Analytic Modeling and Metaheuristic PID Control of a Neutral Time Delay Test Case Central Heating System

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Abstract: - In the present paper, the analytic mathematical model of a test case central heating system is developed in the form of a nonlinear neutral time delay model with time varying delay which in turn is simplified to a neutral time delay model with constant time delay. In the process it is shown that the influence of the delay is significant, thus its incorporation to the model is of high importance while the constant delay approximation imposes only small error. A PID controller is derived to control the temperature of a room, which is modeled as a first order differential equation. The controller parameters are evaluated using a metaheuristic algorithm.

Key-Words: central heating, modeling, neutral time delay system, PID controller, metaheuristic

1 Introduction

The problem of modeling and control of central heating systems has attracted significant attention during the last years (see f.e. [1]-[15]). In particular, significant attention has been given to the modeling, construction and optimization of core components of central heating systems, such as pipe networks and piping elements (see f.e. [4]-[8] and the references therein), radiators and other heating systems (see f.e. [9]-[10]), boilers (see [11]-[12]) etc. Furthermore, different control techniques have been applied to such systems in order to regulate the ambient air temperature in heated areas (see f.e. [13]-[17]).

The present paper is an extended version of [16] where the dynamic model of the system is briefly presented while the room temperature is controlled via a PI controller. In the present paper the mathematical model of the test case central heating system will analytically be presented in the form of a nonlinear neutral time delay model. In particular, separate models will be presented for the core components of the system, i.e. pipe network, radiator and boiler. The separate models will be combined to a nonlinear neutral time delay system (with time varying delay) which will be simplified to a neutral time delay model with constant time delay. It will be shown that the influence of the delay is significant, thus its incorporation to the model is of high importance while the constant delay approximation imposes only a small error to the system. Finally, a PID controller will be derived to control the temperature of a room, which will be modeled as a first order differential equation. The controller parameters will be evaluated using a metaheuristic approach whose efficiency will be investigated. The resulting closed loop response will be compared to the response produced by the same controller for the case where the parameters are evaluated using the first Ziegler Nichols method and the open loop response of the system applying appropriate constant actuatable inputs.

2 Dynamic Model of a Test Case Central Heating System

In what follows, the general dynamic model of a test case central heating system will be produced. The system consists of the piping network, a radiator and a boiler (see Figure 1). The radiator heats up a room, thus the performance output of the system is the power emitted by the radiator which is directly related to the temperature of the ambient air.

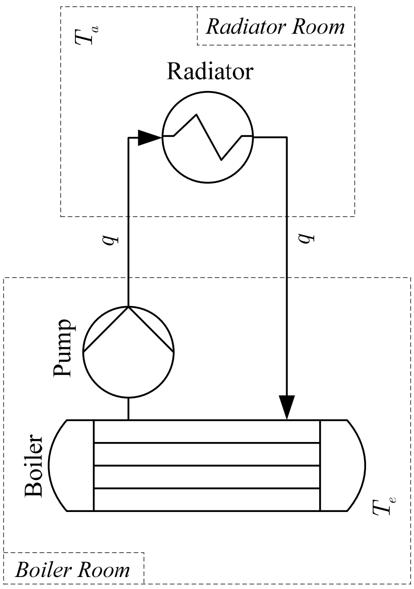

Fig. 1: Layout of the Test Case Central Heating System

2.1 Radiator Modeling

The dynamic model of a radiator is presented in the form of the partial differential equation, [10]

$$C_l \frac{\partial T}{\partial t} = C_p \rho q \frac{\partial T}{\partial x} - \Phi_{0,l} \left(\frac{T - T_a}{\Delta T_{ma,0}} \right)^{n_1} \quad (2.1)$$

where T is the water temperature inside the radiator, T_a is the ambient air temperature, q is the water volumetric flow rate in the radiator, C_l is the heat capacity of water and radiator material (per length), C_p and ρ are the thermal capacity and density of the water respectively, $\Phi_{0,l}$ denotes the nominal power of the radiator (per length), $\Delta T_{ma,0}$ is the arithmetic mean temperature difference at standard conditions and n_1 is an exponent in the range 1.2 to 1.4.

Relation (2.1) can be approximated by a nonlinear system of ordinary differential equations as

$$\frac{C}{N\Delta x} \frac{dT_j}{dt} = C_p \rho q \frac{\Delta T_j}{\Delta x} - \frac{\Phi_0}{N\Delta x} \left(\frac{T_j - T_a}{\Delta T_{ma,0}} \right)^{n_1}$$

or equivalently

$$\frac{dT_j}{dt} = \frac{NC_p \rho q}{C} \Delta T_j - \frac{\Phi_0}{C} \left(\frac{T_j - T_a}{\Delta T_{ma,0}} \right)^{n_1} \quad (2.2)$$

where C is the heat capacity of the water and radiator material, Φ_0 is the nominal heat of the radiator, N is the number of sections the radiator is partitioned and T_j is the radiator temperature in the j[th] section. The temperature difference ΔT_j is given by

$$\Delta T_j = (1-\varphi) T_{j-1} + (2\varphi - 1) T_j - \varphi T_{j+1}$$

where φ denotes a fractional number between 0 and 0.5. In a more analytic manner, the first and j[th] section are given as

$$\frac{C}{N} \frac{dT_1}{dt} = H_q q \left[T_i - (1-\varphi) T_1 - \varphi T_2 \right] - \frac{\Phi_0}{N} \left(\frac{T_1 - T_a}{\Delta T_{ma,0}} \right)^{n_1}$$

and

$$\frac{C}{N} \frac{dT_j}{dt} = H_q q \left[(1-\varphi) T_{j-1} + (2\varphi - 1) T_j - \varphi T_{j+1} \right] - \frac{\Phi_0}{N} \left(\frac{T_j - T_a}{\Delta T_{ma,0}} \right)^{n_1}$$

while the last section is given as

$$\frac{C}{N} \frac{dT_N}{dt} = H_q q \left(T_{N-1} - T_N \right) - \frac{\Phi_0}{N} \left(\frac{T_N - T_a}{\Delta T_{ma,0}} \right)^{n_1}$$

where $H_q = C_p \rho$ and T_i is the influent water temperature. Note that the last section differential equation has been produced setting $\varphi = 0$. This assumption is used in order for the effluent radiator temperature to be equal to the last section temperature.

Applying elementary computations, the nonlinear system of equations can be rewritten as a set of ODEs in the form of a nonlinear state space model as follows

$$\dot{x}_r(t) = f_r\left(x_r(t), u_r(t), \xi_r(t) \right) \quad (2.3a)$$

while the performance of the radiator is considered to be the emitted thermal power

$$y_r(t) = \frac{\Phi_0}{N} \sum_{j=1}^{N} \left(\frac{x_{j,r}(t) - \xi_r(t)}{\Delta T_{ma,0}} \right)^{n_1} \quad (2.3b)$$

where

$$x_r(t) = \left[x_{1,r}(t) \ \cdots \ x_{N,r}(t) \right]^{\mathrm{T}} = \left[T_1(t) \ \cdots \ T_N(t) \right]^{\mathrm{T}}$$

$$u_r(t) = \left[u_{1,r}(t) \quad u_{2,r}(t)\right]^{\mathrm{T}} = \left[q(t) \quad T_i(t)\right]^{\mathrm{T}}$$

$$\xi_r(t) = T_a(t)$$

$$f_r\left(x_r(t), u_r(t), \xi_r(t)\right) = \left[f_{1,r}\left(x_r(t), u_r(t), \xi_r(t)\right) \ldots \right.$$
$$\left. f_{N,r}\left(x_r(t), u_r(t), \xi_r(t)\right)\right]^T$$

$$f_{1,r}\left(x_r(t), u_r(t), \xi_r(t)\right) = \frac{NH_q u_{1,r}(t)}{C} \times$$
$$\left[u_{2,r}(t) - (1-\varphi)x_{1,r}(t) - \varphi x_{2,r}(t)\right]$$
$$-\frac{\Phi_0}{C}\left(\frac{x_{1,r}(t) - \xi_r(t)}{\Delta T_{ma,0}}\right)^{n_1}$$

$$f_{j,r}\left(x_r(t), u_r(t), \xi_r(t)\right) = \frac{NH_q u_{1,r}(t)}{C} \times$$
$$\left[(1-\varphi)x_{j-1,r}(t) + (2\varphi - 1)x_{j,r}(t) - \varphi x_{j+1,r}(t)\right]$$
$$-\frac{\Phi_0}{C}\left(\frac{x_{j,r}(t) - \xi_r(t)}{\Delta T_{ma,0}}\right)^{n_1}, \; j = 2, \ldots, N-1$$

$$f_{N,r}\left(x_r(t), u_r(t), \xi_r(t)\right) = \frac{NH_q u_{1,r}(t)}{C}$$
$$\left(x_{N-1,r}(t) - x_{N,r}(t)\right) - \frac{\Phi_0}{C}\left(\frac{x_{N,r}(t) - \xi_r(t)}{\Delta T_{ma,0}}\right)^{n_1}$$

