An Epistemological Comparison between Fuzzy Logic Engines and Bayesian Filters

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Abstract: In this paper we analyze two methods of artificial intelligence: the Bayesian filter and the Fuzzy Logic engine. In order to do this we present each method and compare them. The mentioned methods have similar backgrounds but from epistemological point of view they are different. The paper ends with three case studies: the first about a Fuzzy Logic engine which is integrated into a Bayesian filter, this give us the possibility to underline the mentioned difference; the second case study about a mobile robot, where we present the main advantage of the Bayesian filter, which is the possibility to compute the degree of true about the model result; and the third case study about human decision modeling with Bayesian reasoning, where we underline the flexibility of the method .

Key-Words: Artificial intelligence methods, Fuzzy Logic, Plausible reasoning, Bayesian filter, Degree of true

1 Introduction

Present paper intends to continue the research program of human knowledge process which was started in [1,2]. We remember that the results of the phenomenological researches on artificial intelligent (AI) collocation [1] were seven questions which allowed the possibility to deep the understanding of the AI and which can drive to intelligent product construction. These seven questions are:

1. Are they known theories that have as object the human knowledge?
2. How can we use them in order to develop a human knowledge model?
3. How can we simulate this model and how can we improve it?
4. What is the technology – the methods and the tools – which can be used in order to copy the model?
5. What are the properties of the object that can be transformed in intelligent object?
6. How can we experiment the intelligent object?
7. What are the ethical aspects of the intelligent object construction?

One possible answer to the first question could be Plausible Reasoning another can be the Fuzzy Logic. The Plausible Reasoning theory [2] is based on reverend Thomas Bayes and Laplace results. The main concept of the Plausible Reasoning and Fuzzy Logic is the degree of truth – the plausibility. It is obvious that after this information we need some clarifications which are the subject of the present paper. We will focus not on the mentioned theories but on two methods which have been developed by these theories: the Bayesian filter and the Fuzzy Engine. The question which we intend to answer can be state in the following way: what is the difference between the degrees of truth used in Bayesian filters and in Fuzzy Engines? The answer will help to better understanding the mentioned concept and will allow new developments.

Our paper structure is composed from the following parts: presentation of Bayesian filters and Fuzzy engines fundaments; the two mentioned methods analyze; three cases study which involves both methods and in the end conclusions about the comparison results.

From the Plausible Reasoning point of view the backgrounds of the present work are E.T Jayne's probability theory [3] and also the related works of Cox. We will mention also the work of E. Yudkowsky [4] where an epistemology based on Thomas Bayesian result is presented and also J. Pearl work on causal reasoning [5]. The bridge between the Bayesian plausible reasoning and modeling a particular phenomenon has been inspired by the work of C. Pradalier, where the navigation of a mobile robot is controlled using Bayesian's filters [6]. The actuality of the subject is underlined also by Amiri in [7].

From the Fuzzy Logic point of view the backgrounds of the present work are [8] where a fuzzy engine construction is presented. We must mention also here the logical developments of I. Rudas [11]. Once again the actuality of the subject underlined in [9] and [10].

2 The Plausible Reasoning

In [2] we have presented, in an axiomatic form, the fundaments of the plausible reasoning theory. The background of a particular theory consists on principles or axioms. The difference between these two concepts consists on the fact that the axioms are self evident fundamental and the principles are accepted fundamental reason. This is the reason why we have chosen to name the next fundamental reasons principles

The principles of plausible reasoning:

The representation for the degree of truth (the plausibility) is given by the plausibility function:

$$p:\Phi \to [0 \quad 1];\ p(A \mid X) = y \tag{1}$$

where:
Φ is a set of sentences;
$p(A \mid X)$ is a continuous and monotonic function which associates a particularly degree of truth for the sentence A in the condition that sentence X is true;

The consistence of the commune sense requires the following property for the function p

$$p(AB \mid X) = p(A \mid X)p(B \mid AX) \tag{2}$$

$$p(A \mid X) + p(\neg A \mid B) = 1 \tag{3}$$

$$p(A + B \mid X) = p(A \mid X) + p(B \mid X) - p(AB \mid X) \tag{4}$$

$$p(A_i \mid X) = \frac{1}{n} \qquad i = 1...n \tag{5}$$

where $\{A_i\}|_{i=1...n}$ is a complete set of mutual excusive sentence

Some comments are necessaries:
by consistence we mean:

- every possible way of reasoning a sentence must lead to the same result;
- the equivalent sentences have an equal degree of truth – the same plausibility;
- in order to obtain the plausibility for a sentence we must take into account all the available evidence;

