

# Output feedback model Predictive Control for nonlinear systems

I. I. SILLER-ALCALÁ, J. JAIMES-PONCE, AND R. ALCÁNTARA-RAMÍREZ.

Departamento de Electrónica, Grupo Control de Procesos  
Universidad Autónoma Metropolitana

Av. San Pablo No. 180, Col. Reynosa Tamaulipas, Del. Azcapotzalco, C. P.0200, México D.F.  
MÉXICO

[sai@correo.azc.uam.mx](mailto:sai@correo.azc.uam.mx)

*Abstract:* - In this paper, an Output Feedback Model Predictive Control for nonlinear systems is presented. The proposed output feedback control consists of the well known robust controller NCGPC (Nonlinear Continuous Time Generalized Predictive Control) and an open loop observer (a simulated model in parallel) in order to estimate the output derivatives and a regulation filter used to account for plant/model mismatch. The main advantages of the new approach are i) that the assumption of full-state feedback inherent in feedback linearization schemes is eliminated, the only measurement required is the output and finally ii) the process output converges to a constant reference in spite of presence of parameter uncertainties and process disturbances. The analysis of the induction motor and simulation results in the numerical example show that the output feedback model predictive control proposed can tolerate certain process uncertainty.

*Key-Words:* - Nonlinear Control, Predictive Control, Output Feedback Control, Internal Model Control, Robust Control.

## 1 Introduction

The feedback linearization control techniques [1] and [2] needs an accurate model as well as many nonlinear model-based control frameworks need full access to the plant state, but often the full state vector is not available, either because there is no access to all states or because the instrumentation required measuring all the states is very expensive. Therefore, it is desired to obtain an approximation of the unavailable states, a nonlinear observer is required [3]-[5]. Robust non linear observers for variables and parameters estimation in sensorless of induction motors in [6]-[8] and [14]-[24] are proposed.

In this paper an Output Feedback Model Predictive Control for nonlinear systems is presented. The proposed output feedback control consists of the well known robust controller NCGPC [9] and [10] (Nonlinear Continuous Time Generalized Predictive Control) a regulation model is added as in [12] and [13], which is used to estimate the process model errors and an open loop observer (model process simulated in parallel) in order to obtain the output derivatives, which are necessary to develop the predictive control. The main advantages offered by the proposed scheme are: the output is the only measurement required, the robustness in spite of plant-model mismatch or disturbances, obtained by addition of the regulation filter and the simplicity of the control law allows a simple and straightforward

implementation. A numerical example is given to illustrate the effectiveness of the controller.

## 2 Description of the Robust NCGPC

The development of the Nonlinear Continuous Time Generalized Predictive Control (NCGPC) [9] and [10] was carried out following the receding horizon strategy of its linear counterpart [11].

### 2.1 System Description

The Nonlinear Continuous Time Generalized Predictive Control (NCGPC) considers nonlinear dynamics systems with the state-space representation:

$$\begin{aligned}\dot{x}(t) &= f(x) + g(x)u \\ y(t) &= h(x)\end{aligned}\quad (1)$$

where  $f$ ,  $g$  and  $h$  are differentiable  $N_y$  times with respect to each argument,  $x \in R^n$  is the vector of the state variables,  $u \in R$  is the manipulated input,  $y \in R$  is the output to be controlled and  $r$  is the relative degree.

The following requirements are necessary:

- The system is stable
- The zero dynamics are stables
- $N_u = N_y - r$

**2.2. Prediction of the output**

The output prediction is approximated as in [11] for a Maclaurin series expansion of the system output as follows.

$$y^*(t, T) = y(t) + \dot{y}(t)T + y^{(2)}(t) \frac{T^2}{2!} + \dots + y^{(N_y)}(t) \frac{T^{N_y}}{N_y!} \quad (2)$$

or

$$y^*(t, T) = T_{N_y} Y_{N_y} \quad (3)$$

where

$$Y_{N_y} = [y \quad \dot{y} \quad y^{(2)} \quad \dots \quad y^{(N_y)}]^T \quad (4)$$

and

$$T_{N_y} = [1 \quad T \quad \frac{T^2}{2!} \quad \dots \quad \frac{T^{N_y}}{N_y!}] \quad (5)$$

The predictor order  $N_y$  is chosen less than the number of the times that the output has to be differentiated in order to obtain terms not linear in  $u$ , in this paper is considered  $N_y = r$ .

