\mathbf{VD}^{-1}\mathbf{U}^T[\mathbf{x}^d-\mathbf{f}(\mathbf{x})]$. (The $\mathbf{U}$ and $\mathbf{V}$ matrices are *orthogonal*, while $\mathbf{D}$ is diagonal.) This equation has the form of $\mathbf{b}=\mathbf{VD}^{-1}\mathbf{U}^T\mathbf{a}$ in the calculation of which the associativity of the matrix product can be utilized in the following manner: at first calculate $(\mathbf{VD}^{-1})$ and $(\mathbf{U}^T\mathbf{a})$, and finally calculate the product of these two terms. Evidently

$$\begin{aligned} &\mathbf{b} = \mathbf{VD}^{-1}\mathbf{U}^T\mathbf{a} = \\ &\left[\mathbf{v}^{(1)} \mid \cdots \mid \mathbf{v}^{(n)} \right] \begin{bmatrix} D_{11}^{-1} & 0 & 0 \\ 0 & \cdots & \ldots \\ 0 & \cdots & D_{nm}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{(1)T} \\ \ldots \\ \mathbf{u}^{(m)T} \end{bmatrix} \mathbf{a} = \\ &\left[D_{11}^{-1}\mathbf{v}^{(1)} \mid \cdots \mid D_{kk}^{-1}\mathbf{v}^{(k)} \right] \begin{bmatrix} \mathbf{u}^{(1)T}\mathbf{a} \\ \ldots \\ \mathbf{u}^{(m)T}\mathbf{a} \end{bmatrix} = \\ &= \left(\mathbf{u}^{(1)T}, \mathbf{a} \right) D_{11}^{-1}\mathbf{v}^{(1)} + \cdots + \left(\mathbf{u}^{(k)T}, \mathbf{a} \right) D_{kk}^{-1}\mathbf{v}^{(k)} \end{aligned} \tag{8}$$

in which k=min(n,m), and in the central line in the place of the three dots following D_{kk}^{-1} in [...$|D_{kk}^{-1}$ |...] either nothing stands or zeros are located. [The parentheses of the form (**a**,**b**) denote real scalar products of vectors.] The geometric interpretation of this expression is straightforward: characteristic pairs of mutually orthogonal directions are found in the input and the output spaces to which characteristic stretch/shrink operations determined by the singular values $D_{ii}\geq 0$ belong. To zero singular values special directions pertain that do not take part in the mapping realized by the linear operator under consideration. Via applying the form of (8) in the expression of the actual error used for calculating the next step with the components of the orthogonal matrix $\mathbf{U}$ in the ideal case it is obtained that

$$\begin{aligned} &\mathbf{x}^d - \mathbf{f}(\mathbf{x}) = \sum_l c_l \mathbf{u}^{(l)},\ c_l = \left(\mathbf{u}^{(l)}, \mathbf{x}^d - \mathbf{f}(\mathbf{x}) \right) \\ &\Delta\mathbf{x} \approx \alpha \sum_l D_{ll}^{-1} c_l \mathbf{v}^{(l)} \end{aligned} \tag{9}$$

For guaranteeing convergence small $\Delta\mathbf{x}$ is needed. Since the SVD for an invertible quadratic matrix yields $D_{11}\geq D_{22}\geq\ldots\geq D_{nn}$ it can be said that $0<D^{-1}_{11}\leq\ldots\leq D^{-1}_{nn}$ so $\|\Delta\mathbf{x}\|\leq\sqrt{n}\,\alpha\max_{l=1,\ldots,n}\{|c_l|\}D^{-1}_{nn}$. By the introduction of the quantity K expressing the maximum allowable step length in the $\mathbf{x}$ space the proper value of α can be estimated as $\alpha \approx KD_{nn}\big/\left(\sqrt{n}\max_{l=1}^{n}|c_l|\right)$. A simple geometric way of thinking can be utilized here as follows: it is not necessary to exactly move in the $\mathbf{x}$ space as it is defined in (7): it is just enough to make a small step "*approximately in the same direction*". Therefore, if we have some approximate model of the Jacobian of $\mathbf{f}$ of our system, only one times executing the SVD on this approximation may be satisfactory to approximate the $\mathbf{U}$, $\mathbf{D}$, and $\mathbf{V}$ matrices that can be used for estimating the factor α, and the system can be directed to the direction of the decreasing error even if not exactly the direction of the "*steepest descent*" according to (7) is achieved.

In the sequel this idea will be applied in the adaptive control of the cart plus double pendulum system considered as a paradigm. It is assumed that the axles of this system suffer from dynamic friction.

3 The Dynamic Model of the Cart Plus Double Pendulum System

The cart + double pendulum system is a typical example for mechanical systems having badly conditioned inertia matrix in the vicinity of certain critical points of the configurational space [30]. Its rough sketch is given in Fig. 1.

It consists of a cart serving as a body rolling on wheels of negligible momentum and inertia having the overall mass M, pendulums assembled on the cart by parallel shafts and arms having negligible masses and lengths L_1 and L_2, respectively. At the end of each arm a ball of negligible size and considerable masse m_1 and m_2 are attached.

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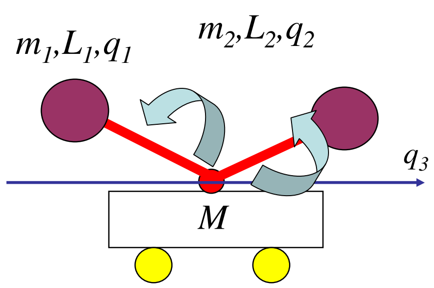

