Abstract: - Mining transportation is an important element of the economic efficiency rising in mining sector, because the transportation average included in the cost of production for one tone of extracted coal, represents approximately half of it. In order to control the transportation system and to have enough information about the below-ground and the above-ground manufacturing process, it is necessary to keep a close look on the transported output through the canvas conveyer. The permanent measurement of the mining material volume controlled by continuous transporting equipment machines implies knowing its transversal section profile at a given moment and the instantaneous velocity of the transporting belt conveyor. The approximation of the material profile section is done by measuring the level of the transporting material in different points and afterwards by approximating the corresponding curve through methods of numerical interpolation. In this paper two approximation (interpolation) methods are presented: Lagrange and the spline functions method. The present study wants to solve the problem by using some unconventional methods, in order to assure reliability, precision, as well as the possibility of integrating it into the mining monitoring system. The constant measurement of the cross section of the transported material through the canvas conveyer, as well as of the belt speed, leads to the determination of the transported volume, and depending on it, knowing the average density, you can determine the output.

Key-Words: Interpolation, Lagrange, Spline, Polynomial, Function, Ultrasound Transducers

1 Numerical Methods of Approximating a Profile

In Figure 1 the profile (2) can be approximated by interpolation, after having measured its height, compared to the overflow head (1) of the belt conveyer with the help of some ultrasound translators (T₁...,Tₙ) set up on the frame (3); the instantaneous velocity is determined by measuring the output tension of a tachogenerator (TG) stimulated by the belt conveyer.

The sizes mentioned above constitute the input data (tensions), which will be read by using a data acquisition card on a numerical computer.

The calculus of the volume is done by integrating the approximate profile of the transported mining mass by its average density we can determine its mass debit.

To integrate the profile of the mining mass so as to calculate the transversal area it is necessary to measure its height compared to the overflow head in several points and approximating the form through numerical methods of interpolation.

The interpolation is a calculus method used for obtaining a simple approximate function, using the known discrete values (resulting from measurements, experiments) of the function of which analytical expression is unknown and which cannot be determined by analytical means.

The function \( f \) can be tabular in calculus, which means that the corresponding values of the argument \( x: f(x₀), f(x₁), ..., f(xₙ) \) are known. Using these values a rather simple function is built for calculus, which in the given points \( x₀, x₁, ..., xₙ \) can take the same value as the given function and in other different...
points of \( x_0, x_1, ..., x_n \) by approximating with a certain degree of precision the values of the function.

Frequently this function which approximates \( f \) is presented like a polynomial, because on short distance intervals of the \( x \) argument the function \( f(x) \) can be approximated successfully by using curves, their analytical representation being polynomials.

If the equation \( f(x) = 0 \) has a real root \( a \) in the \((a,b)\) interval, the successive approximations (approximate values) of \( x \) are given by the formula [5]:

\[
x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, \quad k = 0,1,2,...
\]  

The interpolation is important nevertheless in the cases in which the analytical function is unknown. Moreover, it is an important theoretical instrument being used in the case of other numerical calculus methods as the numerical derivation, the numerical integration and the numerical integration of differential equations.

2 The Lagrange Interpolation Polynomial

Let’s consider \( f: [a, b] \rightarrow \mathbb{R} \) and the distinctive knots \( x_0, x_1, ..., x_n \) from the \( [a, b] \) interval.

Supposing that we also know the values of the function \( y_i = f(x_i), \quad (i = 0,1,...,n) \) in these knots, we want to determine a polynomial of grade \( n \) which can approximate the function \( f \).

For this we will represent the grade \( n \) polynomial according to the \( y_0, y_1, ..., y_n \) ordinates, that is [5]:

\[
L_n(x) = y_0 b_0(x) + y_1 b_1(x) + ... + y_n b_n(x)
\]  

where \( b_0(x), b_1(x), ..., b_n(x) \) represent grade \( n \) polynomials, which will be determined by imposing the interpolation conditions [5]:

\[
L_n(x_i) = y_i, \quad i = 0,1,...,n
\]  

The \( L_n(x) \) polynomial from (2) can be written like this [5]:

\[
L_n(x) = \sum_{k=0}^{n} y_k \frac{(x-x_0)(x-x_1)...(x-x_{k-1})(x-x_{k+1})...(x-x_n)}{(x_k-x_0)(x_k-x_1)...(x_k-x_{k-1})(x_k-x_{k+1})...(x_k-x_n)}
\]  

We can simplify this form of the Lagrange interpolation polynomial by considering the function [5]:

\[
Q(x) = (x-x_0)(x-x_1)...(x-x_n)
\]

Its derivative being for \( x = x_k \)

\[
Q'(x_k) = (x_k-x_0)...(x_k-x_{k-1})(x_k-x_{k+1})...(x_k-x_n)
\]

The Lagrange polynomial can be also written under the form [5]:

\[
L_n(x) = \sum_{k=0}^{n} y_k \frac{Q(x)}{Q'(x_k)}
\]

3 Spline Functions

The interpolation issue can also be expressed on portions for polynomial functions.

