The Influence of Disturbed Measured Values and Counter Pulses on Gas Volume Control

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Abstract: - Firstly the authors explain the nature of contracts between primary suppliers of gas and local suppliers. They then describe and investigate an effect observed frequently in the field of gas volume and flow control. Instead of running a straight line, the volume controller opens or closes the control valve dramatically at the end of the accounting period. The effect turns out to be caused by a relative deviation of the measured value of the gas flow rate from the actual value, and it is explained by solving an easy differential equation. Rounding errors occurring during necessary calculations may lead to the same effect. In section 2 of their paper the authors introduce a more general linear differential equation to describe various kinds of perturbations simultaneously: relative and absolute deviations of the measured values from the actual values of the gas flow rate and pulses disturbing the pulses counting the standard volume flown so far. By solving the differential equation and analyzing the solution, the authors then explain the influence of the perturbations on the behaviour of the control valve. They too discuss the risk of running a peak load, i.e. of exceeding the contracted amount of gas.

Key-Words: - Gas Distribution System, Gas Supplier, Third Party Contract, electronic corrector systems, Standard Volume, Volume Control, Control Valve, flow rate, PID Control, Internal Set Point

1 Introduction

Local gas suppliers, like municipal utility companies or public supply companies, purchase the gas to be distributed from primary suppliers, big companies purchasing the gas from petroleum and gas producing countries or from other primary suppliers. They then sell the gas to final consumers or to sub providers. Local suppliers negotiate contracts with primary suppliers, allowing them to draw gas up to a fixed amount W within a certain accounting period T. Customary accounting periods are T = 1 hour or T = 1 day. It is on this fixed amount that the so called demand charge depends. The demand charge is the price, which the local suppliers have to pay for the provision of the gas. Of course they also have to pay an energy rate. The energy rate is the price to pay for the amount of gas actually purchased.

It is the aim of a local supplier to exhaust its fixed amount W in each period T, i.e. to draw an amount of gas within the period T approaching W as close as possible. In particular, this is important at the end of cold winter nights, when in the morning a high amount of gas is needed to heat the flats and the schools and the factories and so on. If, on the other hand the actual amount of the gas purchased surmounts W, the local supplier risks suffering high losses: The fixed amount W will be raised up to the actual amount of gas purchased, an effect, which may cause an expensive experience. This is what they call “running a peak load”.

To fulfill those different requirements, PLCs (programmable logic controllers) are applied, controlling the flow rate of the gas by means of a control valve. They contain step controllers actuating the drive of the control valve or analogous controllers piloting a valve positioner. Normally a PI controller is sufficient for this task, whereas pressure controllers may require a derivative term (PID controller). In case of some special valves self tuning controllers are necessary. For new developments in the field of self tuning controllers see [7] and [8]whereas the design of PID control “in View of Controller Location in the Plant” is treated in [15] and [16]. The controller compares the actual flow rate with an internal set point and then determines the length of the pulses actuating the valve by pulse-width modulation. It is desirable that the control task is done with a minimum of pulses. Some valve manufacturers use to limit the number of pulses positioning the valve within a certain time period. A frequently observed phenomenon is that at the end of a gas accounting period the volume controller closes the control valve completely to avoid running a peak load. Or, in contrary, it opens the valve widely in order to achieve the contracted amount of gas. Gas suppliers do not like such a behaviour of their valves.

It seems to the authors that, until now, no one tried to explain those effects by mathematical methods. And so it is the aim of this paper to investigate the nature of the phenomena by means of a differential equation. The methods used are described in [4], [5], and [6]. Especially in [4] and [6] a lot of examples can be found were physical, chemical, biological, technical, military and technical processes investigated by means of differential equations or by systems of differential equations.

It will turn out in section 2 that, for example, a small difference between the actual gas flow rate and the measured value of the gas flow rate (coming from an electronic corrector system) may cause such behaviour. Corrector systems or volume converters are microcomputers converting raw data (pulses coming from high frequency turbine-type meters) into standard flow rates and standard volumes at base conditions according to the ideal gas law (USA: pressure 14.73 psi absolute, temperature 60 ° F).

Apart from measured (analog) values counter pulses play an important role in gas volume control. They count the volume of the gas flown through the control valve, and they have to be summed up by the controller in order not to exceed the contracted amount of gas. If there are disturb pulses interfering with the regular pulses, there is a risk of running a peak load. Those phenomena are investigated in section 3, where a more general initial value problem is stated. By solving this problem the risk of running a peak load and the behaviour of the valve are discussed.

Last but not least the calculations, which have to done to perform the control task, may be afflicted by rounding errors. In this case the same behaviour of the control valve can be observed.

There is one more specialty in the gas business (as all over the energy market): Third party contracts. The "third party law" allows a consumer to purchase the energy he needs from a provider different from his local supplier. The gas is then conveyed to the final consumer through the gas pipes of the local supplier. This fact has to be taken care of when calculating the internal set point of the controller.

Other issues are uncontrolled inlets: Apart from the gas inlet equipped with a controlled valve, there may be uncontrolled inlets. This fact too has to be taken in account. A typical distribution system of a local supplier is shown in Fig.1 below.

A detailed analysis of gas distribution systems can be found in [1] or [2]. Methods to integrate data coming from the different inlets and outlets via a DSfG-Interface according to IEC 60870-5-101 can be found in [9]. The consequences of the liberalisation of the energy market in Europe are investigated in [10], [11], [12], [13], and [14]. Approaches to optimize gas distribution system are developed in [3].