Fig. 1: Sketch of the cart plus double pendulum system

The Euler-Lagrange equations of motion of this system are as follows:

$$[Q_1 \quad Q_3 \quad Q_3]^T =$$
$$= \begin{bmatrix} m_1L_1^2 & 0 & -m_1L_1\sin q_1 \\ 0 & m_2L_2^2 & -m_2L_2\sin q_2 \\ -m_1L_1\sin q_1 & -m_2L_2\sin q_2 & (M+m_1+m_2) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \qquad (10)$$
$$+ \begin{bmatrix} -m_1L_1\cos q_1\dot{q}_1\dot{q}_3 - m_1gL_1\cos q_1 \\ -m_2L_2\cos q_2\dot{q}_2\dot{q}_3 - m_2gL_2\cos q_2 \\ -m_1L_1\cos q_1\dot{q}_1^2 - m_2L_2\cos q_2\dot{q}_2^2 \end{bmatrix}$$

in which g denotes the gravitational acceleration, Q_1 and Q_2 denote the driving torques at the rotary shafts, and Q_3 stands for the force moving the cart in the horizontal direction. The appropriate rotational angles are q_1 and q_2, and the linear degree of freedom belongs to q_3. The determinant of the inertia matrix in (10) has the form of

$$\det \mathbf{H} = m_1L_1^2 m_2L_2^2 \times$$
$$\times\left(M + m_1 + m_2 - m_1\sin^2 q_1 - m_2\sin^2 q_2\right) \qquad (11)$$

the minimum value of which is equal to $(\det\mathbf{H})_{\min} = m_1L_1^2 m_2L_2^2 M$. The "critical" points belong to the minimum of the determinant of the inertia matrix in the coincidence of the "critical coordinate values" q_1, $q_2=\pm\pi/2$. On this reason in the present, extended paper, the main idea of the RBFNs was used by "spanning the tent-cloth" over the grid points at $\pm\pi$, $\pm\pi/2$, and 0 for both q_1, and q_2 that means 5×5=25 points with the radial function $d_{ij}(q_1,q_2)=\exp\left(-4\left((q_1-q_{1,ij})^2+(q_2-q_{2,ij})^2\right)\right)$. In the estimation of the **U**, **V**, and **D** matrices these d_{ij} values were used for weighting. The SVD was executed only in the grid-points prior to initiating the control. Calculation of such a weighted average of a few small matrices does not mean considerable computational burden. These grid-points do not concern the Slotine-Li control. For describing the phenomenon of friction the Lund-Grenoble model [24, 25] was used which the deformation of the bristles of some "brushes" are applied to describe the deformation of the surfaces in dynamic contact, so friction is described as a dynamic coupling between two systems having their own equations of motion as

$$\frac{dz}{dt} = v - \frac{\sigma_0|v|z}{F_C + F_S\exp\left(-|v|/v_s\right)},$$
$$F = \sigma_0 z + \sigma_1\frac{dz}{dt} + F_v v \qquad (12)$$

for which the proper direction of F has to be set in the applications. Variable v denotes the relative velocity of the sliding surfaces, F_v describes the viscous friction coefficient, z denotes the deformation as an internal degree of freedom, σ_0 plays the role of some "spring constant" of the internal deformation, and σ_1 is a new parameter pertaining to the effect of the bending bristles. The F_C, F_S, and v_s parameters' role is the description of sticking. This model evidently yields $dz/dt=0$ for $v=0$ that can result finite friction force at even zero velocities. The behaviour of the whole system is described by the dynamic coupling between the hidden internal and the observed degrees of freedom. Though the appropriate quantities in (12) were developed for linear motion and forces, it easily can be generalized for rotary motion in which torques appear in the role of the forces, and rotational velocity is present instead of linear motion's velocity. The model given in (10) evidently can be completed via adding the additional torque of the friction to the appropriate components of **Q** in it. In general it is very difficult to identify the friction parameters. Appropriate steps for identifying the friction model of SISO systems and controlling them on the basis of the identified model were done e.g. in [31]. However, it seems to be more expedient to apply simple adaptive approach that completely evades such identification problems. The here proposed FPT/SVD-based method just corresponds to this idea more or less akin to the idea of "situational control" [32] in the sense that no complete system model has to be built up for control purposes.

In the next section computation results will be presented for the comparative study.

4 Simulation Results

In the present paper the following model inertia and dynamical parameters were used for the controlled system: M=5 kg, m_1=6 kg, m_2=4 kg, L_1=2 m, L_2=3 m, g=9.1 m/s^2. The friction models had the following parameters: σ_{01}=10 Nm/rad, σ_{11}=150 Nms/rad, F_{v1}=1 Nms/rad, F_{C1}=100 Nm, F_{S1}=200 Nm, v_{s1}=0.1 rad/s for the 1st axle, σ_{02}=20 Nm/rad,

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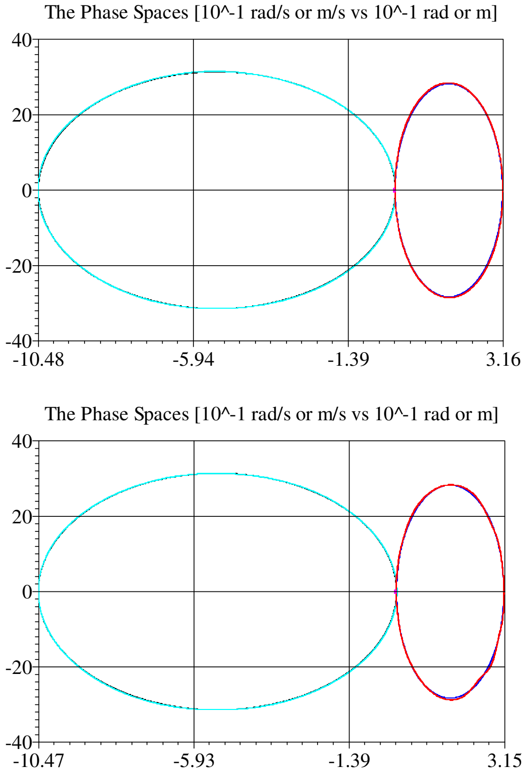

Fig. 2: Fundamental test for the Slotine-Li control: phase trajectory tracking with exact dynamic model without friction and external disturbances: non-adaptive (upper), adaptive (lower) chart

σ_{12}=300 *Nms/rad*, F_{v2}=2 *Nms/rad*, F_{C2}=200 *Nm*, F_{S2}=400 *Nm*, v_{s2}=0.2 *rad/s* for the 2nd axle, and σ_{03}=30 *N/m*, σ_{13}=450 *Ns/m*, F_{v3}=3 *Ns/m*, F_{C3}=300 *N*, F_{S3}=300 *N*, v_{s3}=0.3 *m/s* for the 3rd axle. For numerical computation simple Euler-integration was used with the time resolution of δt=1 *ms*. In the tests concerning the effects of the imprecision of the dynamic models the roughly approximate $\hat{M}=2.5\,kg$, $\hat{m}_1=2.4\,kg$, $\hat{m}_2=1.2\,kg$ values were used.

