

# Multiple Modeling and Fuzzy Predictive Control of A Tubular Heat Exchanger System

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**Abstract:** - In this paper, a novel generalized predictive control (GPC) strategy using multiple models approach has been presented. The proposed strategy is realized based on the Takagi-Sugeno-Kang (TSK) fuzzy-based modeling for control of a tubular heat exchanger system. In this strategy, different operating environments of the system with varying parameters are first identified. Then for each environment, a linear model and its corresponding fuzzy predictive controller are designed. For demonstrating the effectiveness of the proposed approach, simulations are done and the results are compared with those obtained using the single model predictive control approach. The results can verify the validity of the proposed control scheme.

**Key-Words:** - Fuzzy generalized predictive control, TSK fuzzy-based modeling, multiple models control approach, tubular heat exchanger system, the best model identification, operating environments.

## 1 Introduction

The model based predictive control (MBPC) scheme has widely been used in the field of process control, since it has a good performance while we are using an explicit linear model of the system. In most applications of the MBPC family, such as model algorithmic control (MAC), dynamic matrix control (DMC), generalized predictive control (GPC) and other related techniques, the process is presented over its operating environment by a linear model [1],[9]. Here, the proposed GPC controller is realized based on a single fixed linear model or slowly adapting model of the system as long as the operating environment is either time invariant or varies slowly with time. In this case, while the operating environment has large variation with time, the control design based on a linear model may deteriorate the system performance in other operating environments. Hence, the performance of the GPC is reduced and it may not have any satisfactory result. In practical applications such as tubular heat exchanger, due to the parameters variation, the system needs to operate in multiple operating environments, which may change abruptly from one to another [10],[16]. An appropriate method to improve the GPC performance while we are having a complex system is to use Takagi-Sugeno-Kang (TSK) fuzzy-based approach. The main idea of the proposed strategy in this paper, is to realize GPC approach using the TSK fuzzy-based modeling and the obtained linear models of the system. In this way, while we are abruptly

encountered with parameters variation in the system, the best subsystem of the TSK fuzzy-based GPC approach is exactly identified. In fact, for having a good tracking performance, the control action of the proposed strategy is appropriately updated based on both the desired set point and the system parameters variation, at any instant of time.

The remainder of this paper is organized as follows. The tubular heat exchanger modeling is presented in Section 2. The principle of generalized predictive control is briefly investigated in Section 3. The multiple models control approach and the proposed control strategy are also presented in Sections 4 and 5, respectively. Finally, the simulation results and concluding remarks are given in Sections 6 and 7, respectively.

## 2 Tubular Heat Exchanger Modeling

The heat exchanger is a process that is used to change the temperature distribution of two materials, as is shown in Fig. 1.

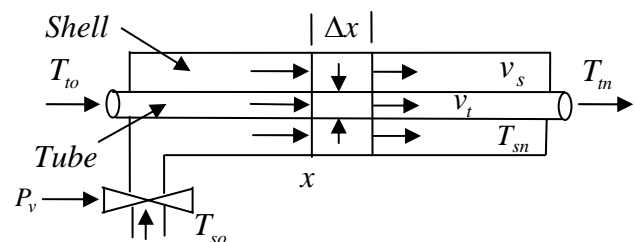


Fig. 1. Diagram of a tubular heat exchanger system.

The tubular heat exchanger system has both the inner and the shell tubes with concurrent reactions when they are in direct or indirect contacts. The fluid flows through the inner tube and its temperature is varied by another fluid that flows concurrently around it. In such a case, the dynamics of the heat exchanger is described by the partial differential equations (PDEs). Thus, it is truly used as infinite dimensional system [17],[26]. In order to model the heat exchanger system, the following parameters can now be defined

- $\alpha$  : Section area of the tube ( $m^2$ )
- $\rho$  : Fluid density ( $kg / m^3$ )
- $v$  : Fluid velocity ( $m / s$ )
- $d$  : Internal diameter of the tube ( $m$ )
- $T_x$  : Temperature of  $x$  ( $K$ )
- $C_p$  : Specific heat capacity ( $J / kgK$ )
- $U$  : Overall heat transfer coefficient ( $W / m^2 K$ )
- $\Delta x$  : Incremental element in the tube ( $m$ )

It should be noted that for modeling the tubular heat exchanger system, the following assumptions are considered

- The velocity variation of the fluids are negligible, i.e., are independent of  $x$ .
- The temperature of the fluid in the shell tube is constant.
- The properties of the fluids are assumed to be constant.