**2.3 Prediction of the reference trajectory**

The objective of the control is to drive the predicted output along a desired smooth path to a set point. Such a path is called a reference trajectory. The reference trajectory following [11] is given by

$$w_r^*(t, T) = [p_0 + p_1 T + p_2 \frac{T^2}{2!} + \dots + p_r \frac{T^{N_y}}{N_y!}] [w - y(t)] + y(t) \quad (6)$$

where  $w$  is the set point, or rewriting this equation

$$w_r^*(t, T) = T_{N_y} w_r + y(t) \quad (7)$$

where

$$w_r = [p_0 \quad p_1 \quad \dots \quad p_r]^T (w - y(t)) \quad (8)$$

and  $T_{N_y}$  is given by (5)

The control law becomes at follows:

$$u(t) = \frac{(w - y) - \sum_{i=1}^r \beta_i L_{f_i}^i h(x)}{\beta_r L_g L_{f_r}^{r-1} h(x)} \quad (9)$$

where

$$\beta_r = 1 / (t_1 p_0 + t_2 p_1 + \dots + t_r p_{r-1}) \quad (10)$$

$$\text{and } \beta_i = t_{i+1} / (t_1 p_0 + t_2 p_1 + \dots + t_r p_{r-1}) \quad (11)$$

where  $t_1, t_2, \dots, t_r$  are the elements of the first row of  $T_{y_{22}}^{-1} T_{y_{21}}$ . This control law is identical of error feedback-GLC [2], which uses an open loop Observer in order to obtain an output feedback.

The transfer function in closed loop is given by

$$G(s) = \frac{1}{\beta_r s^r + \beta_{r-1} s^{r-1} + \dots + \beta_1 s + 1} \quad (12)$$

**2.4 Derivatives emulation**

In order to improve the robustness and performance, a correction will be applied to the output derivatives of the model, taking account the difference between plant and model. The correction is carried out by applying a regulation filter as in [12] and [13]. Then a controller with two degrees of freedom is obtained.

The regulation filter is given by

$$\begin{aligned} \dot{x}_e(t) &= A_e x_e(t) + B_e e(t) \\ y_e(t) &= C_e x_e(t) \end{aligned} \quad (13)$$

where  $x_e \in R^n$ ,  $C_e \in R^{1 \times n}$ ,  $B_e \in R^{n \times 1}$ ,  $A_e \in R^{n \times n}$ ,  $e \in R$ .

The filter has a unity gain, the matrix  $A_e$  has eigenvalues with negative real part and its input is the difference between output process and output model,  $e(t) = y(t) - y_m(t)$ .

To obtain the predictive controller it was necessary to get the derivatives of output of a process model. It has the relative degree equal to the process. This process model (open loop observer or internal model) is simulated in parallel in order to get the states and then obtain the derivatives of output.

$$\begin{aligned} \dot{x}_m(t) &= f_m(x_m) + g_m(x_m)u \\ y_m(t) &= h_m(x_m) \end{aligned} \quad (15)$$

where  $f_m$ ,  $g_m$  and  $h_m$  are differentiable  $N_y$  times with respect to each argument,  $x_m \in R^n$  is the vector of the state variables,  $u \in R$  is the manipulated input and  $y \in R$  is the output to be controlled,  $u$  and  $y$  are the same as the process.

The NCGPC is based in taking the derivatives of the output, which are obtained as follows

$$\begin{aligned} \dot{y}(t) &\approx \dot{y}_e(t) + L_{f_m} h_m(x_m(t)) \\ y^{(2)}(t) &\approx y_e^{(2)}(t) + L_{f_m}^2 h_m(x_m(t)) \\ &\vdots \end{aligned} \quad (16)$$

$$y^{(r)}(t) \approx y_e^{(r)}(t) + L_{f_m}^r h_m(x_m(t)) + L_{g_m} L_{f_m}^{r-1} h_m(x_m(t))u(t)$$

These output derivatives are obtained from the system of Eq. 1 and  $N_y = r$ , where  $r$  is the relative degree. Output and its derivatives can be rewritten by

$$Y_{N_y}(t) = O(x(t)) + H(x(t))u_{N_y} \quad (17)$$

where

$$Y_{N_y} = [y \quad \dot{y} \quad y^{(2)} \quad \dots \quad y^{(N_y)}]^T \text{ and}$$

$$u_{N_y} = [u \quad \dot{u} \quad u^{(2)} \quad \dots \quad u^{(N_y-r)}]^T$$

$$O = \begin{bmatrix} y \\ L_f h(x) \\ L_f^2 h(x) \\ \vdots \\ L_f^r h(x) \\ S_1(x) \\ S_2(x) \\ \vdots \\ S_{(N_y-r)}(x) \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_g L_f^{r-1} h(x) & 0 & \dots & 0 \\ J_1(x) & L_g L_f^{r-1} h(x) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ J_2(x) & I_1(x) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ J_{N_y-r}(x) & I_{N_y-r}(x) & \dots & L_g L_f^{r-1} h(x) \end{bmatrix} \quad (18)$$