For the simulation of the Slotine-Li method the following formal transformation of the original dynamic model (10) had to be introduced:

For the "inertia matrix":

$$\hat{\mathbf{H}}\dot{\mathbf{v}}=$$

$$=\begin{bmatrix} L_1^2\dot{v}_1-L_1\sin q_1\dot{v}_3 & 0 & 0 \\ 0 & L_2^2\dot{v}_2-L_2\sin q_2\dot{v}_3 & 0 \\ -L_1\sin q_1\dot{v}_1 & -L_2\sin q_2\dot{v}_2 & \dot{v}_3 \end{bmatrix}\times \quad (13)$$

$$\times\begin{bmatrix} \hat{m}_1:=\hat{p}_1 \\ \hat{m}_2:=\hat{p}_2 \\ \hat{M}+\hat{m}_1+\hat{m}_2:=\hat{p}_3 \end{bmatrix}$$

For the "**C** matrix":

$$\hat{\mathbf{C}}\mathbf{v}=$$

$$\begin{bmatrix} -L_1\dfrac{cq_1\dot{q}_3v_1+cq_1\dot{q}_1v_3}{2} & 0 & 0 \\ 0 & -L_2\dfrac{cq_2\dot{q}_3v_2+cq_2\dot{q}_2v_3}{2} & 0 \\ -L_1cq_1\dot{q}_1v_1 & -L_2cq_2\dot{q}_2v_2 & 0 \end{bmatrix}\times$$

$$\times\begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \end{bmatrix} \quad (14)$$

For the "gravitational term":

$$\hat{\mathbf{g}}=\begin{bmatrix} -L_1g\cos q_1 & 0 & 0 \\ 0 & -L_2g\cos q_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \end{bmatrix} \quad (15)$$

The matrix determining the speed of the parameter tuning was diagonal **G**=<0.1,0,0;0,0.1,0;0,0,0.001>, in the role of the $\mathbf{K}_D$ matrix the scalar value K_D=1200 was applied (within the matrix the various matrix elements may have different physical dimensions), for matrix $\mathbf{\Lambda}$ also a scalar was chosen as Λ=10 s^{-1}. These values were determined by running tests.

For testing the appropriateness of these values as well as the correctness of the model the Slotine-Li control was tested with exact dynamical model without friction and external perturbations for a test trajectory. As it was expected, the use of the exact model without adaptivity and the adaptive Slotine-Li control resulted in very close phase trajectories and trajectories, i.e. this test was successfully carried out (Fig. 2).

Since the FPT/SVD-based controller *ab ovo* does not use exact model, making a similar test for it was not possible. To make "comparable conditions" for the two different approaches, the same Λ value was applied for this controller, too. However, taking the advantage, that this latter approach does not impose formal restrictions for the prescribed kinematic tracking policy (in strict contrast with the requirements of the Slotine-Li method), for the error compensation the $(\Lambda+d/dt)^3=0$-type prescription was applied, that in this case correspond to a PID-type kinematic control with time-constants originating from this structure: $P=3\Lambda^2$, $I=\Lambda^3$, $D=3\Lambda$ in

$$\ddot{\mathbf{q}}^d=\ddot{\mathbf{q}}^N-D\left(\dot{\mathbf{q}}-\dot{\mathbf{q}}^N\right)-P\left(\mathbf{q}-\mathbf{q}^N\right)-$$
$$-\frac{I}{2}\int dt\left(\mathbf{q}-\mathbf{q}^N\right) \quad (16)$$

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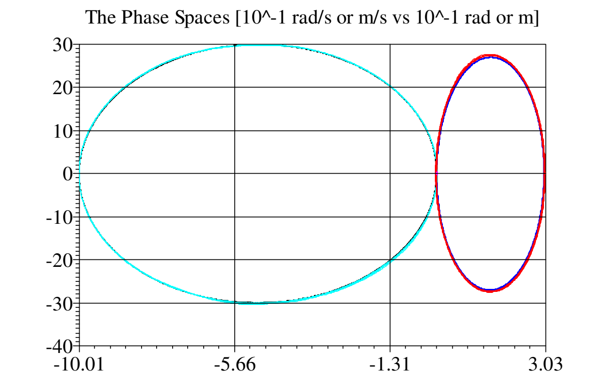

Fig. 3: Fundamental test for the FPT/SVD control: phase trajectory tracking without friction and external disturbances

For the other control value K=20 was chosen, again "experimentally". The "basic" test for this method was a simple adaptive run without friction and perturbations (Fig. 3).

Following the "fundamental tests" carrying out a comparative analysis became possible. The first test aimed at the study of the effects of the imprecise dynamic parameters without friction and external influences (Fig. 4).

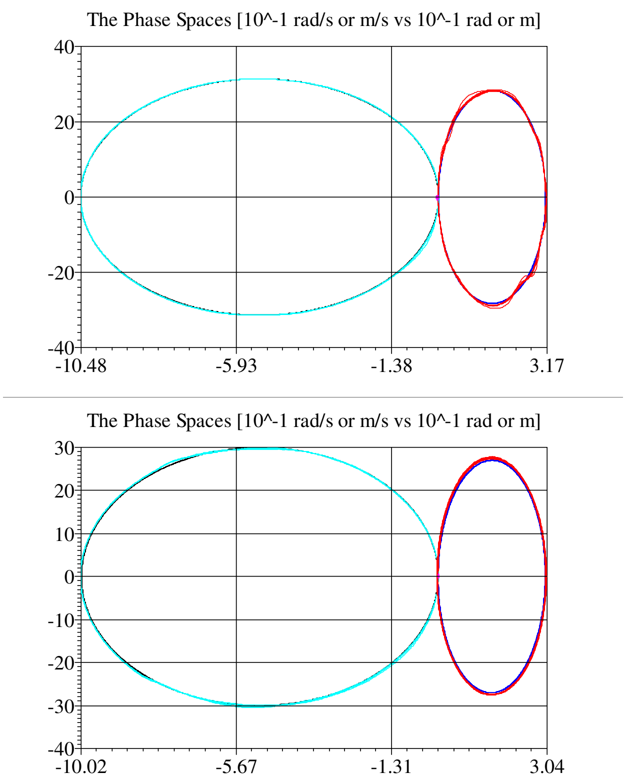

Fig. 4: Comparative test for the effects of imprecise model parameters without friction and external disturbances: phase trajectory tracking Slotine-Li (upper), FPT/SVD (lower) chart

