Solutions for Nonlinear Multivariable Processes Control

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Abstract: - The paper proposes a combined feedforward and feedback control scheme with nonlinear compensators as a solution for multivariable nonlinear processes. This is developed based on classic decentralized control approach for multivariable processes. The design methods for each structure's component are presented. These are based on experimental tests, classic identification and closed loop pole placement methods. The proposed method is compared to the classic solutions given by the decentralized and static decoupling procedures. An analysis on the advantages and the disadvantages of the proposed structure was made. To solve the disadvantages there have been proposed two additional structures. The applicability of the proposed method and of the two classic methods is proved using a real-time structure based on a RST control algorithm. In the end, its software implementation and the obtained results are also showed and commented.

Key-Words: - multivariable control, nonlinear process, real time system

1 Introduction

It is a common industrial practice to reduce a multivariable control problem to SISO control approach [1], [2]. There are a lot of valuable methods, strategies and solutions obtained in researches for solving this problem. Few of these are base on decentralized control strategy and respectively, on static decoupling procedures [3-8].

In this paper there are presented some comparative results of three methods real time implementation for the control of a multivariable process with nonlinear static characteristic. The first two methods are classical and are based on decentralized control and respectively, on static decoupling procedures. For the third method, the authors propose an improved scheme and the corresponding design procedures, based on a combined feedforward - feedback structure [9-12].

Shortly, the decentralized control strategy design the SISO control loops of a MIMO process as totally independent loops, as shown in Fig. 1. Each control algorithm is robust enough to reject the disturbances determined by all other "parallel" loops functioning.

Supplementary, the static decoupling procedures come up with an additional decoupling block, generally introduced between the multivariable process and the N independent control algorithms (Fig. 2). The product between the decoupling block (matrix) and the process's static gain matrix is an identity matrix [7].

$$\begin{bmatrix} k_{11} & k_{21} & \cdots & k_{n1} \\ k_{12} & k_{22} & \cdots & k_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ k_{1n} & k_{2n} & \cdots & k_{nn} \end{bmatrix} \times \begin{bmatrix} kd_{11} & kd_{21} & \cdots & kd_{n1} \\ kd_{12} & kd_{22} & \cdots & kd_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ kd_{1n} & kd_{2n} & \cdots & kd_{nn} \end{bmatrix} = I_n$$

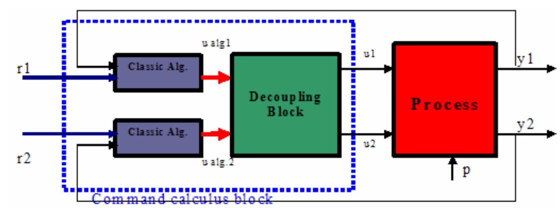

Fig.1. Decentralized control principle

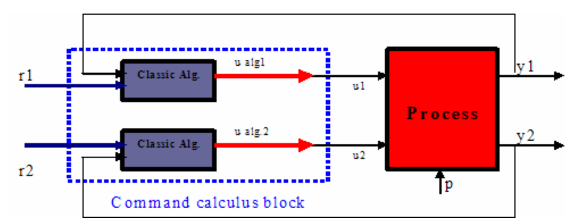

Fig.2. Static decoupling scheme

The two presented methods provide very good results in normal situations, when the process's nonlinearities do not have important effects. If the nonlinearities are important, for real time functioning, there can be observed: decreasing performances or limitations (like bad reference tracking and the reduction of control system range).

To solve these problems, starting with decentralized scheme, the authors propose a control scheme that compensates the process's static nonlinearities. This solution implies that for each control loop two commands should added: "a direct command – feedforward command" generated by the inverse model command generator, and the second, generated by a classic and very simple algorithm (PID, RST etc.).

Several papers and researches on this type of structure, also named *inverse model,* exist. Few of these, with a very fortuity choosing excuse, can be mentioned: [9-11]. According to them, the paper proposes a very simple and efficient version, presented in Fig. 3.