### 3.5 Cost function minimization

The function is not defined with respect current time, but respect a moving frame, which origin is in time  $t$ , where  $T$  is the future variable. Given a predicted output over a time frame the CGPC calculates the future controls. The first element  $u(t)$  of the predicted controls is then applied to the system and the same procedure is repeated at the next time instant. This makes the predicted output depend on the input  $u(t)$  and its derivatives, and the future controls being function of  $u(t)$  and its  $N_u$ -derivatives. The cost function is:

$$J(u_{N_y}) = \int_{T_1}^{T_2} [y^*(t, T) - w_r^*(T, t)]^2 dT \quad (19)$$

With the necessary substitutions the cost function becomes

$$J(u_{N_y}) = \int_{T_1}^{T_2} [T_{N_y} O + T_{N_y} H u_{N_y} - T_{N_y} w_r]^2 dT \quad (20)$$

and the minimization results in

$$u_{N_y} = K(w_r - O) \quad (21)$$

where

$$T_y = \int_{T_1}^{T_2} T_{N_y}^T T_{N_y} dT \quad \text{and} \quad K = [H^T T_y H]^{-1} [H^T T_y] \quad (22)$$

As explained above, just the first element of  $u_{N_y}$  is applied. Then, the first row of, which will be called, the control law is given by

$$u(t) = k[w_r - O]$$

Considering this modification, it is easy to show that the control law NCGPC is given by

$$u(t) = \frac{(w - y) - \sum_{i=1}^r \beta_i (L_{f_m}^i h_m(x_m) + y_e^{(i)}(t))}{\beta_r L_{g_m} L_{f_m}^{r-1} h_m(x_m)} \quad (23)$$

The control structure is shown in the Fig. 1 and is described below:

- The block labeled Process Model is the open loop observer or internal model used to get the model states; these are

needed to emulate the output derivatives of the system.

- The block labeled Regulation Filter is used to counteract the error between the output process derivatives and the output model derivatives. The regulation filter input is the error between the output process and output model.
- The control signal  $u$  is the same for process and model, the control system has an output feedback.

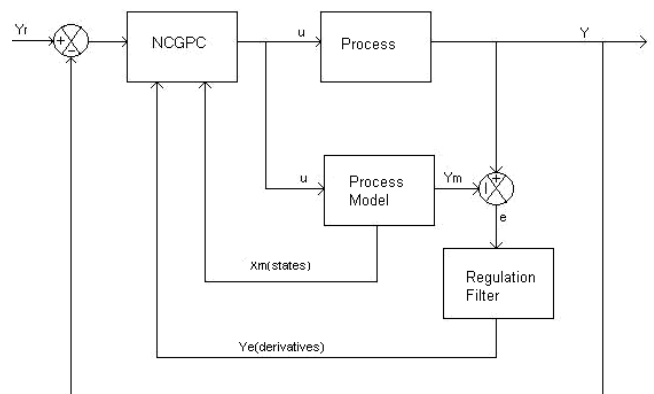


Fig. 1 Control Scheme

### 3 Analysis of Close loop Response

Substituting the control law into the  $r$ th derivative given by Eq. 16 leads to:

$$y_m^{(r)}(t) = 1/\beta_r (w - y(t)) - \sum_{i=1}^{r-1} \beta_i (y_m^{(i)}(t) + y_e^{(i)}(t)) - y_e^{(r)}(t) \quad (24)$$

From the regulation filter output, it can be seen

$$y^{(r)}(t) = e^{(r)}(t) + y_m^{(r)}(t)$$

Substituting the above equation into this equation, the following expression is obtained

$$y^{(r)}(t) = e^{(r)}(t) + 1/\beta_r (y_r - y) - \sum_{i=1}^{r-1} \frac{\beta_i}{\beta_r} (y_m^{(i)} + y_e^{(i)}) - y_e^{(r)}(t) \quad (25)$$

Making some algebraic manipulation and taking the Laplace transform, considering initial conditions equal zero, the following expression is obtained

$$Y_m(s) + Y_e(s) = G(s)W(s) - G(s)(Y(s) - Y_m(s) - Y_e(s)) \quad (26)$$

Where  $G(s)$  was given by Eq. 12, adding  $y(s)$  to both sides of this equation and rewriting:

$$Y(s) = G(s)Y_r(s) + (1 - G(s))(Y(s) - Y_m(s) - Y_e(s)) \quad (27)$$

$Y_e(s)$  can be obtained by the following transfer function

$$Y_e(s) = G_e(s)(Y(s) - Y_m(s)) \quad (28)$$

Substituting  $Y_e(s)$  into the previous equation, gives

$$Y(s) = G(s)W(s) + (1 - G(s))(1 - G_e(s))(Y(s) - Y_m(s)) \quad (29)$$

When the process model is perfect, the response is given as:

$$Y(s) = G(s)W(s) \quad (30)$$

From this equation, it can be deduced that

$$y(t) \rightarrow w \text{ as } t \rightarrow \infty \quad (31)$$

If the process model is not perfect, the response is given by the Eq. 24, it is possible to see that the second term on the right hand side of this equation will tend to zero if

$$y(t) - y_m(t) \rightarrow \text{constant} \text{ when } t \rightarrow \infty \quad (32)$$

