Hydraulically Actuated Active Suspension System with Proportional Integral Sliding Mode Control

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Abstract:— This paper will present a new mathematical model and control of the hydraulically actuated suspension system for the half model. The model presented takes into account all the pressure difference parameters inherent in the hydraulic cylinder. However, from the derived mathematical expression for the systems, it is found that the system may experience a mismatched condition problem due to the nature of the road disturbance which is not in phase with the control input. In order to achieve the desired ride comfort and road handling and to solve the mismatched condition, a proportional-integral sliding mode control (PISMC) technique is presented to deal with the system and uncertainties. Extensive simulations are performed for different road profiles and the results showed that the proposed controller performed well in improving the ride comfort and road handling for the half car model using the hydraulically actuated suspension system. It is shown that the proposed controller is capable to overcome the mismatched condition problem that present in the active suspension system. Furthermore, the results also showed that the system is completely insensitive to the external disturbance due to the road surface irregularities.

*Key-Words:—*Active Suspension System, Hydraulic Actuator, Sliding Mode Control.

1 Introduction

A car suspension system is the mechanism that physically separates the car body from the wheels of the car. The suspension system can be categorized into passive, semi-active and active suspension system according to external power input to the system and/or a control bandwidth [3]. Active suspensions differ from the conventional passive suspensions in their ability to inject energy into the system, as well as store and dissipate it. Various control strategies have been proposed by numerous researchers to improve the trade-off between the ride comfort and road handling. Therefore, this action is directly will suppress vibrations of the car body and car wheel simultaneously [26]. These control strategies are included LQR method [4], [6], [19]; H-infinity control [20]; sliding mode control (SMC) [1], [8], [14] and many others.

The simplest method to model the active suspension system is assuming that the system has a linear force input. Unfortunately, this method does not give an accurate model of the system because the actuator's dynamics have been ignored in the modeling In order to overcome the problem, the hydraulic actuator dynamics are considered in the design of active suspension system. Thus, in this paper, a complete formulation of the active suspension system actuated by hydraulic actuator based on the formulation presented by [21] and [23] which is known as a proportional-integral sliding mode control (PISMC) technique is proposed for half car model. The results also showed that the system is completely insensitive to the external disturbance due to the road surface irregularities.

2 Modeling

Modeling of the active suspension systems in the early days considered that input to the active suspension is a linear force as [4], [7] and [18]. Recently, due to the development of new control theory, the force input to the active suspension systems has been replaced by an input to control the actuator. Therefore, the dynamic of the active suspension systems now consists of the dynamic of suspension system plus the dynamic of the actuator system. Hydraulic actuators are widely used in the vehicle active suspension systems as considered in [2], [5], [10], [11], [21], [22] and [23].

The active suspension system of the half car model is shown in figure 2.1. Let f_f and f_r be the force inputs for the front and rear actuators, respectively. Therefore, the motion equations of the active suspension for the half car model may be determined as follows [16], [23]:

$$\frac{m_b}{L}(L_f\ddot{x}_{bf}+L_r\ddot{x}_{br})+c_{bf}(\dot{x}_{bf}-\dot{x}_{wf})+k_{bf}(x_{bf}-x_{wf})+c_{br}(\dot{x}_{br}-\dot{x}_{wr})+k_{br}(x_{br}-x_{wr})-f_f-f_r=0 \quad (1)$$

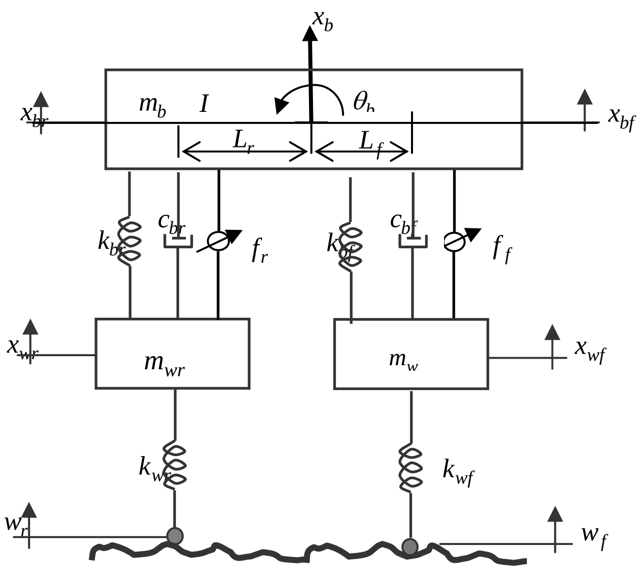

Fig. 1 The active suspension for the half car model

$$\frac{I_b}{L}(\ddot{x}_{bf}-\ddot{x}_{wf})+L_f[c_{bf}(\dot{x}_{bf}-\dot{x}_{wf})+k_{bf}(x_{bf}-x_{wf})-f_f]-L_r[c_{br}(\dot{x}_{br}-\dot{x}_{wr})+k_{br}(x_{br}-x_{wr})-f_r]=0 \quad (2)$$

$$m_{wf}\ddot{x}-c_{bf}(\dot{x}_{bf}-\dot{x}_{wf})-k_{bf}(x_{bf}-x_{wf})+k_{wf}(x_{wf}-w_f)+f_f=0 \quad (3)$$

$$m_{wr}\ddot{x}_{wr}-c_{br}(\dot{x}_{br}-\dot{x}_{wr})-k_{br}(x_{br}-x_{wr})+k_{wr}(x_{wr}-w_r)+f_r=0 \quad (4)$$