Here, the temperature distribution of an incremental element  $\Delta x$ , along  $x$ , based on the principle of conservation of energy, at the time  $t$ , could be given as

$$\alpha \rho C_p \Delta x \frac{\partial T}{\partial t} = \alpha \rho C_p v (T_x - T_{x+\Delta x}) + U \pi d \Delta x \Delta T \quad (1)$$

where

$\alpha \rho C_p \Delta x \frac{\partial T}{\partial t}$  denotes the accumulation of energy into  $\Delta x$ ,  $\alpha \rho C_p v T_x$  denotes the convection flow of the energy into  $\Delta x$ ,  $\alpha \rho C_p v T_{x+\Delta x}$  also denotes the convection flow of the energy out of  $\Delta x$  and finally  $U \pi d \Delta x \Delta T$  represents the heat transfer to  $\Delta x$ . Now, by assuming  $\Delta x \rightarrow \delta x$ , the obtained PDEs describing the system could also be written as

$$\alpha \rho C_p \frac{\partial T}{\partial t} = -\alpha \rho C_p v \frac{\partial T}{\partial x} + U \pi d \Delta T \quad (2)$$

Afterward, by using  $T_t$  and  $T_s$  parameters as the temperature in the inner and the shell tubes, respectively, it could be written as

$$\begin{cases} \frac{\partial T_s}{\partial t} = -v_s \frac{\partial T_s}{\partial x} + a_s T_t - a_s T_s; & a_s = \frac{U \pi d}{\alpha_s \rho_s C_{ps}} \\ \frac{\partial T_t}{\partial t} = -v_t \frac{\partial T_t}{\partial x} + a_t T_s - a_t T_t; & a_t = \frac{U \pi d}{\alpha_t \rho_t C_{pt}} \end{cases} \quad (3)$$

Let us define  $T_t$  and  $T_s$  as outlet and inlet of the system and also by assuming  $s = \frac{\delta}{\delta t}$ , we could have

$$\frac{\partial T_t(x, s)}{\partial x} + \frac{s + a_t}{v_t} T_t(x, s) = \frac{a_t}{v_t} T_s(x, s) \quad (4)$$

Hence, the outlet temperatures in terms of the inlet temperatures and the  $x$  could be deduced as

$$T_t(x, s) = \exp\left(-\frac{x}{v_t}(s + a_t)\right) + \frac{a_t}{s + a_t} T_s(x, s) \quad (5)$$

The system modeling results should uniformly be divided into small incremental elements while boundary conditions are given at  $x = kN$ ;  $k = 0, 1, 2, \dots, n$ . Therefore, system temperature could be represented by

$$T_{tk} = T_t(kN, s), \quad T_{sk} = T_s(kN, s) \quad (6)$$

where  $T_{tk}$  and  $T_{sk}$  are given as the temperature at the  $k^{th}$  point of the inner tube and the shell tube, respectively. Here,  $T_{sk}$  is defined as constant temperature with respect to  $x$ , i.e.,

$$T_{s0} = T_{s1} = \dots = T_{sk} \quad (7)$$

Also by using (3), (6) and

$$\frac{\partial T_{tk}}{\partial x} = \frac{T_{tk} - T_{t(k-1)}}{N} \quad (8)$$

the obtained PDEs result could be expressed as

$$\frac{\partial T_{tk}}{\partial t} = -(a_t + \frac{v_t}{N})T_{tk} + a_t T_{sk} + \frac{v_t}{N} T_{t(k-1)} \quad (9)$$

Meanwhile, by using (5) and (6), the outlet temperature at  $k^{th}$  point could be given as

$$T_{tk} = \exp(-\frac{kN}{v_t}(s + a_t)) + \frac{a_t}{s + a_t} T_{sk} \quad (10)$$

Now, the system transfer functions could also be written as

$$\frac{T_{tk}}{T_{sk}} = \frac{a_t}{s + a_t} (1 - \exp(-\frac{kN}{v_t}(s + a_t))) \quad (11)$$

Moreover, the obtained result could be given in terms of the valve pressure, i.e.,  $\frac{T_{tk}}{P_v}$ . By using

$$K_v \lambda P_v - U\pi d \int_0^{kN} (T_{sk} - T_{tk}) dx = C_s \frac{\partial T_{sk}}{\partial t} \quad (12)$$

and (11), it could be written as

$$T_{sk} - T_{tk} = T_{sk} (1 - \frac{a_t}{s + a_t} (1 - \exp(-\frac{x}{v_t}(s + a_t)))) \quad (13)$$

Hereinafter, by using (12) and (13), we could have

$$K_v \lambda P_v - U\pi d (kN - \frac{a_t kN}{s + a_t} + \frac{a_t v_t}{(s + a_t)^2} (1 - \exp(-\frac{kN}{v_t}(s + a_t)))) = C_s s T_{sk} \quad (14)$$

As a result, by using (11) and (14), the tubular heat exchanger modeling could finally be deduced as follows

$$\frac{T_{tk}}{P_v} = \frac{k_1 \xi(s)}{a a_t^{-1} s^2 + (a + a_t^{-1})s + \frac{a_t}{s + a_t} \xi(s)} \quad (15)$$

where  $P_v$ ,  $\lambda$ ,  $C_s$  and  $K_v$  are given as the valve pressure, the compressed steam temperature of the shell tube, the shell tube capacitance and the valve gain respectively. Also  $\xi(s)$ ,  $k_1$  and  $a$  are given as;

$$1 - \exp(-\frac{kN}{v_t}(s + a_t)), \quad \frac{K_v \lambda}{U k \pi d N} \quad \text{and} \quad \frac{C_s \lambda}{U k \pi d N},$$

respectively.