then

$$y(t) \rightarrow w \text{ as } t \rightarrow \infty \quad (33)$$

When the regulation model is not considered, its response will be given by

$$Y(s) = G(s)W(s) + (1 - G(s))(Y(s) - Y_m(s)) \quad (34)$$

This response is given as well in the controller presented in [2]. It can be seen that in order to reduce the effect that the mismatch between model and process, which is present in the second term of the right hand side of the Eq. 34, it is necessary to increase the band width  $G(s)$ . However this may cause over shoot in the responses and an excessive input signal. In other words  $G(s)$  has influence on the performance and on the robustness. While, when the regulation filter is added, it is possible to see from the Eq. 29 that it is not necessary to increase the band width  $G(s)$ , to reduce the effect that the mismatch between model and process. The band width of  $(1 - G(s))(1 - G_e(s))$  can be increased, if the band width of  $G_e(s)$  is increased. Then the performance is given by  $G(s)$  and the robustness by  $G_e(s)$ .

#### 4 Analysis of Predictive Control for an Induction Motor

In [14] a position predictive control scheme for an induction motor is presented. The non-linear differential equations, which describe the dynamics of the motor, are represented by a d-q model. The design of a Generalised Predictive Control is obtained

as a simplified model. On the other hand, an observer is used in open loop in order to obtain state measurements. The design of the controller was based on the use of the simplified model. Obviously, the model is simpler than the original d-q model. This approximation is sufficient for control design purposes. Thus, the relative degree of the motor and its model are the same, where the output variable is the angular position  $q$ , and the control variable is  $V_s$ . Nevertheless, torque dynamics were neglected, while the average torque is considered. To obtain the predictive controller it was necessary to get the derivatives of output of the simplified model. Obtaining

$$\begin{aligned} y_m &= q_m \\ \dot{y}_m &= w_{rm} \end{aligned} \quad (35)$$

$$\ddot{y}_m = -Dw_{rm} / J + T_{em} / J$$

In this case until the second derivative was gotten, this is the relative degree of the simplified model.

When the predictor is equal to the relative degree, the NCGPC becomes in a state feedback linearization, and the control law derived from the model equation is as follow:

$$u = \frac{-L_f^2 h(x_m) + \{r_0(y_{ref} - y) - \frac{10}{3T} + [r_1(y_{ref} - y) - w_{rm}] \frac{5}{2T} + r_2(y_{ref} - y)\}}{L_g L_f h(x_m)} \quad (36)$$

where

$$L_f^2 h(x_m) = -Dw_{rm} / J \quad (37)$$

$$L_g L_f h(x_m) = T_{em} / J \quad (38)$$

and  $T_{em}$  is given by

$$T_{em} = \frac{2nR_r V_s^2 / (w\phi)}{(R_s + R_r / \phi)^2 + (w(l_s + l_r))^2} \quad (39)$$

Replacing  $u = V_s^2$  we obtain

$$T_{em} = f(w_{r_m})u$$

where

$$f(w_{r_m}) = \frac{2nR_r / (w\phi)}{(R_s + R_r / \phi)^2 + (w(l_s + l_r))^2} \quad (40)$$

The control input is voltage amplitude  $V_s = \sqrt{u}$ . And therefore the mechanical part of the motor is reduced to:

$$\phi = 1 - \|s - 1\|$$

Where  $\phi$  represents a normalisation of the slip  $s$ , which can be written as

$$s = \frac{w_s - w_{r_m}}{w_s} \quad (41)$$

with  $w_s = w / n$

Where  $w_s$  is defined as the synchronous speed of the motor.

Note that the simplified model has to be simulated in parallel (open loop observer), in order to obtain the rotor velocity  $w_{rm}$ .

Substituting the control law  $u$  given by Eq. 32 into the last expression of Eq. 25,

$$\ddot{y}_m = a_2(y_r - y) - a_1 \dot{y}_m \tag{42}$$

where

$$a_2 = [r_0 \frac{10}{3T^2} + r_2 + r_1 \frac{5}{2T}] \quad a_1 = \frac{5}{2T}$$

Making some algebraic manipulations

$$\ddot{y}_m + a_1 \dot{y}_m + a_2 y_m = a_2(y_r - y + y_m) \tag{43}$$

Taking Laplace transform

$$Y_m(s)(s^2 + a_1s + a_2) = a_2(Y_r(s) - Y(s) + Y_m(s)) \tag{44}$$

if

$$G(s) = \frac{a_2}{s^2 + a_1s + a_2} \tag{44}$$

Then

$$Y_m(s) = G(s)(Y_r(s) - Y(s) + Y_m(s)) \tag{45}$$

Rewriting

$$Y(s) = Y_r(s)G(s) + (1 - G(s))(Y(s) - Y_m(s)) \tag{46}$$

When the process model is perfect, it can be deduced that

$$y(t) \rightarrow y_r \text{ as } t \rightarrow \infty \tag{47}$$

If the process model is not perfect, the response is given by the Eq. 46 and the second term on the right hand side of this equation will tend to zero if

$$y(t) - y_m(t) \rightarrow \text{constant} \text{ when } t \rightarrow \infty$$

then

$$y(t) \rightarrow y_r \text{ as } t \rightarrow \infty$$

In order to visualize this, in Fig.2 it is possible to see that the responses of process and model are not the same, as mentioned in [12], the difference is a constant and the model output never reaches the reference, that in this case is  $10 \text{ rad}$ , rewriting Eq. 46 it is easy to see.