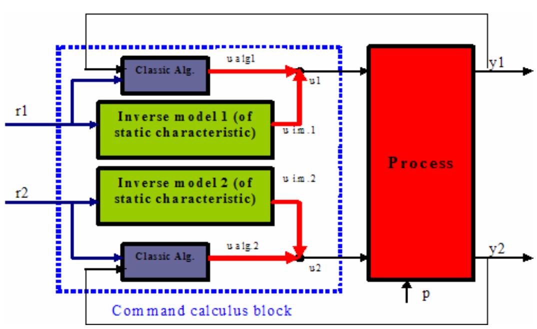

Fig.3. Proposed control scheme for multivariable decupled process, with nonlinearity compensation

On Fig.1, 2, 3 the blocks and variables are:

- Process – physical system to be controlled;
- Command calculus –computes the control law;
- Classic Alg. – control algorithm (PID, RST);
- y – output of the process;
- u – output of the Command calculus block;
- u alg. – output of the classic algorithm;
- u i.m. – output of nonlinearity compensator (the inverse static model) block;
- r – system's set point or reference trajectory;
- p – disturbances of physical process.

Here, the first command *(u i.m.)*, based on the process static characteristics, is dependent on set point value and is designed to generate a corresponding value to drive the process's output close to imposed set point. The second (classic) algorithm *(u alg.)* generates a command that, correct the difference caused by external disturbances and according to set point, by eventual bias error caused by mismatches between calculated inverse process characteristic and situation from real process.

This solution proposes the treatment of these inverse model mismatches that "disturb" the first command, as a second command classic algorithm's (identification) model mismatches. This imposes the design of a classic algorithm with a corresponding robustness reserve. For this reason, designing the second algorithm takes in two steps:

- the design of a classic algorithm based on a model identified in a real functioning point – fortuity selected or, on the middle of process characteristic;
- verification of algorithm's robustness and its improvement, if necessary, using a (re)designing procedure;

Related to classical control loops, inverse model control need addressing some supplementary specific aspects:

- Determination of static characteristic of the process;
- Construction of inverse model;
- Robust control law design.

The structure presented before can be used to control multivariable processes [23-25] that support decentralized procedures. These imply decomposition of an MIMO (N inputs and M outputs) process in a max(N,M) (usually N=M) parallel and independent processes. A singular process has a main "canal" from u_i to y_i and possible, a lot of secondary "canals" from u_i to y_j, where $i \neq j$. All secondary "canals", which represent connections between parallel processes, can be considered as disturbances, nonlinearities, process identification mismatches etc. from the point of view of the main "canal". Fig. 4 presents the main and secondary "canals" for u_1 to y_1 process.

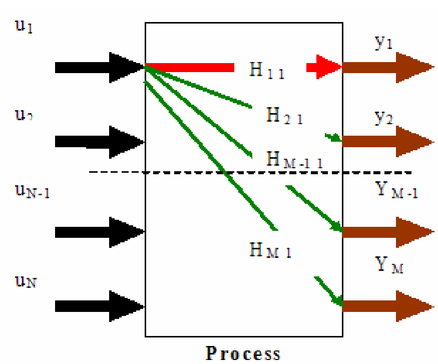

Fig.4. Decentralized or decomposition procedure for an MIMO process

Particular, for a process with 2 inputs and 2 outputs the control algorithm is presented in Fig. 3.

In the next sections, we will focus on the most important aspects met while designing the presented structure.

2 Inverse Model Design Procedure

As mentioned above, for the proposed inverse model control structure, the supplementary specific aspects are: determination of static characteristic of each parallel processes, construction of inverse model and robust control law design. We will present these in the next sections.

2.1 Determination of static characteristic

This operation is based on several experiments of discrete step increasing and decreasing of the command *u(k)* of the main "canal" and measuring the corresponding stabilized process output *y(k)*. The command *u(k)* covers all possibilities (0 to 100% in percentage representation). Because the secondary "canals", witch will have all important combinations during experiments, can affect the main "canal", and because the process is disturbed by noises, usually the static characteristics are not identically. The final static characteristic is obtained by meaning of correspondent position of these experiments. Fig. 5 presents this operation. The graphic between two "mean" points can be obtained using extrapolation procedure.

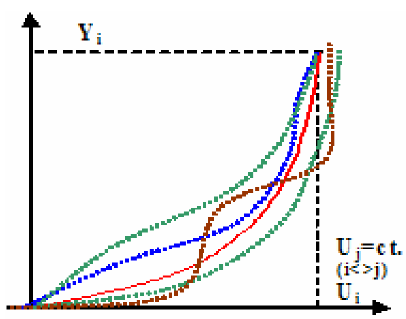

Fig.5. Determination of static characteristic of the process. Continuous line represents the final characteristic.