Now, by having the obtained system modeling results, the next requirement is to define the appropriate operating environments and their linear models, correspondingly. These linear models could be expressed in the form of CARIMA as follows

$$A^i(q^{-1}) y^i(k) = B^i(q^{-1}) u(k-1) + \frac{e(k)}{\Delta(q^{-1})} \quad (16)$$

$$A^i(q^{-1}) = 1 + a_1^i q^{-1} + \dots + a_n^i q^{-n}$$

$$B^i(q^{-1}) = 1 + b_1^i q^{-1} + \dots + b_m^i q^{-m}$$

where  $y^i(k)$ ;  $i = 1, 2, \dots, r$ ,  $u(k)$ ,  $e(k)$ ,  $m$  and  $n$  represent the  $i^{th}$  model output, the control action, uncorrelated random sequence, the delay input and the delay output of the system, respectively. In such a case, the obtained system modeling could be used as the best chosen model of the system at each instant of time, as long as the coefficients are appropriately obtained.

### 3 Generalized Predictive Control

The GPC method proposed by Clark, is one of the most popular MBPC used in both industry and academic centers. The basic idea of the GPC is to calculate a sequence of future control signals in such a way that it minimizes a cost function over a prediction horizon. The index to be optimized now is the expectation of a quadratic function measuring the difference between the predicted system output and the predicted reference sequence over a horizon, plus a quadratic function measuring the control efforts. The  $j$ -step ahead predictor of the GPC algorithm can be given by

$$y(k+j) = H_j(q^{-1})\Delta u(k-1) + G_j(q^{-1})\Delta u(k+j-1) + F_j(q^{-1})y(k) \quad (17)$$

where  $y(k)$  and  $\Delta u(k)$  denote the output variable and the manipulated variable, respectively. In addition,  $\Delta(q^{-1})$  is given as;  $1 - q^{-1}$  while the  $q^{-1}$  represents the backward shift operator. Furthermore, the  $F_j(q^{-1})$ ,  $H_j(q^{-1})$  and  $G_j(q^{-1})$  are all given as

$$F_j(q^{-1}) = [F_{N_1}(q^{-1}), \dots, F_{N_2}(q^{-1})]^T \quad (18)$$

$$H_j(q^{-1}) = [H_{N_1}(q^{-1}), \dots, H_{N_2}(q^{-1})]^T \quad (19)$$

$$G_j(q^{-1}) = \begin{bmatrix} g_{N_1}^j(q^{-1}) & g_{N_1-1}^j(q^{-1}) & \dots & 0 \\ g_{N_1+1}^j(q^{-1}) & g_{N_1}^j(q^{-1}) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ g_{N_2}^j(q^{-1}) & g_{N_2-1}^j(q^{-1}) & \dots & g_{N_2-N_u+1}^j(q^{-1}) \end{bmatrix} \quad (20)$$

Here, the  $g_i^j(q^{-1})$ s are the coefficients of the  $G_j(q^{-1})$  matrix polynomials which correspond to the system step response values. Meanwhile,  $N_2 - N_1 + 1$  and  $N_u$  are given as the prediction horizon and control horizon, respectively. Afterward, the  $F_j(q^{-1})$ ,  $H_j(q^{-1})$  and  $G_j(q^{-1})$  could be obtained by using the following Diophantine equation, i.e.,

$$\begin{cases} 1 = E_j(q^{-1}) A(q^{-1}) \Delta(q^{-1}) + q^{-j} F_j(q^{-1}) \\ F = F_j(q^{-1}) y(k) + H_j(q^{-1}) \Delta u(k-1) \\ E_j(q^{-1}) B(q^{-1}) = G_j(q^{-1}) + q^{-j} H_j(q^{-1}) \end{cases} \quad (21)$$

where  $A(q^{-1})$  and  $B(q^{-1})$  can be obtained using CARIMA model of the system. In addition, by denoting

$$\begin{cases} \tilde{U} = [\Delta u(k), \dots, \Delta u(k + N_u - 1)]^T \\ R = [r(k + N_1), \dots, r(k + N_2)]^T \\ F = [f(k + N_1), \dots, f(k + N_2)]^T \end{cases} \quad (22)$$