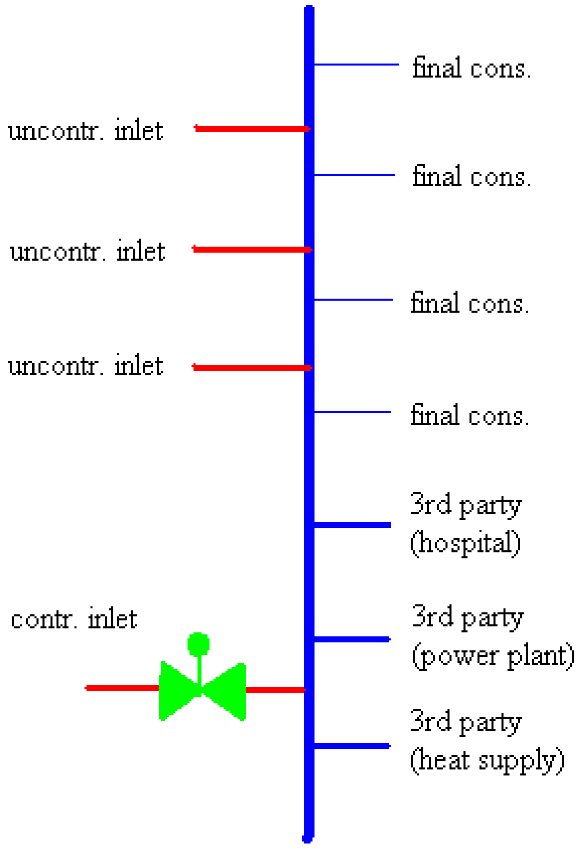

Fig. 1

2 Volume and Flow Control of Gas

2.1 The Internal Set Point

The PLC containing the gas controller receives counter pulses from an electronic corrector system. The pulses are detected either by digital (counter) inputs, or the pulse rate is transmitted by a field bus system. They are counting the standard volume of the gas flown so far at base conditions. We distinguish the number of pulses received at a certain time τ_k according to their origin:

$c_c(\tau_k)$ = Number of pulses from controlled inlets,
$c_{uc}(\tau_k)$ = Number of pulses from uncontrolled inlets,
$c_{tp}(\tau_k)$ = Number of pulses from third party outlets.

Then the volume of gas flown from the beginning of the accounting period up to the time $\tau \le T$ is given by

$$\Sigma(\tau) = \sum_{\tau_k \le \tau} c_c(\tau_k) + \sum_{\tau_k \le \tau} c_{uc}(\tau_k) - \sum_{\tau_k \le \tau} c_{tp}(\tau_k)$$
$$= \Sigma_c(\tau) + \Sigma_{uc}(\tau) - \Sigma_{tp}(\tau).$$

Therefore the remaining volume allowed to be drawn until the end of the accounting period is equal to

$$W - \Sigma(\tau) \ .$$

The internal set point of the gas controller is then determined by the quotient of remaining volume divided by remaining time:

$$W_{int}(\tau) = \frac{W - \Sigma(\tau)}{T - \tau} \ .$$

The functions

$$\Sigma_c(\tau), \Sigma_{uc}(\tau), \Sigma_{tp}(\tau)$$

are step functions. To set up a differential equation, let us replace them by continuously differentiable functions

$$\sigma_c(\tau), \sigma_{uc}(\tau), \sigma_{tp}(\tau)$$

and

$$\sigma(t) = \sigma_c(\tau) + \sigma_{uc}(\tau) - \sigma_{tp}(\tau) \ ,$$

a condition, being always achievable by interpolating the step functions (by spline functions for example). The internal set point is then replaced by

$$\omega_{int}(\tau) = \frac{W - \sigma(\tau)}{T - \tau} \ .$$

2.2 The Differential Equation

The instantaneous value of the gas flow rate at the time τ is equal to the derivative of the volume flown so far

$$\frac{d\sigma(\tau)}{d\tau} \ .$$

The gas controller insures that the instantaneous flow rate is equal to the internal set point

$$\frac{d\sigma(\tau)}{d\tau} = \omega_{int}(\tau) \ .$$

In reality they are not equal; there is always a control deviation depending on the quality of the controller. But for our purposes (a qualitative explanation of some phenomena) we may assume that they are equal. And therefore, because of the definition of the internal set point, $\sigma(\tau)$ has to be a solution of the initial value problem

$$(1) \quad \frac{d\sigma(\tau)}{d\tau} = \frac{W - \sigma(\tau)}{T - \tau}, \sigma(\tau) = 0 \ .$$

The differential equation can be solved by separating the variables (cf. [5]):

$$\int_0^\sigma \frac{ds}{W - s} = \int_0^\tau \frac{dt}{T - t}$$
$$\ln |W - s|\Big|_0^\sigma = \ln |T - t|\Big|_0^\tau$$
$$\ln |W - \sigma| - \ln W = \ln |T - \tau| - \ln T$$
$$\frac{|W - \sigma|}{W} = \frac{|T - \tau|}{T}$$
$$\sigma(\tau) = W - \frac{T - \tau}{T} W \text{ for } 0 \le \tau \le T.$$

The graph of $\sigma(\tau)$ is shown Fig.2. It visualizes the ideal case; in reality $\sigma(\tau)$ is not a straight line. But for our qualitative considerations, we may assume the ideal case.

Now, during the startup of a gas controller and sometimes even during normal operation, the following phenomenon can be observed frequently: There is a difference between the measured value Q of the gas flow rate (coming from the electronic corrector system or determined by the controller

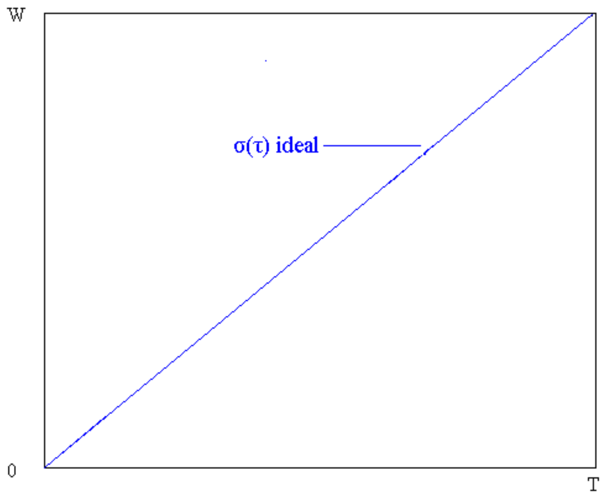

Fig. 2 α =1

itself from the raw data) and the theoretical value dσ/dτ:

$$Q = \alpha \frac{d\sigma(\tau)}{d\tau}, \alpha \neq 1 \qquad .$$

Those differences may result from a bad adjusting of the transducers in operation.