Equations (1) - (4) can be written in the following form:

$$M_{h_l}\ddot{X}_{h_l}(t)+S_{h_l}\dot{X}_{h_l}(t)+T_{h_l}X_{h_l}(t)=D_{h_l}F_{h_l}(t)+E_{h_l}W_{h_l}(t) \quad (5)$$

where

$$X_{h_l}(t)=[\dot{x}_{bf}(t)\quad \dot{x}_{wf}(t)\quad x_{br}(t)\quad x_{wr}(t)]^T$$

$$W_{h_l}(t)=[w_f(t)\quad w_r(t)]^T;$$

$$M_{h_l}=\begin{bmatrix} \frac{L_r m_b}{L} & 0 & \frac{L_f m_b}{L} & 0 \\ \frac{I_b}{L} & 0 & -\frac{I_b}{L} & 0 \\ 0 & m_{wf} & 0 & 0 \\ 0 & 0 & 0 & m_{wr} \end{bmatrix};$$

$$S_{h_l}=\begin{bmatrix} c_{bf} & -c_{bf} & c_{br} & -c_{br} \\ L_f c_{bf} & -L_f c_{bf} & -L_r c_{br} & L_r c_{br} \\ -c_{bf} & c_{bf} & 0 & 0 \\ 0 & 0 & -c_{br} & c_{br} \end{bmatrix};$$

$$T_{h_l}=\begin{bmatrix} k_{bf} & -k_{bf} & k_{br} & -k_{br} \\ L_f k_{bf} & -L_f k_{bf} & -L_r k_{br} & L_r k_{br} \\ -k_{bf} & k_{bf}+k_{wf} & 0 & 0 \\ 0 & 0 & -k_{br} & k_{br}+k_{wr} \end{bmatrix};$$

$$E_{h_l}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ k_{wf} & 0 \\ 0 & k_{wr} \end{bmatrix};\ D_{h_l}=\begin{bmatrix} 1 & 1 \\ L_f & -L_r \\ -1 & 0 \\ 0 & -1 \end{bmatrix};\ F_{h_l}(t)=[f_f\quad f_r]^T \quad (6)$$

The state space representation of the motion equations may be written in the following form:

$$\dot{x}_{h_l}(t)=A_{h_l}x_{h_l}(t)+G_{h_l}w_h(t) \quad (7)$$

where

$$x_{h_l}(t)=[x_1\quad x_2\quad x_3\quad x_4\quad x_5\quad x_6\quad x_7\quad x_8]^T$$
$$=[\dot{x}_{bf}\quad \dot{x}_{wf}\quad \dot{x}_{br}\quad \dot{x}_{wr}\quad x_{bf}\quad x_{wf}\quad x_{br}\quad x_{wr}]^T \quad , \quad (8)$$

$$A_{h_l}=\begin{bmatrix} a_{h11} & a_{h12} & a_{h13} & a_{h14} & a_{h15} & a_{h16} & a_{h17} & a_{h18} \\ a_{h21} & a_{h22} & 0 & 0 & a_{h25} & a_{h26} & 0 & 0 \\ a_{h31} & a_{h32} & a_{h33} & a_{h34} & a_{h35} & a_{h36} & a_{h37} & a_{h38} \\ 0 & 0 & a_{h43} & a_{h44} & 0 & 0 & a_{h47} & a_{h48} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

$$G_{h_l}=\begin{bmatrix} 0 & 0 \\ g_{h21} & 0 \\ 0 & 0 \\ 0 & g_{h42} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\ w_{h_l}(t)=\begin{bmatrix} w_f \\ w_r \end{bmatrix} \quad (10)$$

where the non-zero elements of A_{h_l} and G_{h_l} matrices are:

$$a_{h11}=\frac{-c_{bf}}{m_b}-\frac{L^2{}_f c_{bf}}{I_b}\ ,\quad a_{h12}=\frac{c_{bf}}{m_b}+\frac{L_f{}^2 c_{bf}}{I_b}$$

$$a_{h13}=\frac{-c_{br}}{m_b}+\frac{L_f L_r c_{br}}{I_b}\ ,\quad a_{h14}=\frac{c_{br}}{m_b}-\frac{L_f L_r c_{br}}{I_b}$$

$$a_{h15} = \frac{-k_{bf}}{m_b} - \frac{{L_f}^2 k_{bf}}{I_b} \ , \quad a_{h16} = \frac{k_{bf}}{m_b} + \frac{{L_f}^2 k_{bf}}{I_b}$$

$$a_{h17} = \frac{-k_{br}}{m_b} + \frac{L_f L_r k_{br}}{I_b} \ , \quad a_{h18} = \frac{k_{br}}{m_b} - \frac{L_f L_r k_{br}}{I_b}$$

$$a_{h21} = \frac{c_{bf}}{m_{wf}} \ , \quad a_{h22} = \frac{-c_{bf}}{m_{wf}} , \quad a_{h25} = \frac{k_{bf}}{m_{wf}} ,$$

$$a_{h26} = \frac{-(k_{bf} + k_{wf})}{m_{wf}}$$

$$a_{h31} = \frac{-c_{bf}}{m_b} + \frac{L_f L_r c_{bf}}{I_b} \ , \quad a_{h32} = \frac{c_{bf}}{m_b} - \frac{L_f L_r c_{bf}}{I_b}$$

$$a_{h33} = \frac{-c_{br}}{m_b} - \frac{{L_r}^2 c_{br}}{I_b} \ , \quad a_{h34} = \frac{c_{br}}{m_b} + \frac{{L_r}^2 c_{br}}{I_b}$$

$$a_{h35} = \frac{-k_{bf}}{m_b} + \frac{L_f L_r k_{bf}}{I_b} \ , \quad a_{h36} = \frac{k_{bf}}{m_b} - \frac{L_f L_r k_{bf}}{I_b}$$

$$a_{h37} = \frac{-k_{br}}{m_b} - \frac{{L_r}^2 k_{br}}{I_b} \ , \quad a_{h38} = \frac{c_{br}}{m_b} + \frac{L^2{}_r c_{br}}{I_b}$$

$$a_{h43} = \frac{c_{br}}{m_{wr}} , \quad a_{h44} = \frac{-c_{br}}{m_{wr}} , \quad a_{h83} = \frac{k_{br}}{m_{wr}} \ ,$$

$$a_{h84} = \frac{-(k_{br} + k_{wr})}{m_{wr}}$$

$$g_{h21} = \frac{k_{wf}}{m_{wf}} \ , \quad g_{h42} = \frac{k_{wr}}{m_{wr}} \tag{11}$$