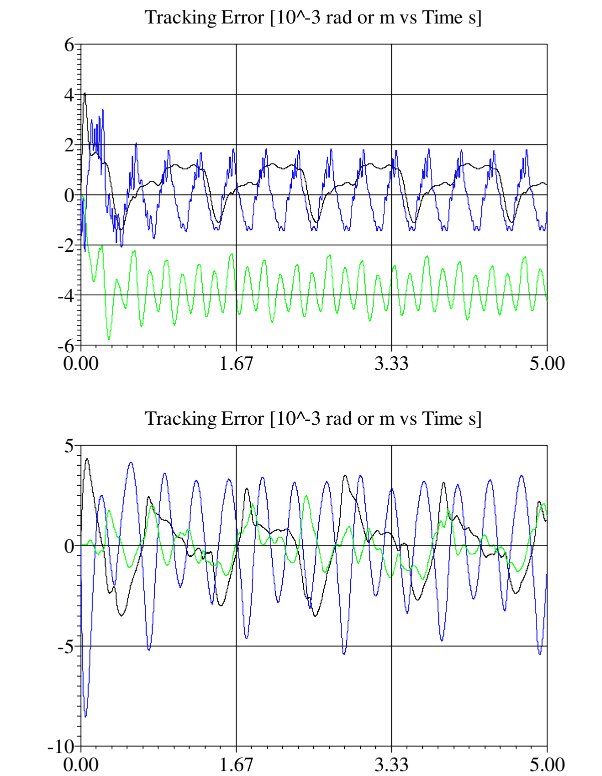

Fig. 5: Comparative test for the effects of imprecise model parameters without friction and external disturbances: trajectory tracking Slotine-Li (upper), FPT/SVD (lower) chart

Fig. 4 reveals that the two methods worked with comparable precision as it was originally expected. The Slotine-Li method is appropriately designed to compensate such modeling imprecision, and the FPT/SVD-based ab ovo has to compensate such errors, too. Subtle details of the "trajectory tracking errors" described in Fig. 5 reveal that the two different methods considered work in quite different manner. While the FPT/SVD approach keeps the center of the error fluctuation at zero, the Slotine-Li approach allows a kind of bias (the axes are denoted as follows: 1: black, 2: blue, 3: green).

It is very interesting to see the details of the parameter tuning of the Slotine-Li method. At the parameter settings investigated the three different inertia parameters are tuned in quite different manner. The most interesting is the behavior of p_1 and p_2 (Fig. 6): the first one slowly fluctuates around a mean value that physically can well be interpreted; similar but far more hectic fluctuation happens to the 2nd parameter, but essentially it also is settled around some mean value that has possible physical interpretation. However, an initial "transient phase" can well be identified in its fluctuation that is damped in the average.

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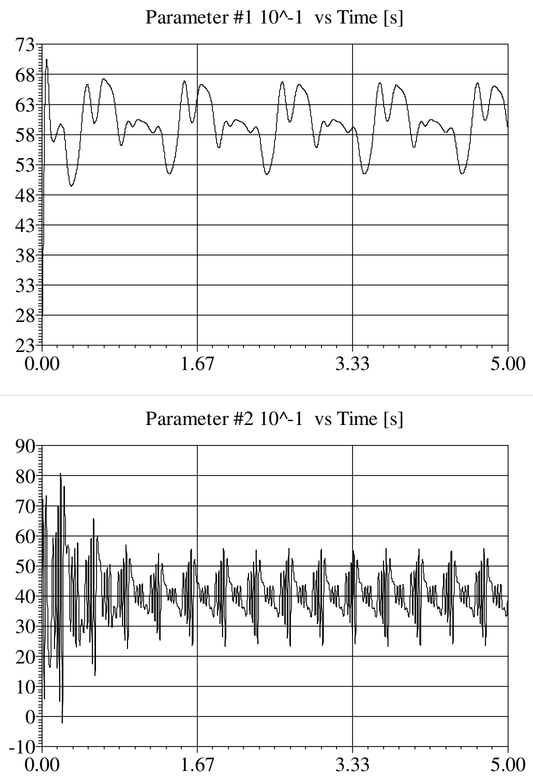

Fig. 6: Tuning p_1 and p_2 in case of Fig. 5 in the Slotine-Li control

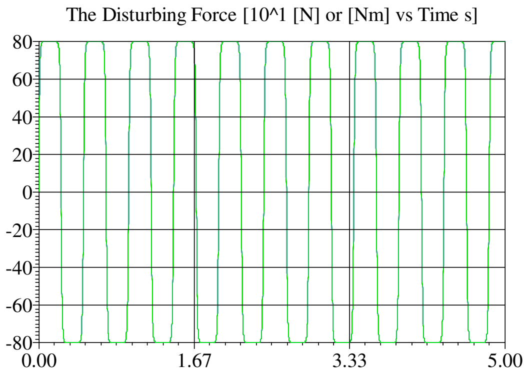

Fig. 7: The disturbance torques or forces simultaneously applied for each axle vs. time

The combined effects of imprecise dynamic model with disturbance forces (depicted in Fig. 7) without friction is described. In the run considered each axle had been disturbed simultaneously. The FPT/SDV-controller works well, but in the operation of the Slotine-Li controller considerable deficiencies can be observed. This observation is confirmed by Fig. 8, too, that describes the appropriate phase trajectory tracking and the trajectory tracking errors versus time. It can well be seen, that the disturbance forces are well "mirrored" in the generalized forces exerted by the FPT/SDV-controller (Fig. 9).