$p(AB \mid X)$ means the plausibility of sentence A and B in the condition that sentence X is true; $\neg A$ means non A; $p(A + B \mid X)$ means the plausibility of sentence A or B in the condition that sentence X is true;

The theoretical results:

Analyzing the mentioned principles theoretical results can be deduced. From the beginning we will mention that because the probability function has the same properties (1…5) it can be accepted that the plausibility function is synonymous with the probability function. This is the only reasons that theoretical results from probability theory can be transferred to the theory of plausible reasoning.
It is obvious that we do not intend to present exhaustive theoretical results. We will resume presenting the Bayesian theorem which can easily obtained [3] from (1).
This theorem tells us that the degree of trough for sentence A in condition of knowing O, is proportionally with degree of truth for the sentence A and with the degree of truth for the sentence O in condition that A is true.

$$p(A \mid O) = p(A)\frac{p(O \mid A)}{p(O)} \tag{6}$$

The objective of the theories is the knowledge improvement. In science the theories become operational by constricting models. In order to converge to the model construction inside the Plausible Reasoning the Bayesian filter method have been developed [6]:
A Bayesian filter allows to estimate the state X_t for a Markovian system in condition of knowing the observation Z_1,..Z_t. In order to solve this problem several steps are necessary:

- variable definition:

$\{X_i\}_{0 \le i \le t}$ the system states; $\{Z_i\}_{0 \le i \le t}$ observations;

- decomposition

$$p(X_0...X_t, Z_0...Z_t) = \prod_{i=0}^{t} p(X_i \mid X_{i-1})p(Z_i \mid X_i) \tag{7}$$

- initial knowledge:
  the initial state distribution;

$$p(X_0) \tag{8}$$

the transition **model** from state i-1 to state i

$$p(X_i \mid X_{i-1}) \tag{9}$$

- the sensor **model**;

$$p(Z_i \mid X_i) \tag{10}$$

- the question

$$p(X_t \mid Z_t...Z_0) \tag{11}$$

Some comments are necessary:
The Bayesian filter concept systematizes a plausible reasoning problem construction;
The Bayesian filter supposes two levels: the problem description and the question;
The first level consists also from two parts: specification of the model and identification of the

parameters.

3 The Fuzzy Engine

By Fuzzy Engine we understand a method - a theoretical result - of the Fuzzy Logic theory. The fuzzy engine is able to construct an operational system of a priori knowledge: a model.
In order to construct this model we use the following algorithm [11]:

- The heuristically description of the phenomena;
- Choice of the input-output variable;
- Definition of fuzzy sets and of the linguistic value associate to these sets;
- Selection the inference rules:
  - membership function;
  - logic operations;
  - implication rules (If_Them rules);
- Definition of fuzzification and defuzzification rules;
- Definition of adaptive rules in order to initiate the teaching and improve the fuzzy engine performances;

If we compare the arithmetical form modeling (classical models) and the fuzzy modeling we can observe that the first simplifies the phenomena by introducing hypothesis as the second is able to cover the phenomena by more and more heuristically descriptions. At the same time we must mention that modeling with fuzzy logic is not (yet) theoretical substantiated. More precisely, when we model a particular phenomenon, we don't have the theoretical mechanism in order to choose the fuzzy sets or the inference rules.
This means that we have the possibility to select particulars inference rules, we're defining what are known as the fuzzy intersection or conjunction (AND), fuzzy union or disjunction (OR), and fuzzy complement (NOT). In general these functions are arbitrary.
We will remember that the membership function is a continuous function defined on the fuzzy set:

$$\mu : A \to [0,1] \tag{12}$$

The intersection of two fuzzy sets A and B is specified in general by a binary mapping T, which aggregates two membership functions as follows:

$$\mu A \cap B(x) = T\left(\mu A(x), \mu B(x)\right) \tag{13}$$

These fuzzy intersection operators, which are usually referred to as T-norm (Triangular norm) operators T(.,.), meet the following basic requirements:

- boundary: $T(0,0) = 0, \quad T(a,1) = T(1,a) = a$
- monotonicity: $T(a,b) \le T(c,d)$ if $a \le c$ and $b \le d$
- commutativity: $T(a,b) = T(b,a)$;
- associativity: $T\left(a, T(b,c)\right) = T\left(T(a,b), c\right)$.