The new situation is described by the initial value problem

$$(2) \quad \alpha \frac{d\sigma(\tau)}{d\tau} = \frac{W - \sigma(\tau)}{T - \tau}, \alpha \neq 1, \sigma(\tau) = 0 .$$

When solving this differential equation, we get

$$\alpha \cdot \int_0^\sigma \frac{ds}{W - s} = \int_0^\tau \frac{dt}{T - t}$$

$$\alpha \cdot \ln |W - s| \|_0^\sigma = \ln |T - t| \|_0^\tau$$

$$\alpha \cdot (\ln |W - \sigma| - \ln W) = \ln |T - \tau| - \ln T$$

$$\frac{|W - \sigma|^\alpha}{W^\alpha} = \frac{T - \tau}{T}$$

$$\sigma(\tau) = W - \left(\frac{T - \tau}{T} \right)^{1/\alpha} W \quad \text{for } 0 \leq \tau \leq T .$$

As was to be expected

$$\lim_{\tau \to T} \sigma(\tau) = W ,$$

i.e. the contracted amount of gas within the accounting period is held. A typical graph of the solution (α = 0,5) is shown in Fig. 3.

Next we shall investigate the derivative of σ(τ):

$$\frac{d\sigma(\tau)}{d\tau} = \frac{1}{\alpha} \left(\frac{T - \tau}{T} \right)^{1/\alpha - 1} W \quad \text{for } 0 \leq \tau \leq T .$$

The case α < 1 means that the actual flow rate of gas is always a little bit higher than the measured flow rate (actual value of the controller): The controller passes a too high amount of gas per each time unit. Therefore, at the end of the period, the controller closes the valve in order to not violate the contracted bound.

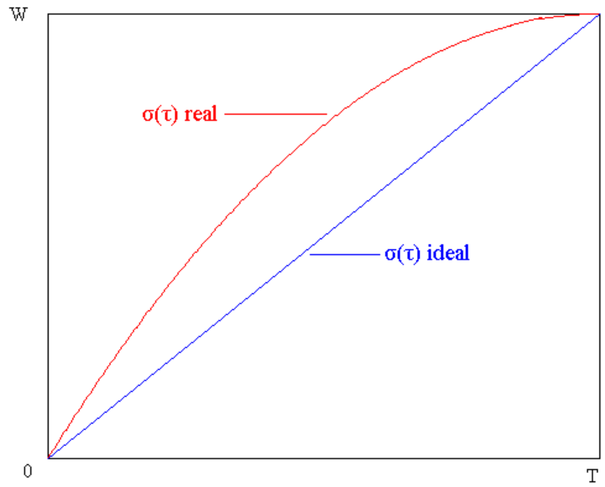

Fig. 3 α =0.5

At time T we have

$$\frac{d\sigma(T)}{d\tau} = \frac{1}{\alpha} \left(\frac{T - \tau}{T} \right)^{1/\alpha - 1} \cdot W \Bigg|_{\tau = T} ,$$

$$= 0$$

meaning that the controller has to close the valve completely. This effect, though extremely undesirable, can be observed frequently when operating a gas control system. As for Fig.3, we took the somewhat disproportionate value of α = 0,5 to demonstrate the effect more dramatically. But the effect itself remains the same, even if the deviation is very small.

If, on the other hand, we have α > 1, than the actual flow rate of gas is always a little bit lower than the measured flow rate: The controller passes a too low amount of gas. Therefore, at the end of the period, the controller opens the valve in order to achieve the contracted amount. Contrarily to the above case, we have

$$\lim_{\tau\to T}\frac{d\sigma(\tau)}{d\tau}=\lim_{\tau\to T}\frac{1}{\alpha}\left(\frac{T-\tau}{T}\right)^{1/\alpha-1}\cdot W,$$
$$=+\infty$$

meaning that the controller has to open the valve completely. A typical graph of the solution in the case of $\alpha = 2$ is shown in Fig.4.

Let us now finally take a glance at the calculations providing the internal set point. Those calculations always are afflicted by rounding errors. But suppose now for the moment that these rounding errors are of such a kind that they cause a permanent deviation of the calculated set point from the actual set point: either always in the positive direction or always in the negative one. Then the same effect can be observed and explained by equation (2). Rounding errors of this kind may also be observed, when the actual standard flow rate is calculated by the PLC and not by volume converter.

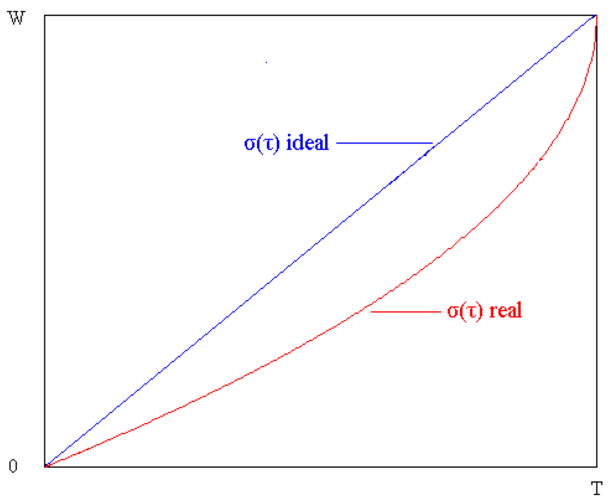

Fig. 4 α =2

But however bad the behaviour of the control valve is, in each case of this section the contracted amount of gas is held. This situation will change in the next section.