According to system identification theory the dispersion of process trajectory can be finding using expression (1):

$$\sigma^2[n] \cong \frac{1}{n-1}\sum_{i=1}^{n} y^2[i],\ \forall n \in N^* \setminus \{1\} \qquad (1)$$

This can express a measure of superposing of secondary "canals", noise that action onto process, process's nonlinearity etc. and is very important on control algorithm designed robustness. Other possibility is to find the position and the value *mg* of the maximal distance from "mean" characteristic.

2.2 Construction of inverse model

This step deals with the „transposition" operation of the means process's static characteristic. Figure 6 presents this construction. According to this, *u(k)* is dependent to *r(k)*. This characteristic is stored in a table; thus we can conclude with this, for the inverse model based controller, selecting a new set point *r(k)* will impose finding in this table the corresponding command *u(k)* that determines a process output *y(k)* close to the reference value.

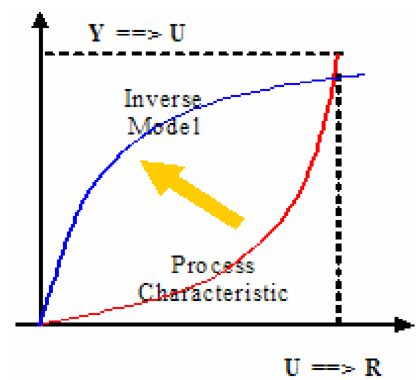

Fig.6. Construction of inverse model

2.3 Control law design

Control algorithm's duty is to eliminate the disturbance and differences between inverse model computed command and real process behavior.

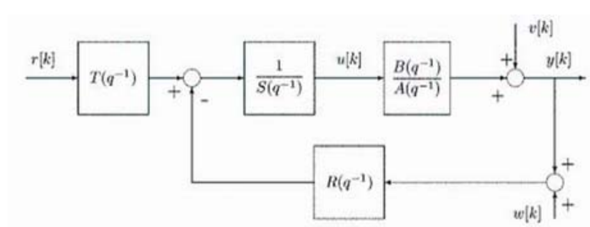

Fig.7. RST control algorithm structure

A large variety of control algorithms can be used here, PID, RST, fuzzy etc., but the goal is to have a very simplified one [1], [2], [13].

For this study we use a RST algorithm. This is designed using pole placement procedure [1]. Fig. 7 presents a RST algorithm.

The R, S, T polynomials are:

$$\begin{aligned} R\left(q^{-1}\right) &= r_0 + r_1 q^{-1} + \ldots + r_{nr} q^{-nr} \\ S\left(q^{-1}\right) &= s_0 + s_1 q^{-1} + \ldots + s_{ns} q^{-ns} \\ T\left(q^{-1}\right) &= t_0 + t_1 q^{-1} + \ldots + t_{nt} q^{-nt} \end{aligned} \tag{2}$$

Algorithm pole placement design procedure is based on identified process's model.

$$y(k) = \frac{q^{-d} B(q^{-1})}{A(q^{-1})} u(k) \tag{3}$$

where

$$\begin{aligned} B\left(q^{-1}\right) &= b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{nb} q^{-nb} \\ A\left(q^{-1}\right) &= 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na} \end{aligned} \tag{4}$$

The identification [1], [14], [15] is made in a specific process operating point and can use recursive least square algorithm exemplified in next relations developed in [1]:

$$\begin{aligned} &\hat{\theta}(k+1) = \hat{\theta}(k) + F(k+1)\phi(k)\varepsilon^0(k+1), \forall k \in N \\ &F(k+1) = F(k) - \frac{F(k)\phi(k)\phi^T(k)F(k)}{1+\phi^T(k)F(k)\phi(k)}, \forall k \in N \\ &\varepsilon^0(k+1) = y(k+1) - \hat{\theta}^T(k)\phi(t), \forall k \in N, \end{aligned} \tag{5}$$

with the following initial conditions:

$$F(0) = \frac{1}{\delta} I = (GI)I, 0 < \delta < 1 \tag{6}$$

The estimated $\hat{\theta}(k)$ represents the parameters of the polynomial plant model and $\phi^T(k)$ represents the measures vector.