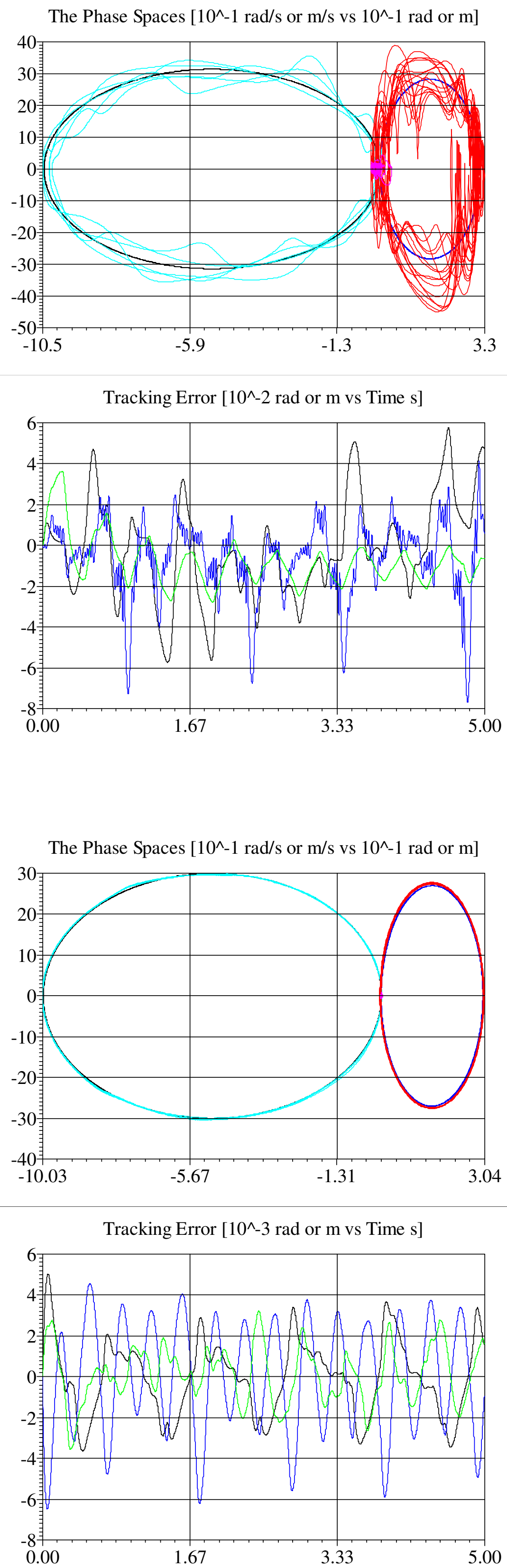

Fig. 8: Effects of imprecise dynamic model with disturbance forces without friction: the Slotine-Li controller (the first two charts), the FPT/SDV-based controller (the second two charts)

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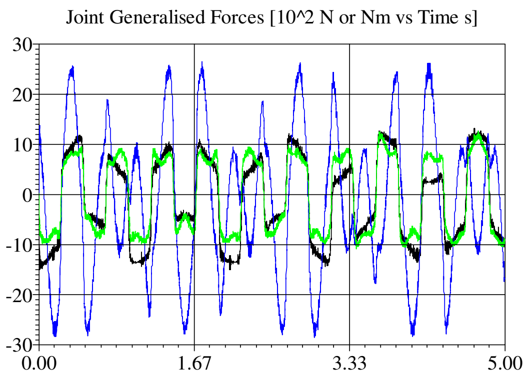

Fig. 9: Imprecise dynamic model with disturbance forces without friction: the joint generalized forces exerted by the FPT/SDV-controller

It is very interesting to see what happens to the tuned parameters of the Slotine-Li controller. As it can well be observed in Fig. 10 the amplitude of the fluctuation of the parameters is considerably increased. Furthermore, these parameters can take values that do not have possible physical interpretation (the masses cannot take negative values). Via combination of the equations (2) and (3) it can qualitatively be understood that for very big $\mathbf{K}_D$ the Slotine-Li controller in short time-scale works as a PD-type one with very strong feedback. The effects in learning appear only on a larger scale.

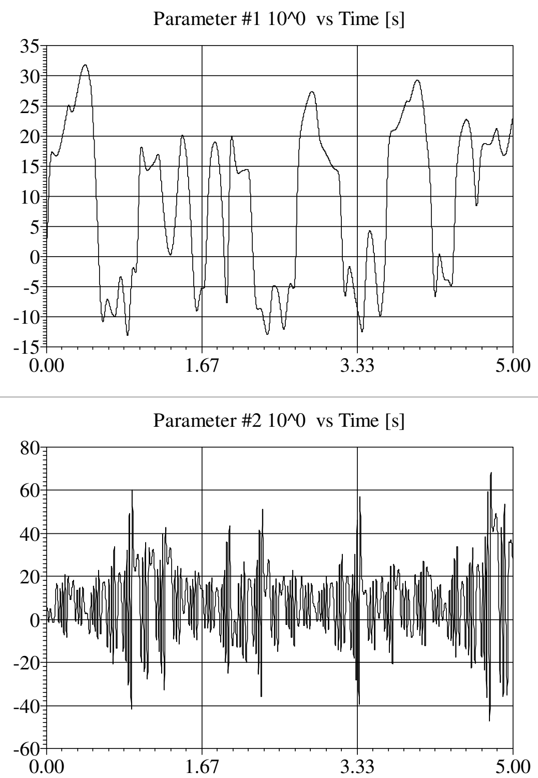

Fig. 10: Imprecise dynamic model with disturbance forces without friction: parameter tuning in the Slotine-Li controller