3 The General Case

3.1 The Differential Equation

Let us, in this section, assume that the input values of the controller are affected by more than only one kind of perturbations. Suppose, in detail, that:

a) The measured value Q of the actual gas flow rate is disturbed by a relative deviation $\alpha(\tau)$ (similar to section 2) and, in addition, by an absolute deviation $-\gamma(\tau)$:

$$Q=\alpha(\tau)\frac{d\sigma(\tau)}{d\tau}-\gamma(\tau)$$

b) The sum $\Sigma(\tau)$ of the counter values is interfered by disturb pulses. For our purposes only the sum of these pulses at time τ from the beginning of the accounting period is important. We call this sum $\beta(\tau)$. Hence

$$\Sigma(\tau)=\sigma(\tau)+\beta(\tau)$$

is the sum of all pulses registered and processed by the controller to perform the control task.

Therefore, in total, the situation is described by the initial value problem

$$(3)\ \alpha(\tau)\frac{d\sigma(\tau)}{d\tau}=\frac{W-\sigma(\tau)-\beta(\tau)}{T-\tau}+\gamma(\tau),\ \sigma(\tau)=0,$$

replacing (2).

The initial value problem (3) is solvable under rather general conditions on $\alpha(\tau)$, $\gamma(\tau)$ and $\beta(\tau)$ ($\alpha(\tau)$, $\gamma(\tau)$ and $\beta(\tau)$ continuous, $\alpha(\tau)\neq 0$ for example). But to get an qualitative idea of some situations, which may occur, and to simplify matters, it is sufficient to investigate the following special case:

$$\alpha(\tau)=\alpha,\alpha\neq 0$$
$$\beta(\tau)=\beta\cdot\tau$$
$$\gamma(\tau)=\gamma$$

The condition on $\beta(\tau)$ means that the flow of disturbing counter pulses is constant in time.

Hence, the differential equation we have to solve is

$$(4)\ \alpha\frac{d\sigma(\tau)}{d\tau}=\frac{W-\sigma(\tau)-\beta\cdot\tau}{T-\tau}+\gamma,\sigma(\tau)=0$$

or, after division by α,

$$(5)\quad \frac{d\sigma(\tau)}{d\tau}=-\frac{1}{\alpha}\frac{\sigma(\tau)}{T-\tau}+\frac{1}{\alpha}\frac{W-\beta\cdot\tau}{T-\tau}+\frac{\gamma}{\alpha},\alpha\neq 0.$$

The general solution of (5) is given by

$$\sigma(\tau)\quad=\sigma_h(\tau)+\sigma_p(\tau),$$

where σ_h is the general solution of the homogenous equation

$$(6)\ \frac{d\sigma_h(\tau)}{d\tau} = -\frac{1}{\alpha}\frac{\sigma_h(\tau)}{T-\tau}$$

and σ_p is a particular solution of the inhomogeneous equation (5) (see [5]).

The homogenous equation (6) can be solved by separating the variables:

$$\begin{aligned}
\frac{d\sigma_h(\tau)}{d\tau} &= -\frac{1}{\alpha}\frac{\sigma_h(\tau)}{T-\tau}\\
\frac{d\sigma_h}{\sigma_h} &= -\frac{1}{\alpha}\frac{d\tau}{T-\tau}\\
\int\frac{d\sigma_h}{\sigma_h} &= (-1/\alpha/\int\frac{d\tau}{T-\tau}\\
\ln|\sigma_h| &= (-1/\alpha/\alpha)|T-\tau| + \text{const}\\
\sigma_h(\tau) &= C(T-\tau)^{1/\alpha}
\end{aligned}$$

A particular solution of (5) can be found by variation of the constant:

$$\begin{aligned}
\sigma_p(\tau) &= C(\tau)(T-\tau)^{1/\alpha}\\
\sigma_p'(\tau) &= C(\tau)(-\frac{1}{\alpha})(T-\tau)^{1/\alpha-1} + C'(\tau)(T-\tau)^{1/\alpha}.
\end{aligned}$$

Inserting these expressions in (5) yields

$$\begin{aligned}
C(\tau)(-\frac{1}{\alpha})(T-\tau)^{1/\alpha-1} + C'(\tau)(T-\tau)^{1/\alpha}&\\
&= -\frac{1}{\alpha}\frac{C(\tau)(T-\tau)^{1/\alpha}}{T-\tau} + \frac{1}{\alpha}\frac{W-\beta\cdot\tau}{T-\tau} + \frac{\gamma}{\alpha}\\
&= -\frac{1}{\alpha}C(\tau)(T-\tau)^{1/\alpha-1} + \frac{1}{\alpha}\frac{W-\beta\cdot\tau}{T-\tau} + \frac{\gamma}{\alpha},
\end{aligned}$$

hence

$$C'(\tau)(T-\tau)^{1/\alpha} = \frac{1}{\alpha}\frac{W-\beta\cdot\tau}{T-\tau} + \frac{\gamma}{\alpha},$$

and therefore

$$\begin{aligned}
C'(\tau) &= \frac{1}{\alpha}\frac{W-\beta\cdot\tau}{(T-\tau)^{1/\alpha+1}} + \frac{\gamma}{\alpha}\frac{1}{(T-\tau)^{1/\alpha}}\\
&= \frac{1}{\alpha}\frac{W}{(T-\tau)^{1/\alpha+1}} + \frac{1}{\alpha}\frac{\beta\cdot T-\beta\cdot\tau}{(T-\tau)^{1/\alpha+1}}\\
&\quad -\frac{1}{\alpha}\frac{\beta\cdot T}{(T-\tau)^{1/\alpha+1}} + \frac{\gamma}{\alpha}\frac{1}{(T-\tau)^{1/\alpha}}\\
&= \frac{1}{\alpha}\frac{W}{(T-\tau)^{1/\alpha+1}} + \frac{1}{\alpha}\frac{\beta}{(T-\tau)^{1/\alpha}}\\
&\quad -\frac{1}{\alpha}\frac{\beta\cdot T}{(T-\tau)^{1/\alpha+1}} + \frac{\gamma}{\alpha}\frac{1}{(T-\tau)^{1/\alpha}}.
\end{aligned}$$