This approach allows the users to verify, and if is necessary, to calibrate algorithm's robustness [1], [16], [17]. Next expression and Fig. 8 present "disturbance-output" sensibility function.

$$\begin{aligned} &S_{vy}(e^{j\omega}) \overset{def}{=} H_{vy}(e^{j\omega}) = \\ &= \frac{A(e^{j\omega})S(e^{j\omega})}{A(e^{j\omega})S(e^{j\omega}) + B(e^{j\omega})R(e^{j\omega})}, \quad \forall \omega \in R \end{aligned} \tag{7}$$

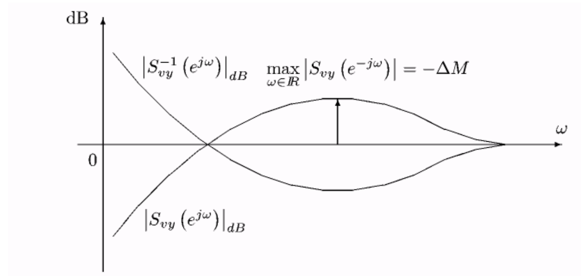

Fig.8. Sensibility function graphic representation

In the same time, the negative maximum value of sensibility function represents the module margin.

$$\Delta M\Big|_{dB} = -\max_{\omega \in R} \left| S_{vy}(e^{j\omega}) \right\|_{dB} \tag{8}$$

Base on this value, in an "input-output" representation [1], process nonlinearity can be bounded inside of "conic" sector, presented in Fig. 9, where a_1 and a_2 are calculated using next expression:

$$\frac{1}{1-\Delta M} \geq a_1 \geq a_2 \geq \frac{1}{1+\Delta M} \tag{9}$$

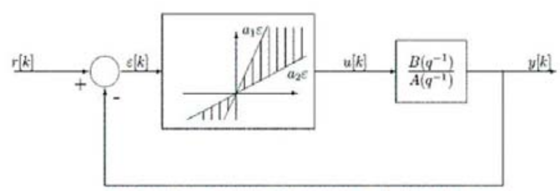

Fig.9. Robust control design procedure

Finally, if is imposed that all nonlinear characteristics to be (graphically) bounded by the two gains, or gain limit to be great or equal to process static characteristic maximal distance $\Delta G \geq mg$, a controller that has sufficient robustness was designed.

3 Analysis of Proposed Structure

In this section we will present a few advantages, disadvantages or limitations and some possible developments of the presented structure.

3.1 Advantages of proposed structure

The main advantage consists in using a classic procedure for designing the control algorithm and determination of the inverse command blocks, comparative to multivariable control design procedures. Well know procedure for identification and control law design are used. As it will be shown in experimental tests, all procedures for the inverse model characteristic identification can be included in a real time software application.

The system is very stabile due to the global command that contains a "constant" component generated by an inverse model command block, accordingly to set point value. This component is not influenced by the noise.

A fuzzy logic bloc that can "contain" human experience about some nonlinear processes can replace the inverse model command generator.

Being not very complex in terms of real time software and hardware implementation, the control law doesn't need important resources.

3.2 Disadvantages or limitations of structure

The main limitation is that this procedure can be applied just for the processes that support decupling control scheme.

This structure is very difficult to be used for the system that doesn't have a bijective static characteristic and for systems with different functioning regimes.

Another limitation is that this structure can be used only for stabile processes. In the situations where the process is "running", the global command is likely not to have enough flexibility to control it.

The increased number of experiments for the determination of a correct static characteristic can be another disadvantage.

3.3 Possible solutions

In the situation when the control law becomes very complex, situation caused by very "sinuous" process characteristics, these can be "divided" in two or more components. The global control system becomes a "multivariable multiple inverse model system" [18-20]. Fig. 10 presents the way such a process characteristic is divided in three sections.

Selection between models can be made based on set point or filtered set point position. Fig. 11 presents the scheme of a system which has a multiple model structure for one of controllers [20].