the quadratic cost function can be given by

$$J = (M\tilde{U} + F - R)^T (M\tilde{U} + F - R) + \lambda \tilde{U}^T \tilde{U} \quad (23)$$

where  $R$  and  $F$  denote the desired set points and the system free responses, respectively. In addition,  $\lambda$  is given as the control coefficient and  $M$  matrix could be obtained from the output prediction, i.e.,

$$Y = [y_p(k + N_1), \dots, y_p(k + N_2)]^T = M\tilde{U} + F \quad (24)$$

Moreover, the optimal control law could be obtained using

$$\delta J / \delta U = 0 \quad (25)$$

Now, the GPC control action could clearly be deduced as

$$\tilde{U} = K^{GPC} (R - F) \quad (26)$$

where the controller gain can be given as

$$K^{GPC} = (M^T M + \lambda I)^{-1} M^T \quad (27)$$

#### 4 Multiple Models Control Approach

The multiple models control strategy is an approach for controlling the complex systems. This control strategy operates in multiple operating environments, which may change abruptly from one to another. On the other hand, the system has extended operating environment regions, or the operating environments are time variant. As it can be seen, the process behavior is often nonlinear and a linear fixed model may not really lead to the expected performance. In this case, a good method for realizing the linear controller to cope with complex systems is to use multiple models control strategy. This strategy improves the linear controller performance. For realization of such a controller, some models for covering the different environments are needed and consequently, an appropriate controller is designed for each one of them. If the models are not correctly chosen, then the results corresponding to the required performance cannot be accepted. Multiple models strategy is briefly described as follows

- Defining some models corresponding to operating environments of the complex system.
- Designing the local controllers corresponding to each of the predefined models.
- Identification of the best model and selecting the appropriate control signal at each instant.

The multiple models control strategy can be divided into two groups: classical multiple models control and intelligent multiple models control schemes. Classical type of the method works based on the switching criteria. So, according to some criteria, the model with the smallest error with respect to the process is chosen. In another words, the switching is used to identify the model with the closest response to the process output. When the number of models and the system operating points are infinite, then an adaptive control strategy could be used. Otherwise, for the finite number of operating points, using some tuning algorithms, a multiple models control scheme can be used. The block diagram of a classical multiple models control is shown in Fig. 2. According to this figure,  $M\#1, \dots, M\#r$  are given as the  $r$  models, which are used in parallel with the process. Also  $y(t)$  and  $u(t)$

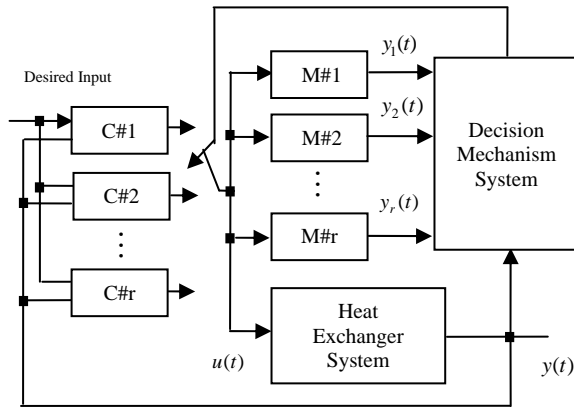


Fig. 2. The classical multiple models control scheme.

are given as the output of the process and the final control action, respectively. In designing multiple models control strategy, two separated parts are considered: identification and control [27]-[31].

Identification part is composed of predefined models which cover different environments of the process and the decision mechanism, in order to decide which model is closest to the process. The classical multiple models control is realized by defining some performance indices, where the switching can occur when the performance indices reaches its minimum value. The specific performance indices can be given by

$$J_i(t) = \alpha e_i^2(t) + \beta \int_0^t e^{-\lambda(t-\tau)} e_i^2(\tau) d\tau \quad (28)$$

$$; \alpha \geq 0; \beta, \lambda > 0; i = 1, 2, \dots, r$$

where

$$e_i(t) = y(t) - y_i(t) \quad (29)$$

The design parameters for the switching part of the control system are given as;  $\alpha, \beta, \lambda$  and  $T_d$ , where  $\alpha$  and  $\beta$  denote the weighting factors on the instantaneous measures and the long term accuracy, respectively. In this way,  $\lambda$  denotes a forgetting factor which assures the bounded ness of the criterion for bounded  $e_i(t)$ . Meanwhile,  $T_d$  denotes the minimum time delay between two switches, and it plays an important role on the stability of the above classical scheme. This parameter plays significant role in having quick response in subsystem changing. By choosing larger values for  $\alpha/\beta$  and  $\lambda$  more quick response can be achieved. But such values may lead to unwanted switching in the presence of disturbances and as a result, it deteriorates the system performance.