2.2 Boiler Modeling

The dynamic model of the boiler can be described as a first order differential equation of the form (2.4), [11]-[12]

$$C_b \frac{dT_w}{dt} = n_{com}(T_w) Q_{burner} -$$
$$\rho C_p q\left(T_{w_s} - T_{w_r}\right) - a_j\left(T_w - T_e\right) \quad (2.4)$$

where T_w is the lumped water temperature, T_{w_s} and T_{w_r} are the effluent and influent boiler water temperature, T_e is the temperature of the boiler room, Q_{burner} is the energy supply to the boiler, C_b is the thermal capacity of the boiler, a_j is the rate of heat loss from the boiler jacket to the environment (i.e. boiler room) and n_{com} denotes the combustion efficiency of the boiler. The combustion efficiency is given by a polynomial of the form

$$n_{com}(T_w) = \sum_{j=1}^{\infty} a_j \left(T_w / T_{w,\max}\right)^{j-1} \quad (2.5)$$

where the coefficients can be obtained from experimental data. In the present paper it will be assumed that $a_i = 0$ for $i \geq 3$. The lumped water temperature T_w can be connected to the influent and effluent temperatures through

$$T_w = aT_{w_r} + (1-a)T_{w_s} \quad (2.6)$$

a is the coefficient connecting the lumped water temperature of the boiler to the inlet and outlet temperatures.

Using relations (2.4) to (2.6), the nonlinear model of the boiler can be expressed in state space form as

$$\frac{dx_b(t)}{dt} = f\left(x_b(t), u_b(t)\right) \quad (2.7)$$

where

$$x_b(t) = T_{w_s}(t)$$

$$u_b(t) = \left[u_{1,b}(t) \quad u_{2,b}(t) \quad u_{3,b}(t) \quad u_{4,b}(t) \quad u_{5,b}(t)\right]^{\mathrm{T}}$$
$$= \left[Q_{burner}(t) \quad q(t) \quad T_{w_r}(t) \quad \dot{T}_{w_r}(t) \quad T_e(t)\right]^{\mathrm{T}}$$

$$f\left(x_b(t), u_b(t)\right) =$$
$$\frac{1}{C_b(1-a)} n_{com}\left(x_b(t), u_{3,b}(t)\right) u_{1,b}(t) -$$
$$-\frac{\rho C_p}{C_b(1-a)} u_{2,b}(t)\left[x_b(t) - u_{3,b}(t)\right] -$$
$$\frac{a_j}{C_b(1-a)}\left[a u_{3,b}(t) + (1-a)x_b(t) - u_{5,b}(t)\right] -$$
$$-\frac{a}{1-a} u_{4,b}(t)$$

2.3 Pipe Network Modeling

Given a straight circular pipe of length l, diameter d and roughness e, the dynamic model of one-dimensional incompressible flow q of a fluid of density ρ, driven by the pressure $\Delta p = p_{in} - p_{out}$ across the pipe, can be expressed as (see [10])

$$\frac{dq(t)}{dt} = f_{j,1}(q)\left[p_{in}(t) - p_{out}(t)\right] - f_{j,2}(q) \quad (2.8)$$

It holds that

$$f_{j,1}(q) = \pi d^2 / 4\rho l \quad (2.9)$$

while

$$f_{j,2}(q) = 2\lambda(q,d) q(t)^2 / \pi d^3 \quad (2.10)$$

where $\lambda(q,d)$ is the friction factor. The friction factor $\lambda(q,d)$ depends upon the conditions of the flow inside the pipe, i.e. weather the flow is laminar, transient or turbulent. The flow conditions can be examined using the Reynolds number, defined as

$$\mathrm{Re}(q,d) = \frac{4\rho q}{\pi d \mu}$$

where μ is the viscosity of the fluid. For the case of laminar flow, i.e. if $\mathrm{Re}(q,d) \leq 2300$, it holds that

$$\lambda(q,d) = 64 \big/ \mathrm{Re}(q,d) \qquad (2.11)$$

while for the turbulent region, i.e. if $\mathrm{Re}(q,d) > 3000$, it holds that the friction factor is the solution of relation (2.12) with respect to λ

$$1 \big/ \sqrt{\lambda(q,d)} = -2\log_{10}\left[\frac{2.51}{\mathrm{Re}(q,d)\sqrt{\lambda(q,d)}} + \frac{e}{3.71d}\right] \qquad (2.12)$$

With respect to the transient region, i.e. for $2300 < \mathrm{Re}(q,d) \leq 3000$, the friction factor can be approximated linearly, [5], as long as the linear function satisfies (2.11) and (2.12) for $\mathrm{Re}(q,d) = 2300$ and $\mathrm{Re}(q,d) = 3000$ respectively. With respect to the turbulent region, it can be divided into smaller regions. For example when the Reynolds number is less than 10^5 and the pipe is smooth, i.e. the pipe roughness is small, the friction factor is described by the Blasius equation

$$\lambda(q,d) = 0.316 \big/ \mathrm{Re}(q,d)^{0.25} \qquad (2.13)$$

Approximation (2.13) is valid for typical flow and geometry conditions for single family houses. Hence, assuming that the pipes used are smooth relation (2.12) can be substituted by (2.13). Consequently, the linear approximation for $2300 < \mathrm{Re}(q,d) \leq 3000$ takes on the form

$$\lambda(q,d) = = 4 \cdot 10^{-5}\left[0.5333\,\mathrm{Re}(q,d) - 530.4027\right] \qquad (2.14)$$

Fitting pressure losses are sometimes presented in terms of the equivalent length of straight pipe that would have the same pressure loss as the fitting. If a fitting is to be replaced by an equivalent length L_{eq} of pipe, then it must hold that $L_{eq} = K_f d \big/ \lambda(q,d)$, where K_f denotes the pressure loss coefficient for fittings. This relationship shows the fundamental shortcoming with the equivalent-length approach. It must be noted that even though K_f and d are constants for a given pipe under various flow conditions, the friction function is not. The same assumptions can be used for the case where a radiator is present in the pipe network. Additionally, an extra turbulent pressure drop will exist due to the fitting present to entrance of the radiator connecting it to the network. This pressure drop will be considered to be of the form

$$\delta P_t(q) = K_t q^2 \qquad (2.15)$$

where K_t is a turbulent pressure drop factor.

According to the above presented formulae the nonlinear model of the piping network can be written in state space form as

$$\frac{dx_p}{dt} = \left[\frac{1}{f_{11}(x_p)} + \frac{1}{f_{21}(x_p)} + \frac{1}{f_{31}(x_p)}\right]^{-1} \times \left[u_p(t) - \delta P_t(x_p) - f_{12}(x_p)\big/ f_{11}(x_p) + - f_{22}(x_p)\big/ f_{21}(x_p) - f_{32}(x_p)\big/ f_{31}(x_p)\right] \qquad (2.16)$$

where

$$x_p(t) = q(t),\; u_p(t) = \Delta P(t)$$

$$\delta P_t(x_p) = -K_t x_p^2,\; f_{11}(x_p) = \frac{\pi d^2}{4\rho L},$$

$$f_{21}(x_p) = \frac{\pi d_r^2}{4\rho L_r}\; f_{31}(x_p) = \frac{\pi d^2}{4\rho L},$$

$$f_{1,2}(x_p) = \frac{2\lambda(x_p,d)}{\pi d^3} x_p^2$$

$$f_{2,2}(x_p) = \frac{2\lambda(x_p,d_r)}{\pi d_r^3} x_p^2 \quad f_{3,2}(x_p) = \frac{2\lambda(x_p,d)}{\pi d^3} x_p^2$$

and where d_r and L_r are the hydraulic diameter and length of the radiator respectively, d and L are the diameter and length of the pipes connecting the boiler to the radiator respectively. The term ΔP stands for the pressure added to the pipe network by the pump (actuatable input).