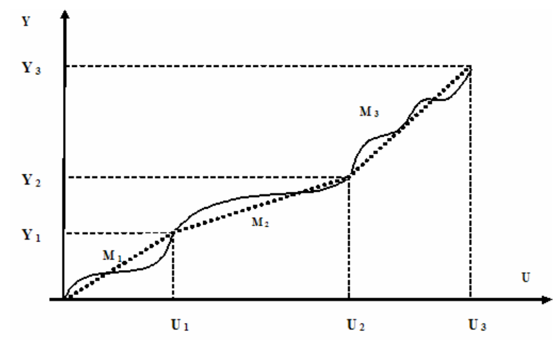

Fig.10. Dividing of process characteristic in three sections. Continuous line represents the process characteristic.

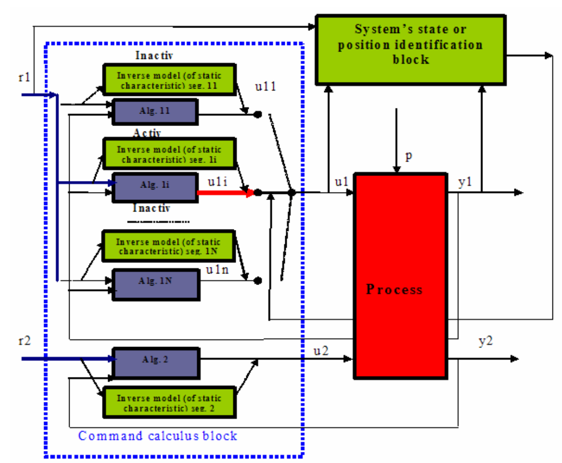

Fig.11. Multivariable multiple inverse model system – solutions for very "sinuous" process characteristics

Another situation when the control law becomes very complex is caused by multiple "parallel" process characteristics. For this situation, the process can be considered to have multiple functioning regimes. For each regime determined by a command or process output domain interval U_{jk1} - $U_{jk1,}$ the corresponding model M_j is considered as shown in Fig. 12.

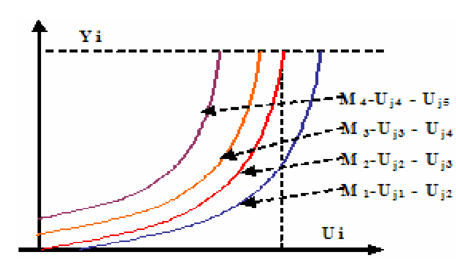

Fig.12. Process with multiple functioning regimes

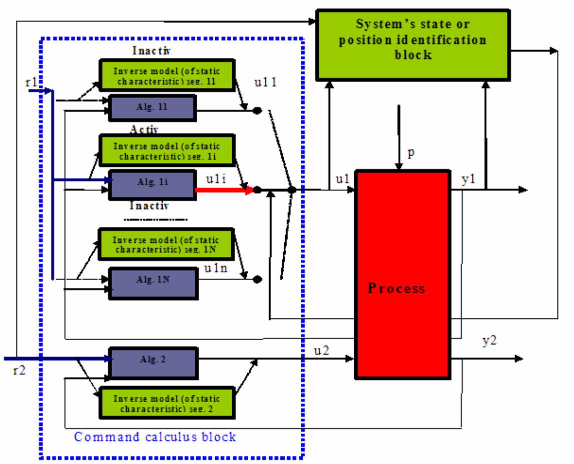

Fig.13. Multivariable multiple inverse model system – solutions for multiple functioning regimes

Here, the global control system becomes a "multivariable multiple (inverse) regimes system" [19].

Selection between regimes/models can be made based on set point or filtered set point position. Fig. 13 presents the scheme of a system, which has a multiple model structure for one of controllers [19], [20], [21].

For this solution, the set points that select the model come from "parallel" controller.

One of multiple model specific problems is that shocks can appear when switching between controllers. To solve this unwished situation, the set point value is filtered by a digital system [1], [22], [26].

$$rd(k) = \frac{Bm(q^{-1})}{Am(q^{-1})} r(k) \tag{10}$$

The output of this filter is forced to touch all intermediary models (if any) during set point changes. The filter's coefficients are obtained by sampling a first or second order continuous system.

There are a lot of solutions for algorithms switching. A few of them, which can be applied on real time systems, are presented in [19], [20].

These proposed structures can be easily implemented on PLC structures.

4 Experimental Results

We have evaluated the performances of the proposed schemes (Fig. 1, 2, 3) on an experimental software simulator presented in Fig. 14.