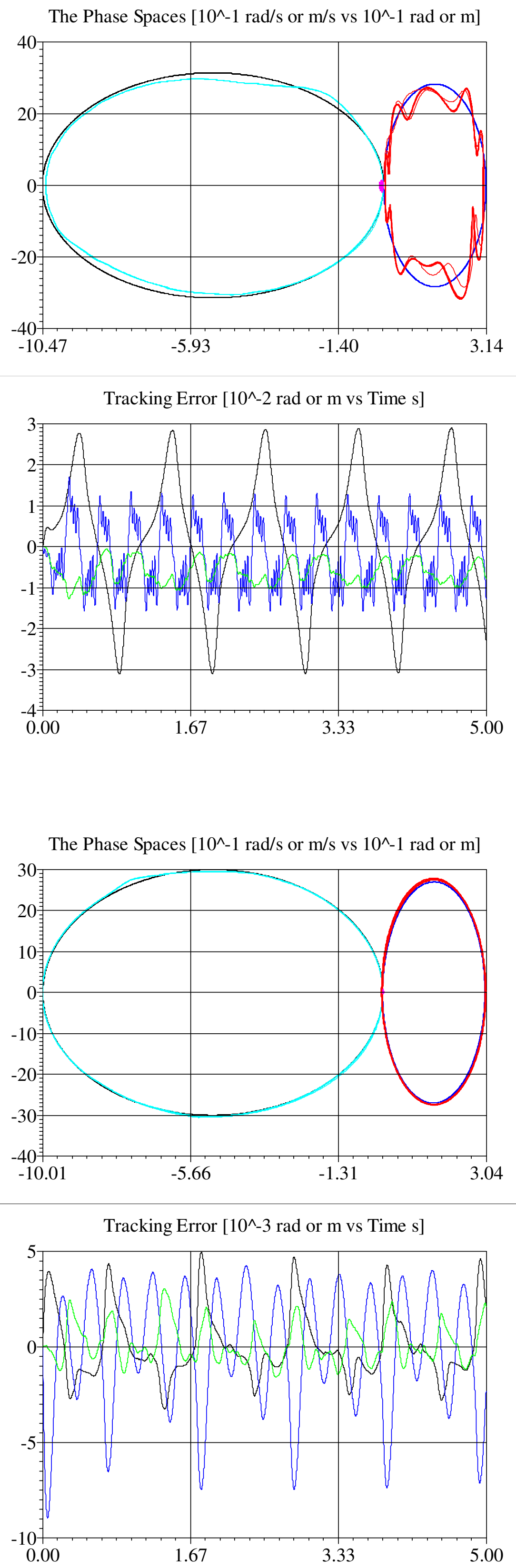

Fig. 11: Effects of imprecise dynamic model with friction and without disturbance forces: the Slotine-Li controller (the first two charts), the FPT/SDV-based controller (the second two charts)

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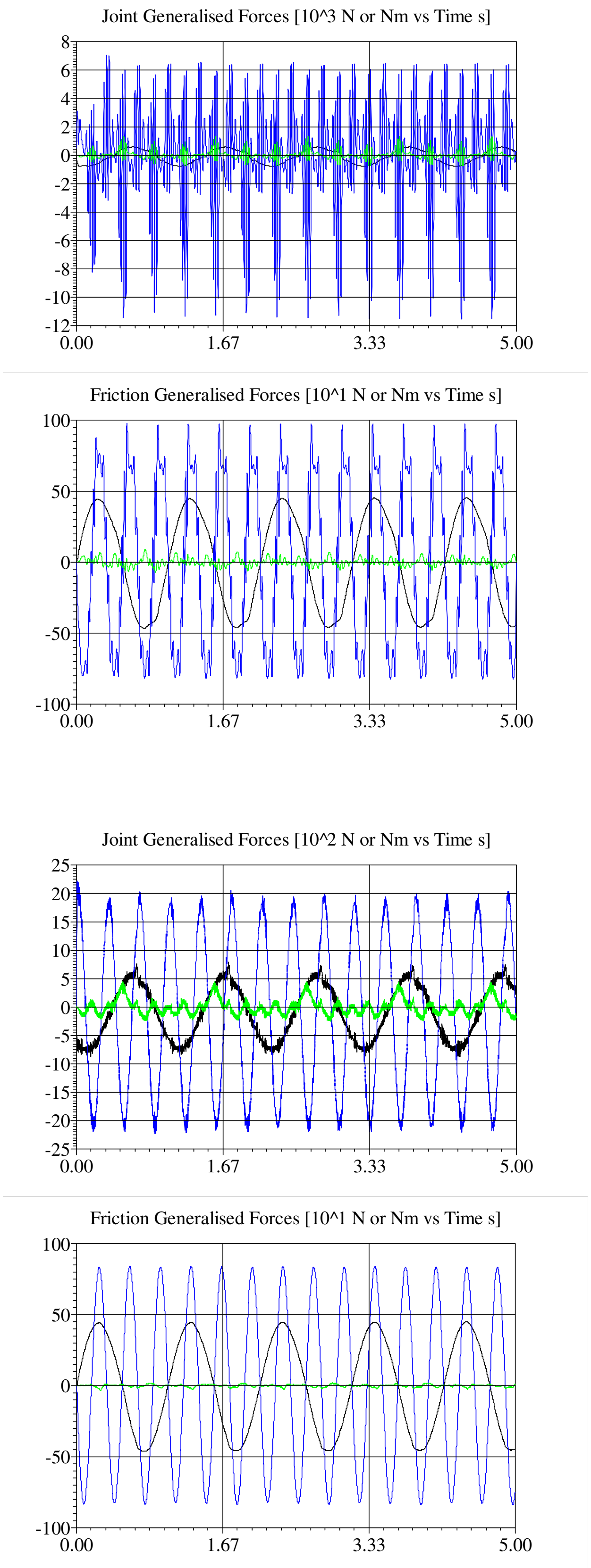

Fig. 12: The joint generalized forces and the friction torques/forces in the case of imprecise dynamic model with friction and without disturbance forces: the Slotine-Li controller (the first two charts), the FPT/SDV-based controller (the second two charts)

It also is very interesting to what happens if the disturbance forces are switched off but friction comes into effect. In Fig. 11 the phase trajectories and the trajectory tracking errors are displayed. As in the case of the unknown external perturbations, the Slotine-Li controller results in seriously distorted phase trajectories and degraded tracking precision. In contrast to that, the FPT/SVD-based controller yields nice phase trajectory tracking and precise trajectory tracking, too.

In this context it is worth noting that there are essential differences between the effects of the external disturbances here considered and that of the friction. The disturbance forces were “explicitly applied” independently of the state propagation of the controlled system. In contrast to that, according to the LuGre model the presence of the friction has more complicated effects: it establishes strong nonlinear coupling in the dynamics of the controlled system. Since the Slotine-Li controller cannot adequately compensate these effects, due to that coupling it generates more uneven generalized forces and friction forces than the FPT/SVD-based approach. This is well illustrated by Fig. 12. According to Fig. 13 the parameter-tuning process of the Slotine-Li controller is seriously disturbed by the friction.