2.4 Nonlinear Model of the Overall Plant

In order to construct the nonlinear model of the overall plant, it suffices to establish the necessary algebraic equations standing for the "connections" between the different elements of the plant. Define the composite state, input and disturbance vectors, as defined in Sections 2.1 to 2.3

$$x(t) = \left[x_1(t) \;\vdots\; x_2(t) \;\; \cdots \;\; x_{N+1}(t) \;\vdots\; x_{N+2}(t)\right]^T = \left[q_r(t) \;\vdots\; T_1(t) \;\; \cdots \;\; T_N(t) \;\vdots\; T_{w_s}(t)\right]^T$$

$$u(t) = \left[u_1(t) \quad u_2(t)\right]^T = \left[\Delta P(t) \quad Q_{burner}(t)\right]^T$$

$$\xi(t) = \left[\xi_1(t) \quad \xi_2(t)\right]^T = \left[T_a(t) \quad T_e(t)\right]^T$$

From Figure 1, it can readily be observed that

$$u_{1,r}(t) = x_p(t) = x_1(t) \tag{2.17a}$$

$$u_{2,r}(t) = x_b(t - \tau(t)) = x_{N+2}(t - \tau(t)) \tag{2.17b}$$

$$u_{2,b}(t) = x_p(t) = x_1(t) \tag{2.17c}$$

$$u_{3,b}(t) = x_{N,r}(t - \tau(t)) = x_{N+1}(t - \tau(t)) \tag{2.17d}$$

$$u_{4,b}(t) = \frac{du_{3,b}(t)}{dt} \tag{2.17e}$$

Applying elementary computations, the nonlinear model of the overall plant, takes on the form

$$\frac{dx(t)}{dt} + \tilde{E}_1 \frac{d}{dt} x(t - \tau(t)) = \tilde{f}\left(x(t), u(t), x(t - \tau(t)), \xi(t)\right) \tag{2.18a}$$

$$\int_{t-\tau(t)}^{t} x_1(\rho)\, d\rho = \frac{\pi d^2 L}{4} \tag{2.18b}$$

$$y(t) = \frac{\Phi_0}{N} \sum_{j=2}^{N+1} \left(\frac{x_j(t) - \xi_1(t)}{\Delta T_{ma,0}} \right)^{n_1} \tag{2.18c}$$

where

$$\tilde{f}\left(x(t), u(t), x(t - \tau(t)), \xi(t)\right) = \begin{bmatrix} \tilde{f}_1\left(x(t), u(t), x(t - \tau(t)), \xi(t)\right) \\ \vdots \\ \tilde{f}_{N+2}\left(x(t), u(t), x(t - \tau(t)), \xi(t)\right) \end{bmatrix}$$

$$\tilde{f}_1\left(x(t), u(t), x(t - \tau(t)), \xi(t)\right) = \left[\frac{1}{f_{11}(x_1)} + \frac{1}{f_{21}(x_1)} + \frac{1}{f_{31}(x_1)} \right]^{-1} \times \left[u_1(t) + \delta P_t(x_1) - \frac{f_{12}(x_1)}{f_{11}(x_1)} - \frac{f_{22}(x_1)}{f_{21}(x_1)} - \frac{f_{32}(x_1)}{f_{31}(x_1)} \right]$$

$$\tilde{f}_2\left(x(t), u(t), x(t - \tau(t)), \xi(t)\right) = \frac{NH_q x_1(t)}{C} \left[x_{N+2}(t - \tau(t)) - (1-\varphi) x_2(t) - \varphi x_3(t) \right] - \frac{\Phi_0}{C} \left(\frac{x_2(t) - \xi_1(t)}{\Delta T_{ma,0}} \right)^{n_1}$$

$$\tilde{f}_j\left(x(t), u(t), x(t - \tau(t)), \xi(t)\right) = \frac{NH_q x_1(t)}{C} \left[(1-\varphi) x_{j-1}(t) + (2\varphi - 1) x_j(t) - \varphi x_{j+1}(t) \right] - \frac{\Phi_0}{C} \left(\frac{x_j(t) - \xi_1(t)}{\Delta T_{ma,0}} \right)^{n_1} \quad \text{for } j = 3, \ldots, N$$

$$\tilde{f}_{N+1}\left(x(t), u(t), x(t - \tau(t)), \xi(t)\right) = \frac{NH_q x_1(t)}{C} \left(x_N(t) - x_{N+1}(t) \right) - \frac{\Phi_0}{C} \left(\frac{x_{N+1}(t) - \xi_1(t)}{\Delta T_{ma,0}} \right)^{n_1}$$

$$\tilde{f}_{N+2}\left(x(t), u(t), x(t - \tau(t)), \xi(t)\right) = \frac{1}{C_b(1-a)} n_{com}\left(x_{N+2}(t), x_{N+1}(t - \tau(t)) \right) u_2(t) - \frac{\rho C_w}{C_b(1-a)} x_1(t) \left[x_{N+2}(t) - x_{N+1}(t - \tau(t)) \right] - \frac{a_j}{C_b(1-a)} \left[a x_{N+1}(t - \tau(t)) + (1-a) x_{N+2}(t) - \xi_2(t) \right]$$

$$\tilde{E}_1 = \begin{bmatrix} 0_{(N+1)\times N} & 0_{(N+1)\times 1} & 0_{(N+1)\times 1} \\ 0_{1\times N} & \dfrac{a}{1-a} & 0 \end{bmatrix}$$

Note that the pump pressure and the energy supply to the boiler are considered to be actuatable inputs while the room and boiler room ambient air temperatures act as disturbances upon the system. The time delay $\tau(t)$ stands for the transport delay from the output of the boiler to the input of the radiator and the output of the radiator to the input of the boiler. It must be noted that these delays are in general different. In the present paper, it is assumed that the length and diameter of the pipes from the boiler to the radiator and vice versa are equal hence the respective time delays, let $\tau_1(t)$ and $\tau_2(t)$, are also equal between themselves, i.e. $\tau_1(t) = \tau_2(t) = \tau(t)$.

3 Influence of the Delay to the Nonlinear Model of the Overall Plant

In what follows, the influence of the delay to the response of the overall plant will be examined, through computational experiments. Assume that the model (2.18) operates on certain operating conditions, let $\bar{u}_1$, $\bar{u}_2$, $\bar{\xi}_1$ and $\bar{\xi}_2$ for the actuatable inputs and disturbances yielding to the respective nominal values for the state variables, let $\bar{x}_j$ for $j = 1,\ldots,N+2$. Without loss of generality, assume that at $t = 0$ the actuatable inputs and disturbances become $u_1(t) = p_{u_1}\bar{u}_1 u_s(t)$, $u_2(t) = p_{u_2}\bar{u}_2 u_s(t)$, $\xi_1(t) = p_{\xi_1}\bar{\xi}_1 u_s(t)$ and $\xi_2(t) = p_{\xi_2}\bar{\xi}_2 u_s(t)$, where $p_{u_1} \in \left[(p_{u_1})_{\min},(p_{u_1})_{\max}\right]$, $p_{u_2} \in \left[(p_{u_2})_{\min},(p_{u_2})_{\max}\right]$, $p_{\xi_1} \in \left[(p_{\xi_1})_{\min},(p_{\xi_1})_{\max}\right]$ and $p_{\xi_2} \in \left[(p_{\xi_2})_{\min},(p_{\xi_2})_{\max}\right]$ and where $u_s(t)$ denotes the unit step function, driving the system to new operating conditions, let $\bar{x}'_j$ for $j = 1,\ldots,N+2$, through the respective responses, let $x_j(t)$ for $j = 1,\ldots,N+2$. The same experiment can be carried out for the system (2.18) assuming that the time delay $\tau(t)$ is equal to zero, leading to different responses, let $\tilde{x}_j(t)$ for $j = 1,\ldots,N+2$. In order to examine the influence of the time delay to the system a Euclidean norm type of error will be used, defined as

$$p\left(x_j,\tilde{x}_j\right) = 100\% \times \frac{\left\|x_j - \tilde{x}_j\right\|_2}{\left\|x_j - \bar{x}_j\right\|_2} \qquad (3.1)$$

The same investigation will be carried out for the performance variable. Note that since the flow rate response is not influenced by the delay, in both cases, with or without the presence of the delay, it will be equal, i.e. the error will be by definition zero. Furthermore, for the case where $p_{u_1} = 1$, $p_{u_2} = 1$, $p_{\xi_1} = 1$ and $p_{\xi_2} = 1$ both systems remain in the operating point, hence both numerator and denominator become zero. In that case the error is defined as zero.