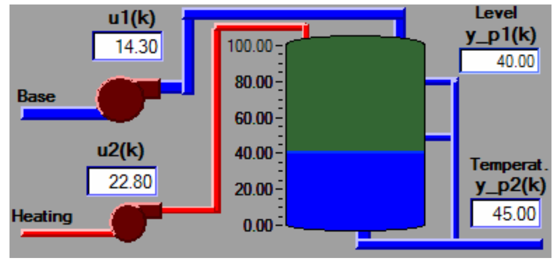

Fig.14. Experimental process simulator

On this installation, the user can control two parameters: level and temperature. According to decentralized principles, the level is controlled by the base quantity input and the temperature respectively, by heating quantity input. Both level and temperature processes have nonlinear characteristics caused by the real physics phenomenon and installation particularities (Fig. 15 medium characteristics).

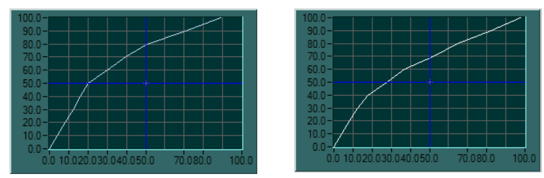

Fig.15. Nonlinear characteristics level – left, right - temperature

To acquire the data form the software simulator, a real time software application presented in Fig. 16 was developed. The application was developed using National Instruments's LabWindows/CVI. With this application 2x2 (two inputs and two outputs) processes can be tested. The acquired data are used for identification of the process parameters. For this reason, on each input – output pairs, the tester has the next functionalities:

- connection with the software simulator,
- setting/applying the manual command,
- setting/applying an additional pseudo random binary sequence (PRBS),
- setting the sampling period value,
- displaying the real time evolution curves,
- storage in files the process 's acquired input – output pairs values.

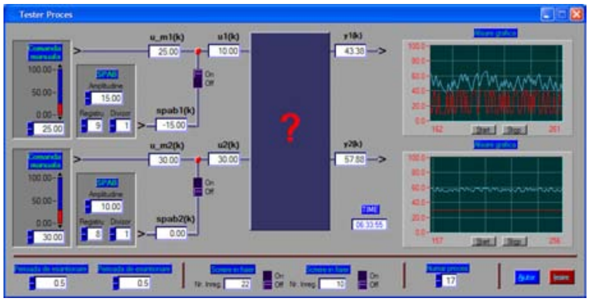

Fig.16. Interface of real-time data acquisition software application for 2x2 multivariable processes

Between the main "canals" of the process there are (two) secondary influences: "canal" from level to temperature, and "canal" from temperature to level. The influence can be express by static 2x2 gain matrix.

By making some tests:

a) u1=20.0, u2=0.0 we obtain K11=2.0, K21=0.245

b) u1=0.0, u2=20.0 we obtain K22=1.9, K12=0.5.

For these values, the coefficients of the decoupling matrix are:

Kd11=0.516655, Kd21=-0.06662
Kd22=0.543848, Kd12=-0.13596

By increasing the command u1 from 0.0 to 30.0 and keeping u2=10.0 constant the raising time Ts1 is 4.5s implying the sampling period Te1of 0.5 s.

By keeping u1=10.0 constant and increasing the command u2 from 0.0 to 30.0 we obtain the raising time Ts2 = 1.8s that makes the sampling period Te2=0.2 s.

The models for the level and the temperature processes, obtained by using the recursive least square procedure [1] from WinPIM software are:

$$M_{level} = \frac{0.60592 + 0.07053q^{-1}}{1. \; -0.55799q^{-1} -0.0358q^{-2}}$$

$$M_{temp} = \frac{0.12993 + 0.43005q^{-1} + 0.14257q^{-2}}{1. \; -0.75371q^{-1} -0.15158q^{-2} + 0.03953q^{-3}}.$$

These two models were identified in the lower (40%) region of the static characteristic.