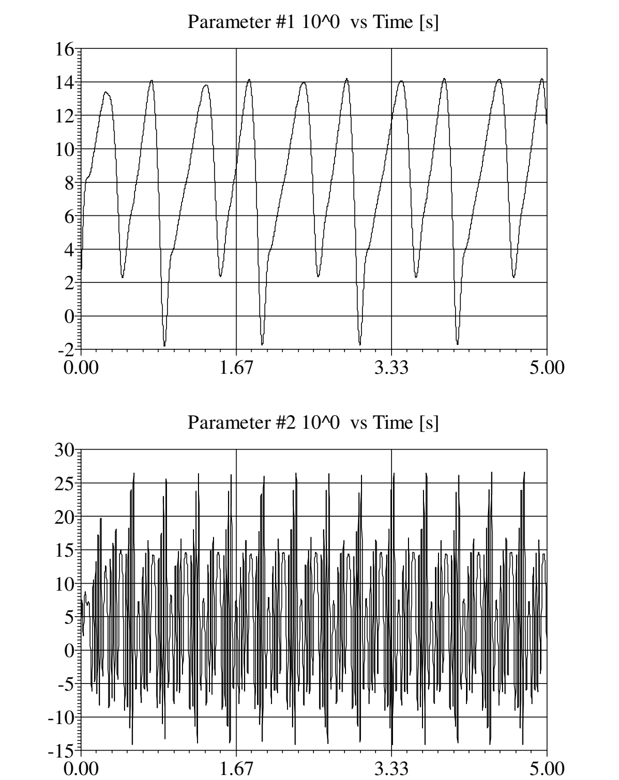

Fig. 13: Imprecise dynamic model with friction and without disturbance forces: parameter tuning in the Slotine-Li controller

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As in the case of the disturbance forces, negative values lacking physical interpretation appear.

Since the rest of this paper is devoted to the analysis of the novel FPT/SVD-based controllers, it is expedient to reveal its subtle analytical details in this paragraph. Within the control cycle it used the very simple affine system model instead of (10) as $\mathbf{Q}=10d^2\mathbf{q}/dt^2+[10;10;10]^T$. The formally correct analytical model was used only outside of the control cycle, for calculating the SVD decomposition of $\frac{\partial \mathbf{f}}{\partial \ddot{\mathbf{q}}}=10\hat{\mathbf{M}}^{-1}$ in the 25 grid points. In order to evade the occurrence of very drastic transients "ancillary tricks" also were applied as follows: instead of K its slowly decreasing value calculated as $K_n=K[0.6+0.4\times100/(n+100)]$ in the n^{th} control step was applied. Instead using the α_n parameter directly calculated from K_n its "smoothed" value was utilized by using the content of a forgetting integrating buffer as $\alpha_n^{buf}(1-\beta)$ where the buffer's content for the $(n+1)^{\text{th}}$ step was refreshed as $\alpha_{n+1}^{buf}=\beta\alpha_n^{buf}+\alpha_n$ with β=0.2. Finally, a regulating factor was also applied that reduced the too big steps by measuring the absolute value of the necessary step via the variable $\xi_n:=\left\|\ddot{\mathbf{q}}d(t_n)-\ddot{\mathbf{q}}(t_{n-1})\right\|$ through a linear interpolation determined by two "limit parameters" ε_1=0.05, $\varepsilon_2=10^{-5}$, and a "shape factor" s=0.5 defined by $\lambda_n:=(1+\varepsilon_1)+(\varepsilon_2-1-\varepsilon_1)s\xi_n/(1+s\xi_n)$, and with the modified desired tracking given to the iteration as $\ddot{\mathbf{q}}_n^{d*}:=(1-\lambda_n)\ddot{\mathbf{q}}(t_{n-1})+\lambda_n\ddot{\mathbf{q}}_n^d(t_n)$. For very small ξ_n $\ddot{\mathbf{q}}_n^{d*}\approx-\varepsilon_1\ddot{\mathbf{q}}(t_{n-1})+(1+\varepsilon_1)\ddot{\mathbf{q}}_n^d(t_n)\approx\ddot{\mathbf{q}}_n^d(t_n)$ since $\lambda_n\approx(1+\varepsilon_1)\approx1$, that is practically no "reduction" happens, i.e. the original control strategy is used. For very big ξ_n $\lambda_n\approx\varepsilon_2$, and $\ddot{\mathbf{q}}_n^{d*}\approx(1-\varepsilon_2)\ddot{\mathbf{q}}(t_{n-1})+\varepsilon_2\ddot{\mathbf{q}}_n^d(t_n)$ that means "strong reduction of the goal", that is no very big jumps in the accelerations are allowed. (These features were in operation in the simulations resulting the previously presented charts, too.)

In the next simulation the complex effects of both the dynamic model inaccuracies, external disturbance forces and internal friction of the axles can be analyzed. According to the results displayed in Fig. 14 the friction and disturbance effects fob the Slotine-Li controller but the FPT/SVD-based one still works quite accurately. On this reason it is interesting to trace what happens with the internal variables of this controller.