Let $L = 25[\mathrm{m}]$, $d = 0.015[\mathrm{m}]$, $L_r = 2[\mathrm{m}]$, $d_r = 0.0096153[\mathrm{m}]$, $K_t = 0.00001$, $C = 36[\mathrm{KJ/K}]$, $N = 4$, $\varphi = 0$, $\Phi_0 = 2500[\mathrm{W}]$, $n_1 = 1.25$ $a = 2/9$, $a_1 = 1$, $a_2 = -0.12$, $T_{w,\max} = 100[°\mathrm{C}]$, $a_j = 5.06[\mathrm{W/K}]$, $C_w = 4180[\mathrm{J/K\cdot Kgr}]$, $C_b = 42400[\mathrm{J/K}]$, $\rho = 971.81[\mathrm{Kgr/m^3}]$, $\mu = 0.0003547[\mathrm{Pa\cdot s}]$, $\Delta T_{ma,0} = 60[°\mathrm{C}]$ be the parameters of the nonlinear model (2.18). Assuming that $\bar{u}_1 = 3000[\mathrm{Pa}]$, $\bar{u}_2 = 2300[\mathrm{W}]$, $\bar{\xi}_1 = 19.111[°\mathrm{C}]$ and $\bar{\xi}_2 = 10[°\mathrm{C}]$ the starting nominal points are evaluated to be $\bar{x}_1 = 3.6287\cdot 10^{-5}[m^3/\mathrm{s}]$, $\bar{x}_2 = 69.6437[°\mathrm{C}]$, $\bar{x}_3 = 66.4807[°\mathrm{C}]$, $\bar{x}_4 = 63.5596[°\mathrm{C}]$, $\bar{x}_5 = 60.8587[°\mathrm{C}]$, $\bar{x}_6 = 73.0728[°\mathrm{C}]$, while the nominal emitted power by the radiator can be easily evaluated to be $\bar{y} = 1796.1[\mathrm{W}]$. In Figures 2 to 6 contour plots of the criterion (3.1) are presented while in Figure 7 the same criterion is presented for the performance variable for a wide range of p_{u_1} and p_{u_2}, indicatively for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$. It can readily be observed that the influence of the delay to the system is significant, especially to the performance output. Hence, it can be safely stated that the incorporation of the delay to the system makes it more accurate. It must be noted that the Euclidean Norm Error plots have been zoomed to a particular area in order to demonstrate specific details.

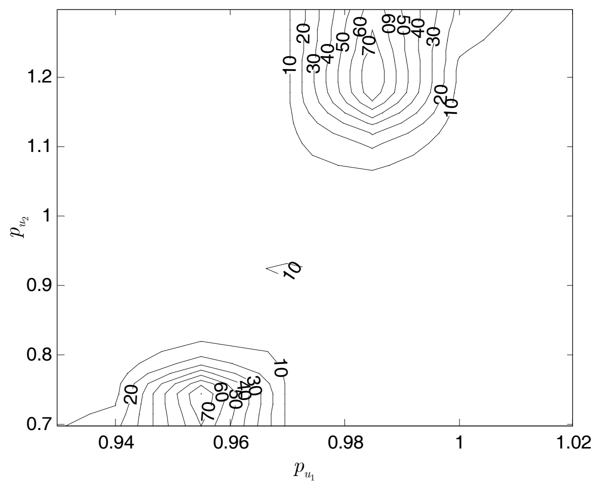

Fig. 2: Euclidian Norm Error $p(x_2,\tilde{x}_2)$ for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$

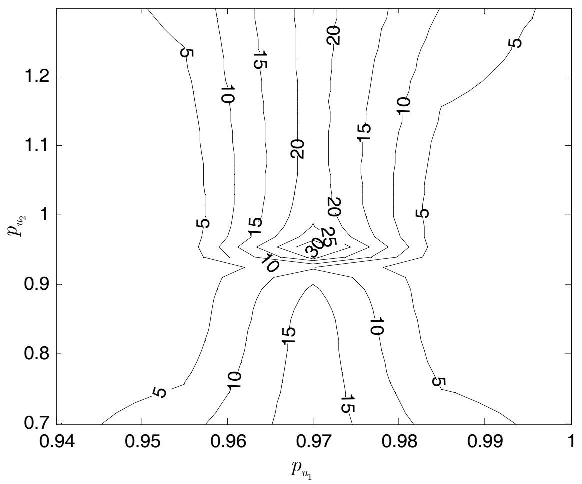

Fig. 3: Euclidian Norm Error $p\left(x_3,\tilde{x}_3\right)$ for

$p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

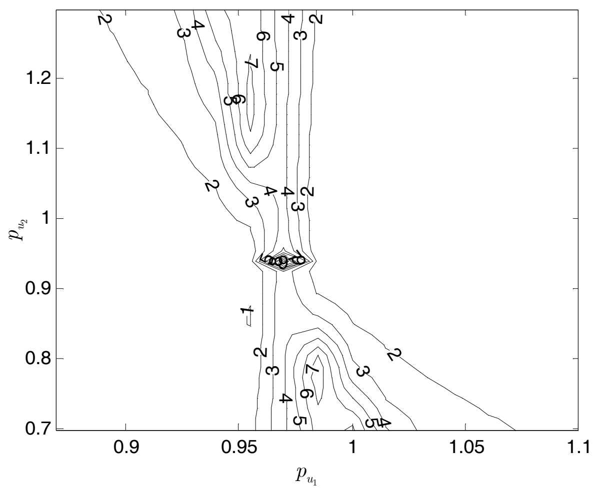

Fig. 4: Euclidian Norm Error $p\left(x_4,\tilde{x}_4\right)$ for

$p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

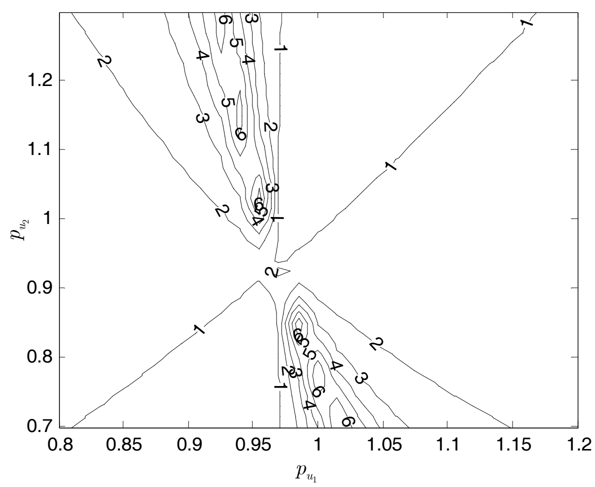

Fig. 5: Euclidian Norm Error $p\left(x_5,\tilde{x}_5\right)$ for

$p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

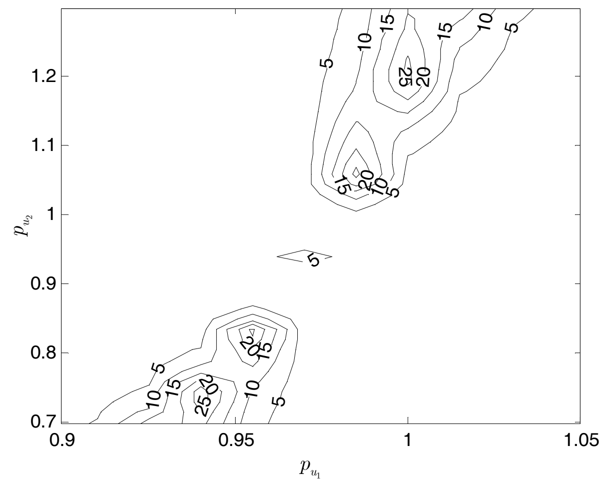

Fig. 6: Euclidian Norm Error $p\left(x_6,\tilde{x}_6\right)$ for

$p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

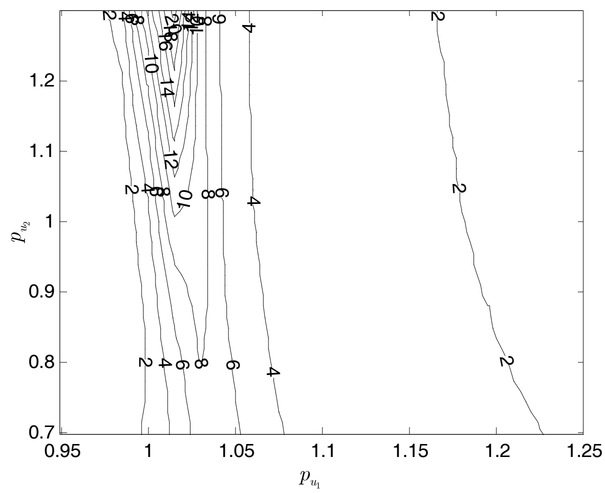

Fig. 7: Euclidian Norm Error $p\left(y,\tilde{y}\right)$ for

$p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

3.1 Approximation of the Nonlinear Model of the Plant

In order to simplify the nonlinear model of the plant it will be approximated using constant delays, instead of the time varying ones, i.e. the dynamic model (2.18) becomes

$$\frac{dx\left(t\right)}{dt}+\tilde{E}_1\frac{d}{dt}x\left(t-\overline{\tau}\right)=\tilde{f}\left(x\left(t\right),u\left(t\right),x\left(t-\overline{\tau}\right),\xi\left(t\right)\right)\qquad(3.2a)$$

$$y\left(t\right)=\frac{\Phi_0}{N}\sum_{j=2}^{N+1}\left(\frac{x_j\left(t\right)-\xi_1\left(t\right)}{\Delta T_{ma,0}}\right)^{n_1}\qquad(3.2b)$$