The first requirement imposes the correct generalization to crisp sets. The second requirement implies that a decrease in the membership values in A or B cannot produce an increase in the membership value in A intersection B. The third requirement indicates that the operator is indifferent to the order of the fuzzy sets to be combined. Finally, the fourth requirement allows us to take the intersection of any number of sets in any order of pair wise groupings.
Like fuzzy intersection, the fuzzy union operator is specified in general by a binary mapping S:

$$\mu A \cup B(x) = S\left(\mu A(x), \mu B(x)\right) \tag{14}$$

These fuzzy union operators, which are often referred to as T-conorm (or S-norm) operators S(.,.), must satisfy the following basic requirements:

- boundary: $S(1,1) = 1, \quad S(a,0) = S(0,a) = a$;
- monotonicity: $S(a,b) \le S(c,d)$ if $a \le c$ and $b \le d$;
- commutativity: $S(a,b) = S(b,a)$;
- associativity: $S\left(a, S(b,c)\right) = S\left(S(a,b), c\right)$.

For example H. Reichenbach [6] has proposed the following mathematical models for the inference rules (AND, OR, NO, Implication)

$$T\left(\mu A(x), \mu B(x)\right) = \mu A(x) \cdot \mu B(x) \tag{15}$$
$$S\left(\mu A(x), \mu B(x)\right) = \mu A(x) + \mu B(x) - \mu A(x) \cdot \mu B(x) \tag{16}$$
$$\mu \neg A(x) = 1 - \mu A(x) \tag{17}$$
$$\mu A \to B(x) = 1 - \mu A(x) + \mu A(x) \cdot \mu B(x) \tag{18}$$

4 The fuzzy engine versus Bayesian filter

The Plausible Reasoning and the Fuzzy Logic are working with the same concept, the degree of truth. In both theories the degree of truth - the plausibility and the membership functions - is a number which is included in the [0,1] domain. This detail incites to compare these theories and find the differences and the similarity.
If we analyze the mentioned principles of Plausible Reasoning we will recognize that the logic operations which are used are similar with those which are used in Fuzzy Logic: (2) with (15) and (4) with (16).
We will resume comparing, from cognition point of view, the tow results of the mentioned theories: the Bayesian filter and the Fuzzy Engine.
The Fuzzy Engine is used to model phenomena. It is

important to underline that these phenomena include the human decision process. A model is an approximation even it become the subject of an adaptive process. Here, the degree of truth – the membership function – doesn't refer to the difference between the reality and the model results but to an internal aspect of the model, the belonging to an arbitrary set. More precisely, after the adaptation process, we will not involve the difference between the model result and the reality in order to obtain the degree of truth. By adaptation we intend to obtain a better approximation, but we can not eliminate the unknown or the eluded aspects (the perturbations).
A contrary the Bayesian filter adopts a new strategy by accepting that the used model is an approximation. We can measure, statistically, this approximation and based on this knowledge we can establish the degree of truth – the plausibility – of our results. In the Bayesian filter the adaptation process – the learning – is replaced by observations which correct the output and increase its plausibility. Using the Bayesian filter implies to use a model and this model can be a classical or a fuzzy model. Because both, Fuzzy Logic and Plausible reasoning, use inference rules it is important to underline the variety of these rules developed in Fuzzy Logic [11]. Using these rules can be a good development for new Plausible Reasoning principles. In the same time the plausibility functions, which are in fact statistical distributions can be used like a good examples for the membership functions used in Fuzzy Logic.
In the end we will mention that modeling with Fuzzy Logic is very intuitive and the created models are used in many industrial applications. A contrary using Plausible Reasoning is not intuitive and for many engineers this is the main drawback of this theory.
In the next section we will present three cases studies: of a mechanical pendulum; of a mobile robot and about a human decision process.

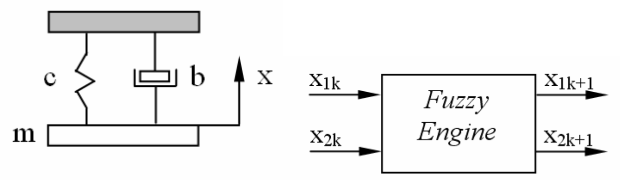
5 A first case study

The mechanical pendulum is presented in figure 1a. The dynamical model (result of the classical modeling) is presented with equations (19).

$$m\ddot{x} + b\dot{x} + cx = 0 \qquad (19)$$

where:
m is the pendulum mass (1kg);
b is the viscous friction of the damper (1.5Ns/m);
c is the spring stiffness (2N/m)

We will mention that in order to construct the fuzzy engine of this mechanical system equation (19) is not necessary. According to the presented algorithm we need heuristically information which can be obtained by observing the pendulum vibration.
After these observations we have decided that the input/output of the fuzzy engine are the position x_1 and the velocity of the pendulum x_2 (see figure 1b).
Before we define the fuzzy sets and the used linguistic values some comments are necessary. Our intention is to design a fuzzy engine which corresponds to a state space model obtained by classical modeling. More precisely, the fuzzy engine will model the transition from state *k* to state *k+1*.



a), b)

Figure 1.
a). The mechanical pendulum;
b) the input/output of the fuzzy engine

In figure 2 we have presented the defined sets for the input/output.