To integrate the hydraulic actuators dynamics to the half car suspension system, the following mathematical approach is proposed. The derivation starts with rewriting the motion equation of half car active suspension in equation (5) into the following form,

$$\dot{X}_{h_l} + {M_{h_l}}^{-1} S_{h_l} \dot{X}_{hl} + {M_{h_l}}^{-1} T_{h_l} X_{h_l} = {M_{h_l}}^{-1} D_{h_l} F_{h_l} + {M_{h_l}}^{-1} E_{h_l} W_{h_l} \tag{12}$$

Defining the new state vector x_h ,

$$\begin{aligned} x_h &= [\dot{X}_{h_l} \quad X_{h_l} \quad f_h]^T \\ &= [\dot{x}_{bf} \quad \dot{x}_{wf} \quad \dot{x}_{br} \quad \dot{x}_{wr} \quad x_{bf} \quad x_{wr} \quad x_{br} \quad x_{wr} \quad f_f \quad f_r]^T \end{aligned} \tag{13}$$

Let the rate of change of the control forces for the front and rear hydraulic actuators can be written as:

$$\dot{f}_h = F_{1h} f_h - F_{2h} \dot{x}_h + F_{3h} u \tag{14}$$

where

$f_h = [f_f \quad f_r]^T$; $x_h = [x_{bf} \quad x_{wf} \quad x_{br} \quad x_{wr}]^T$ and $u = [u_f \quad u_r]^T$;

$$F_{1h} = \begin{bmatrix} -\frac{1}{A_{ef}} & 0 \\ 0 & -\frac{1}{A_{er}} \end{bmatrix} ; \ F_{2h} = \begin{bmatrix} -\frac{A_{yf}}{A_{ef}} & \frac{A_{yf}}{A_{ef}} & 0 & 0 \\ 0 & 0 & -\frac{A_{yr}}{A_{er}} & \frac{A_{yr}}{A_{er}} \end{bmatrix} ;$$

$$F_{3h} = \begin{bmatrix} \frac{1}{A_{ef}} & 0 \\ 0 & \frac{1}{A_{er}} \end{bmatrix} \tag{15}$$

Therefore, by augmenting equation (12) and equation (14) the state space representation of the half car active suspension system with the hydraulic dynamics may be obtained as follows:

$$\begin{bmatrix} \ddot{X}_{h_l} \\ \dot{X}_{h_l} \\ \dot{f}_h \end{bmatrix} = \begin{bmatrix} -{M_{h_l}}^{-1} S_{h_l} & -{M_{h_l}}^{-1} S_{h_l} & -{M_{h_l}}^{-1} F_{h_l} \\ I & 0 & 0 \\ -F_{2h} & 0 & F_{1h} \end{bmatrix} \begin{bmatrix} \dot{X}_{h_l} \\ X_{h_l} \\ f_h \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ F_{3h} \end{bmatrix} u + \begin{bmatrix} {M_{h_l}}^{-1} E_{h_l} \\ 0 \\ 0 \end{bmatrix} w_h \tag{16}$$

$$\dot{x}_h = A_h x_h + B_h u_h + G_h w_h$$

where

$$A_h = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{110} \\ a_{21} & a_{22} & 0 & 0 & a_{25} & a_{26} & 0 & 0 & a_{29} & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} & a_{39} & a_{310} \\ 0 & 0 & a_{43} & a_{44} & 0 & 0 & a_{47} & a_{48} & 0 & a_{410} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{91} & a_{92} & 0 & 0 & 0 & 0 & 0 & 0 & a_{99} & 0 \\ 0 & 0 & a_{103} & a_{104} & 0 & 0 & 0 & 0 & 0 & a_{1010} \end{bmatrix} \tag{17}$$

where the non-zero elements of the matrix A_h are as given in Appendix A, and

$$B_h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{A_{ef}} & 0 \\ 0 & \frac{1}{A_{er}} \end{bmatrix} \tag{18}$$

$$G_h = \begin{bmatrix} 0 & 0 \\ \frac{k_{wf}}{m_{wf}} & 0 \\ 0 & 0 \\ 0 & \frac{k_{wr}}{m_{wr}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (19)$$

It can be observed from equations (18) and (19) that the non-zero elements of the disturbance matrix G_h are not in phase with the input matrix B_h. Therefore the system suffers from the mismatched condition.

The parameter values used for the active suspension system are listed as shown in Table I [4];

Table I Parameter values for the half car suspension

Mass of the car body, m_b	:	1794.4 Kg
Moment of inertia for the car body, I_b	:	3443.05 Kgm^2
Mass of the front wheel, m_{wf}	:	87.15 Kg
Mass of the rear wheel, m_{wr}	:	140.04 Kg
Stiffness of the front car body spring, k_{bf}	:	66824.2 N/m
Stiffness of the front car body spring, k_{br}	:	18615.0 N/m
Stiffness of the front car tire, k_{wf}	:	101115 N/m
Stiffness of the rear car tire, k_{wr}	:	101115 N/m
Damping of the front car damper, c_{bf}	:	1190 Ns/m
Damping of the rear car damper, c_{br}	:	1190 Ns/m

The parameter values for the front and rear hydraulic actuators are $A_{ef} = A_{er} = 1\, s$ and $A_{yf} = A_{yf} = 10\ kNs/m$ as presented in [23].

The dynamic equation for the hydraulically actuated active suspension system for the half car model in state space form as follows:

$$\begin{bmatrix} \ddot{x}_{bf} \\ \ddot{x}_{wf} \\ \ddot{x}_{br} \\ \ddot{x}_{wr} \\ \dot{x}_{bf} \\ \dot{x}_{wf} \\ \dot{x}_{br} \\ \dot{x}_{wr} \\ f_f \\ f_r \end{bmatrix} = \begin{bmatrix} -1.22 & 1.22 & 0.07 & -0.07 & -68.6 & 68.6 & 1.4 & -1.4 & 0.001 & -7.5x10^{-5} \\ 13.7 & -13.7 & 0 & 0 & 767 & -1930 & 0 & 0 & -0.015 & 0 \\ 0.09 & -0.09 & -1.41 & 1.41 & 5.02 & -5.02 & -26.2 & 26.24 & -7.5x10^{-5} & 0.0014 \\ 0 & 0 & 7.14 & -7.14 & 0 & 0 & 132.98 & -854.97 & 0 & -0.0071 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13.27 & 13.7 & 0 & 0 & 0 & 0 & 0 & 0 & -0.885 & 0 \\ 0 & 0 & 13.27 & 13.27 & 0 & 0 & 0 & 0 & 0 & -0.885 \end{bmatrix} \begin{bmatrix} \dot{x}_{bf} \\ \dot{x}_{wf} \\ \dot{x}_{br} \\ \dot{x}_{wr} \\ x_{bf} \\ x_{wf} \\ x_{br} \\ x_{wr} \\ f_f \\ f_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.885 & 0 \\ 0 & 0.885 \end{bmatrix} \begin{bmatrix} u_f \\ u_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1160 & 0 \\ 0 & 0 \\ 0 & 0.72 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_f \\ w_r \end{bmatrix}$$