In order to evaluate the constant delay, it will be assumed that the system operates on its nominal points. It can easily be verified that $\overline{\tau}=\pi d^2L\Big/4\overline{x}_1$

In order to check the accuracy of the proposed simplification a cost criterion similar to that in (3.1) will be used. Assume that the model (2.18) operates on certain operating conditions, let $\overline{u}_1$, $\overline{u}_2$, $\overline{\xi}_1$ and $\overline{\xi}_2$ for the actuatable inputs and disturbances yielding to the respective nominal values for the state variables, let $\overline{x}_j$ for $j = 1,\ldots,N+2$. Without loss of generality, assume that at $t = 0$ the actuatable inputs and disturbances become $u_1\left(t\right) = p_{u_1}\overline{u}_1 u_s\left(t\right)$, $u_2\left(t\right) = p_{u_2}\overline{u}_2 u_s\left(t\right)$, $\xi_1\left(t\right) = p_{\xi_1}\overline{\xi}_1 u_s\left(t\right)$ and $\xi_2\left(t\right) = p_{\xi_2}\overline{\xi}_2 u_s\left(t\right)$, where $p_{u_1} \in \left[\left(p_{u_1}\right)_{\min},\left(p_{u_1}\right)_{\max}\right]$, $p_{u_2} \in \left[\left(p_{u_2}\right)_{\min},\left(p_{u_2}\right)_{\max}\right]$, $p_{\xi_1} \in \left[\left(p_{\xi_1}\right)_{\min},\left(p_{\xi_1}\right)_{\max}\right]$, $p_{\xi_2} \in \left[\left(p_{\xi_2}\right)_{\min},\left(p_{\xi_2}\right)_{\max}\right]$ and where $u_s\left(t\right)$ denotes the unit step function, driving the system to new operation conditions, let $\overline{x}_j'$ for $j = 1,\ldots,N+2$, through the respective responses, let $x_j\left(t\right)$ for $j = 1,\ldots,N+2$. The same experiment can be carried out for the system (3.2), leading to different responses, let $\widehat{x}_j\left(t\right)$ for $j = 1,\ldots,N+2$. Note that the starting and ending nominal points are equal to both cases, with constant or time varying delay. In order to examine the influence of the constant time delay to the system a Euclidean norm type of error will be used, similar to that defined in (3.1). Consider the data presented in the previous subsection. In Figures 8 to 12 contour plots of the cost criterion are presented while in Figure 13 the same criterion is presented for the performance variable for a wide range of p_{u_1} and p_{u_2}, indicatively for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$. It can readily be observed that the influence of the time varying delay to the system as compared to the constant delay case is not significant. Hence, the system can safely be simplified using constant delay rather than time varying. It must be noted that the Euclidean Norm Error plots have been zoomed to a particular area in order to demonstrate specific details.

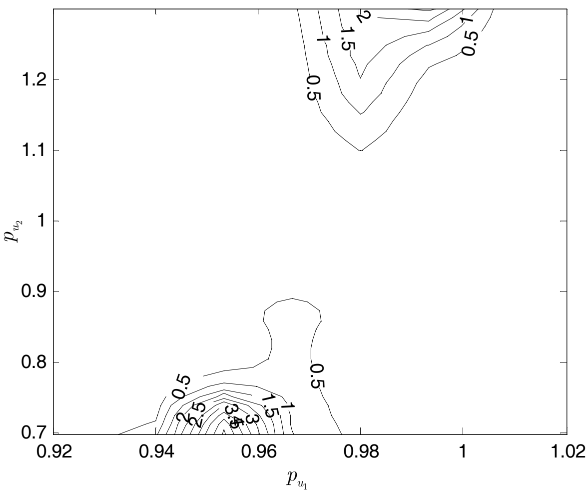

Fig. 8: Euclidian Norm Error $p\left(x_2,\widehat{x}_2\right)$ for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$

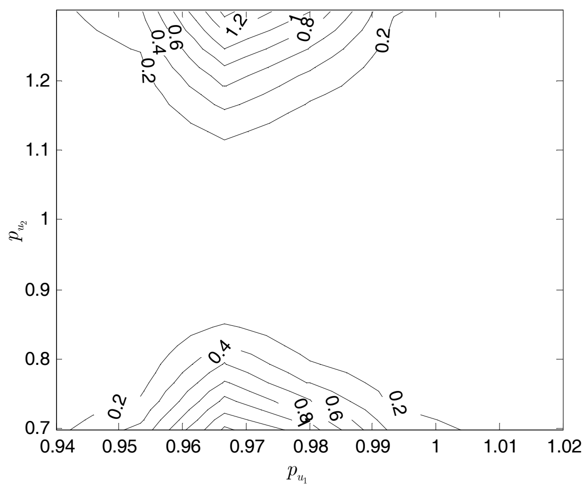

Fig. 9: Euclidian Norm Error $p\left(x_3,\widehat{x}_3\right)$ for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$

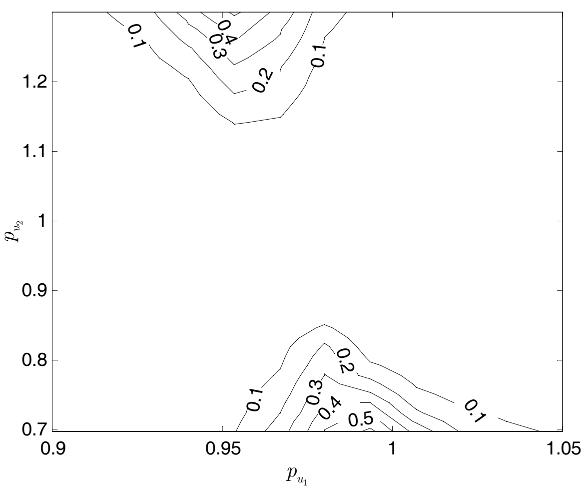

Fig. 10: Euclidian Norm Error $p\left(x_4,\widehat{x}_4\right)$ for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$

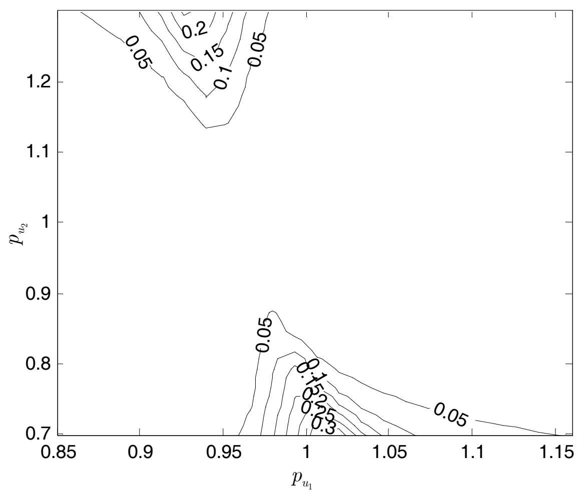

Fig. 11: Euclidian Norm Error $p\left(x_5,\hat{x}_5\right)$ for $p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

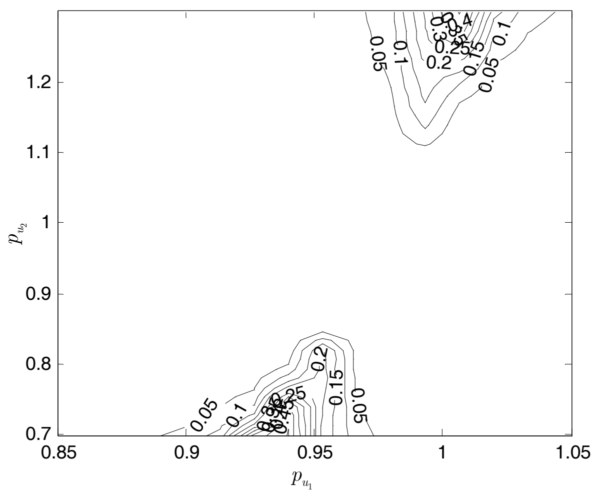

Fig. 12: Euclidian Norm Error $p\left(x_6,\hat{x}_6\right)$ for $p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