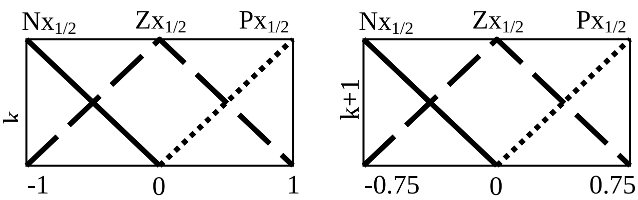


Figure 2.
The input (k) output (k+1) fuzzy sets

Using the mentioned sets we have defined the following rules:
If (inputX1 is NX1) and (inputX2 is NX2) then (outputX1 is MX1)(outputX2 is ZX2);
If (inputX1 is ZX1) and (inputX2 is NX2) then (outputX1 is MX1)(outputX2 is ZX2);
If (inputX1 is PX1) and (inputX2 is NX2) then (outputX1 is ZX1)(outputX2 is NX2);
If (inputX1 is NX1) and (inputX2 is ZX2) then (outputX1 is ZX1)(outputX2 is PX2);
If (inputX1 is ZX1) and (inputX2 is ZX2) then (outputX1 is ZX1)(outputX2 is ZX2);
If (inputX1 is PX1) and (inputX2 is ZX2) then (outputX1 is ZX1)(outputX2 is NX2);
If (inputX1 is NX1) and (inputX2 is PX2) then (outputX1 is ZX1)(outputX2 is PX2);
If (inputX1 is ZX1) and (inputX2 is PX2) then

(outputX1 is PX1)(outputX2 is ZX2);
If (inputX1 is PX1) and (inputX2 is PX2) then (outputX1 is PX1)(outputX2 is ZX2);
We have chosen the logic operations: for AND the minim method and for Or the maxim method. The designed fuzzy engine gives us the pendulum behavior – the output values – in a discrete form with a 1 s sampling time. In figure 3 we have presented the fuzzy engine simulation, compared with the classical model simulation.

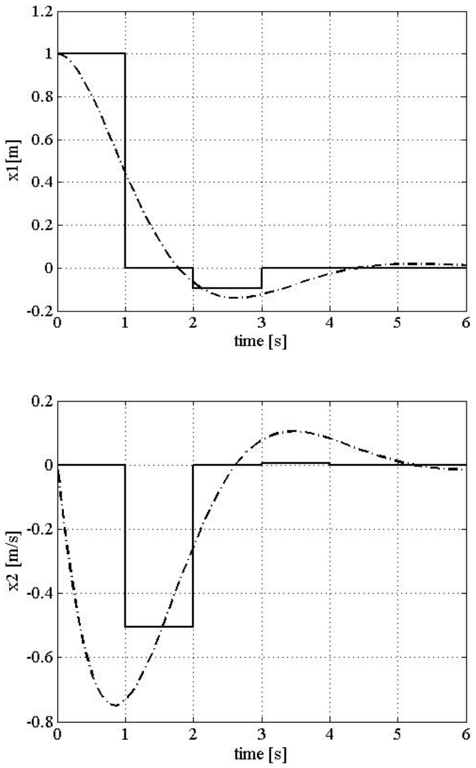


Figure 3
The fuzzy engine output compare with the classical model output
(_)fuzzy model output(-.) classical model output

Some comments are necessary: it can be seen that our fuzzy model is rudimentary, the sampling time is too big and the error between the results of the classical model (here the trusted model) and the results of fuzzy model are too large. Starting from this point two developments – knowledge improvements – are possible:

- developing our fuzzy model eventual by adaptation;
- constructing the Bayesian filter over the fuzzy engine .

We have chosen the second possibility which can be mathematical described by the following equations:

$$x_k^{est} = x_k + \pi^{est} \tag{20}$$

where: $x_k^{est} = \left[x_{1k}^{est} \quad x_{2k}^{est}\right]^T$ is the outputs estimations; $x_k = \left[x_{1k} \quad x_{2k}\right]^T$ is the fuzzy engine output; π^{est} is the model perturbations.