Let \( a = x_1 < x_2 < ... < x_n = b \) be a division of the \([a,b]\) interval. A spline polynomial function of grade \( p \) with the knots in the points \( x_i, i = 1, ..., n \) is an \( S \) function with the following characteristics: on each interval \([x_i, x_{i+1}]\), \( i = 1, ..., n - 1 \), \( S(x) \) is a polynomial of grade \( p \) and \( S(x) \) and its first \( p - 1 \) derivatives are continuous on \([a, b]\).

We will analyze the interpolation with the help of cubic spline functions by determining the number of parameters, which characterize a cubic spline function. According to the \( n - 1 \) subintervals we have \( n - 1 \) fragments of polynomials of grade three each one characterized by 4 parameters, so we have \( 4n - 4 \) parameters to determine in all.

The fact that the \( S \) function and its \( S' \) and \( S'' \) derivatives must be continuous in the interior \( x_i \) knots \( (i = 2, ..., n - 1) \) gives us 3\((n - 2)\) conditions.

Then \( S(x_i) = y_i \) for each knot gives us again \( n \) conditions, so in all \( 4n - 6 \) conditions. As a result, we still need two more supplementary conditions to determine in a single way the spline cubic function. These can be specified in several ways.

For instance we can claim that [5]:

\[
S''(x_1) = S''(x_n) = 0
\]

Constructing a spline cubic function represents a simple, stable numerical procedure. Let’s consider the subinterval \((x_i, x_{i+1})\) and let [5]:

\[
h_i = x_{i+1} - x_i
\]

\[
\omega = \frac{x - x_i}{h_i}
\]

\[
\omega' = 1 - \omega
\]
If \( x \) covers this subinterval then \( \omega \) varies from 0 to 1 and \( \overline{\omega} \) from 1 to 0. It is not difficult to notice that on the considered subinterval we could have represented \( S(x) \) like this [5]:

\[
S(x) = \omega y_{i+1} + \overline{\omega} y_i + h_i \left( \left( \omega^3 - \omega \right) \sigma_i + \left( \overline{\omega}^3 - \overline{\omega} \right) \right) \tag{12}
\]

where \( \sigma_i \) and \( \sigma_{i-1} \) are constants that are still undetermined. The first two terms correspond to a customary linear interpolation and the last term represents a cubic correction which will allow us to ensure the smoothness of \( S(x) \).

For the spline polynomial function, which verifies the conditions of limit, the \( \sigma_i \) coefficients verify the next \( n \) linear equation system with \( n \) unknown factors.

\[
\begin{bmatrix}
-h_1 & h_1 & 0 & 0 \\
h_1 & 2(h_1 + h_2) & h_3 & 0 \\
0 & h_2 & 2(h_2 + h_3) & h_4 \\
\vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 2(h_{n-2} + h_{n-1}) & h_n \\
& & & -h_{n-1}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\vdots \\
\sigma_{n-1} \\
\sigma_n
\end{bmatrix}
= 
\begin{bmatrix}
h_1^2 \Delta_1^{(3)} \\
\Delta_2 - \Delta_1 \\
\Delta_3 - \Delta_2 \\
\vdots \\
\Delta_{n-1} - \Delta_{n-2} \\
-h_{n-1}^2 \Delta_{n-3}^{(3)}
\end{bmatrix}
\tag{13}
\]

The matrix of the system is tridiagonal, symmetrical, non-singular, and dominantly diagonal.