On the other hand, small values for aforementioned parameters make the unwanted switches to be reduced, but the model selection becomes slow and consequently, the system variations cannot be tracked. Here, for choosing the model in the corresponding operating environment, at each instant of time,  $\alpha/\beta$  and  $\lambda$  could be chosen in maximum value, i.e.,  $\alpha=1, \beta=0$  and  $\lambda > 1$ . It means, the corresponding model is strictly chosen in the its given operating environment.

In the second group, the intelligent type multiple models method is defined. This control methodology is illustrated in Fig. 3, assuming  $r$  explicit linear models. In this scheme, the decision mechanism could be realized based on the fuzzy logic, the neural network and the other related approaches. Realization of the intelligent multiple models; identification of the best model and its appropriate control strategy, is described as follows.

#### 4.1 Appropriate Control Action

In the intelligent multiple models control strategy, the  $C\#1, \dots, C\#r$  are used as the local controllers. The controller design is realized based on the predefined linear model. The final controller;  $u(t)$ , is also realized based on the intelligent control strategy as shown in Fig. 3, i.e., the linear combination of the local controllers could be given by

$$u(t) = \sum_{k=1}^r P_k(t) u_k(t) \quad (30)$$

where

$$\sum_{k=1}^r P_k(t) = 1 \quad (31)$$

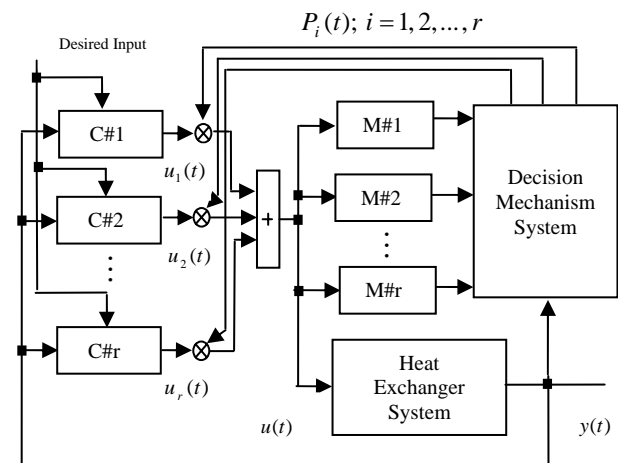


Fig. 3. The intelligent multiple models control scheme.

As it can be seen,  $r$ ,  $P_k(t)$ ,  $u_k(t)$  and  $u(t)$  denote the number of appropriate local controllers, the appropriate weight of the  $k^{th}$  local controller, the  $k^{th}$  local output and the final control output, respectively.

### 4.2 Best Model Identification

In intelligent multiple models control, the identification of the best model of the system is represented using intelligent decision mechanism. This mechanism could easily be realized based on the fuzzy logic, the neural network and the other related approaches. Now, as an example, if the 2<sup>nd</sup> model is identified as the best model, then the weight parameters can be varied as

$$P_2(t) \rightarrow 1, P_j(t) \rightarrow 0 ; \forall j \neq 2 \quad (32)$$

and the final control output could also be deduced as

$$u(t) = \sum_{k=1}^3 P_k(t) u_k(t) \cong P_2(t) u_2(t) \quad (33)$$

where

$$\sum_{k=1}^3 P_k(t) = 1 \quad (34)$$

## 5 The Proposed Control Strategy

As mentioned before, the TSK fuzzy-based GPC scheme is considered as the control strategy of the tubular heat exchanger system. Figure 4 represents the proposed fuzzy control scheme, where  $M\#i$ ;  $i=1,2, \dots, r$  denotes the  $i^{th}$  CARIMA model of the system as described in (16). As we know, the conventional GPC scheme cannot be used for the control of complex systems. Hence, the identification of the system using multiple linear models are needed, as long as, if the operating environments could distinctly be defined [32],[43]. Subsequently, based on the obtained linear models of the system, the fuzzy GPCs are correspondingly designed. Now, by assuming  $N_2 - N_1 + 1 = N_u = n$ , the  $i^{th}$  rule of the proposed TSK fuzzy-based GPC approach could be written as

$$\begin{aligned} \text{GPC rule}^i : & \text{IF } y(k) \text{ is } C^i \text{ THEN } \Delta u^i(k) \\ & = \sum_{p=1}^n s_p^i r(k+p) + \sum_{p=0}^n f_p^i y_i(k-p) + t_0^i \Delta u(k-1) \end{aligned}$$