We don't know a priori the model perturbation (even if in figure 3 we have a certain image of this perturbation) but we can obtain, by experiments, the statistical distribution of π^{es}: $p(\pi^{est})$. This distribution accomplishes (1) so we can define the estimation plausibility like the degree of truth for the following sentence: *"the estimated output k for our model is* x_k^{est} *"*.
From (20) we have:

$$p(\pi^{est}) = p(x_k^{est} - x_k) \tag{21}$$

We must note that using the Fuzzy Engine we will obtain the state k from state k-1 so we can rewrite (21)

$$p(\pi^{est}) = p(x_k^{est} - x_k) \equiv p(x_k^{est} \mid x_{k-1}) \qquad (21)'$$

Using the Bayesian rule (5) we can write:

$$p(x_k^{est}) \propto \sum_{x_{k-1}} p(x_{k-1}) p(x_k^{est} \mid x_{k-1}) \tag{22}$$

where:
$p(x_k^{est})$ is the plausibility of the output estimation;
$p(x_{k-1})$ is the plausibility of state x_{k-1};
$p(x_k^{est} \mid x_{k-1})$ is the plausibility of the estimation when we know the state x_{k-1};
$\propto$ means proportional.

If during the vibration we measure (we make observations) we can describe this process in the following mathematical form:

$$x_k^{mes} = x_k^{est} + \pi^{mea} \tag{23}$$

where: x_k^{mes} is the output measurement;
π^{mea} is the measurement perturbation

Once again we don't know a priori the value of the measurement perturbation but if we experiment our sensor we can obtain a statistical distribution of these values. We can write:

$$p(\pi^{mes}) = p(x_k^{mes} - x_k^{est}) \equiv p(x_k^{mes} \mid x_k^{est}) \tag{24}$$

Using (5) we obtain:

$$p(x_k^{mes}) \propto p(x_k^{est})p(x_k^{mes} \mid x_k^{est}) \quad (25)$$

If we use normalized distribution we can transform (22) and (25) in equations.
For the purpose of the Bayesian filter constructing we will return to relation (7-11).

Variable definition:
$\{x_k\}_{k\in\{0,\ldots n\}}$ the system states are the position and the velocity of the mass *m* (see figure 1a) ; $\{x_k^{mes}\}_{k\in\{0,\ldots n\}}$ we will measure both the position and the velocity;

Decomposition

$$p(x_1^{est},\ldots x_n^{est}, x_1^{mea},\ldots x_n^{mea}) = \prod_{i=0}^{t} p(x_k^{est} \mid x_{k-1})p(x_k^{mes} \mid x_k^{est}) \quad (7)'$$

Initial knowledge:
The initial state distribution, is obtained after experiments, in this case we have chosen the following Gaussian distribution;

$$p(x_0) \propto \exp\left(-\frac{(x-x_0)^2}{2\cdot 0.1^2}\right) \quad (8)'$$

The transition model from state *k-1* to state *k*, is presented in (21)', the mathematical form of this distribution can be obtained from experimental measurement, once again we have chosen a Gaussian distribution:

$$p(\pi^{est}) = p(x_k^{est} \mid x_{k-1}) \propto \exp\left(-\frac{(\pi^{est})^2}{2\cdot 0.2^2}\right) \quad (9)'$$

Sensor model:
The sensor model is presented in (24), the mathematical form of this distribution can be obtained from experimental measurement, and once again we have chosen a Gaussian distribution:

$$p(\pi^{mes}) = p(x_k^{mes} \mid x_k^{est}) \propto \exp\left(-\frac{(\pi^{mes})}{2\cdot 0.05^2}\right) \quad (10)'$$

The question:
The question is about the plausibility of the each state when we know the transition plausibility and the measurement (sensor) plausibility; in order to compute this results we have used (22) and (25):

$$p(x_k^{mes}) \propto \left(\sum_{x_{k-1}} p(x_{k-1})p(x_k^{est} \mid x_{k-1})\right)\cdot p(x_k^{mes} \mid x_k^{est}) \quad (11)'$$

The response is a distribution for each *k=0…n*.

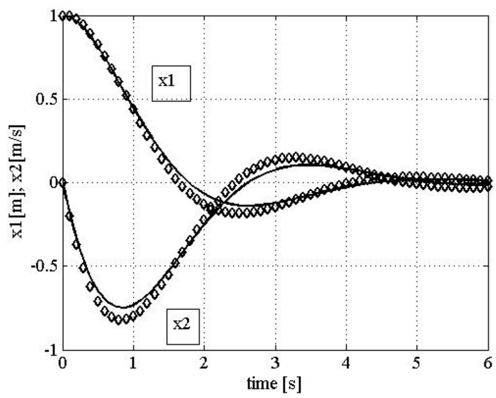


Figure 4
The classical model output (_),compare with the Bayesian filter results(◊)

This distribution has a maximum value which is the most plausibile answer to the question. More precisely, each itteration we obtaine a 2 component information: the most plausible answer (the pendulum output) and the value of its plausibility. In figure 4 we have presented the first answer.

6 The second case study

In order to exemplify the mentioned theoretical results we will consider the case of a mobile robot which modifies his state (position) and - from time to time- make observations (measure his position), see figure 5.