The corresponding RST controllers, determined by employing the WinREG software are:

- for the level process:

$$R(q^{-1}) = 0.552689 \; -0.332786q^{-1} + 0.029291q^{-2}$$
$$S(q^{-1}) = 1.0 \; -1.057657q^{-1} + 0.057657q^{-2}$$
$$T(q^{-1}) = 1.478306 \; -1.893358q^{-1} \; + \; 0.664246q^{-2}$$

- for the temperature process:

$$R(q^{-1}) = 0.492338 \; -0.388487q^{-1} + 0.108353q^{-2} + \\ + 0.027731q^{-3}$$
$$S(q^{-1}) = 1.0 \; -0.591022q^{-1} -0.308964q^{-2} \; -0.100014q^{-3}$$
$$T(q^{-1}) = 1.423386 \; -1.823019q^{-1} \; + \; 0.639569q^{-2}$$

The close loop simulations made on WinPim software for the two model/controller pair: level and temperature are presented in Fig. 17 and respectively Fig. 18.

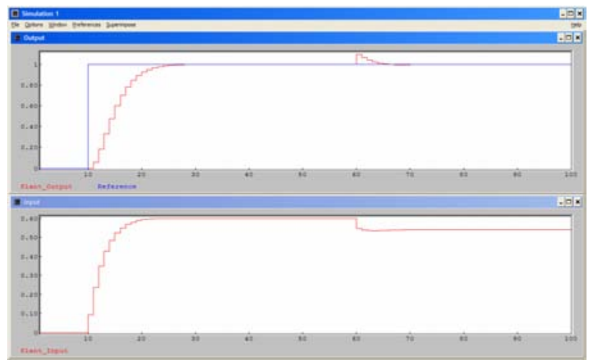

Fig.17. Simulation of closed loop for the model and controller for the level process

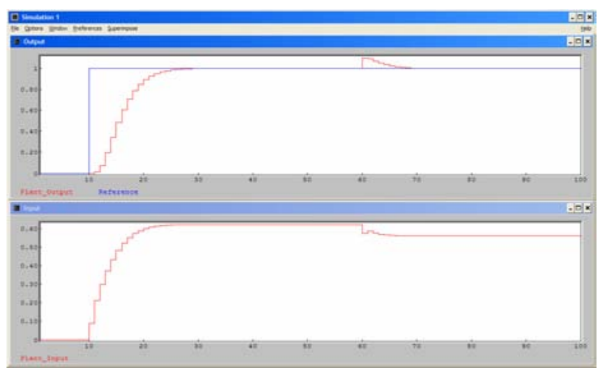

Fig.18. Simulation of closed loop for the model and controller for the temperature process

It can be observed that these figures are very similar. The reason of this similarity, especially for references tracking zones, is imposing the same performances for both closed loops.

For real time tests with the proposed controllers solutions (Fig. 1, 2, 3), a second real time software application, presented in Fig. 19, was developed. This was developed in the same National

Instruments's LabWindows/CVI.

This can control the 2x2 (two inputs and two outputs) processes. For this reason, for each of the two control loops, the application has the next functionalities:

- connection with the software simulator,
- setting/applying the manual command,
- setting/applying the automat command,
- setting/applying the set point value,
- switching between manual/automat command,
- setting the sampling period value,
- loading of process model and control algorithm parameters,
- loading of inverse model command generator parameters,
- displaying the real time evolution curves,

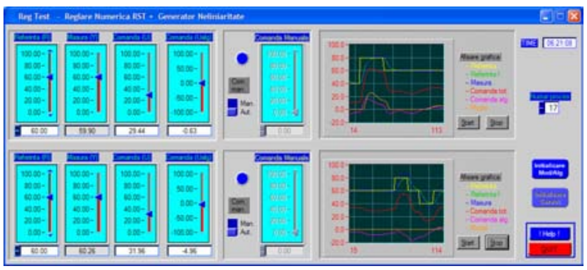

Fig.19. Inverse-model multivariable controller real-time software application – main window

The second application's window of great functionality importance is presented in Fig. 20.