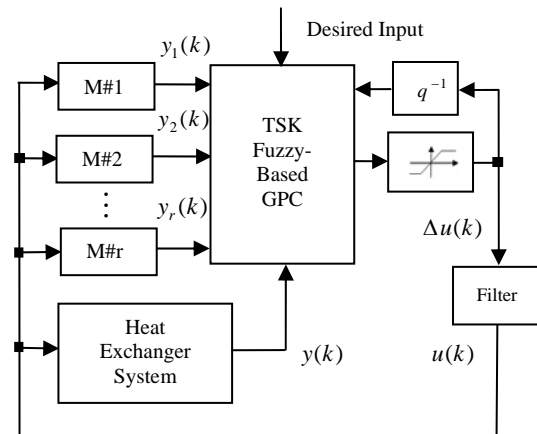


Fig. 4. The proposed TSK fuzzy-based GPC scheme.

where  $\Delta u(k)$ ,  $r(k)$ ,  $y(k)$  and  $n$  are previously defined. Also  $s_p$ ,  $f_p$  and  $t_0$  denote the control coefficients which must be obtained from the GPC algorithm. Meanwhile, after several experiments, the fuzzy sets used in the proposed strategy ( $C^i$ ;  $i=1, 2, \dots, r$ ) are obtained, as shown in Fig. 5.

Now, by using the centroid defuzzification and the product type of inference, the TSK fuzzy-based GPC approach could be obtained as

$$\Delta u(k) = \sum_{i=1}^r w^i \Delta u^i(k) \quad (35)$$

where

$$w^i = \frac{\mu_{C^i}(\Delta u(k))}{\sum_{i=1}^r \mu_{C^i}(\Delta u(k))} \quad (36)$$

Afterwards, the TSK fuzzy-based GPC approach could be used to cope with the complex system, while changes in the system coefficients are allowed. In another words, when the operating environments

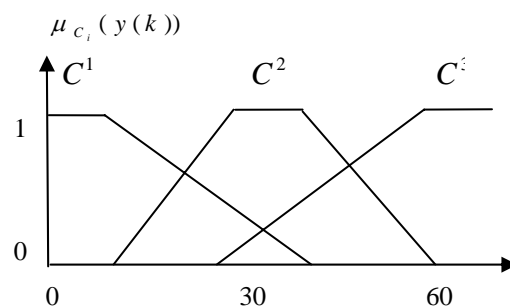


Fig. 5. The fuzzy sets used in the TSK fuzzy-based GPC strategy scheme ( $r=3$ ).

of the system are abruptly changed from one to another, the best subsystem is identified, for having a good tracking performance. In this control strategy, the manipulated variable signal, i.e.,  $\Delta u(k)$  is constrained and the control action, i.e.,  $u(k)$  is also obtained from the following discrete filter

$$H_f(q^{-1}) = \frac{1}{1 - q^{-1}} = \frac{u(k)}{\Delta u(k)} \quad (37)$$

### 6 Simulation Results

To illustrate the applicability of the proposed approach, the water tubular heat exchanger at  $x = nN = 1m$  is now considered. Here, the water temperature is adjusted by commanded valve pressure ( $P_v$ ) on the shell tube, and the tracking performance uses both the desired set point between  $0^\circ\text{C}$  and  $50^\circ\text{C}$ , and the following parameters variation

$$\underline{\rho}_t \leq \rho_t(t) \leq \overline{\rho}_t \quad (38)$$

$$\underline{C}_{p_t} \leq C_{p_t}(t) \leq \overline{C}_{p_t} \quad (39)$$

where

$$\underline{\rho}_t = \alpha_1 \cdot \overline{\rho}_t, \quad \overline{\rho}_t = \alpha_2 \cdot \underline{\rho}_t \quad (40)$$

$$\underline{C}_{p_t} = \alpha_3 \cdot \overline{C}_{p_t}, \quad \overline{C}_{p_t} = \alpha_4 \cdot \underline{C}_{p_t} \quad (41)$$

$$\rho_t = 1000 \text{ kg/m}^3, \quad C_{p_t} = 4.0 \text{ kJ/kg}^\circ\text{C} \quad (42)$$

and the system coefficients can also be defined, as tabulated in Table 1 [44]. Now, based on the three predefined system operating environments, i.e.,

- $\rho_t(t) = \overline{\rho}_t, \quad C_{p_t}(t) = \overline{C}_{p_t} \quad (43)$

- $\rho_t(t) = \frac{\overline{\rho}_t + \underline{\rho}_t}{2}, \quad C_{p_t}(t) = \frac{\overline{C}_{p_t} + \underline{C}_{p_t}}{2} \quad (44)$

- $\rho_t(t) = \underline{\rho}_t, \quad C_{p_t}(t) = \underline{C}_{p_t} \quad (45)$

the three following CARIMA models of the system

Table 1. The system coefficients.

coefficients	values	coefficients	values
$\alpha_1$	0.9520	$\alpha_2$	1.010
$\alpha_3$	1.041	$\alpha_4$	1.064