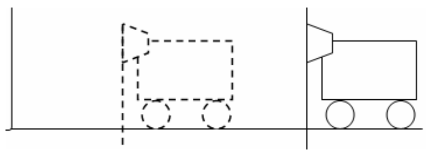


Figure 5
The mobile robot

Variable definition:
$\{x_k\}_{k\in\{0,\ldots n\}}$ the robot position; $\{x_k^{mes}\}_{k\in\{0,\ldots n\}}$ the position measurement.

Decomposition

$$p(x_1^{est},\ldots x_n^{est}, x_1^{mea},\ldots x_n^{mea}) = \prod_{i=0}^{t} p(x_k^{est} \mid x_{k-1})p(x_k^{mes} \mid x_k^{est}) \quad (7)''$$

Initial knowledge:
For the initial position we have use the distribution (8)", see also figure 6

$$p(X_0) \propto \exp\left(-\frac{(x_0)^2}{2\cdot 0.5^2}\right) \quad (8)''$$

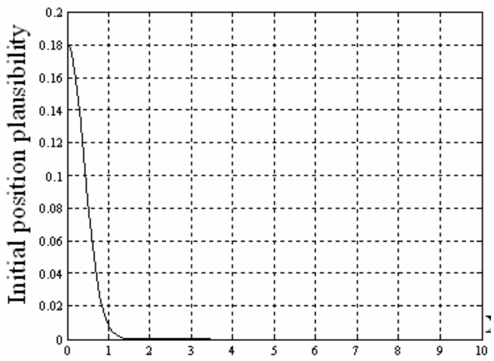


Figure 6
The initial position plausibility distribution

Initial knowledge:
According to the Bayesian filter definition, in order to answer to question (11) preliminary models are needed
For the transition model (9) we have proposed the following normalized distribution

$$p(x_i^{est} \mid x_{i-1}) \propto \exp\left(-\frac{\left(x_i - \left(x_{i-1} + 0.5\right)\right)^2}{2 \cdot 0.5^2}\right) \qquad (9)”$$

Sensor model:
For the sensor model (10) we have proposed the following normalized distribution:

$$p(x_i^{mes} \mid x_i^{est}) \propto \exp\left(-\frac{\left(x_i^{mes} - \left(x_i^{est} + 0.5\right)\right)^2}{2 \cdot \left(1 + 0.1 x_i^{mes}\right)^2}\right) \qquad (10)”$$

The question:
The question is about the plausibility of the each position (we know the transition plausibility and the measurement plausibility).

Using these models we have imagined and simulate the following situations:

The first situation:
The robot has several state transition and no observations are made during this transitions. This situation is computed with equation (26).

$$P(x_i^{est}) \propto \sum_{x_{i-1}} P(x_{i-1}^{est}) P(x_i \mid x_{i-1}^{est}) \qquad (26)$$

Simulation results are presented in figure 7. If we analyze this result the main conclusion is that even the translation value - according to (11) – remains constant, the degree of plausibility has decreased continuously from translation to translation. This means that the degree of trust decrees continuously.

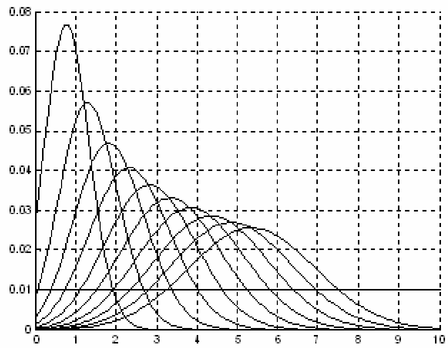


Figure 7
The plausibility distribution during the simulation: because the perturbation the plausibility decrees

The second situation:
The robot performs several observations – without performing any transition. This situation is computed with equation (27)

$$p(x_k^{mes}) \propto p(x_k^{est}) \cdot p(x_k^{mes} \mid x_k^{est}) \qquad (27)$$

From figure 8 and 9 where we have presented the results of this simulation we can see that the degree of plausibility increases continuously and converges to value 1 (absolute trust).
Figure 8 illustrate the situation of two different measurements: when the measurement confirm or infirm the estimated value of the position. In the first case the plausibility rising is bigger then in the second case.

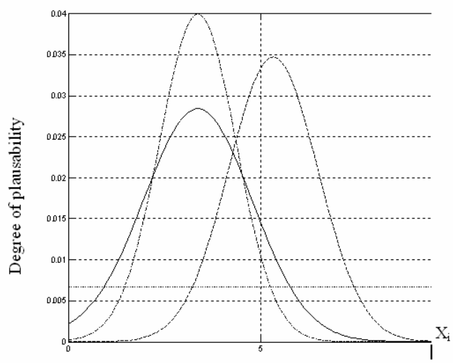


Figure 8.
Two observation (.-.- and ---) which starts from the same state (-)

Figure 9 present the situation of several measurements which confirm the estimated value of the position. The plausibility increase permanently.