To find the unique solution of the system we can use the Gauss method without pivotation. The Gauss elimination reduces the system at the form [5]:

\[
\begin{bmatrix}
\alpha_1 & h_1 & 0 & 0 & \cdots & 0 \\
0 & \alpha_2 & h_2 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & \cdots & \cdots & \alpha_n \\
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\vdots \\
\sigma_{n-1} \\
\sigma_n
\end{bmatrix}
= 
\begin{bmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_{n-1} \\
\beta_n
\end{bmatrix}
\tag{14}
\]

where the diagonal elements are calculated in the following way [5]:

\[
\alpha_i = -h_i \\
\alpha_i = 2(h_{i-1} + h_i) \frac{h_i^2}{\alpha_{i-1}}, \quad i = 2, 3, \ldots, n-1
\]

\[
\alpha_n = -h_{n-1} \frac{h_n^2}{\alpha_{n-1}}
\]

and the free \( \beta \) terms [5]:

\[
\beta_1 = h_1^2 \Delta_1^{(3)} \\
\beta_i = (\Delta_{i-1} - \Delta_{i-2}) \frac{h_{i-1} \beta_{i-1}}{\alpha_{i-1}}, \quad i = 2, 3, \ldots, n-1
\]

\[
\beta_n = -h_{n-1} \Delta_{n-3}^{(3)} - h_n \frac{\beta_{n-1}}{\alpha_n}
\]

As a result, the \( \alpha_i \) coefficients are calculated by regressive substitution [5]:

\[
\sigma_n = \frac{\beta_n}{\alpha_n} = -h_i \sigma_{i+1}, \quad i = n-1, n-2, \ldots, 1
\tag{17}
\]

In some applications it is advisable to keep \( \alpha_i \) and to use them directly for the calculus of the spline function. But if we need to calculate several times the spline function then the terms must be put in order. For several reasons it is advisable to calculate and to keep the coefficients \( b_i, c_i \) and \( d_i \) for representing the cubical spline function [5]:

\[
S(x) = y_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3
\tag{18}
\]

on each subinterval \([x_i, x_{i+1}]\), \( i = 1, 2, \ldots, n - 1 \). These coefficients are represented according to the formulae [5]:

\[
\sigma_i = \frac{y_{i+1} - y_i}{h_i} - h_i(\sigma_{i+1} + 2\sigma_i), \quad i = 1, 2, \ldots, n - 1
\tag{19}
\]

\[
c_i = 3\sigma_i, \\
d_i = \frac{\sigma_{i+1} - \sigma_i}{h_i}, \quad i = 1, 2, \ldots, n - 1
\]

4 A comparative study of methods
To compare the two interpolation methods presented above (polynomial interpolation through Lagrange interpolation polynomial and interpolation through spline cubical functions) we have used the C programming language.

For evaluation we have taken into account two
calculus variants. The first synthetizes a transversal section corresponding to an uniform arranging of the material on the belt conveyor in the overflow point area. The considered measured values (in relative units) are shown in Table 1.

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>0</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>1.5</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Through linear interpolation the measured profile is the one from Figure 2:

Fig.2 The profile of the linear interpolation (first variant)

The Lagrange interpolation polynomial leads to the profile shown in Figure 3:

Fig. 3 The Lagrange interpolation profile (first variant)

Using the spline functions, the profile from Figure 4 resulted:

Fig. 4 The Spline interpolation profile (first variant)

The second calculus variant synthetizes a transversal section corresponding to an unregular arranging of the material on the belt conveyor in the overflow point. As in the proceeding case we will show in Table 2 the relative values of x and y.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>1</td>
<td>2.5</td>
<td>1</td>
<td>1.5</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Now through linear interpolation the measured profile is the one from Figure 5:

Fig. 5 The profile of linear interpolation (second variant)

The Lagrange interpolation polynomial leads to a profile like the one in Figure 6:

Fig. 6 The Lagrange interpolation profile (2nd variant)

Using spline functions the profile in Figure 7 resulted:

Fig. 7 The Spline interpolation profile (2nd variant)

5 The ultrasound transducer

Microsonar UT-212

In order to measure the height of the profile of the transported material, at the head of the canvas conveyer, it is used ultrasound transducers. Microsonar UT-212 (figure 8) is a ultrasound transducer suitable for detection of the position or measuring distance of objects. The measurement can only be accomplished if the space between the unit
and the target is free of any obstacles for making the way of the ultrasound beam and good reflection on the target.

Fig. 8 Ultrasound transducer Microsonar UT-212

In order to avoid such interference (figure 9) between two or more units of this type, depending on the installation, the units should be installed at a minimum distance of \( B_{\text{min}} > 0.25 \) m. Consequently the number of disturbing points will be limited. This allows the transducers to determine the profile of the transported material.

The transfer characteristic of the Microsonar UT-212 transducer is shown in Figure 10 and 11.

Fig. 9 Interference depending on the installation

Fig. 10 Transfer characteristic of units with current output

According to the targets position (figure 12) the result of the measure will be the one shown in Table 3.