( $r=3$ ) by assuming  $n=m=p$  in (16), could be obtained using the RLS method, as the resulted coefficients are tabulated in Table 2. Now, for analyzing the model validation, the  $i^{th}$  model error with respect to the system, could be expressed as

$$e_i(k) = y(k) - y_i(k); \quad i = 1, 2, 3 \quad (46)$$

The results can verify the validity of the chosen models, as tabulated in Table 3. Finally, by using the obtained system models, the TSK fuzzy rule-based GPC approach is realized. The realization of the three rules of the TSK fuzzy-based GPC approach, is summarized in Table 4. The criterion for choosing the rule number in the proposed strategy is realized based on the system performance with respect to minimum number of the rule. At first, maximum rules were used and then number of the used rules was experimentally decreased to three rules. According to the obtained results, the used rule number is deduced as the appropriate rule number for having a good system tracking performance. Furthermore, the manipulated variables are constrained between  $-0.2 \text{ V}$  and  $0.2 \text{ V}$ . Hereinafter, Fig. 6 shows the tracking performance of the proposed fuzzy approach, called by the authors the MMFPC (Multiple Modeling and Fuzzy Predictive Control), while Fig. 7 represents the performance of the SMGPC (Single Model GPC) that is realized using the first obtained model of the system. The manipulated variable and the control signal are also shown in Figs. 8 and 9, respectively. In this simulation, both the system coefficients and the set point are abruptly varied at several times. The variation in the system coefficients can distinctly be seen at 45 sec., 85 sec. and 185 sec., respectively. By using both the MMFPC and the SMGPC approaches

Table 2. The coefficients of the CARIMA models.

$i$	$p$	$a_p^i$	$b_p^i$
1	1	-0.9933	0.2506e-3
	2	-0.4342	0.3519e-3
	3	0.0069	0.5283e-3
	4	0.4219	0.1830e-3
2	1	-0.9947	0.2469e-3
	2	-0.4327	0.3426e-3
	3	0.0083	0.5208e-3
	4	0.4204	0.1738e-3
3	1	-0.9960	0.2434e-3
	2	-0.4313	0.3336e-3
	3	0.0097	0.5135e-3
	4	0.4189	0.1637e-3

Table 3. The model validation.

	M#1	M#2	M#3
$\overline{\sum e^2}$ (V)	3.82e-3	7.41e-3	5.12e-3
$\overline{\sum  e }$ (V)	5.14e-2	9.89e-2	6.46e-2

Table 4. The coefficients of the TSK fuzzy-based GPC.

$i$	$p$	$s_p^i$	$f_p^i$	$t_p^i$
1	0	-	-0.0483	1.123e-4
	1	1.404e-4	0.1394	-
	2	5.460e-4	-0.1625	-
	3	1.195e-4	0.0881	-
	4	5.060e-4	-0.0184	-
2	0	-	-0.2125	5.035e-4
	1	6.290e-4	0.6097	-
	2	2.430e-3	-0.7081	-
	3	5.300e-3	0.3823	-
	4	2.030e-3	-0.0797	-
3	0	-	-0.01275	2.763 e-4
	1	3.45e-5	0.03721	-
	2	1.37e-4	-0.04396	-
	3	3.058e-4	0.02414	-
	4	1.07e-4	-0.005126	-

in several simulations, with the same conditions, the performance improvement of the MMFPC approach is easily observed. In these cases, as it can be seen,

the SMGPC approach does not perform well, when changes in system coefficients are abruptly occurred. For another example, the proposed fuzzy control strategy is also implemented where the obtained results are shown in Figs. 10 to 13, respectively.

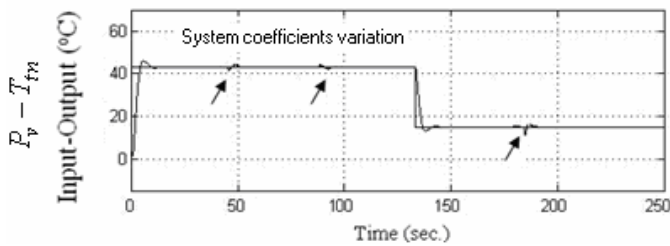


Fig. 6. The MMFPC tracking performance.

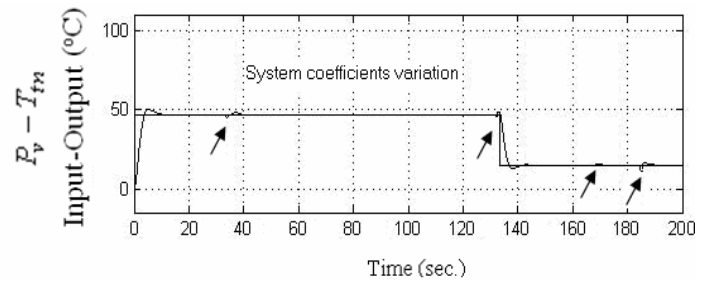


Fig. 10. The MMFPC tracking performance.