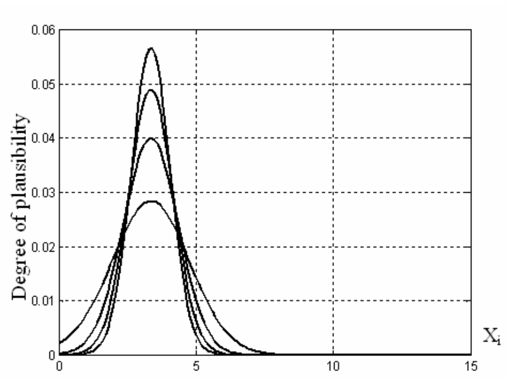


Figure 9.
Increasing the plausibility by
several observations

The third situation:
The robot performs transitions and after each transition performs observations. We have presented in figure 10 two situations. The first involves two observations after each transition, and the second only one observation after each transition. It can be observed that the first strategy increases the degree of plausibility for the current state of the robot.

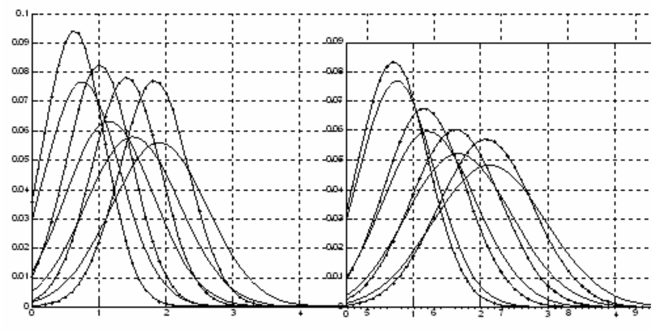


Figure 10
Transitions (-) followed by two or
one observations (-.-)

7 The third case study
The intention of the third case study is to prove the ability of Plausible Reasoning in human reasoning modeling. For this purpose we will tray to model the famous story of Sun Tzu: “Advance to Chencang by a hidden path” [9].
The story that we intend to explain by Bayesian model is the following:
This stratagem took place towards the end of the Qin dynasty. Xiang Yu appointed Liu Bang as king of Hanzhong, effectively making him leave China. To further ensure that Liu Bang does not return to China from the East, Xiang Yu divided Guanzhong into three principilities and put three people in charge, informing them to be alert against Liu Bang.
Liu Bang said, "In order to placate Xiang Yu and the three kings, we must destroy the mountain plank road to show that we've no intention of returning to China."
After nine years of preparations, Liu Bang's army became powerful and was ready to march eastwards. Liu Bang ordered his generals to take 10,000 men and horses and repair the plank road within three months.
Meanwhile, his enemies were greatly perturbed. One of the kings even led his forces to block the plank road exit.
Liu Bang then led his generals and several thousand troops to overrun Guanzhong by the old roundabout route through Chencang
We intend to model this story by using the Bayesian theorem (6). At first sight the victory of Liu Bang is based on his ability to increase the plausibility of the likelihood that he will attack on the plank road.
If we analyze more deeply the story we will find that there are two stage of the conflict: the first when Liu Bang must decide about the reaction concerning the Xiang Yu actions, and the second when Liu Bang shows his attack intention but he must choose the attack direction.
The story scenario is presented in figure 11. It can be see that in the first stage of the conflict, by destroying the road Liu Bang have increased the peace (non attack) likelihood and in this way manipulate Xiang You. In the second stage of the conflict by restoring the road Liu Bang have increased the mountain direction attack (Am) and manipulate once again his enemy.
From mathematical point of view this scenario can be describe in the following way:

- in the initial moment Xiang You can not decide the intention of Liu Bang:

  $P(A) = P(\neg A) = 50\%$; where A is the sentence “Liu Bang will attack”
- after seeing that Liu Bang destroyed the road Xiang You decides that:
  - $P(O_1 \mid \neg A) > P(O_1 \mid A)$; where O_1 is the observation of the destroyed road;
  - in consequence (6) $P(\neg A \mid O_1) > P(A \mid O_1)$;
- in the initial moment Xiang You can not decide the attack direction of Liu Bang:
  - $P(Am) = P(\neg Am) = 50\%$; where Am is the sentence “Liu Bang will attack from the mountain”;
- after seeing that Liu Bang constructs the road Xiang You decides that:
  - $P(O_2 \mid Am) > P(O_2 \mid \neg Am)$; where O_2 is the observation of the constructed road;