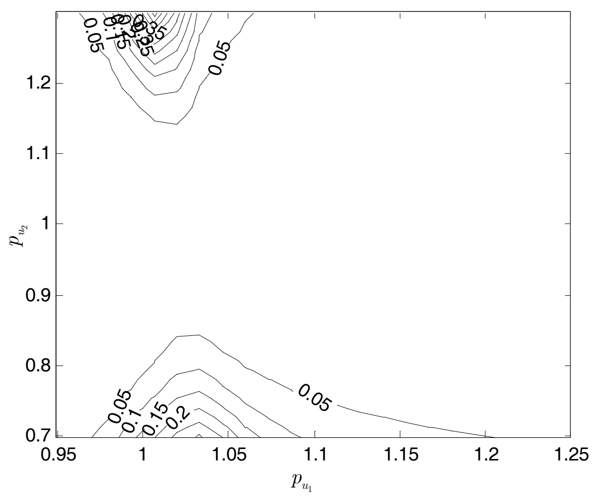

Fig. 13: Euclidian Norm Error $p\left(y,\hat{y}\right)$ for $p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

4 Temperature Control

In the present section, a PID controller will be designed in order to regulate the temperature of a room in which the central heating system presented previously is installed. To do so, the room will be modeled as a first order differential equation, [10], under the assumptions that the walls of the room are subject to the same external temperature, there is no influence of the weather (wind or rain) on the thermal resistance of the walls, radiative heat transfer is negligible, there is no ventilation and no influence from the humidity of the air, there are no heat losses from the ceiling and the floor and there are no heat sources in the room besides the radiator. Under the above assumptions the dynamic model of the room takes on the form

$$\frac{dT_r\left(t\right)}{dt}=\frac{1}{C_rR_w}\left[T_{out}\left(t\right)-T_r\left(t\right)\right]+\frac{1}{C_r}P_{rad}\left(t\right) \quad (4.1)$$

where T_r is the room temperature, T_{out} is the environment temperature, P_{rad} is the thermal power emitted by the radiator, C_r is the thermal capacity of the room and R_w is the thermal resistance of the outer walls. For simulation purposes it will be assumed that $C_r=72.1\left[\mathrm{KJ/K}\right]$ and $R_w=7.85\cdot10^{-3}\left[\mathrm{K/W}\right]$. It can readily be observed that the environment temperature acts as a disturbance while the radiator power is the “actuatable” input.

In the overall system, i.e. room plus central heating unit, it will be assumed that the only measurable variable is the room temperature while the actuatable input is fuel, in the form of power, supplied to the boiler.

The controller is chosen of the form [18]

$$u_2\left(t\right)=f_pe\left(t\right)+f_i\int_0^t e\left(\tau\right)d\tau+f_d\dot{e}\left(t\right)+\overline{u}_2 \quad (4.2)$$

where $e\left(t\right)=T_r\left(t\right)-r\left(t\right)$ and r is the room set point temperature.

In order to choose the controller parameters, two techniques will be used. First, a classical Ziegler-Nichols approach will be applied, see f.e. [16]. Using the data presented in previous sections, the controller parameters using the first Ziegler-Nichols method can be found to be

$$\left(f_p\right)_{ZN}=780.2586,\ \left(f_i\right)_{ZN}=0.1689$$

$$\left(f_d\right)_{ZN} = 800970$$

Second, a metaheuristic approach, similar to that presented in [19] and [20], will be applied. In particular, the metaheuristic algorithm, applied to the present control scheme, takes on the form

---

Initial data of the algorithm

- Center values and half widths for the initial search area of the controller parameters $f_{p,c}$, $f_{i,c}$, $f_{d,c}$, $f_{p,w}$, $f_{i,w}$ and $f_{d,w}$.
- Desired properties of the closed-loop system
- Actuator, state and/or output variable constraints
- Sampling period T
- Time window range N
- Performance criterion $J\left(x,u,\xi\right)$
- Loop repetition parameters n_{loop}, n_{rep} and n_{tot}
- Search algorithm thresholds λ_{f_p}, λ_{f_i} and λ_{f_d}
- External command used for simulation

Search algorithm

Step 1. Set the numbering index $i_{\max} = 0$. Set $J_{total\min} = \infty$.

Step 2. Set the numbering index $i_1 = 0$.

Step 3. Determine a search area $\Im$ for the controller parameters. The search area is bounded according to the inequalities

$$f_{p,\min} \le f_p \le f_{p,\max}\,,\ f_{i,\min} \le f_i \le f_{i,\max}$$
$$f_{d,\min} \le f_d \le f_{d,\max}$$

where

$$f_{p,\min} = f_{p,c} - f_{p,w}\,,\ f_{i,\min} = f_{i,c} - f_{i,w}$$
$$f_{d,\min} = f_{d,c} - f_{d,w}\,,\ f_{p,\max} = f_{p,c} + f_{p,w}$$
$$f_{i,\max} = f_{i,c} + f_{i,w}\,,\ f_{d,\max} = f_{d,c} + f_{d,w}$$

Step 4. Set $i_{\max} = i_{\max} + 1$. If $i_{\max} > n_{total}$ go to Step 17.

Step 5. Set the numbering index $i_2 = 0$.

Step 6. Select randomly a set of controller parameters within the search area $\Im$.

Step 7. Check if the closed-loop system satisfies the properties. If no, set $J = \infty$ and go to Step 10.

Step 8. Perform simulation of the closed-loop system resulting by applying controller (4.2) to the system (3.2). Use the simulation results for a sufficiently large time window $0 \le t \le NT$ and with an appropriate sampling period T, to check if the control input variables $u\left(kT\right)$, the state variables $x\left(kT\right)$ and the output variable $y\left(kT\right)$, $k = 0,\ldots,N$ satisfy the actuator, state and/or output constraints. If no, set $J = \infty$ and go to Step 10.

Step 9. Use the simulation results of Step 8 to compute the value of $J\left(x,u,\xi\right)$

Step 10. Set $i_2 = i_2 + 1$. If $i_2 \le n_{loop}$ go to Step 6.

Step 11. Use the results of the last n_{loop} repetitions of Steps 6 - 9 to determine the suboptimal controller that resulted in the smallest value J_{i+1} of the cost criterion.

Step 12. Set $i_1 = i_1 + 1$. If $i_1 \le n_{rep}$ go to Step 5.

Step 13. Find

$$J_{\min} = \min\left\{J_i, i = 1,\ldots,n_{rep}\right\}$$
$$J_{\max} = \max\left\{J_i, i = 1,\ldots,n_{rep}\right\}$$

and the corresponding controller parameters $\left(f_p\right)_{J_{\min}}$, $\left(f_i\right)_{J_{\min}}$, $\left(f_d\right)_{J_{\min}}$, $\left(f_p\right)_{J_{\max}}$, $\left(f_i\right)_{J_{\max}}$ and $\left(f_d\right)_{J_{\max}}$.

Step 14. If $J_{\min} = \infty$ then set

$$f_{p,w} = 2f_{p,w}\,,\ f_{i,w} = 2f_{i,w}\,,\ f_{d,w} = 2f_{d,w}$$

and go to Step 2. If $J_{\min} < J_{total\min}$, set $J_{total\min} = J_{\min}$ and

$$f_{p,total\min} = \left(f_p\right)_{J_{\min}}, f_{i,total\min} = \left(f_i\right)_{J_{\min}}$$
$$f_{d,total\min} = \left(f_d\right)_{J_{\min}}$$

Otherwise, set $J_{\min} = J_{total\min}$ and

$$\left(f_p\right)_{J_{\min}} = f_{p,total\min}, \left(f_i\right)_{J_{\min}} = f_{i,total\min}$$
$$\left(f_d\right)_{J_{\min}} = f_{d,total\min}$$

Step 15. Define

$$d_{f_p} = \left|\left(f_p\right)_{J_{\min}} - \left(f_p\right)_{J_{\max}}\right|,\ d_{f_i} = \left|\left(f_i\right)_{J_{\min}} - \left(f_i\right)_{J_{\max}}\right|$$
$$d_{f_d} = \left|\left(f_d\right)_{J_{\min}} - \left(f_d\right)_{J_{\max}}\right|$$

Let

$$f_{p,\min} = \left(f_p\right)_{J_{\min}} - d_{f_p}\,,\ f_{i,\min} = \left(f_i\right)_{J_{\min}} - d_{f_i}$$
$$f_{d,\min} = \left(f_d\right)_{J_{\min}} - d_{f_d}\,,\ f_{p,\max} = \left(f_p\right)_{J_{\min}} + d_{f_p}$$
$$f_{i,\max} = \left(f_i\right)_{J_{\min}} + d_{f_i}\,,\ f_{d,\max} = \left(f_d\right)_{J_{\min}} + d_{f_d}$$

If $\left|f_{p,\max} - f_{p,\min}\right| > \lambda_{f_p}$ or $\left|f_{i,\max} - f_{i,\min}\right| > \lambda_{f_i}$ or $\left|f_{d,\max} - f_{d,\min}\right| > \lambda_{f_d}$, go to Step 2

Step 16. End of the algorithm. If $J_{total\min} < \infty$, use the controller parameter values $f_{p,total\min}$, $f_{i,total\min}$ and $f_{d,total\min}$. Otherwise the algorithm has failed.