$$Y_m(s) = G(s)Y_r(s) - G(s)(Y(s) - Y_m(s)) \tag{48}$$

Meanwhile, the process output tends to the reference as explained it above in Eq. 46.

This response is given as well in the controller presented in [2]. It can be seen that in order to reduce the effect that the mismatch between model and process, which is present in the second term of the right hand side of the Eq. 46, it is necessary to increase the band width  $G(s)$ , but the parameters are function of  $T$  and the parameters of the reference trajectory. However this may cause over shoot in the

responses and an excessive input signal. In other words  $G(s)$  has influence on the performance and on the robustness. While, when the regulation filter is added, as explained in the previous section, it is not necessary to increase the band width  $G(s)$ , to reduce the effect that the mismatch between model and process. The band width of  $(1-G(s))(1-G_e(s))$  can be increased, if the band width of  $G_e(s)$  is increased. Then the performance is given by  $G(s)$  and the robustness by  $G_e(s)$ .

The Fig. 3 shows the outputs when  $T=4,5,1$ ,  $a_1=5, a_2=6$  and the Fig. 4 shows the control signals. Also, it is necessary to remark that the choice of the parameters  $r_0, r_1, r_2$  and  $T$  must be done very carefully, because the system can become unstable, the poles real part of  $G(s)$  must be negative, then  $a_1, a_2$  must be positives. If  $r_0 = 0, r_1 = 2, r_2 = -4$  from Eq. 42 it can be deduced that  $T \leq \frac{10}{8}$  in order to the closed loop system will be stable.

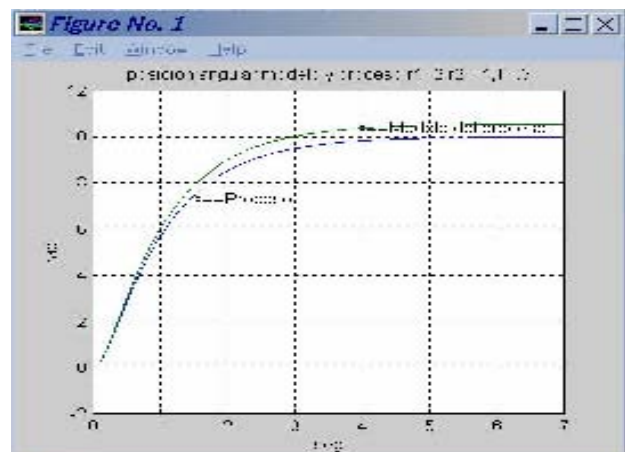


Fig. 2 Model Output and Process Output

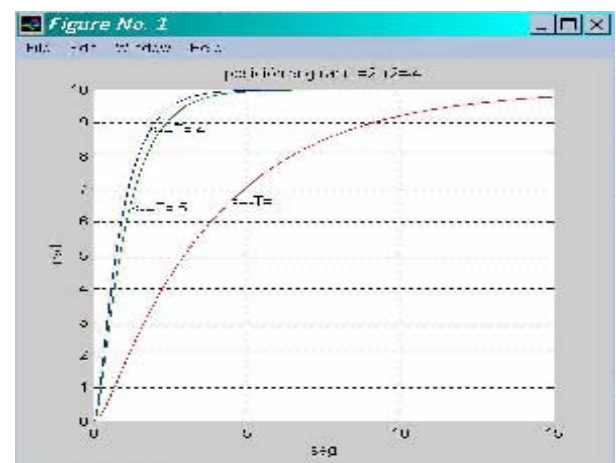


Fig. 3 The effects on process output (angular position) when T is varied.

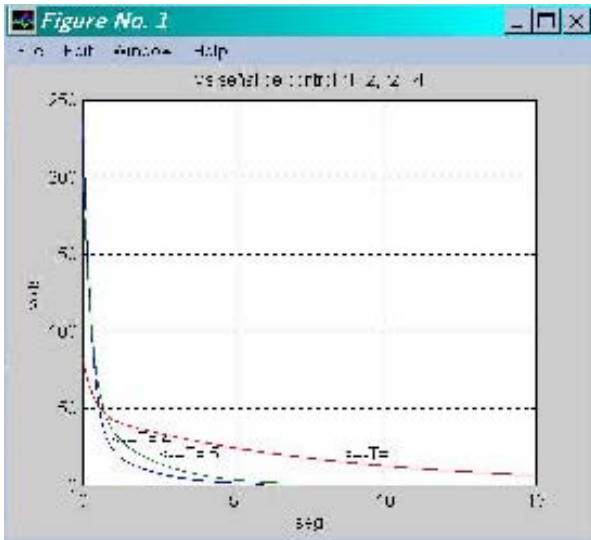


Fig. 4 The effects on  $u$  control input the voltage amplitude  $V_s = \sqrt{u}$  . when  $T$  is varied.