For the rest of the calculations, have to consider to different cases $\alpha \neq 1$ and $\alpha = 1$.
In the case $\alpha \neq 1$, by integration, we find a primitive of C'

$$\begin{aligned}
C(\tau) &= \frac{W}{(T-\tau)^{1/\alpha}} + \frac{1}{\alpha}\frac{1}{1/\alpha-1}\frac{\beta}{(T-\tau)^{1/\alpha-1}}\\
&\quad -\frac{\beta\cdot T}{(T-\tau)^{1/\alpha}} + \frac{\gamma}{\alpha}\frac{1}{1/\alpha-1}\frac{1}{(T-\tau)^{1/\alpha-1}}\\
&= \frac{W}{(T-\tau)^{1/\alpha}} + \frac{1}{1-\alpha}\frac{\beta}{(T-\tau)^{1/\alpha-1}}\\
&\quad -\frac{\beta\cdot T}{(T-\tau)^{1/\alpha}} + \frac{\gamma}{1-\alpha}\frac{1}{(T-\tau)^{1/\alpha-1}},
\end{aligned}$$

and a particular solution of (5) is therefore given by

$$\begin{aligned}
\sigma_p(\tau) &= C(\tau)(T-\tau)^{1/\alpha}\\
&= \left\{\frac{W}{(T-\tau)^{1/\alpha}} + \frac{1}{1-\alpha}\frac{\beta}{(T-\tau)^{1/\alpha-1}}\right.\\
&\quad \left. -\frac{\beta\cdot T}{(T-\tau)^{1/\alpha}} + \frac{\gamma}{1-\alpha}\frac{1}{(T-\tau)^{1/\alpha-1}}\right\}(T-\tau)^{1/\alpha}\\
&= W + \frac{\beta}{1-\alpha}(T-\tau) - \beta\cdot T + \frac{\gamma}{1-\alpha}(T-\tau).
\end{aligned}$$

Hence the general solution of (5) is

$$\begin{aligned}
\sigma(\tau) &= C(\mathrm{T}-\tau)^{1/\alpha}\\
&\quad + W + \frac{\beta}{1-\alpha}(T-\tau) - \beta\cdot T + \frac{\gamma}{1-\alpha}(T-\tau),
\end{aligned}$$

and setting $\tau = 0$ yields the value of the constant C:

$$\sigma(0) = 0$$

$$0 \quad = CT^{1/\alpha} + W + \frac{\beta}{1-\alpha}T - \beta \cdot T + \frac{\gamma}{1-\alpha}T$$

$$C \quad = -WT^{-1/\alpha} - \frac{\beta}{1-\alpha}T^{1-1/\alpha} + \beta \cdot T^{1-1/\alpha} - \frac{\gamma}{1-\alpha}T^{1-1/\alpha}$$

I.e., in the case $\alpha \neq 1$ the initial value problem (4) is solved by

$$\begin{aligned}\sigma(\tau)_{\alpha\neq 1} = & -(WT^{-1/\alpha} + \frac{\beta}{1-\alpha}T^{1-1/\alpha} \\ & - \beta \cdot T^{1-1/\alpha} + \frac{\gamma}{1-\alpha}T^{1-1/\alpha})(T-\tau)^{1/\alpha} \\ & + W + \frac{\beta}{1-\alpha}(T-\tau) - \beta \cdot T + \frac{\gamma}{1-\alpha}(T-\tau),\end{aligned}$$

or in a more compact version

$$\begin{aligned}\sigma(\tau)_{\alpha\neq 1} = & -(W + \frac{\beta}{1-\alpha}T - \beta \cdot T + \frac{\gamma}{1-\alpha}T)\frac{(T-\tau)^{1/\alpha}}{T^{1/\alpha}} \\ & + W - \beta \cdot T + \frac{\beta+\gamma}{1-\alpha}(T-\tau)\end{aligned}$$

In the case $\alpha = 1$ we get

$$\begin{aligned}C'(\tau) &= \frac{W}{(T-\tau)^2} + \frac{\beta}{T-\tau} - \frac{\beta \cdot T}{(T-\tau)^2} + \frac{\gamma}{T-\tau} \\ &= (W - \beta \cdot T)\frac{1}{(T-\tau)^2} + (\beta+\gamma)\frac{1}{T-\tau},\end{aligned}$$

and as a primitive of C' we find

$$C(\tau) = (W - \beta \cdot T)\frac{1}{T-\tau} - (\beta+\gamma)\ln(T-\tau).$$

A particular solution of (5) is therefore given by

$$\begin{aligned}\sigma_p(\tau) &= C(\tau)(T-\tau) \\ &= ((W - \beta \cdot T)\frac{1}{T-\tau} - (\beta+\gamma)\ln(T-\tau))(T-\tau) \\ &= W - \beta \cdot T - (\beta+\gamma)(T-\tau)\ln(T-\tau)\end{aligned}$$

Hence the general solution of (5), in the case $\alpha = 1$, is

$$\begin{aligned}\sigma(\tau) = & C(\mathrm{T}-\tau) \\ & + W - \beta \cdot T - (\beta+\gamma)(T-\tau)\ln(T-\tau).\end{aligned}$$