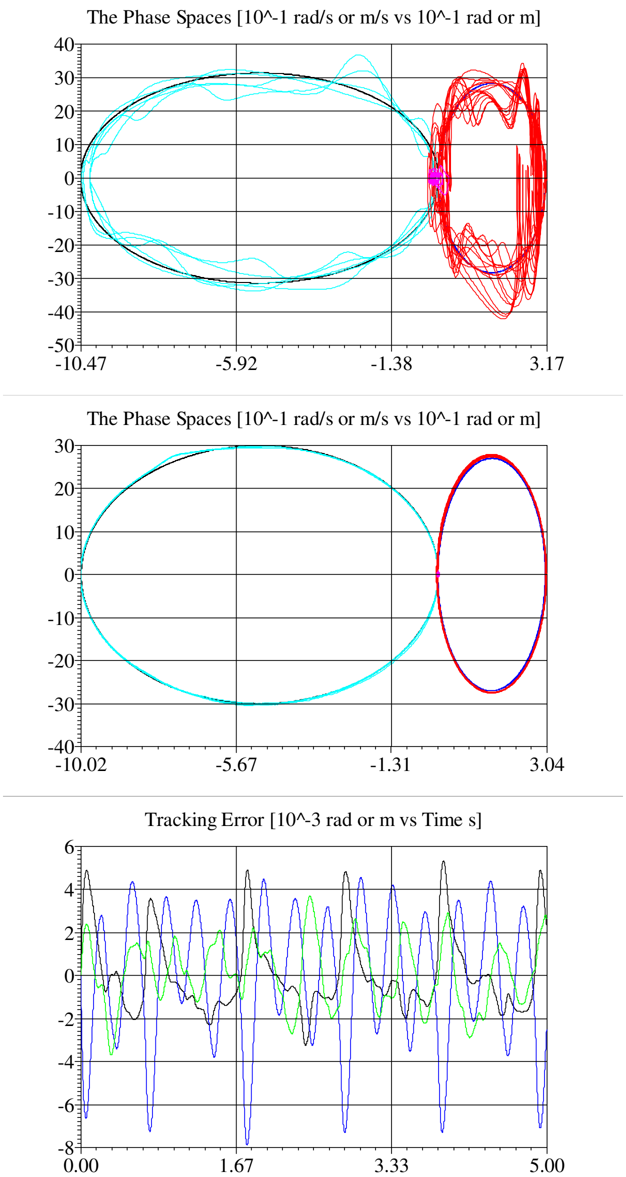

Fig. 14: The case of imprecise dynamic model with friction and disturbance forces: the phase trajectory tracking of the Slotine-Li controller (1st chart), that of the FPT/SVD-based controller (2nd chart), and the trajectory tracking error of the latter controller (3rd chart)

As it can well be seen in Fig. 15 the generalized forces again are exempt of rough fluctuations. The little fluctuation observable in the chart originates from that of the parameter $\alpha_n(t)$. These fluctuations much probably originate from the equation used for its estimation $\alpha\approx KD_{nn}/\left(\sqrt{n}\max_{l=1}^{n}|c_l|\right)$: finding the maximum in different c_l elements really can cause small discontinuities. However, as it is revealed on the chart depicting the components of $\mathbf{Q}$, these small discontinuities are quite negligible. The variation of the regulating factor $\lambda_n(t)$ also is considerable.

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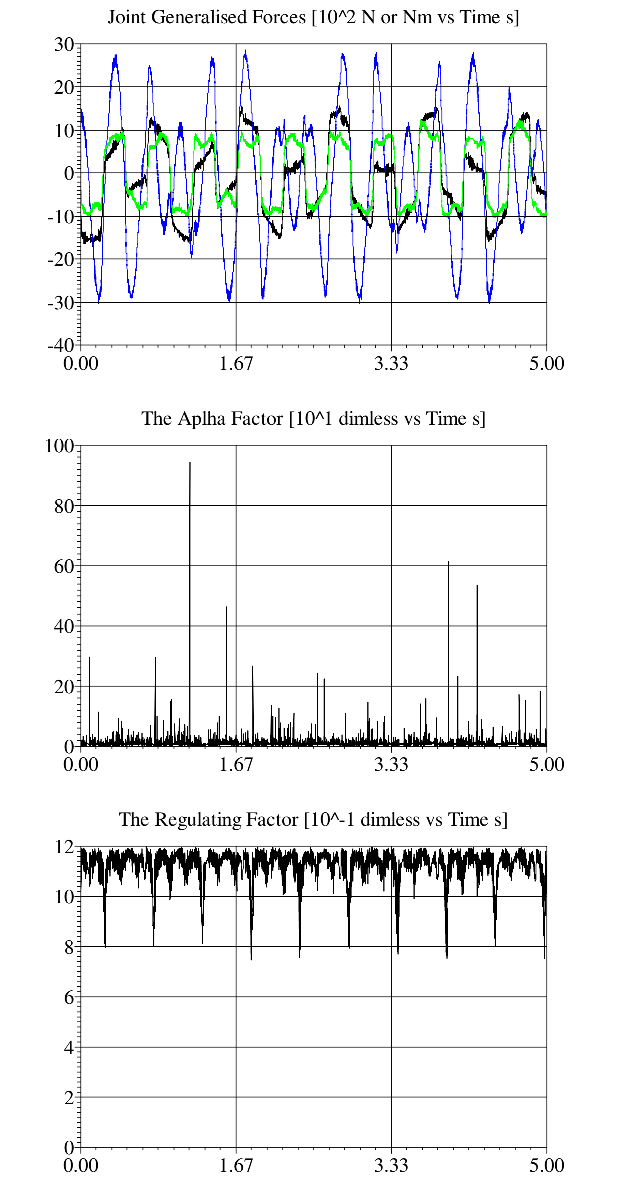

Fig. 15: The operation of the FPT/SVD-based controller in the case of imprecise dynamic model with friction and disturbance forces: the exerted generalized forces vs. time (1st chart), the α_n factors vs. time (2nd chart), and the regulating factor λ_n vs. time (3rd chart)

5 Conclusions

In this paper a comparative analysis of the operation of the FPT/SVD-based adaptive controller and that of the Slotine-Li approach was given. For numerical computations the cart + double system with internal dynamic friction as an application paradigm was used. The nominal trajectory investigated required some sinusoidal swinging of the pendulums around different central angular positions with different amplitudes and frequencies while the cart's nominal position was fixed. The effects of modeling imprecision, external disturbance forces acting on each driven axle, and the unmodeled internal friction simulated on the basis of the LuGre model were considered.

It was shown, that in accordance with the expectations both controllers well compensated the effects of the imprecise dynamic model. However, unmodeled internal friction and unknown external disturbances can completely fob or "mislead" the parameter tuning process of the Slotine-Li controller, but scarcely concern the operation and the internal variables of the SVD-based method. This latter has the deficiency that it may get out of its region of convergence. As a consequence its design and use needs preliminary numerical simulation investigations. However, the simulation examples exemplify that its convergence can be guaranteed for practical purposes. Furthermore, the use of this simple type of adaptive technique seems to far more viable way to friction compensation than trying to identify the parameters of some complicated friction model.

In the future we plan to investigate the possible application of the FPT/SVD-based method for trajectories that asymptotically approach some constant position, since it is well known that certain controllers are apt to produce some fluctuation as a limit cycle along such trajectories.

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