In order to initialize the metaheuristic algorithm, it is assumed that $f_{p,c} = 1000$, $f_{i,c} = 0.25$, $f_{d,c} = 500000$, $f_{p,w} = 1000$, $f_{i,w} = 0.25$, $f_{d,w} = 500000$, $T = 60[\sec]$, $N = 120$, $n_{rep} = 5$, $n_{loop} = 50$, $n_{tot} = 4000$, $\lambda_{f_p} = 0.0001$, $\lambda_{f_i} = 0.0001$ and $\lambda_{f_d} = 0.0001$. The actuatable input is assumed to be constrained by $0 \le u_2(kT) \le 7500[\mathrm{W}]$ while the effluent temperature of the boiler is constrained by $0 \le x_6(kT) \le 100[°\mathrm{C}]$. The external command is chosen to be of the form $r(t) = \bar{\xi}_1 + u_s(t)$ where $u_s(t)$ is a unit step function while the cost criterion is chosen of the form $J(x,u,\xi) = \left\| \xi_1(kT) - r(kT) \right\|_2$. Using the above presented data, the controller parameters have been found to be

$$\left(f_p\right)_{MH} = 2003.619,\ \left(f_i\right)_{MH} = 0.37187$$
$$\left(f_d\right)_{MH} = 362699.8191$$

With respect to efficiency of the search algorithm, it has been observed that it had converged to the controller parameters after 3500 simulations. Nevertheless, it must be noted that the search algorithm thresholds are quite strict and the controller has practically converged much sooner while the closed loop performance response remains practically unchanged. Indicatively, in Table 1 the center values and half widths of the controller parameters are presented, while the overshoot, rise time, settling time and cost criterion for the optimal controller up to that point are also presented. Indeed, it can be observed that the response characteristics remain practically unchanged after the fourth loop. Note that the all response parameters are evaluated using the sampled data.

$f_{p,c}$	$f_{i,c}$	$f_{d,c}$	$f_{p,w}$	$f_{i,w}$	$f_{d,w}$	Overshoot (%)	Rise Time (sec)	Settling Time (sec)	Cost
1968.6988	0.4672	413900.7486	20.25557	0.075123	176582.4	6.149637	540	4620	2.70145
1988.4861	0.4002	377274.1091	11.83431	0.021945	44307.62	6.250459	540	2280	2.69142
1999.8603	0.3804	356804.9286	3.605856	0.016848	6385.115	6.321436	540	1920	2.68825
2003.3968	0.3693	362891.6232	0.220001	0.001008	130.5197	6.31093	540	2640	2.68749
2003.6155	0.3703	362820.5299	0.006162	0.001038	121.3849	6.312557	540	2640	2.68744
2003.6190	0.3713	362715.4942	6.3E-06	0.00051	15.90379	6.313546	540	2640	2.68743
2003.6190	0.3718	362700.0480	1.98E-06	0.000107	0.3515	6.313927	540	2640	2.68743
2003.6190	0.3719	362699.9183	3.58E-06	3.63E-06	0.037753	6.314005	540	2640	2.68743
2003.6190	0.3719	362699.8867	1.6E-06	3.34E-07	0.039941	6.314008	540	2640	2.68743
2003.6190	0.3719	362699.8475	4E-08	1.71E-07	0.024149	6.314008	540	2640	2.68743
2003.6190	0.3719	362699.8241	5.6E-08	7E-09	0.004569	6.314008	540	2640	2.68743
2003.6190	0.3719	362699.8202	4E-09	2E-09	0.000942	6.314008	540	2640	2.68743
2003.6190	0.3719	362699.8192	1E-09	1E-09	0.000186	6.314008	540	2640	2.68743
2003.6190	0.3719	362699.8191	1E-09	1E-09	4.99E-05	6.314008	540	2640	2.68743

Table 1: Metaheuristic Algorithm Parameters and Closed Loop Response Characteristics

In order to demonstrate the efficiency of the proposed control scheme, consider the data presented in previous sections. Note that the environment nominal temperature has been chosen to be $5[°\mathrm{C}]$ producing the same nominal temperature $\bar{\xi}_1$ for the room. Using the data presented in previous sections an assuming that the external command is chosen to be of the form $r(t) = \bar{\xi}_1 + 2u_s(t)$, the two controllers will be compared among themselves as well as the open loop response, choosing the actuatable input to be constant and equal to $\overline{u}_2 = 2649.6756\left[\mathrm{W}\right]$ producing the same steady state for the room temperature. In particular in Figures 14 to 17 the temperature in the sections of the radiator is presented, in Figure 18 the boiler effluent temperature is presented, in Figure 19 the room temperature is presented, while in Figures 20 and 21, the emitted radiator power and energy supply to the boiler is presented. With respect to the room temperature (see Figure 19), the controller produced by the metaheuristic algorithm performs significantly better than the controller produced using the 1st Ziegler-Nichols method. In particular the metaheuristic controller is much faster (rise time 11.4627 minutes and settling time 27.7704 minutes), than the Ziegler Nichols (ZN) controller (rise time 32.4572 minutes and settling time 189.4270 minutes). For the open loop case, it can be observed that the response is even slower (rise time 147.3314 minutes and settling time 189.9147 minutes). Note that in the present case rise time is defined as the time where the response reaches 95% of the steady state value while settling time the time after which the system diverges less than 2% from the steady state value. Finally, the metaheuristic controller presents much smaller overshoot (6.3%) than the ZN controller (13.74%). For the open loop case there is no overshoot. With respect to the actuatable input (see Figure 21), even though for the metaheuristic controller it is much higher than the ZN controller, it remains within acceptable limits. The same observation can be made for all state variables as well as the emitted power by the radiator.

With respect to the stability of the closed loop system, consider the linearized approximation of the nonlinear model (3.2) presented in [17]. Substituting the derived PID controller to the linearized approximation it is that the roots of the characteristic polynomial of the resulting closed loop system are placed on the left half complex plan and their real part is less than -0.0001851.

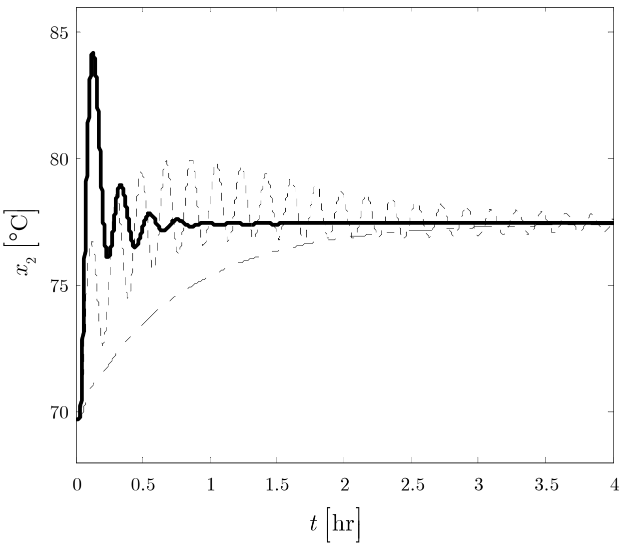

Fig. 14: Radiator Temperature (1st partition)
(*cont.* – metaheuristic, *dotted* – ZN, *dotted dashed* – open loop)

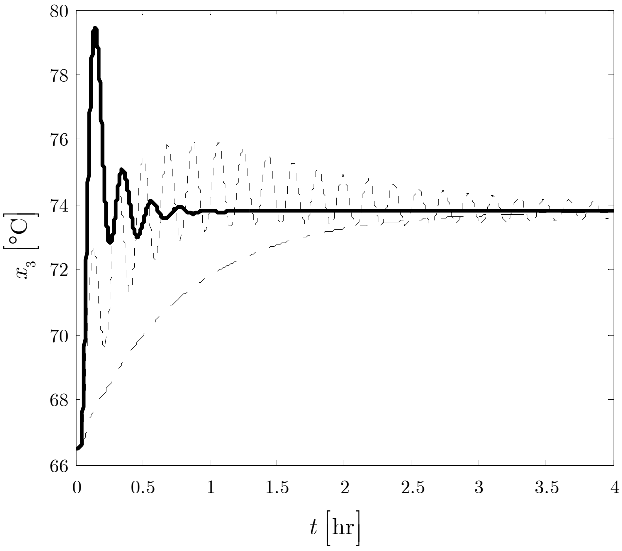

Fig. 15: Radiator Temperature (2nd partition)
(*cont.* – metaheuristic, *dotted* – ZN, *dotted dashed* – open loop)