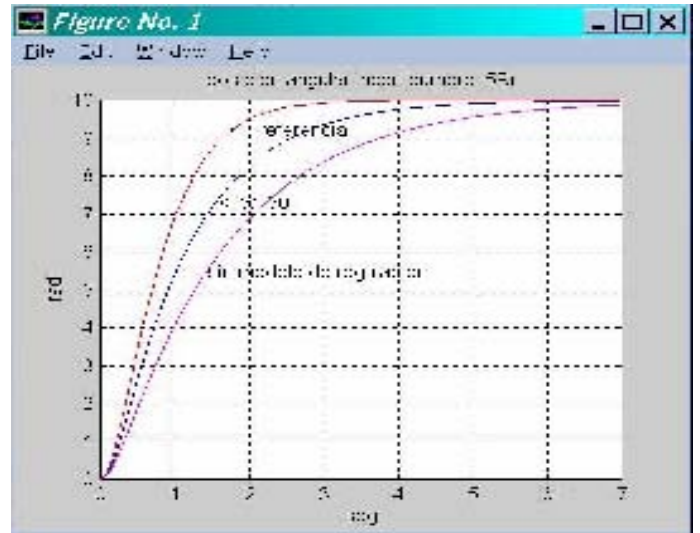


Fig. 5 The process output with and without regulation filter.

In order to improve the performance and robustness, an regulation filter is added as the proposed controller in this paper, the control structure can be seen in the Fig.1.

The emulated output derivatives of the motor are given by:

$$\begin{aligned} \dot{y} &= w_{rm} + y_e(t) \\ \ddot{y} &= -Dw_{rm} / J + f(w_{rm})u / J + \ddot{y}_e(t) \end{aligned} \quad (49)$$

Considering this modification, it is easy to show that the control law NCGPC is given by

$$u = \frac{-L_f^2 h(x_m) - \ddot{y}_e + \{r_0(y_{ref} - y)\frac{10}{3T^2} + [r_1(y_{ref} - y) - w_{rm} - \dot{y}_e]\frac{5}{2T} + r_2(y_{ref} - y)\}}{L_g L_f h(x_m)} \quad (50)$$

Figure 5 shows the process output with and without regulation filter when the control parameters are chosen as  $T=0.5$ ,  $a_1=5$ ,  $a_2=6$  and the measurement uncertainty of the rotor resistance  $R_r$  is 50%. Figure 6 shows the control signals  $u$  using a regulation filter with bandwidth  $50rad/sec$  and the control signal without filter. The improvement of the system performance when a regulation model is used and an excessive input signal is not required as can be seen in Fig. 6

In motors control is usual to find that the required reference trajectories are similar as the trajectory in Fig. 7, the output response is good and the control signal shown in Fig. 8 has a remarkable attenuation.

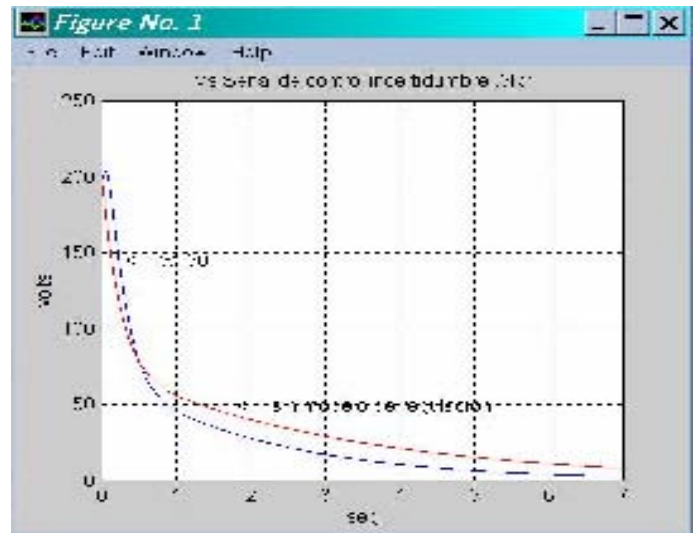


Fig. 6 The control signal  $u$  with and without regulation filter.