Setting $\tau = 0$ yields

$$\sigma(0) = 0$$

$$0 \quad = CT + W - \beta \cdot T - (\beta+\gamma)T \cdot \ln T$$

$$C \quad = \frac{1}{T}(-W + \beta \cdot T + (\beta+\gamma) \cdot T \cdot \ln T),$$

and thus, in the case $\alpha = 1$, the initial value problem (4) is solved by the function

$$\text{(7)} \quad \begin{aligned}\sigma(\tau)_{\alpha=1} = & (-W + \beta \cdot T + (\beta+\gamma) \cdot T \cdot \ln T)\frac{T-\tau}{T} \\ & + W - \beta \cdot T - (\beta+\gamma) \cdot (T-\tau) \cdot \ln(T-\tau).\end{aligned}$$

3.2 The Contracted Amount of Gas

Let us now first check, in which manner the contracted amount of gas W is affected by the perturbations investigated by us, i.e. we have to determine

$$\lim_{\tau \to T} \sigma(\tau).$$

In the case $\alpha \neq 1$ we get

$$\begin{aligned}& \lim_{\tau \to \mathrm{T}} \sigma(\tau)_{\alpha\neq 1} = \\ & = \lim_{\tau \to T}\left\{ -(W + \frac{\beta}{1-\alpha}T - \beta \cdot T + \frac{\gamma}{1-\alpha}T)\frac{(T-\tau)^{1/\alpha}}{T^{1/\alpha}} \right. \\ & \left. + W - \beta \cdot T + \frac{\beta+\gamma}{1-\alpha}(T-\tau) \right\} \\ & = W - \beta \cdot T,\end{aligned}$$

whereas in the case $\alpha = 1$

$$\begin{aligned}& \lim_{\tau \to T} \sigma(\tau)_{\alpha=1} = \\ & = \lim_{\tau \to T}\{(-W + \beta \cdot T + (\beta+\gamma) \cdot T \cdot \ln T)\frac{T-\tau}{T} \\ & + W - \beta \cdot T - (\beta+\gamma) \cdot (T-\tau) \cdot \ln(T-\tau)\} \\ & = W - \beta \cdot T.\end{aligned}$$

Hence, in case of $\beta < 0$, the contracted amount of gas is not held, which causes a peak load in the actual accounting period. This was to be expected, because $\beta < 0$ means that counter pulses are lost, and the controller registers a too low amount of standard volume drawn from the primary supplier.

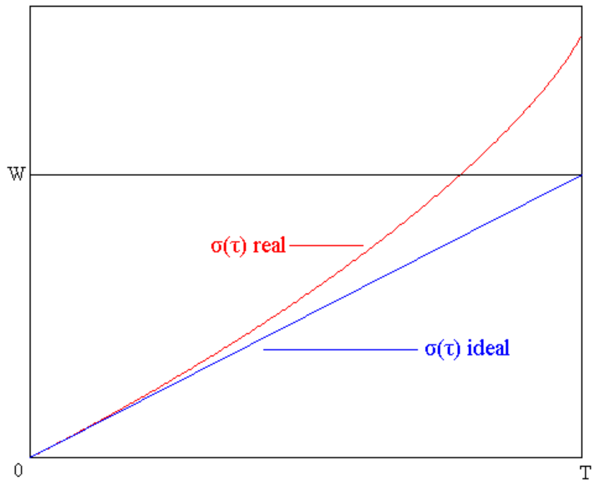

Fig. 5 $\alpha = 1$, $\beta = -0.5$, $\gamma = 0$

The typical graph of such a solution is shown in Fig.5 above.

If, however, $\beta > 0$, than additional counter pulse are registered and processed by the controller. Hence less gas then the allowed amount is drawn. This may cause a peak load after a cold winter night, if there is not gas enough in the distribution system to guarantee the supply of all the consumers. The distributor may then be forced to draw more gas than its contract allows. Fig.6 shows the typical graph of such a solution:

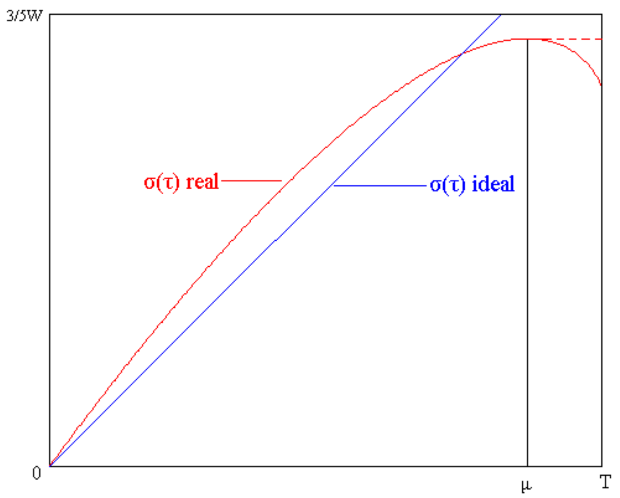

Fig.6 $\alpha = 1$, $\beta = 0.5$, $\gamma = 0$

Evidently $\sigma(\tau)$ has a maximum in the interval $[0, T]$. This behaviour is characteristic in the case $\alpha = 1$, $\beta > 0$, $\gamma = 0$. To see this let us calculate the roots of the derivative of $\sigma(\tau)$. To simplify the calculations, we assume that $T = 1$. The equation

$$\frac{d\sigma(\tau)_{\alpha=1}}{d\tau} = W + \beta \cdot \ln(1-\tau) = 0$$

yields

$$\mu = 1 - e^{-\frac{W}{\beta}},$$

and we have

$$\frac{d\sigma(\tau)_{\alpha=1}}{d\tau} \begin{cases} > 0 \text{ for } \tau < \mu \\ = 0 \text{ for } \tau = \mu \\ <= \text{for } \tau > \mu \end{cases}.$$

I.e. $\sigma(\tau)$ has a local maximum in μ.