To study the robustness of the active suspension subject to various road profiles irregularities, two set of road profiles will be used in this study. The first road disturbance $w(t)$ representing a single bump may be given by the following equation [5]:

$$w(t) = \begin{cases} a(1-\cos 8\pi t), & 1.25\sec \le t \le 1.5\sec \\ 0 & otherwise \end{cases} \qquad (20)$$

where $a = 0.11$. While the other road profile representing the road surface with multiple bumps of uniform height. The height of each bump is 11 cm. The mathematical model of bump for such road profile is given as:

$$w(t) = \begin{cases} a(1-\cos 8\pi t), & 1.25 \sec \le t \le 1.5 \sec, \\ & 3.00\sec \le t \le 3.25\sec, \\ & 5.00\sec \le t \le 5.25\sec, \\ & 7.00\sec \le t \le 7.25\sec, \\ & 9.00\sec \le t \le 9.25\sec. \\ 0, & otherwise \end{cases} \qquad (21)$$

where $a = 0.11$. The road profiles are as shown in Figures 2 and 3, respectively.

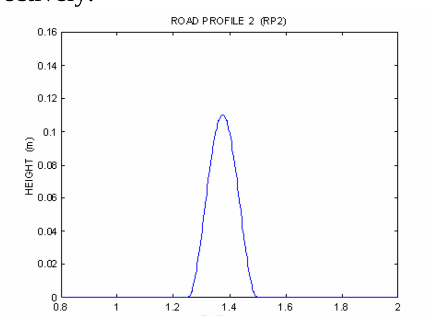

Fig. 2 Road profiles that represented 11 cm bump

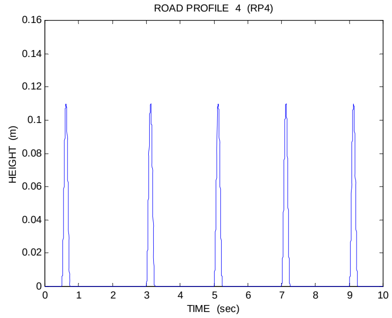

Fig. 3 Road profile that represented by multiple 11 cm bumps

3 Controller

The model can be written in the following form:

$$\dot{x}_h(t) = A_h x_h(t) + B_h u_h(t) + f_h(x,t) \tag{22}$$

where $x_h(t) \in \Re^n$ is the state vector, $u_h(t) \in \Re^m$ is the control input, and the continuous function $f(x,t)$ represents the uncertainties with the mismatched condition.

Some assumptions that are taken into account in this paper;

i) The state vector $x_h(t)$ is fully observable.

ii) There exist a known positive constant β_h such that $\| f_h(t) \| \le \beta_h$

iii) The pair (A_h, B_h) is controllable and the input matrix B_h has full rank.

In this study, the proportional integral sliding surface for a half car suspension model is defined as follows:

$$\sigma_h(t) = C_h x_h(t) - \int_0^t [C_h A_h + C_h B_h K_h] x_h(\tau) d(\tau) \tag{23}$$

where $B_h \in \Re^{m\times n}$ is the input matrix for a half car model, $C_h \in \Re^{m\times n}$ and $K_h \in \Re^{m\times n}$ are the constant matrices, respectively, m is the number of inputs and n is the number of system states. Thus, it can be seen that the active suspension system for the half car model has two sliding surfaces. Let the sliding surface for front and rear suspensions be defined as $\sigma_{hf}(t)$ and $\sigma_{hr}(t)$, respectively.

For the half car model, $m = 2$ and hence the matrix C_h has the following structure:

$$C_h = \begin{bmatrix} c_{f1} & c_{f2} & \ldots\ldots & c_{fn} \\ c_{r1} & c_{r2} & \ldots\ldots & c_{rn} \end{bmatrix} \tag{24}$$

The matrix C_h is chosen such that $C_h B_h \in \Re^{m\times m}$ is nonsingular, while the matrix K_h is chosen such that

$$\lambda_{\max}(A_h + B_h K_h) < 0 \tag{25}$$

The condition forced by equation (24) guarantees that all the desired closed loop poles are located in the left half of the s-plane to ensure stability. The gain matrix K_h may be determined by using pole placement method with the pre-specified pole locations. In this work, all parameters are obtained by using the sensitivity test. These parameters are also can be obtained using a Linear Matrix Inequalities as suggested in [24].

The control input of the sliding mode control can be written as

$$u_h(t) = u_{h_{eq}}(t) + u_{h_s}(t) \tag{26}$$

The switching control $u_{h_s}(t)$ is selected as follows:

$$u_{h_s}(t) = (C_h B_h)^{-1} \rho_h \, \mathrm{sgn}(\sigma_h(t)) \tag{27}$$

It can be seen from equation (26) that the switching control $u_{h_s}(t)$ is nonlinear and discontinuous. The chattering effect caused by the $\mathrm{sgn}(\sigma_h(t))$ function may be replaced by the continuous function. Hence the switching control becomes:

$$u_{h_s}(t) = (C_h B_h)^{-1} \rho_h \frac{\sigma_h(t)}{\|\sigma_h(t)\| + \delta_h} \tag{28}$$

where δ_h is the boundary layer thickness which is selected to reduce the chattering problem and ρ_h is the design parameter which is specified by the designer. The chattering reduction for a nonlinear system has been discussed in [25]. Therefore, the proposed proportional sliding mode controller for the half car active suspension model is given as follows:

$$u_h(t) = K_h x_h(t) - (C_h B_h)^{-1} C_h f_h(t) - (C_h B_h)^{-1} \rho_h \frac{\sigma_h(t)}{\|\sigma_h(t)\| + \delta_h} \tag{29}$$

where $\rho_h > 0$.