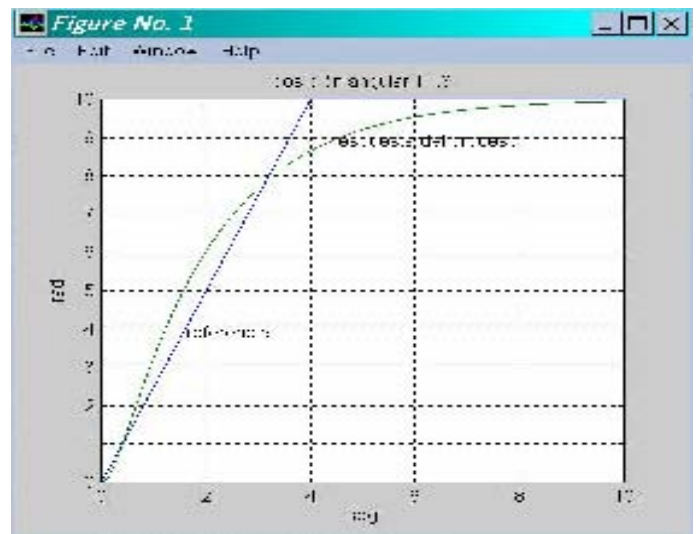


Fig.7 The process output when a reference trajectory is specified.

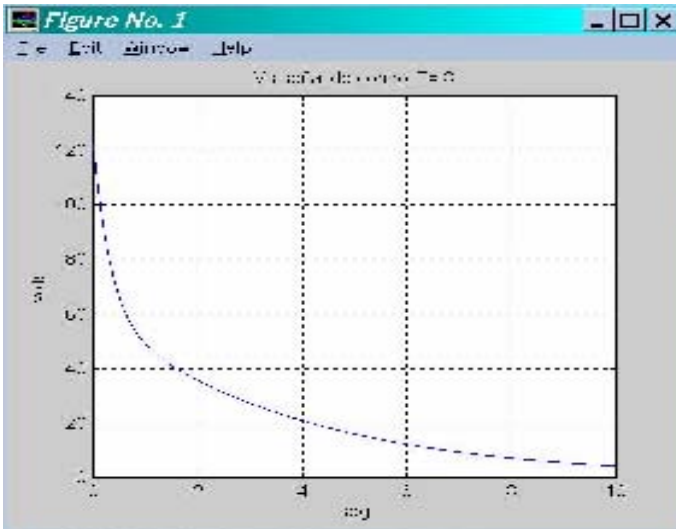


Fig. 8 The control signal when a reference trajectory is specified.

### 5 Simulation Results

In this section simulations are presented in order to show the effectiveness of the proposed controller and illustrate the effects of the regulation filter.

The example used in the simulation is given by

$$\begin{aligned} \dot{x}_1(t) &= -x_1 - a_1 x_2 \\ \dot{x}_2(t) &= \exp(-a_2 x_2) - 1 - a_3 u \end{aligned} \quad (30)$$

The system output is  $y = x_1$

The process model is

$$\begin{aligned} \dot{x}_{1m}(t) &= -x_{1m} - x_{2m} \\ \dot{x}_{2m}(t) &= \exp(-x_{2m}) - 1 - u \end{aligned} \quad (31)$$

The model output is  $y_m = x_{1m}$

The regulation filter is

$$\begin{aligned} \dot{y}_r(t) &= y_{1r} \\ \dot{y}_{1r}(t) &= -a_{0r} y_r - a_{1r} y_{1r} + a_{0r} e \end{aligned}$$

where  $e = y - y_m$

For this study  $T$  is chosen as  $T=0.1$  and the initial conditions are  $x_1(0) = -0.1$  and  $x_2(0) = -0.2$ . If the process model is perfect,  $a_1 = 1$ ,  $a_2 = 1$  and  $a_3 = 1$ . Once again parameter uncertainties, are modeled by setting, these parameters as  $a_1 = 1.4$ ,  $a_2 = 0.7$  and  $a_3 = 1.1$ . The variations of the regulation filter bandwidth are chosen as  $5 \text{ rad/sec}$  and  $15 \text{ rad/sec}$  and one case without this filter. Figures 2 to 5 illustrate the effects due to the variations of regulation filter is used. In fact it can be concluded from these figures

that in order to improve the performance this filter is useful.

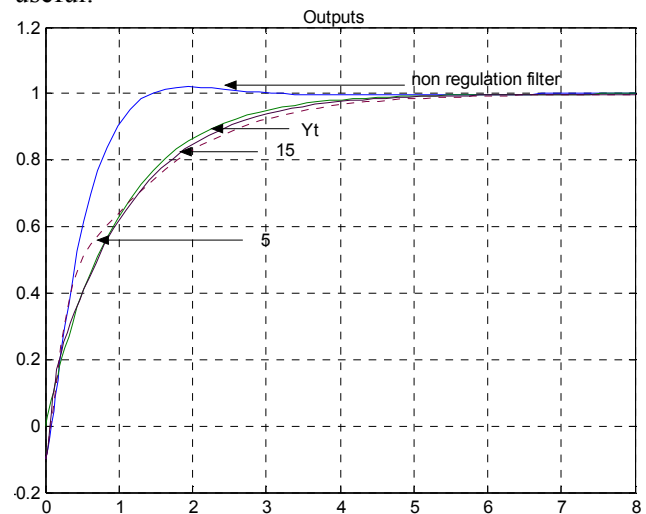


Fig. 9 The effects on  $y_p$  process output when the regulation filter is varied