The sum $\sigma(\mu)$ is the sum of pulses caused by gas actually drawn from the primary supplier, whereas $\beta\mu$ is the sum of pulses coming from fictitiously drawn gas. The pulses contained in $\beta\mu$ are only disturb pulses and do not register gas which was actually purchased. Nevertheless the controller takes them into account when calculating the internal set point of the system. For the sum of all counter pulses at time μ the following equation holds.

$$\begin{aligned}\sigma(\mu)_{\alpha=1} + \beta \cdot \mu &= (-W + \beta)(1-\mu) \\ &\quad + W - \beta - \beta(1-\mu)\ln(1-\mu) + \beta \cdot \mu \\ &= (-W + \beta)e^{-\frac{W}{\beta}} \\ &\quad + W - \beta - \beta e^{-\frac{W}{\beta}}\left(-\frac{W}{\beta}\right) + \beta(1 - e^{-\frac{W}{\beta}}) \\ &= W.\end{aligned}$$

I.e., at time μ the controller closes the valve completely because the contracted amount of gas W is reached by $\sigma(\mu) + \beta\mu$.

Because normally a negative flow of gas is impossible, the valve remains closed for $\tau > \mu$ until the end of the accounting period. Hence, for $\tau > \mu$, the function $\sigma(\tau)$ is no longer a solution of the initial value problem (4), and it is no longer represented by equation (7). In Fig. 6 this situation is demonstrated by the horizontal dashed red line on the right hand side of μ.

Actually in a few old gas distribution systems it may happen that there is a negative flow of gas, i.e.

a backward flow from the end consumers into the net of the local distributor. In those distribution systems special care has to be taken that this situation never occurs, for example by minimum pressure control. Otherwise it could happen that air is flowing back from the environment into the gas distribution system and this would cause a highly explosive mixture.

3.3 The Behaviour of the Valve

To investigate the behaviour of the valve towards the end of the accounting period, we now determine

$$\lim_{\tau\to T}\frac{d\sigma(\tau)}{d\tau}\ .$$

In the case $\alpha \neq 1$ we get

$$\begin{aligned}
&\frac{d\sigma(\tau)_{\alpha\neq 1}}{d\tau}= \\
&-\frac{d}{d\tau}\left(W+\frac{\beta}{1-\alpha}T-\beta\cdot T+\frac{\gamma}{1-\alpha}T\right)\frac{(T-\tau)^{1/\alpha}}{T^{1/\alpha}} \\
&+\frac{d}{d\tau}\left(W-\beta\cdot T+\frac{\beta+\gamma}{1-\alpha}(T-\tau)\right) \\
&=\frac{1}{\alpha}\left(W+\frac{\beta}{1-\alpha}T-\beta\cdot T+\frac{\gamma}{1-\alpha}T\right)\frac{(T-\tau)^{1/\alpha-1}}{T^{1/\alpha}} \\
&-\frac{\beta+\gamma}{1-\alpha},
\end{aligned}$$

and therefore

$$\begin{aligned}
&\lim_{\tau\to T}\frac{d\sigma(\tau)_{\alpha\neq 1}}{d\tau}= \\
&\lim_{\tau\to T}\left\{\frac{1}{\alpha}\left(W+\frac{\beta}{1-\alpha}T-\beta\cdot T+\frac{\gamma}{1-\alpha}T\right)\frac{(T-\tau)^{1/\alpha-1}}{T^{1/\alpha}}\right\} \\
&-\frac{\beta+\gamma}{1-\alpha} \\
&=\begin{cases}-\dfrac{\beta+\gamma}{1-\alpha} & \text{if } \alpha<1 \\ +\infty & \text{if } \alpha>1\end{cases}.
\end{aligned}$$

This result resembles the result described in subsection 2.2: If $\alpha > 1$ the controller opens the valve completely towards the end of the period. If $\alpha < 1$ the behaviour of the valve depends on the modulus and the sign of $\beta + \gamma$. Here, as elsewhere, it has always to be kept in mind that in regions with a non positive derivative of $\sigma(\tau)$ (non positive flow of gas) the valve is completely closed and remains closed until the end of the period. In such regions $\sigma(\tau)$ is no longer a solution of the initial value problem (4) (see the end of subsection 3.2).

In the case $\alpha = 1$ we get:

$$\begin{aligned}
&\frac{d\sigma(\tau)_{\alpha=1}}{d\tau}= \\
&\qquad\frac{d}{d\tau}(-W+\beta\cdot T+(\beta+\gamma)\cdot T\cdot\ln T)\frac{T-\tau}{T} \\
&\qquad+\frac{d}{d\tau}(W-\beta\cdot T-(\beta+\gamma)\cdot(T-\tau)\cdot\ln(T-\tau)) \\
&\qquad=-(-W+\beta\cdot T+(\beta+\gamma)\cdot T\cdot\ln T)\frac{1}{T} \\
&\qquad+(\beta+\gamma)\cdot(\ln(T-\tau)+1),
\end{aligned}$$

and therefore

$$\begin{aligned}
&\lim_{\tau\to T}\frac{d\sigma(\tau)_{\alpha=1}}{d\tau}= \\
&\lim_{\tau\to T}\{-(-W+\beta\cdot T+(\beta+\gamma)\cdot T\cdot\ln T)\frac{1}{T} \\
&+(\beta+\gamma)\cdot(\ln(T-\tau)+1)\} \\
&=\begin{cases}-\infty & \text{if } \beta+\gamma>0 \\ (W-\beta\cdot T)\dfrac{1}{T} & \text{if } \beta+\gamma=0 \\ +\infty & \text{if } \beta+\gamma<0\end{cases}.
\end{aligned}$$

I.e., in this case too the behaviour of the valve depends strongly on the sign of $\beta + \gamma$. As above

$$\frac{d\sigma(\tau)_{\alpha=1}}{d\tau}\leq 0$$

means that the valve is closed and remains closed.