4 Simulation Results

Simulation task is performed using Matlab/Simulink software. Figures 4.1- 4.6 show the comparison between passive suspension, linear state feedback (LSF) and PISMC.

The first step in the design is to determine the relevant parameters value for the sliding surface equation (23) and the PISMC equation (29). These are carried out according to the following steps:

Step 1: Determine the pre-specified (desired) closed loop poles location.

Step 2: Determine the gain matrix K_h in equations (23) and (29) by using the pole placement method with respect to the desired pole locations as defined in Step 1.

Step 3: Choose the appropriate value for the matrix C_h in equations (23) and (29) by trial and error approach.

Step 4: Select the appropriate values for ρ_h and δ_h in equation (29).

Step 5: Run the simulation and observe the performance of the system. Repeat Steps 3 and 4 if the performance is not satisfactory.

The simulations are performed with the following parameters:

The pre-specified poles location is set at:

$[-30 \;\; -20 \;\; -40 \;\; -40 \;\; -60 \;\; -60 \;\; -80 \;\; -80 \;\; -100 \;\; -100]$

By utilizing poles placement method, based on the given poles location, the gain K_h can be calculated as follows,

$$\begin{bmatrix} -5.05\times10^7 & -1.58\times10^6 & -5.47\times10^5 & 5.42\times10^5 & -4.88\times10^8 \\ 3.81\times10^6 & 5.06\times10^5 & -5.07\times10^7 & -5.12\times10^6 & 8.34\times10^7 \end{bmatrix}$$

$$\begin{bmatrix} 9.83\times10^7 & 2.37\times10^7 & -3.62\times10^5 & -330.34 & 2.59 \\ -6.32\times10^6 & -4.56\times10^8 & 1.61\times10^8 & 7.21 & -330.49 \end{bmatrix}$$

By using trial and error method, the following C_h matrix is used in the simulation:

$$C_h = \begin{bmatrix} 2 & 4 & 3 & 5 & 6 & 8 & 9 & 2 & 1 & 4 \\ 7 & 1 & 3 & 5 & 4 & 2 & 7 & 5 & 4 & 3 \end{bmatrix}.$$

The following values of ρ_h and δ_h in the proposed controller have been used:

$$\rho_h = \begin{bmatrix} 100 \\ 10 \end{bmatrix} \text{ and } \delta_h = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

For comparison purposes, the performance of the PISMC is compared to the active suspension system for the half car model using linear state feedback (LSF) control approach and also passive suspension. The following linear state feedback law is used:

$$u_{LSF}(t) = K_h x_h(t) \qquad (30)$$

The suspension travel for the front and rear suspensions for the active suspension system using PISMC and LSF controllers and also the passive suspension system are shown in Figures 4 and 5. The results show that the PISMC technique perform better as compared to the others especially for the rear suspension performance. Furthermore, the suspension travel of both controllers is within the distance of $\pm 8cm$. Figures 6 and 7 describe the response of the wheel deflection for the passive suspension system and also for the active suspension system using the PISMC and SLF controllers.

The simulation results show that the active suspension system with the PISMC approach has a better tyre to road surface contact, hence directly improved the car handling as compared to the LSF method and the passive suspension system. Figures 8 and 9 show that the body acceleration of the proposed controller is slightly reduced as compared to the LSF method and the passive suspension system.

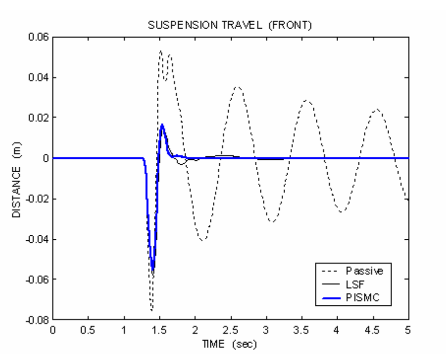

Fig. 4 Suspension travel of the half car front suspension (single bump)

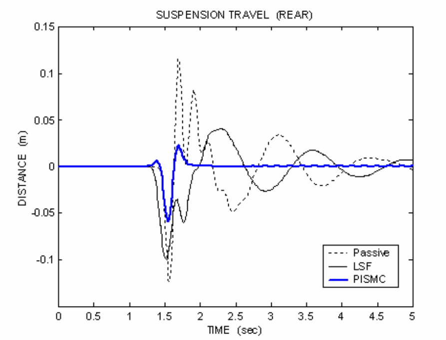

Fig. 5 Suspension travel of the half car rear suspension (single bump)

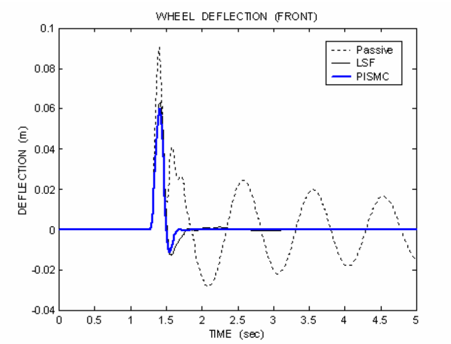

Fig. 6 Wheel deflection of the half car front suspension (single bump)

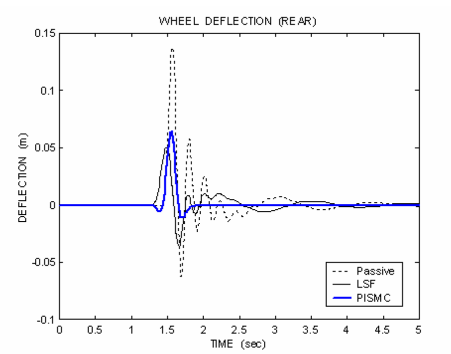

Fig. 7 Wheel deflection of the half car rear suspension (single bump)