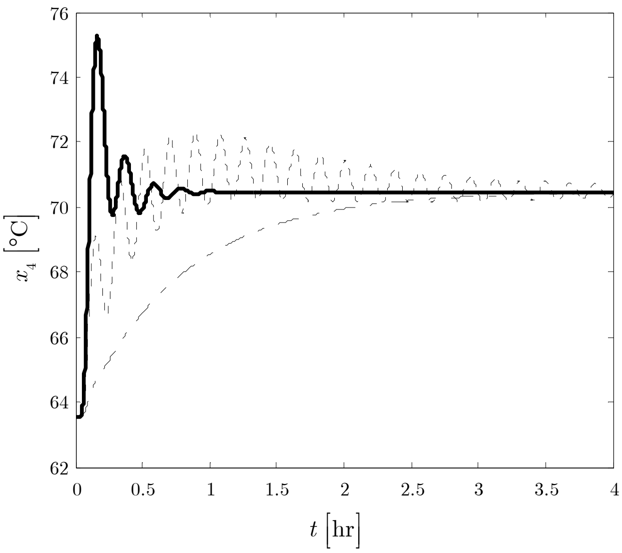

Fig. 16: Radiator Temperature (3d partition)
(*cont.* – metaheuristic, *dotted* – ZN, *dotted dashed* – open loop)

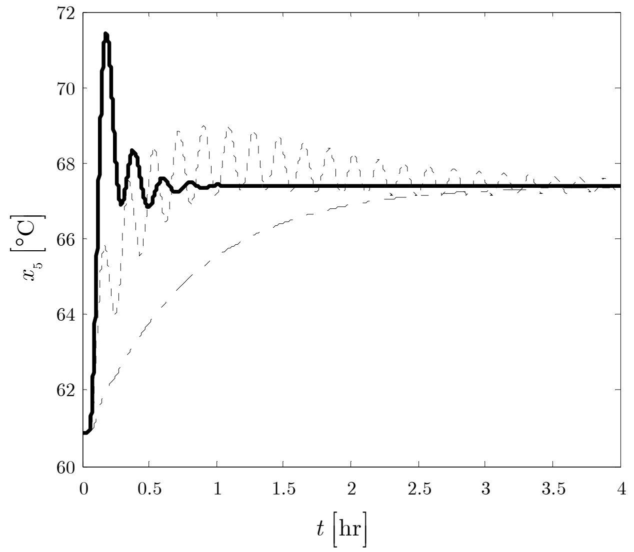

Fig. 17: Radiator Temperature (4th partition)
(*cont.* – metaheuristic, *dotted* – ZN, *dotted dashed* – open loop)

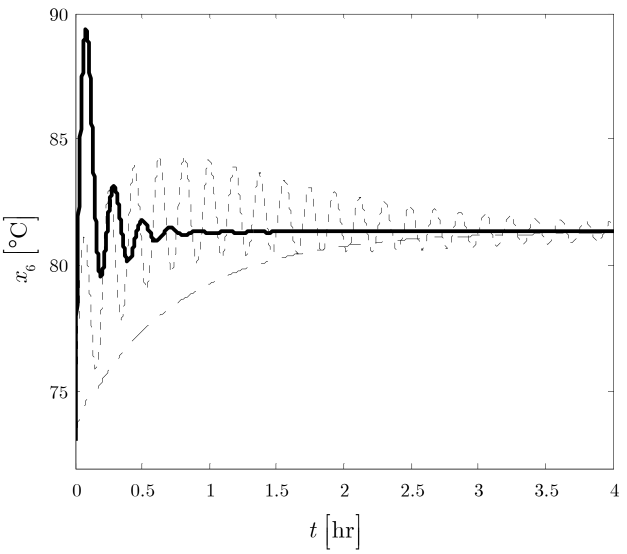

Fig. 18: Boiler Effluent Temperature
(*cont.* – metaheuristic, *dotted* – ZN, *dotted dashed* – open loop)

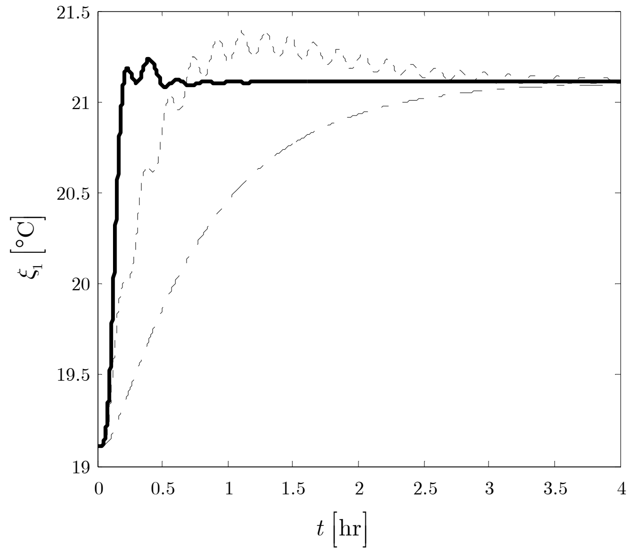

Fig. 19: Room Temperature
(*cont.* – metaheuristic, *dotted* – ZN, *dotted dashed* – open loop)

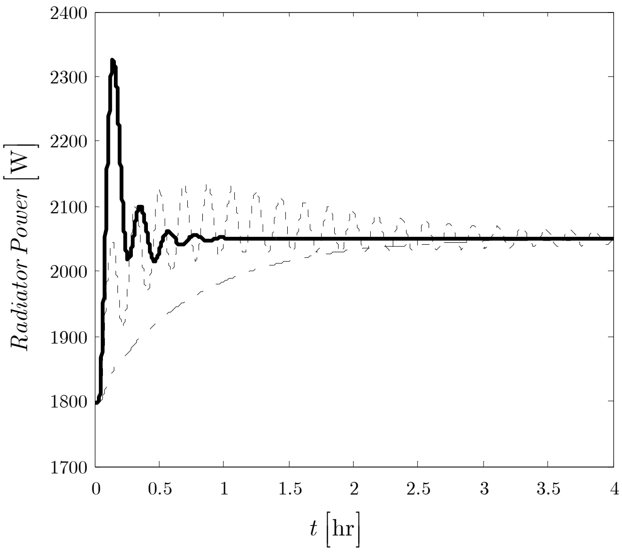

Fig. 20: Emitted Radiator Power
(*cont.* – metaheuristic, *dotted* – ZN, *dotted dashed* – open loop)

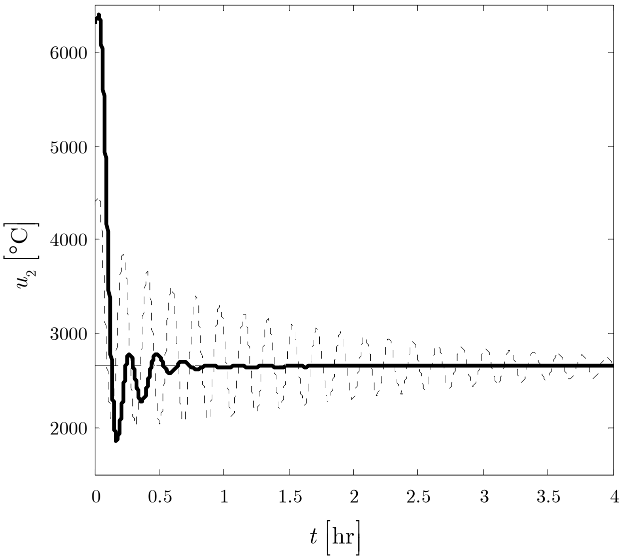

Fig. 21: Energy Supply to the Boiler
(*cont.* – metaheuristic, *dotted* – ZN, *dotted dashed* – open loop)

5 Conclusions

In the present paper the mathematical model of the test case central heating system has analytically been presented in the form of a nonlinear neutral time delay model. In particular, separate models have been presented for the core components of the system, i.e. pipe network, radiator and boiler. The separate models have been combined to a nonlinear neutral time delay system (with time varying delay) which has been simplified to a neutral time delay model with constant time delay. It has been shown that the influence of the delay is significant, thus its incorporation to the model is of high importance while the constant delay approximation imposes only a small error to the system. Finally, a PID controller has been derived to regulate the temperature of a room, which has been modeled as a first order differential equation. The controller parameters have been evaluated using a metaheuristic approach whose efficiency has been investigated. It has been shown that the search algorithm produces fast the controller parameters. The resulting closed loop response has been compared to the response produced by the same controller for the case where the parameters have been evaluated using the first Ziegler Nichols method and the open loop response of the system applying appropriate constant actuatable inputs. It has been shown that the metaheuristic controller produces by far better results than the controller produced by the Ziegler Nichols method and the open loop case.

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