3.4 The Case of a Disturbed Ground

The case $\beta = 0$ can be handled similar to the initial value problems (1) and (2) examined in subsection 2.2. Nevertheless, we shall throw a short glance onto the case $\alpha = 1$ and $\beta = 0$. It describes the situation, where the measured value Q of the gas flow rate is disturbed by an absolute deviation $-\gamma(\tau)$, i.e. the situation of a disturbed ground. In this case we get

$$\sigma(\tau) = (-W + \gamma \cdot T \cdot \ln T)\frac{T - \tau}{T}$$
$$+ W - \gamma \cdot (T - \tau) \cdot \ln(T - \tau)$$

For reasons of simplicity let again $T = 1$, i. e.

$$\sigma(\tau) = W \cdot \tau - \gamma \cdot (1 - \tau) \cdot \ln(1 - \tau)$$

As in subsection 3.2, it turns out that for $\gamma > 0$ $\sigma(\tau)$ has a maximum in the interval [0, 1]: The roots of the derivative of $\sigma(\tau)$ are given by the equation

$$\frac{d\sigma(\tau)_{\alpha=1}}{d\tau} = W + \gamma \cdot (\ln(1 - \tau) + 1)$$
$$= 0,$$

and as in subsection 3.2 we have

$$\frac{d\sigma(\tau)_{\alpha=1}}{d\tau} \begin{cases} > 0 \text{ for } \tau < 1 - e^{-\frac{W}{\gamma} - 1} \\ = 0 \text{ for } \tau = 1 - e^{-\frac{W}{\gamma} - 1} \\ <= \text{for } \tau > 1 - e^{-\frac{W}{\gamma} - 1} \end{cases}.$$

I.e. $\sigma(\tau)$ is maximal in

$$\mu = 1 - e^{-\frac{W}{\gamma} - 1}$$

But this maximum is greater than W:

$$\sigma(\mu) = W \cdot (1 - e^{-\frac{W}{\gamma} - 1})$$
$$- \gamma \cdot (1 - (1 - e^{-\frac{W}{\gamma} - 1})) \cdot \ln(1 - (1 - e^{-\frac{W}{\gamma} - 1}))$$
$$= W \cdot (1 - e^{-\frac{W}{\gamma} - 1}) - \gamma \cdot e^{-\frac{W}{\gamma} - 1} \cdot (-\frac{W}{\gamma} - 1)$$
$$= W + \gamma \cdot e^{-\frac{W}{\gamma} - 1}$$
$$> W$$

Consequently, in this case, a peak load can not be avoided. The contracted amount W is surmounted at

$$\lambda = 1 - e^{-\frac{W}{\gamma} - 1}$$

The typical graph of such a solution is shown in Fig.7. Here again, we chose the somewhat disproportionate value of γ = 1,5, because otherwise the effect would have been difficult to observe. But the effect itself remains the same, even if γ is very small.

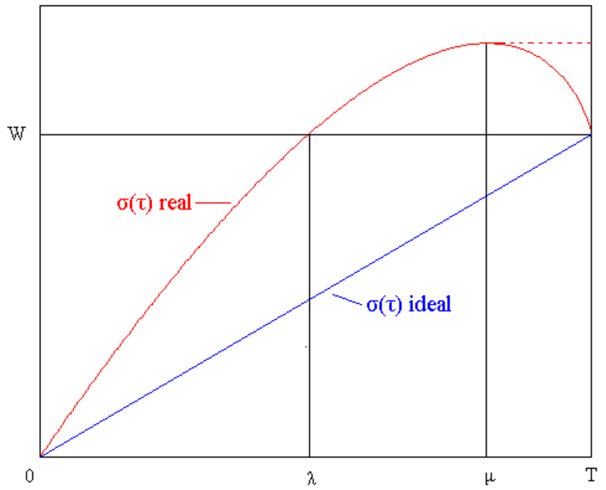

Fig.7 α = 1, β = 0, γ = 1.5

As in subsection 3.2 at time μ, the controller closes the valve completely, and the valve remains closed for $\tau > \mu$ until the end of the accounting period. In Fig. 7 this situation is demonstrated by the dashed red line. In the case of $\gamma < 0$, there is no danger of a peak load. This case is similar to the case $\alpha > 1$, $\beta = 0$, $\gamma = 0$. The typical graph of such a situation is shown in Fig.8.

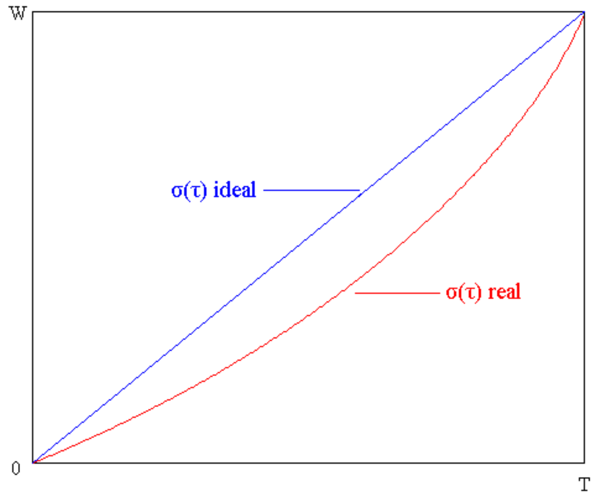

Fig.8 α = 1, β = 0, γ = -0.5

4 Conclusion

The authors investigated phenomena observed frequently when operating gas flow and volume control systems. The effect has been explained by introducing a differential equation and solving it. The reason may be a false measured value of the actual gas flow rate. Then the effect of disturb pulses was examined interfering the counter pulses coming from a volume converter. To this end a more general differential equation was introduced. Rounding errors may lead to the same effect. It could be an issue of future investigations to replace the present differential equation (1) by an equation with lagging argument, taking account of the fact that the actual value of the flow follows the set point with a time delay. May be that the methods described in [17] or [18] will help to attack this problem.

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