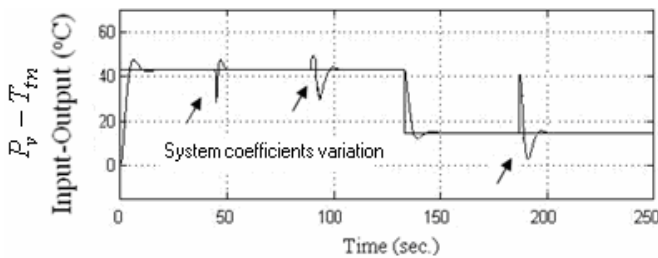


Fig. 7. The SMGPC tracking performance.

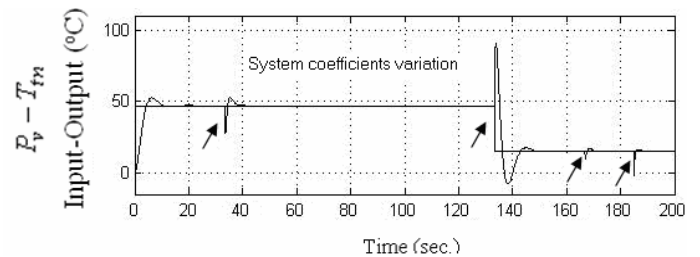


Fig. 11. The SMGPC tracking performance.

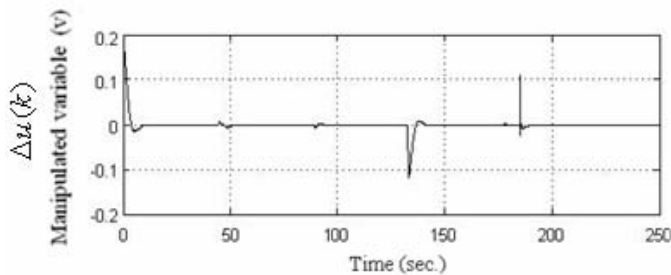


Fig. 8. The MMFPC manipulated variable signal.

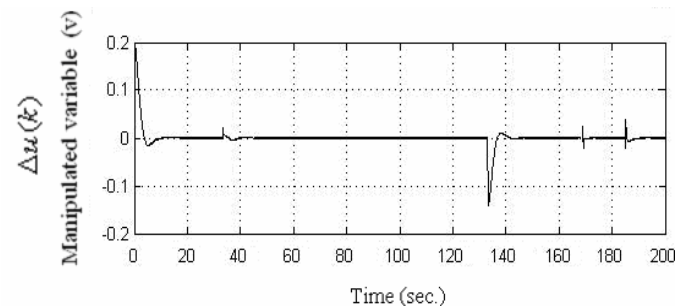


Fig. 12. The MMFPC manipulated variable signal.

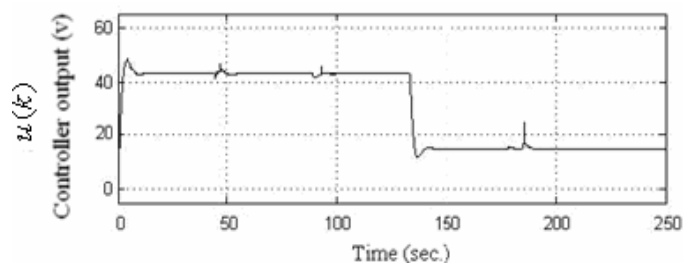


Fig. 9. The MMFPC control signal.



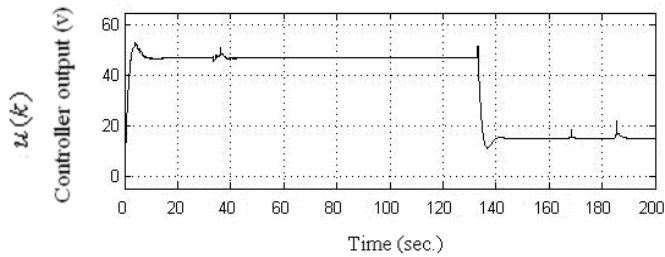


Fig. 13. The MMFPC control signal.

## 7 Conclusion

A novel multiple models control strategy using the generalized predictive control and the Takagi-Sugeno-Kang fuzzy-based approach for control of the water tubular heat exchanger system, with varying parameters is introduced in this paper. In this strategy, the results are compared with those obtained using a single model GPC scheme. As it can be seen, the proposed fuzzy control strategy outperforms the classical single model GPC scheme.

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