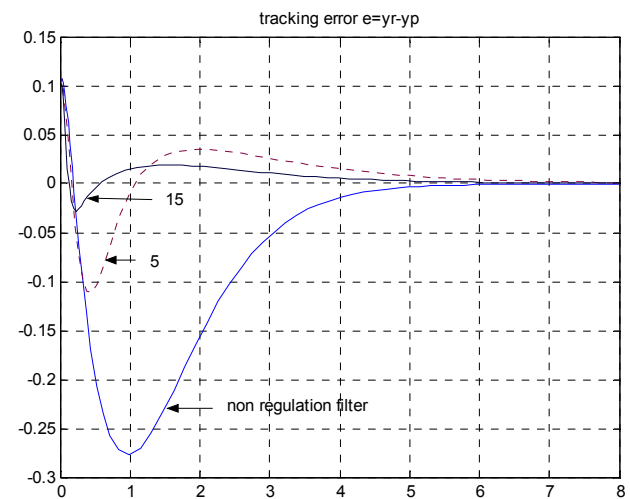


Fig. 10 The effects on tracking error  $e_r = y_r - y_p$  when the regulation filter is varied

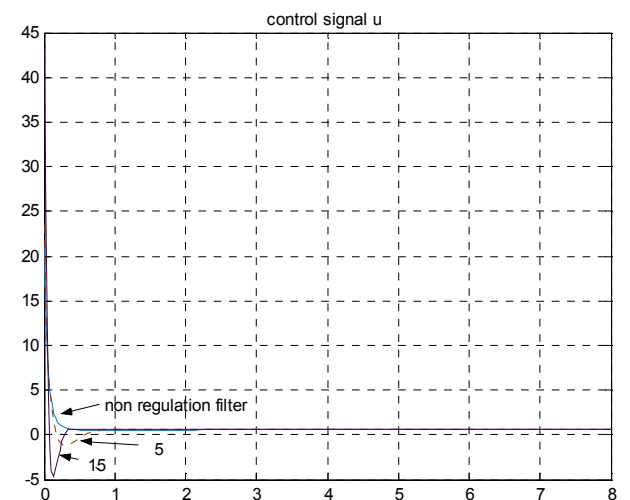


Fig. 11 The effects on  $u$  control signal when the regulation filter is varied

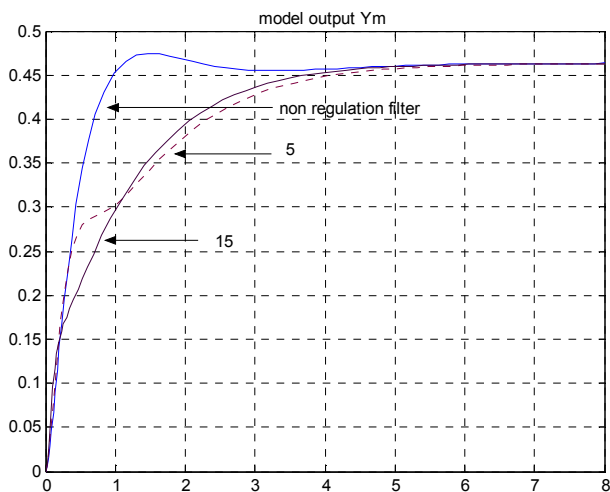


Fig. 12 The effects on  $y_m$  model output when the regulation filter is varied

#### 4 Conclusions

A generalized predictive control with internal model for nonlinear systems is presented, which has the advantage that the output is the only measurement required and the simplicity of the control law allows a simple and straightforward implementation and the robustness obtained by addition of the regulation filter, understanding as robust that stability and the performance specification are achieved, when there are uncertainties in the model. In order to improve the performance and the robustness separately, a correction is applied to the model output derivatives, in order to take account the difference between the process model and plant. The theoretical analysis closed loop response shows that the proposed controller can reduce the effect of modelling uncertainties such that the performance of the control system is greatly improved. The correction is carried out by adding a regulation filter as in [12] and [13]. A controller with two degrees of freedom is achieved by adding the regulation filter. The analysis and simulation results show that the output feedback model predictive control proposed can tolerate certain process uncertainty.

It has been shown by the simulation and the analysis developed in section 4, that the predictive control for an induction motor developed in [14] has been improved by adding the regulation filter. Also it is shown that the output has a good tracking of a reference trajectory that is very usual in motors. The authors consider that the analysis is very helpful, because shown the range of controller parameters in order to get a good performance, stability and robustness of the control system.

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