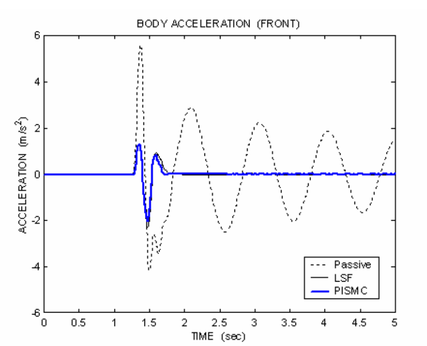

Fig. 8 Body acceleration of the half car front suspension (single bump)

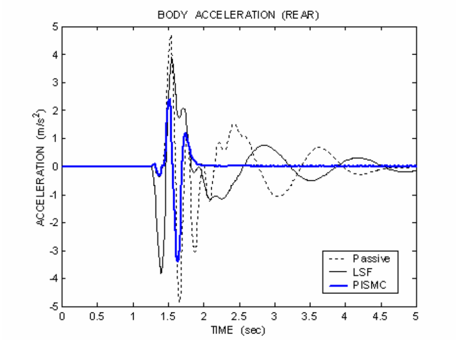

Fig. 9 Body acceleration of the half car rear suspension (single bump)

Figures 10-15 illustrate the performance of the proposed controller in overcoming the disturbances in the form of uniform multiple bumps. It can be seen that the wheel deflections and body accelerations of the front and rear suspensions for the half car active suspension system have been improved as compared to the passive suspension system for both road profiles. It means that, the PISMC has significantly improved the ride comfort and road handling qualities for the half car active suspension model.

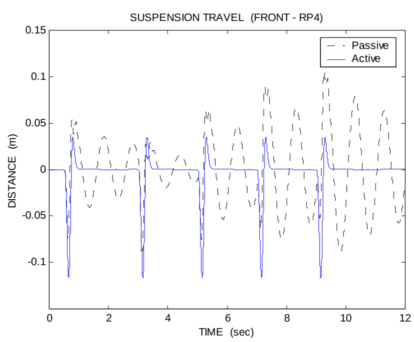

Fig. 10 Suspension travel of the half car front suspension (multiple bumps)

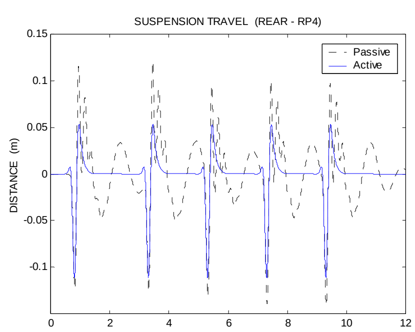

Fig. 11 Suspension travel of the half car rear suspension (multiple bumps)

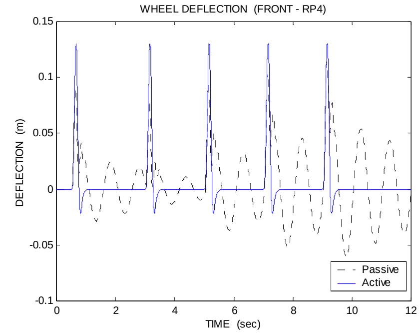

Fig. 12 Wheel deflection of the half car front suspension (multiple bumps)

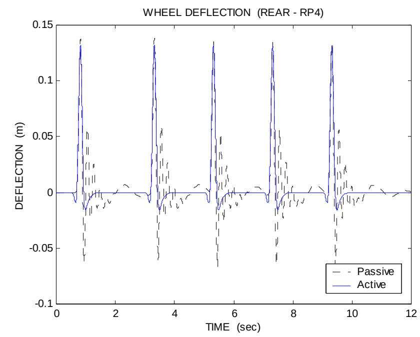

Fig. 13 Wheel deflection of the half car rear suspension (multiple bumps)

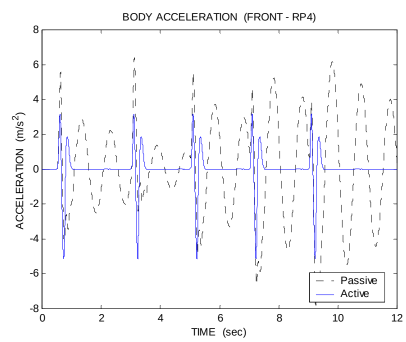

Fig. 14 Body acceleration of the half car front suspension (multiple bumps)

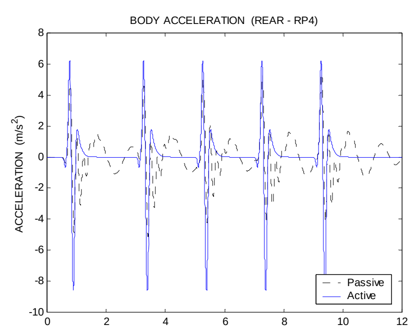

Fig. 15 Body acceleration of the half car rear suspension (multiple bumps)

In the following simulations, the effect of varying the constant ρ_h in the proportional sliding mode controller is studied. For the half car active suspension model, ρ_h is represented by the matrix $[\rho_{h_f} \quad \rho_{h_r}]^T$ where ρ_{h_f} and ρ_{h_r} are the sliding gains for the front and rear suspensions, respectively. The reaching mode condition is satisfied if $\rho_{h_f} > 0$ and $\rho_{h_r} > 0$, and not satisfied if $\rho_{h_f} < 0$ and $\rho_{h_r} < 0$.

The following values of the sliding gains ρ_{h_f} and ρ_{h_r} have been considered in the simulation: Case 1: $\rho_h = [1\times10^5 \quad 1\times10^5]^T$ (reaching mode condition is satisfied)

Case 2: $\rho_h = [-1\times10^5 \quad -1\times10^5]^T$ (reaching mode condition is not satisfied)

The other parameter constants are similar as before.

Figures 16 and 17 exhibit the sliding surfaces of both cases.

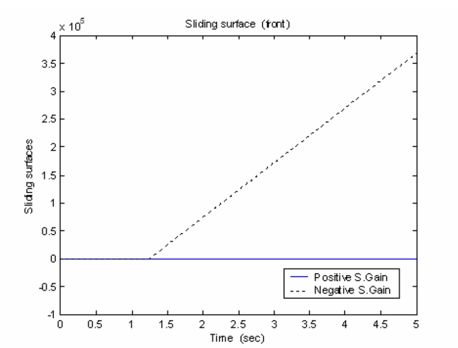

Fig. 16 Sliding surface for varying the parameter ρ_{h_f}

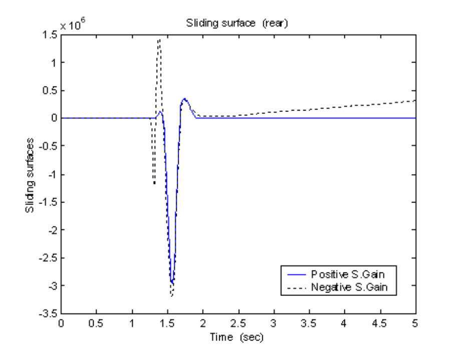

Fig. 17 Sliding surface for varying the parameter ρ_{h_r}

It can be observed from both figures that for the positive sliding gains, the state trajectories for the front and rear active suspensions slide onto the sliding surfaces. In contrary, for the negative sliding gains, the state trajectories do not slide onto the sliding surfaces. Thus, the reaching mode condition is satisfied when the sliding gains are both positive. Figures 18-25 illustrate the effects of varying the sliding gain on the suspension travel, wheel deflection, body acceleration and actuator force for front and rear suspensions.

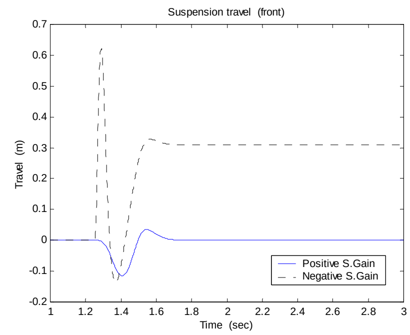

Fig. 18 Suspension travel for varying the parameter ρ_{h_f}

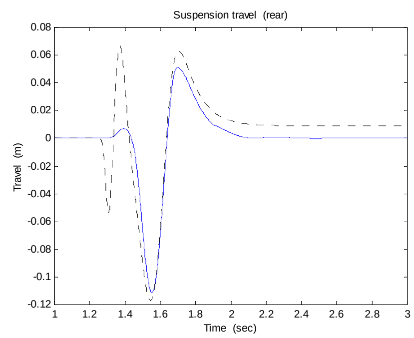

Fig. 19 Suspension travel for varying the parameter ρ_{h_r}

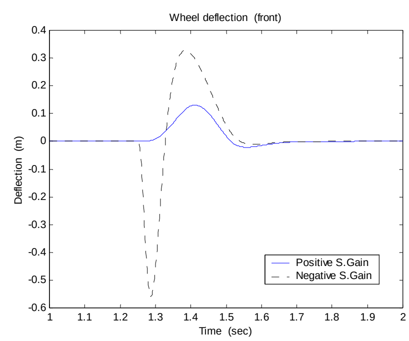

Fig. 20 Wheel deflection for varying the parameter ρ_{h_f}

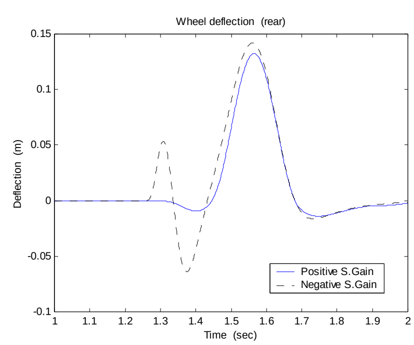

Fig. 21 Wheel deflection for varying the parameter ρ_{h_r}

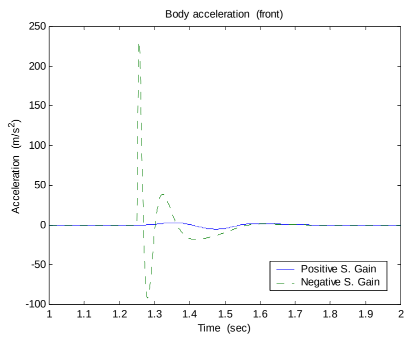

Fig. 22 Body acceleration for varying the parameter ρ_{h_f}

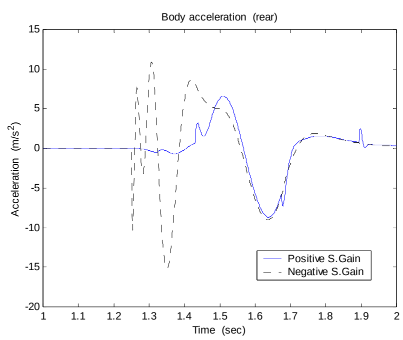

Fig. 23 Body acceleration for varying the parameter ρ_{h_r}

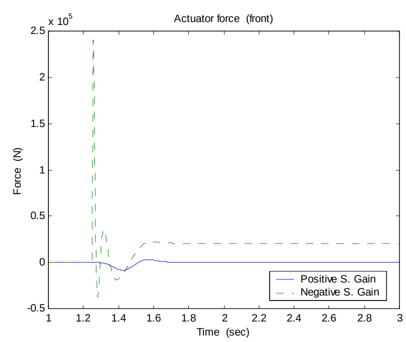

Fig. 24 Actuator force for varying the parameter ρ_{h_f}

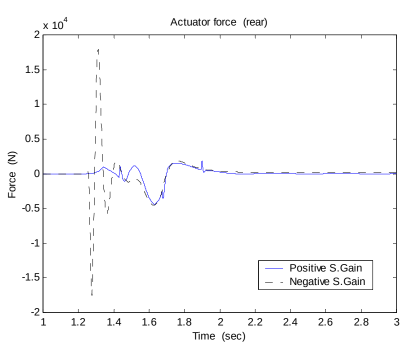

Fig. 25 Actuator force for varying the parameter ρ_{h_r}

5 Conclusion

A modeling and a control technique known as a proportional integral sliding mode control for an active suspension system have been proposed and analyzed. One of the most attractive advantages of the sliding mode control is that the controller is robust to any uncertainties while it is in the sliding mode condition.

As compared to the linear state feedback control technique and passive suspension system, the proposed controller showed great riding comfort and road handling quality without sacrificing the limitation on the suspension travel. As a conclusion, it is shown that the proposed proportional integral sliding mode controller improves the performance of the hydraulically actuated active suspension systems for half car model with the ability to keep the suspension systems stable and robust even on rough roads.

References:

[1] A. Alleyne, P.D. Neuhaus, and J.K. Hedrick, "Application of Nonlinear Control Theory to Electronically Controlled Suspensions," *Vehicle System Dynamics*. 1993. 22: 309-320.

[2] A. Alleyne, and J. K. Hedrick, "Nonlinear Adaptive Control of Active Suspensions," *IEEE Transactions on Control Systems Technology*. 1995. 3(1): 94-101.

[3] M. Appleyard, and P.E. Wellstead, "Active suspension: some background. *IEE Proc. Control Theory Application*," 1995. 142(2): 123 – 128.

[4] E. Esmailzadeh, and H.D. Taghirad, "Active Vehicle Suspensions with Optimal State-Feedback Control," Journal *of Mechanical Science*. 1996. 200(4): 1-18.

[5] I. Fialho, and G.J. Balas, "Road Adaptive Active Suspension DesignUsing Linear Parameter-Varying Gain-Scheduling," *IEEE Transactions on Control Systems Technology*. 2002. 10(1): 43-54.

[6] D. Hrovat, "Influence of Unsprung Weight on Vehicle Ride Quality," Journal *of Sound and Vibration*. 1988. 124(3): 497-516.

[7] S. J. Huang, and H. C Chao, "Fuzzy Logic Controller for a Vehicle Active Suspension System," *Proc. Instn. Mech. Engrs Part D*. 2000. 214: 1-12.

[8] C. Kim, and P.I. Ro, "A Sliding Mode Controller for Vehicle Active Suspension Systems with Non-Linearities," *Proc. Instn. Mech. Engrs Part D*. 1998. 212: 79-92.

[9] F. J. Lin, S. L. Chiu, and K. K. Shyu, "Novel Sliding Mode Controller for Synchronous Motor Drive," *IEEE Transactions on Aerospace and Electrical Systems*. 1998. 34(2): 532-542.

[10] J. S. Lin, and I. Kanellakopoulos, "Nonlinear Design of Active Suspensions," *IEEE Control Systems Magazine*. 1997a. 17: 1-26.

[11] J. S. Lin, and I. Kanellakopoulos, "Road-Adaptive Nonlinear Design of Active Suspensions," *Proc. of the 1997 American Control Conference*. June, 1997. Albuquerque, New Mexico: IEEE. 1997b. 714-718.

[12] Y. Liu, and H. Handroos, "Sliding Mode Control for A Class of Hydraulic Position Servo," *Mechatronics*. 1999. 9: 111-123.

[13] H. E. Merritt, "*Hydraulic Control Systems*," New York: John Wiley & Sons, Inc. 1967.

[14] J. H. Park, and Y. S. Kim, "Decentralized Variable Structure Control for Active Suspensions Based on a Full-Car Model," *IEEE Proc. Int. Conf. On Control Applications*. September 1-4, 1998. Trieste, Italy: IEEE. 1998. 383-387.

[15] N. Olgac, B. E. Platin, and J. M. Chang, "Sliding Mode Control of Remotely Operated Vehicles for Horizontal Plane Motions," *IEE Proceedings Part D*. 1991. 138(5): 469-473.

[16] Y. M. Sam, and J. H. S. Osman, "Modeling and Control of the Active Suspension using Proportional-Integral Sliding Mode Approach," *Asian Journal of Control.*2005. 2(7):91-98.

[17] Y. M. Sam, Osman J. H. S. and Ghani M.R.A.A, "Class of Proportional-Integral Sliding Mode Control with Application to Active Suspension System. *System and Control Letters.* 2004. 51:217-223.

[18] C. S. Ting, T. H. Li, and F. C. Kung, "Design of Fuzzy Controller for Active Suspension System," *Mechatronics*. 1995. 5(4): 365-383.
[19] T. Tseng, and D. Hrovat, "Some Characteristics of Optimal Vehicle Suspensions Based on Quarter-Car Model," *Proceedings of the 29th IEEE Conference on Decision and Control*. December, 1990. Honolulu, Hawaii: IEEE. 1990. 2232-2237.
[20] M. Yamashita, K. Fujimori, K. Hayakawa, and H. Kimura, "Application of H-infinity Control to Active Suspension Systems," *Automatica*. 1994. 30(11): 1717-1792.
[21] T. Yoshimura, Y. Isari, Q. Li, and J. Hino, "Active Suspension of Motor Coaches using Skyhook Damper and Fuzzy Logic Control," *Control Eng. Practice*. 1997. 5(2): 175-184.
[22] T. Yoshimura, A. Kume, M. Kurimoto, and J. Hino, "Construction of An Active Suspension System of A Quarter Car Model Using The Concept of Sliding Mode Control," *Journal of Sound and Vibration*. 2001. 239(2): 187-199.
[23] T. Yoshimura, K. Nakaminamani, M. Kurimoto, and J. Hino, "Active Suspension of Passenger Cars Using Linear and Fuzzy Logic Controls," *Control Engineering Practice*. 1999. 7(2): 41-47.
[24] Y. Xu, W. Tang and Z. Yuan, "Global Sliding Mode Variable Structure Controller for Uncertain Systems Using Linear Matrix Inequalities," *WSEAS Transactions On Circuits And Systems*. 2006. 5(5): 664-670.
[25] A. M. Loh and L.F. Yeung, "Chattering Reduction in Sliding Mode Control: An Improvement for Nonlinear Systems," *WSEAS Transactions On Systems*. 2004. 9(3): 2878-2886.
[26] L. Li dan Q. Li, "Vibration Analysis Based on Full Multi-Body Model and the Active Control of Commercial Vehicle Suspension System," *WSEAS Transactions on Systems and Control*. 2007. 2(2): 230-236.