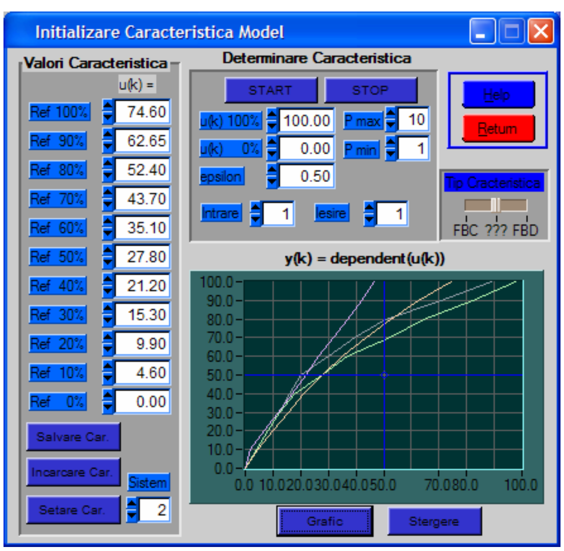

Fig.20. Inverse-model multivariable controller real-time software application – nonlinear characteristics identification window

This window allows to load/set the nonlinear process characteristics and supplementary, the identification of these characteristics.

The procedure of identifications, accordingly to imposed parameters, is based on searching of corresponding process input value that determines the output to be equal to 0%, 10%, 20%, …90%, 100%. The searching algorithm allows an imposed error value. This error value is set depending on the process noises level.

The graphic chart of this control application's window can display one or more process's characteristics.

For the three proposed solutions, a set of tests was performed. The references value was changed both in the region where the models were identified (40%), as well as in the superior zone, where the nonlinearities are profound.

In next figures the evolution curves are represented using next color code: yellow – set point; green – filtered set point; blue – process output; red – control structure output (total command); purple – RTS algorithm output; orange – identified model output;

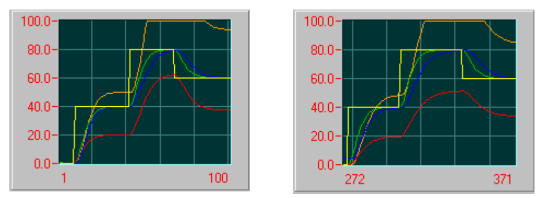

Fig.21. Real time performance for decentralized algorithm solution: level – left, right - temperature

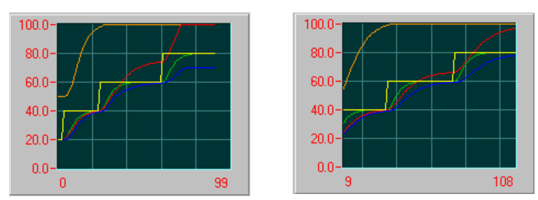

Fig.22. Real time performance for decoupling algorithm solution: level – left, right - temperature

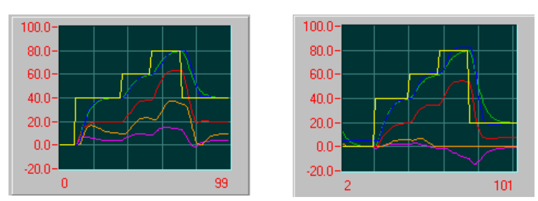

Fig.23. Real time performance for proposed algorithm solution: level – left, right - temperature

In these tests it can be observed that:

- all three solutions are stable;
- the first solution presents the inconvenient that the two loops influence each other, conducting to a slightly instability;
- the decoupling solution presents a command saturation problem;
- the last solution is the only one that can track the reference while it is varied on the entire domain (0-100%);
- for the last solutions, the references tracking performances are much closer to those obtained in simulation (Fig. 17 and Fig. 18), independent to the evolution zones. This proves that this structure can ensure and maintain the same performances for exploitation as those imposed or designed in simulation.

5 Conclusion

The paper proposes an inverse model structure as a solution for multivariable nonlinear processes. This is developed based on classic decentralized control approach for multivariable processes. For this structure, for each component, there are presented the design methods. These are based on experimental tests, classic identification and closed loop pole placement method.

The proposed method is compared to classic solutions of decentralized and static decoupling procedures.

An analysis on the advantages and the disadvantages of the proposed structure was made.

To solve the disadvantages caused by increasing of control algorithms complexity, there are proposed two additional schemes. These are based on multiple model/algorithm principle.

The experimental results section presents the evaluated results obtained using a real time software implementation. The tests are made on a software simulator and on a real time control application. These prove that, according to the class of considered processes, the solution offers good stability performances and that can track the reference while it is varied on the entire domain (0-100%). The exploitation performances are very similar to the imposed or designed performances for simulation.

During exploitation the inverse model solution does not impose complex operations, it is very easy to use and offers superior performances compared to classical structures.

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