Distance parameters \( X_A \) and \( X_B \) can be changed with programming by placing a good reflecting target at the distance to be programmed and by stepping to the relevant Menu Point. The unit will measure and store (learn) the distance.

MICROSONAR will accept the echo coming from the reflecting surface (within the range) nearest to the unit. The range of the unit with Factory Setting extends between \( X_{\text{min}} \) and \( X_{\text{max}} \). (Nominal range).

If the target is moving within narrower range it is advised to reduce the range by far-end blocking which should be done with programming \( X_T \). Factory default:
The signal processing of the unit can be adapted for the most different requirements and conditions of the application. The two programmable parameters influencing signal processing are the averaging number and the number of discarded echoes.

Averaging number: \( a \) (1, 4, 8 or 16).

To avoid mistaken measurements the unit will not provide output on the basis of a single measurement but by taking the average of the last \( a \) number of distance samples.

Increasing averaging number will reduce small oscillation of the output signal born by the indefinite movement of the target or measurement error (caused by noise). On the other hand this will cause a target tracking failure depending on the speed of the target which will be mended after a \( T_b = a \cdot T_p \) settling time or switching delay.

Number of discarded echoes: \( k \) (1, 3, 5 or 10)

Under disadvantageous conditions (air movement, not perpendicular or bad reflecting surface) some of the echoes may miss the sensor. Giving immediate attention to this, might lead to frequent error indication or measurement failures. Therefore the unit would check the measured distance for verifying that it is within the range.

Measured distance outside the range will be disregarded while calculating the average and leaves output signal unchanged. The unit can disregard \( k \) number of consecutive distance samples. After that error will be indicated. If due to bad reflection substantial number of echoes go astray and the number of invalid (incorrect) echoes, between two valid ones, is smaller than \( k \) the unit will work continuously i.e. without indicating error. The greater the value of \( k \) programmed, the less sensitive the unit to invalid echoes but the reaction time for indication of error will increase. To maintain continuous operation the programmed range is advised to keep as narrow as possible.

The greater the speed of the target the smaller should the averaging number be chosen. The worse the reflection of the target the higher value should be chosen for \( k \) (number of discarded echoes).

5.1 Touch-Magnet Programming

The magnetic screwdriver (with its cap removed) should be put to the points on the enclosure marked up with A or B according to Figure 13. These steps will in the following be indicated by A or B. Touch-Magnet Programming is only available if it is not disabled and the PRG wire is free. Disabling can be programmed both by Touch-Magnet Programming or by cable contacting, but it can only be released by cable contacting.

5.2 Programming with cable contact

The steps A or B A will be represented by connecting wire PRG with +US or GND respectively. These connections can be established by the use of switch or push buttons in accordance with Figure 14. or by simply connecting cables.

Different states within the programming procedure will be indicated by the three LEDs. Steps A and B (magnet touch, wire connection) should be maintained till the effect will be indicated by the relevant change of the LED status (figure 15).
Fig. 15 Arrangement of the LEDs and interpretation of markings

Figure 16 represents the procedure of the programming.

For entering programming mode: step \( A \) for 2 s.

For quitting programming mode: step \( B \).

Programming mode entered: will be indicated by blinking of the red and going off the yellow and green LED s. While being in programming mode (with red LED blinking) every step \( A \) will change the Menu Points. The six different Menu Points are indicated by the combination of the states of the yellow and green LEDs. Selecting the Menu Point the unit will after 5 s automatically (without step \( A \) or \( B \)) be ready for programming, which will be indicated by the red LED. In the first four Menu Points learning should be initiated by step \( A \). Blinking of the green LED indicates the measurement during the learning. On getting a valid echo the yellow LED will lit and the relevant numeric value of the measured distance appears on the output (for instance with measured distance of 0.4 m the output will be 0.4 mA or 0.4 V!) With repeated step \( A \) new distance can be learned and the old one will be overwritten. Step \( B \) will finalise learning and result in return to the Menu Point. Step \( B \) represents quitting Programming Mode.

Parameters \( a \) and \( k \) can be programmed in Menu Points 4 and 5.

Touch-Magnet programming can be enabled or disabled in Menu Point 6 by step \( A \). Disabling of the Magnet-Touch Programming by the use of the magnet will be completed after quitting Programming Mode.

During Touch-Magnet Programming the Wire-Contact Programming is disabled and vice versa. If the unit is left in Programming Mode by mistake it will automatically quit after 10 seconds.