in consequence (6)
$P(Am \mid O_2) > P(\neg Am \mid O_2)$

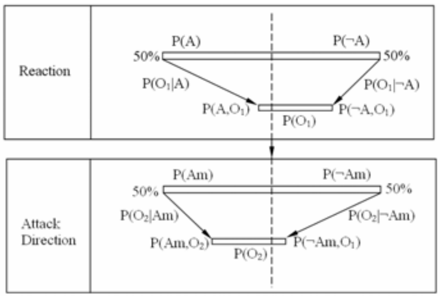


Figure 11
The story scenario

The famous story can be continued with a problem: have had Xiuag You the chance to react at his opponent ability? There are several solutions of this problem the first consist on increasing the number of hypothesis of attack direction and find new observations (spy). The second solution is presented in figure 12 and is based on changing the causal network by introducing a new decision step. More precisely it can be see that after the second observation O_2 Xiuang You becomes able to decide the tactic that Liu Bang will use. This observation increases the likelihood that his opponent uses his ability to manipulate him.

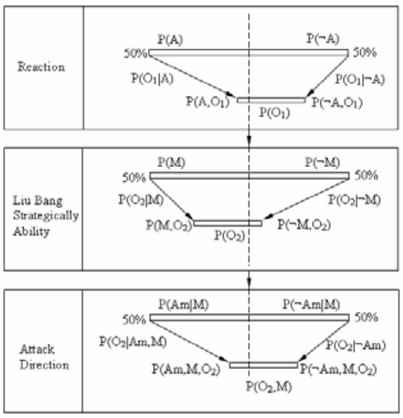


Figure 12.
A possible solution

From mathematical point of view this solution can be described in the following way:

- in the initial moment Xiang You can not decide the intention of Liu Bang:
  - $P(A) = P(\neg A) = 50\%$; where A is the sentence "Liu Bang will attack"
- after seeing that Liu Bang destroyed the road Xiang You decides that:
  - $P(O_1 \mid \neg A) > P(O_1 \mid A)$; where O_1 is the observation of the destroyed road;
  - in consequence (6) $P(\neg A \mid O_1) > P(A \mid O_1)$;
- in the initial moment Xiang You can not decide about the strategically ability of his opponent:
  - $P(M) = P(\neg M) = 50\%$; where M is the sentence "Liu Bang is able to manipulete"
- after seeing that Liu Bang intends to attack Xiang You decides that:
  - $P(O_2 \mid M) > P(O_2 \mid \neg M)$; where O_2 is the observation of the constructed road; M is the sentence "Liu Bang is able to manipulate"
  - in consequence $P(M \mid O_2) > P(\neg M \mid O_2)$;
- in the initial moment Xiang You can not decide the attack direction of Liu Bang:
  - $P(Am) = P(\neg Am) = 50\%$; where Am is the sentence "Liu Bang will attack from the mountain";
- after seeing that Liu Bang constructs the road, and knowing that his opponent can manipulate Xiang You decide that:
  - $P(O_2 \mid Am, M) < P(O_2 \mid \neg Am, M)$; in consequence $P(Am \mid O_2, M) < P(\neg Am \mid O_2, M)$

Conclusions

Present work continues the research program, of the human knowledge, by comparing two methods of AI theories. More precisely, we have compared the Bayesian filter and the Fuzzy Engine. This analyze is important because each of the mentioned methods uses the concept of degree of truth and each of these methods is used to build models. The main difference is epistemological More precisely the difference is the way of the inherent approximations (errors) management. In order to increase the accuracy the Fuzzy Engine needs to run adaptive algorithms. After this process the degree of truth is not more linked to the phenomena behavior. A contrary the Bayesian filter accept and model statistically the errors and link the phenomena model (classical or even fuzzy) to this model. The Bayesian filter develops also a mechanism of observations modeling – learning - in order to correct the a priori knowledge.

We consider that the main advantage of the Bayesian filter consist in fact that it allows epistemological model which contains both inductive and deductive process. The presented examples underline this aspect. Increasing the plausibility of a sentence by performing observation means to perform the induction. We will underline also two aspects which have been obtained from simulation. We will mention firstly the diminution of the trust, during repeated use of a theoretical model and secondly the possibility to increase the plausibility by performing observations. We can develop this conclusion by proposing a minimal value of plausibility where from, in order to use de model, we must perform observations (measurements).

The third case study shows the possibilities of plausible reasoning to model human decisions. For this reason we have modeled one of the famous stories of Sun Tzu.

The main drawback of the presented work consists on the absence of the experimental example. This is the reason that future work intends to develop experiment in order to confirm the presented theoretical results

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