6 Problem Solution

The experiments were made using two transducers Microsonar UT-212, which were installed on a plate surface, which allows to change the distance in order to set up their transfer characteristic. In Figure 17 it is shown the transducer transfer characteristic Microsonar UT-212, as well as its transfer equation.

The measured distance was taken by changing the plate surface distance, on which two transducers had been installed, with a pitch of 50mm and by measuring the frequency of the transducers.

The equation was used in an application which allows to measure the distance to the target.

\[ y = 0.0169x - 3.4359 \]

Fig. 17 Transducer transfer characteristic

The power supplied, by this ultrasound transducer was accepted by a numerical calculator through a data acquisitioning card (Figure 18).
This type of card allows 2 digital-analogical outputs (unipolar and bipolar) and 16 analog-digital simple inputs or 8 analog-digital differential inputs, all of these having a 12 bits resolution. The Technical data of this card are the following:

**D/A**
- resolution: 12 bits
- number of channels: 2
- power output:
  - unipolar output: 0 at 2.5 V, 0 at 5 V, 0 at 10 V;
  - bipolar output: -2.5 at 2.5 V, -5 at 5 V, -10 at 10 V;
- conversion time: < 2 μs
- output impedance: > 2 kΩ

**A/D**
- resolution: 12 bits
- power input:
  - unipolar input: 0 at 2.5 V, 0 at 5 V, 0 at 10 V;
  - bipolar input: -2.5 at 2.5 V, -5 at 5 V, -10 at 10 V;
- conversion time: < 22 μs (for each channel);
- maximum power allowed: 12 V.

The reading, interpreting and display of monitored values program was realized in C language on a DOS platform using the next algorithm:

**Analogue-digital conversion:**
1) select the channel to be read
   OUT port, channel
2) cancel the parameters
   OUT (port+1), 1
3) start conversion

```c
FOR I=1 to 6
    A=INP (port+12)
    NEXT I
FOR I=1 to 8
    A=INP (port+8)
    NEXT I
4) read the above octet (4 bits)
    C=INP (port+3)
    HB=(C/16 – INT(C/16))*16
5) read the below octet (8 bits)
    LB=INP(port+2)
6) measure the value
   A/D=HB*256+LB
```

**Analogue-digital conversion:**
1) write the above octet on the channel 1
   (4 bits)
   OUT (port + 5), Hdata
2) write the below octet on channel 1 (8 bits)
   OUT(port+4),Ldata
3) write the above octet on channel 2 (4 bits)
   OUT (port + 7), Hdata
4) write the below octet on channel 2 (8 bits)
   OUT(port+6),Ldata

The programming has the following functions:
- reads the frequencies of the transducers, at different intervals chosen by the user;
- displays analogically the value of the read frequency, and digitally its value and the value according to its distance (figure 19);
- at the user request writes in a folder the value of the read distances;
- by pressing a button allows to drive a pitch by pitch motor in order to measure a specific target.

Two transducers were installed on the plate surface (figure 20). The target to be measured driven with a pitch by pitch motor is shown in Figure 21. The profile obtained after running the programming is shown in Figure 22.
7 Conclusions

By analyzing the two variants of interpolation taken into account we can conclude that the best approximation of a profile is done by using the spline functions.

The interpolation through Lagrange polynomial has an oscillatory character and the form of the resulting curve using this procedure especially in the case of the second variant is far from physical reality, if we take into account that this approximation would model the profile of the transversal section of the transported material in the overflow point of a belt conveyor. We can also notice that on some intervals the Lagrange polynomial has negative values, which mathematically speaking do not raise any problems regarding the calculus of the area, but practically speaking they do not correspond to reality.

The best approximation can be obtained by using spline functions. Mathematically speaking this approximation is efficient because it determines a cubical polynomial function for each pair of points. At the same time a great number of measurement points do not determine approximations that can surpass the physical context.

To measure the volume of the transported material means to approximate its profile by interpolation and measuring its momentary velocity. The researches done conclude that it is possible to measure with a transducer the profile height of the transported material at the head of the canvas conveyor. The effective measurement of the volume
is done by taking information from the transducers to a computer and a data acquisition card. Taking into account the speed variation of the measured materials (the profile variation and the speed variation) we can say that the programming in C language can give us instantaneous information regarding both the volume of the transported material and its discharge.

As seen in Figure 22, where 2 determinations of the k parameter have been made (1 and 10), the higher the k value is, the less sensitive the transmitter is to invalid echoes.

References: