Synthesis Of Attitude Motion Of Variable Mass Coaxial Bodies

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Abstract: - This work involves the research of a motion of a mechanical system consisting from coaxial bodies of variable mass in a translating coordinate system. The author gives a theorem on the change in the angular momentum of a system of variable mass coaxial bodies with respect to translating axes. The dynamic equations of motion are constructed using the example of two coaxial bodies. The modification of mass parameters of coaxial bodies (mass, moments of inertia) causes nontrivial changes of system angular motion. The article describes a special developed method for qualitative phase space analysis, based on the evaluation of a phase trajectory curvature. The method suggested makes it possible to determine the phase trajectory shape and to synthesize conditions for special motion modes realization (for example, nutation angle monotonous diminution or magnification). The results obtained can be used to describe the motion of a coaxial dual-spin spacecraft, performing active maneuvers with a change in mass.

Key-Words: - Coaxial Bodies, Unbalanced Gyrostat, Dual-Spin Spacecraft, Variable Mass, Active Maneuvers, Attitude Motion, Qualitative Method, Phase Trajectory Curvature

1 Introduction

Research of attitude motion of a system of coaxial rigid bodies and gyrostats always was and still remains one of the important problems of theoretical and applied mechanics. The dynamics of the attitude motion of rotating rigid bodies and gyrostats is a classic mechanical research object. Basic aspects of such motion were studied by Euler, Lagrange, Kovalevskaya, Zhukovsky, Volterra, Wangerin, Wittenburg. The main results of the attitude motion research can be found in appropriate treatises [1-5].

However, the study of the dynamics of rotating bodies and gyrostats is still very important in modern science and engineering. Among the basic directions of modern research within the framework of the indicated problem it is possible to highlight the following points: deriving of exact and approximated analytical and asymptotic solutions [1-5, 25, 28], research of a stability of motion conditions [6-16], the analysis of motion under the influence of external regular and stochastic disturbance, research of dynamic chaos [17-22], research of non-autonomous systems with variable parameters [23-29].

N. Ye. Zhukovsky studied the motion of a rigid body containing cavities filled with homogeneous capillary liquid. The research showed that the equations of motion in such case can be reduced to the equations of the attitude motion of a gyrostat. Also analytical solutions of some special modes of motion gyrostat were found.

The ordinary differential equations of a gyrostat attitude motion with constant angular momentum were solved analytically by Volterra. Volterra Solution has generalized a similar analytical solution for a rigid body in case of Euler. In the works of Wangerin and Wittenburg solution of Volterra is reduced to the convenient parametrization expressed in elliptic integrals.

The analytical solution for a attitude motion of heavy dynamically symmetric gyrostat, colligating a classic solution for a heavy solid body in the case of Lagrange, is given in a paper [25]. In the indicated published solutions for all Euler angles (precession, nutation, and intrinsic rotation) are found in elliptic functions and integrals. Also modes of motion with constant and variable relative angular momentum are considered.

The issues of the rotational motion dynamics of a gyrostat are very important for numerous applications such as the dynamics of satellite-gyrostat, spacecraft and aircraft.

The attitude dynamics of gyrostat satellites and double-rotation satellites has been studied by a number of scientists [6-22]. Most of these efforts were aimed on finding the equilibrium states in the presence of external disturbance torques [6-9], on analysis of the stability of spinning satellites under energy dissipation [10-16]. Some authors recently have investigated bifurcation and chaos in the gyrostat satellites [17-22].

Despite the above mentioned and a wide spectrum of research results available in the known literature the stated problem is still studied insufficiently, especially for the variable structure dynamical systems. The possibility of change in some mass and inertia parameters and the variability of structure can be explained by the fact that a space vehicle performs active maneuvers with the use of the jet engine.

Any spacecraft (SC) in orbit is affected by external disturbances of different kind, e.g. the solar radiation pressure, the gravity gradient torque, the magnetic torque caused by the Earth's magnetic field, or the aerodynamic torque due to the action of a resisting medium like the Earth's atmosphere. However all these external disturbances are not large in comparison with the jet engine thrust of the spacecraft on the active motion stage (e.g. inter-orbital transfer, orbit correction, attitude reorientation). Moreover, variability of mass parameters (mass and moments of inertia) has a considerable influence on attitude dynamics. The change of the moments of inertia entails change of angular momentum, which is the basic characteristic of attitude motion. Thereupon mass (structure) variations of SC is one of the primary factors determining attitude motion.

For the purposes of better understanding the essence of this problem it is important to give a brief overview of the main considered engineering peculiarities of SC's active motion. An SC in order to perform an active maneuver (e.g. inter-orbital transfer) should create a jet engine thrust and thus obtain acceleration or braking momentum $\Delta\mathbf{V}$ (reorbit/ deorbit burn).

This momentum should be generated exactly in a pre-calculated direction. Engine thrust is usually focused along the SC's direct-axis, therefore it is necessary to stabilize the direct-axis in order to ensure the accurate momentum generation. Stabilization of a direct-axis can be carried out in a gyroscopic mode when SC spins around a direct-axis which is oriented in the calculated direction.

Momentum generation is not instantaneous, it demands a continuous operation of the jet engine within several seconds (or minutes). During this period of time a SC performs two motions: trajectory motion of a center of mass and an angular motion around it. Such angular motion obviously changes the location of a direct-axis and, hence, a direction of thrust.

The time history of a thrust direction strongly affects the value and direction of a transfer momentum deviation. Consequently, the transfer is performed to the orbit different from the desired one. There is a "scattering" of thrust (fig. 1). Therefore, it is very important to take SC angular motion into account during the analysis of the powered trajectory motion.

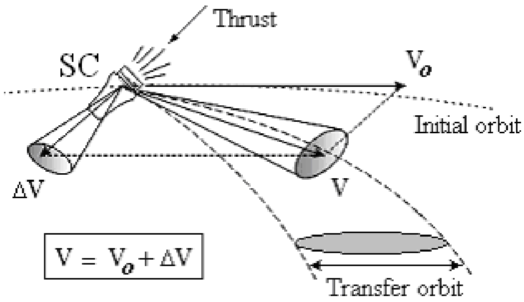

Fig.1

It is necessary to obtain the angular motion which ensures that SC's direct-axis (and the thrust vector) performs precessional motion with monotonously decreasing nutation angle. Thus the direct-axis travels inside an initial cone of nutation and the thrust vector naturally comes nearer to an axis of a precession which is a desired direction of transitional momentum output ("is focused" along a necessary direction).

When the angular motion does not provide a monotonous decrease in nutation angle the direct-axis moves in a complicated way. In such case the thrust vector also makes complicated motion and "scatters" the transitional momentum. A transfer orbit scatters as well.

Among the works devoted to the systems with variable mass and inertia parameters it is possible to mark the following [17, 18, 23, 24, 28, 29]. The works of [18] contain the analysis of chaotic behavior of a spacecraft with a double rotation and time-dependent moments of inertia during it's free motion. The main investigation results of variable mass system dynamics could be found in the monographies [23, 24]. This results include Ivan V. Meschersky theory of motion of bodies with variable mass, theory of “short-range interaction” and “solidification (freezing)”.

The equations of variable mass coaxial bodies system were developed in papers [28]. Also in [27] the attitude motion of coaxial bodies system and double rotation spacecraft with linear time-dependent moments of inertia were analyzed and conditions of motion with decreasing value of nutation were found. The results [27] can be used for the analysis of attitude motion of a dual-spin spacecraft with an operating solid-propellant rocket engine.

This paper represents continuation of the research described in [27-29] and is devoted to variable mass coaxial bodies system and unbalanced gyrostat dynamics.

This paper has the following structure: Section 1 – introduction of the primary theoretical and physical background, Section 2 – mathematical definition of the problem in terms of the angular momentum, Section 3 – main equations of attitude motion of variable mass coaxial bodies system and unbalanced gyrostat, Section 4 – analysis of the attitude motion of variable mass unbalanced gyrostat, Section 5 – conclusion.

2 Problem Definition

Below we will derive the equations of motion of a system of k coaxial bodies of variable mass with respect to noninertial frame. The motion of the system is analyzed with respect to the following coordinate systems (Fig. 2): $P\xi\eta\zeta$ is a system of coordinates, fixed in absolute space, *OXYZ* is a moving noninertial coordinate system with origin at the point *O*, the axes of which remain collinear with the axes of the fixed system during the whole time of motion, and $Ox_iy_iz_i$ are systems of coordinates with a common origin, rigidly connected to the *i*-th body (i = 1, 2, ... , k), rotating with respect to the system *OXYZ*. *OXYZ* system has its origin in a point lying on the common axis of rotation of the bodies and matching with the initial position of the centre of mass. Points in the different parts of the system are distinguished by the body they belong to, and hence in all expressions they are indicated by the subscript ν_i (where i is a number of an appropriate body).

To construct the equations of motion we use the "short-range" hypothesis – particles which obtain a relative velocity when separated from the body no longer belong to the body and have no effect on it. In such case the theorem on the change in the angular momentum of a system of variable mass [23], written with respect to the fixed system of coordinates $P\xi\eta\zeta$, has the following form:

$$\frac{d\mathbf{K}_P}{dt} = \mathbf{M}_P^e + \mathbf{M}_P^R + \sum_{i=1}^{k} \mathbf{S}_i^e\,, \qquad \mathbf{S}_i^e = \sum_{\nu_i} \mathbf{r}_{\nu_i} \times \frac{dm_{\nu_i}}{dt}\mathbf{v}_{\nu_i} \tag{1}$$

where $\mathbf{v}_{\nu_i} = \dot{\mathbf{r}}_{\nu_i}$, $\mathbf{M}_P^e$ is the principal moment of the external forces, $\mathbf{M}_P^R$ is the principal moment of the reactive (jet) forces, and $\mathbf{S}_i^e$ is the sum of the angular momentum of the particles of body i, rejected in unit time in their motion with respect to the fixed system of coordinates.

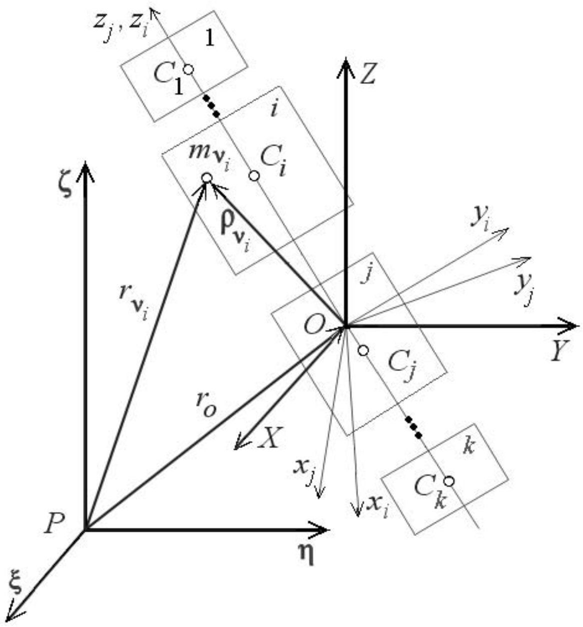

Fig.2

The angular momentum of a system of k bodies in *OXYZ* (Fig. 2) coordinates is defined by the following expression:

$$\begin{aligned}\mathbf{K}_P &= \sum_{i=1}^{k}\sum_{\nu_i} \mathbf{r}_{\nu_i} \times m_{\nu_i}\mathbf{v}_{\nu_i} = \\ &= \sum_{i=1}^{k}\sum_{\nu_i}\left[(\mathbf{r}_0 + \boldsymbol{\rho}_{\nu_i}) \times m_{\nu_i}(\mathbf{v}_0 + \boldsymbol{\omega}_i \times \boldsymbol{\rho}_{\nu_i})\right]\end{aligned} \tag{2}$$

where $\boldsymbol{\omega}_i$ is the angular velocity of body i (and the $Ox_iy_iz_i$ coordinate system is connected with it) relative to movable point *O*.

In order to write the theorem of change in the angular momentum in the *OXYZ* coordinate system we need to implement some auxiliary expressions:

$$\frac{d\boldsymbol{\rho}_{C_i}}{dt} = \boldsymbol{\omega}_i \times \boldsymbol{\rho}_{C_i} + \mathbf{q}_{C_i}$$

$$\sum_{\nu_i}\frac{dm_{\nu_i}}{dt}\left(\boldsymbol{\omega}_i \times \boldsymbol{\rho}_{\nu_i}\right) = \boldsymbol{\omega}_i \times \left[\frac{dm_i}{dt}\boldsymbol{\rho}_{C_i} + m_i\mathbf{q}_{C_i}\right]$$

where $\boldsymbol{\rho}_{C_i}$ is the radius vector of the centre of mass C_i of body i in the *OXYZ* system and $\mathbf{q}_{C_i}$ is the relative velocity of the centre of mass C_i due to a change in its position with respect to the bodies, due to the variability of their masses. Using this expressions and grouping the points of the system according to the body they belong to, we can write the theorem on the change in the angular momentum with respect to $OXYZ$ system [28]:

$$\sum_{i=1}^{k} \frac{d\mathbf{K}_{i,O}}{dt} = \mathbf{M}_O^e + \mathbf{M}_O^R +$$
$$+\sum_{i=1}^{k}\sum_{\nu_i} \boldsymbol{\rho}_{\nu_i} \times \frac{dm_{\nu_i}}{dt}\left(\boldsymbol{\omega}_i \times \boldsymbol{\rho}_{\nu_i}\right) - \boldsymbol{\rho}_C \times m\mathbf{w}_O, \quad (3)$$
$$\mathbf{K}_{i,O} = \sum_{\nu_i} m_{\nu_i} \boldsymbol{\rho}_{\nu_i} \times \boldsymbol{\omega}_i \times \boldsymbol{\rho}_{\nu_i}$$

where $\mathbf{K}_{i,O}$ is angular moments, and $\mathbf{M}_O^e$, $\mathbf{M}_O^R$ are the principal moments of the external and reactive (jet) forces with respect to the point O, m is mass of the system, $\boldsymbol{\rho}_C$ is vector of center of mass of the system, $\mathbf{w}_O$ is the acceleration of point O. For example, external forces correspond to actions of gravitational and magnetic fields.

Using the idea of a local derivative for the angular momentum vector of each body in the system of coordinates $Ox_iy_iz_i$ connected with the body, rotating with respect to $OXYZ$ with angular velocity $\boldsymbol{\omega}_i$ Eq. (3) can be rewritten as follows:

$$\sum_{i=1}^{k}\left[\left(\frac{\tilde{d}\mathbf{K}_{i,O}}{dt}\right)_{Ox_iy_iz_i} + \boldsymbol{\omega}_i \times \mathbf{K}_{i,O}\right] = \mathbf{M}_O^e + \mathbf{M}_O^R + \quad (4)$$
$$+\sum_{i=1}^{k}\sum_{\nu_i} \boldsymbol{\rho}_{\nu_i} \times \frac{dm_{\nu_i}}{dt}\left(\boldsymbol{\omega}_i \times \boldsymbol{\rho}_{\nu_i}\right) - \boldsymbol{\rho}_C \times m\mathbf{w}_O$$

The subscript outside the brackets of the local derivatives indicates a coordinate system in which they were taken. It is necessary to note, what this approach (local derivative using) makes it possible to write equation in native terms of the angular velocity of body.

Equation (4) expresses a vector-based form of the theorem of the change in the angular momentum of bodies of variable mass with respect to the movable axes.

3 Attitude Motion of Two Variable Mass Coaxial Bodies System

We will consider the motion of a system of two bodies, where only 1st has a variable mass. Body 2 does not change its inertial and mass characteristics, calculated in the system of coordinates $Ox_2y_2z_2$ connected to the body, and, consequently, produces no jet forces due to the mass change. The centre of mass of the system, due to the change in the mass of body 1, is shifted with a certain velocity $\mathbf{q}_C$. Fig. 3 shows the case when, at the initial moment of time, the mass of the second body is greater than the mass of the first one.

We will write the angular velocities and the angular momentum of the bodies in projections onto the axes of their connected systems of coordinates

$$\boldsymbol{\omega}_i = p_i\mathbf{i}_i + q_i\mathbf{j}_i + r_i\mathbf{k}_i, \quad \mathbf{K}_{i,O} = \mathbf{I}_i \cdot \boldsymbol{\omega}_i$$
$$\mathbf{I}_i = \begin{bmatrix} I_{i,xx} & -I_{i,xy} & -I_{i,xz} \\ -I_{i,xy} & I_{i,yy} & -I_{i,yz} \\ -I_{i,xz} & -I_{i,yz} & I_{i,zz} \end{bmatrix} \quad (5)$$

where $\mathbf{I}_i$ are inertia tensors of bodies; $\{\mathbf{i}_i, \mathbf{j}_i, \mathbf{k}_i\}$ are the unit vectors of the system $Ox_iy_iz_i$.

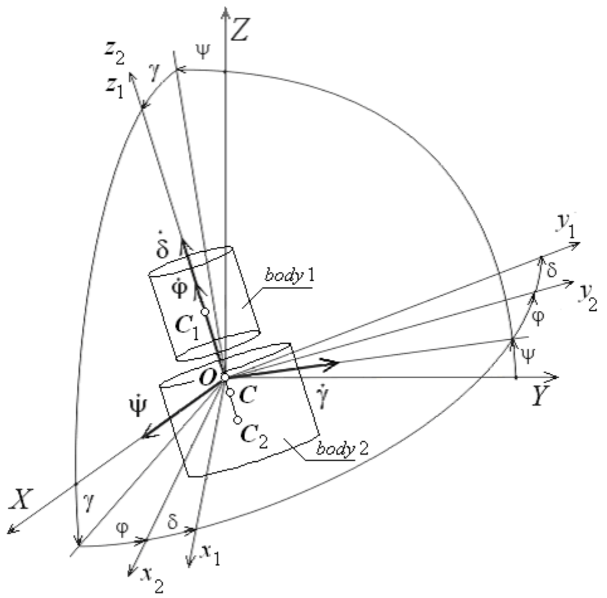

Fig.3

If both tensors are general then angular momentum of the bodies in projections onto the axes of their connected systems of coordinates is defined by

$$\mathbf{K}_{1,O} = A_1(t)p_1\mathbf{i}_1 + B_1(t)q_1\mathbf{j}_1 + C_1(t)r_1\mathbf{k}_1$$
$$\mathbf{K}_{2,O} = A_2p_2\mathbf{i}_2 + B_2q_2\mathbf{j}_2 + C_2r_2\mathbf{k}_2$$

where A_i, B_i, C_i are general moments of inertia of body i, calculated in the corresponding system of coordinates connected to the body.

The bodies of the system can only rotate with respect to one another in the direction of the common longitudinal axis, which coincides with Oz_2 (and with Oz_1). Here we will denote the angle and velocity of twisting of body 1 with respect to body 2 in the direction of the longitudinal axis Oz_2 by δ $\left(\delta = \angle\left(Ox_1, Ox_2\right)\right)$ and $\sigma = \dot\delta$ respectively. The angles $\{\psi, \gamma, \varphi\}$ of spatial orientation of the coaxial bodies with respect to the movable system of coordinates $OXYZ$ are indicated in Fig. 3. The ratio between the angular velocities and the angular accelerations of two bodies in vector form are defined by

$$\boldsymbol{\omega}_1 = \boldsymbol{\omega}_2 + \boldsymbol{\sigma}, \qquad \boldsymbol{\varepsilon}_1 = \boldsymbol{\varepsilon}_2 + \dot{\boldsymbol{\sigma}} \tag{6}$$

where $\boldsymbol{\sigma} = (0,0,\dot{\delta})$ is the vector of the relative angular velocity of the bodies, which has the only projection – onto common axis of rotation Oz_2. The ratio between the components of the angular velocities for the two bodies is expressed by the following equiations

$$\begin{aligned} p_1 &= p_2 \cos\delta + q_2 \sin\delta, \\ q_1 &= q_2 \cos\delta - p_2 \sin\delta, \\ r_1 &= r_2 + \sigma \end{aligned} \tag{7}$$

The theorem on the change in the angular momentum (4) in movable system of coordinates $OXYZ$ can be rewritten in the form

$$\begin{aligned} &\left[\left(\frac{\tilde{d}\mathbf{K}_{1,O}}{dt} \right)_{Ox_1y_1z_1} - \sum_{\nu_1} \boldsymbol{\rho}_{\nu_1} \times \frac{dm_{\nu_1}}{dt} \left(\boldsymbol{\omega}_1 \times \boldsymbol{\rho}_{\nu_1} \right) \right] + \\ &+ \left(\frac{\tilde{d}\mathbf{K}_{2,O}}{dt} \right)_{Ox_2y_2z_2} + \sum_{i=1}^{2} \boldsymbol{\omega}_i \times \mathbf{K}_{i,O} = \\ &= \mathbf{M}_O^e + \mathbf{M}_O^R - \boldsymbol{\rho}_C \times m\mathbf{w}_O \end{aligned} \tag{8}$$

where

$$\boldsymbol{\rho}_{\nu_1} = x_{\nu_1}\mathbf{i}_1 + y_{\nu_1}\mathbf{j}_1 + z_{\nu_1}\mathbf{k}_1$$

By projecting the expression inside the square brackets (Eq. (8)) onto the axes of the system $Ox_1y_1z_1$ and using expressions (5) we obtain

$$\begin{aligned} \mathbf{L} &= \left[\left(\frac{\tilde{d}\mathbf{K}_{1,O}}{dt} \right)_{Ox_1y_1z_1} - \sum_{\nu_1} \boldsymbol{\rho}_{\nu_1} \times \frac{dm_{\nu_1}}{dt} \left(\boldsymbol{\omega}_1 \times \boldsymbol{\rho}_{\nu_1} \right) \right] = \\ &= \left(I_{1,xx}(t)\dot{p}_1 - I_{1,xy}(t)\dot{q}_1 - I_{1,xz}(t)\dot{r}_1 \right)\mathbf{i}_1 + \\ &+ \left(-I_{1,yx}(t)\dot{p}_1 + I_{1,yy}(t)\dot{q}_1 - I_{1,yz}(t)\dot{r}_1 \right)\mathbf{j}_1 + \\ &+ \left(-I_{1,zx}(t)\dot{p}_1 - I_{1,zy}(t)\dot{q}_1 + I_{1,zz}(t)\dot{r}_1 \right)\mathbf{k}_1 \end{aligned} \tag{9}$$

During the simplification of equation (9) terms containing the derivatives of time-varying moment of inertia cancel out with terms following from the sum in square brackets (vector **L**). This is vividly reflected in the projection of **L** onto the connected axis Ox_1

$$\begin{aligned} L_{x_1} &= \frac{d\left(\mathbf{K}_{1,O}\right)_{x_1}}{dt} - \sum_{\nu_1} \left[\boldsymbol{\omega}_1 \rho_{\nu_1}^2 - \boldsymbol{\rho}_{\nu_1} \left(\boldsymbol{\rho}_{\nu_1} \cdot \boldsymbol{\omega}_1 \right) \right]_{x_1} \dot{m}_{\nu_1} = \\ &= \frac{d\left(\mathbf{K}_{1,O}\right)_{x_1}}{dt} - p_1 \sum_{\nu_1} \frac{dm_{\nu_1}}{dt} \left(y_{\nu_1}^2 + z_{\nu_1}^2 \right) + \\ &+ q_1 \sum_{\nu_1} \frac{dm_{\nu_1}}{dt} x_{\nu_1} y_{\nu_1} + r_1 \sum_{\nu_1} \frac{dm_{\nu_1}}{dt} x_{\nu_1} z_{\nu_1} = \\ &= \frac{d}{dt}\left(I_{1,xx}(t)p_1 - I_{1,xy}(t)q_1 - I_{1,xz}(t)r_1 \right) - \\ &- p_1 \frac{dI_{1,xx}(t)}{dt} + q_1 \frac{dI_{1,xy}(t)}{dt} + r_1 \frac{dI_{1,xz}(t)}{dt} = \\ &= I_{1,xx}(t)\dot{p}_1 - I_{1,xy}(t)\dot{q}_1 - I_{1,xz}(t)\dot{r}_1 \end{aligned}$$

If tensors of inertia remain general for each moment of time$\left(I_{1,ij}(t) \equiv 0, \ i \neq j \right)$, then vector **L** may be rewritten

$$\mathbf{L} = A_1(t)\dot{p}_1\mathbf{i}_1 + B_1(t)\dot{q}_1\mathbf{j}_1 + C_1(t)\dot{r}_1\mathbf{k}_1 \tag{10}$$

Taking expressions (9) and (7) into account, we can write Eq. (8) in terms of projections onto the axis of $Ox_2y_2z_2$ system, connected with body 2. When changing from system $Ox_1y_1z_1$ to system $Ox_2y_2z_2$ we will use an orthogonal matrix $[\boldsymbol{\delta}]$ of rotation by angle δ. As a result we obtain

$$\begin{aligned} &[\boldsymbol{\delta}]\left(\mathbf{L} + \boldsymbol{\omega}_1 \times \mathbf{K}_{1,O} \right)_{Ox_1y_1z_1} + \left(\frac{\tilde{d}\mathbf{K}_{2,O}}{dt} + \boldsymbol{\omega}_2 \times \mathbf{K}_{2,O} \right)_{Ox_2y_2z_2} = \\ &= \mathbf{M}_O^e + \mathbf{M}_O^R - \boldsymbol{\rho}_C \times m\mathbf{w}_O \end{aligned} \tag{11}$$

where

$$[\boldsymbol{\delta}] = \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The motion of center of mass of body 1 is easier to analyze as compound motion, where the motion of body 2 is translational. Then we obtain following accelerations of centers of mass

$$\begin{aligned} &\mathbf{w}_{C_2} = \mathbf{w}_O + \boldsymbol{\varepsilon}_2 \times \boldsymbol{\rho}_{C_2} + \boldsymbol{\omega}_2 \times \boldsymbol{\omega}_2 \times \boldsymbol{\rho}_{C_2}, \\ &\mathbf{w}_{C_1} = \mathbf{w}^r + \mathbf{w}^e + \mathbf{w}^c, \\ &\mathbf{w}^e = \mathbf{w}_O + \boldsymbol{\varepsilon}_2 \times \boldsymbol{\rho}_{C_1} + \boldsymbol{\omega}_2 \times \boldsymbol{\omega}_2 \times \boldsymbol{\rho}_{C_1}, \\ &\mathbf{w}^r = \dot{\boldsymbol{\sigma}} \times \boldsymbol{\rho}_{C_1} + \boldsymbol{\sigma} \times \boldsymbol{\sigma} \times \boldsymbol{\rho}_{C_1}, \quad \mathbf{w}^c = 2\boldsymbol{\omega}_2 \times \left(\boldsymbol{\sigma} \times \boldsymbol{\rho}_{C_1} \right), \end{aligned}$$

where $\mathbf{w}^e$ – acceleration of translation, $\mathbf{w}^r$ – relative acceleration, $\mathbf{w}^c$ – Coriolis acceleration.

From the theorem on the motion of the centre of mass of a system of variable mass [23] with help of the last expression for accelerations, the following expressions must hold

$$\begin{aligned} &m_1(t)\mathbf{w}_{C_1} = \mathbf{F}_1^e + \boldsymbol{\Phi}_1^R + \mathbf{N}_{12}^i; \quad m_2\mathbf{w}_{C_2} = \mathbf{F}_2^e + \mathbf{N}_{21}^i; \\ &m_1(t)\mathbf{w}_{C_1} + m_2\mathbf{w}_{C_2} = \mathbf{F}^e + \boldsymbol{\Phi}_1^R; \ \mathbf{F}^e = \mathbf{F}_1^e + \mathbf{F}_2^e; \ \mathbf{N}_{12}^i = -\mathbf{N}_{21}^i; \\ &m_1(t)\mathbf{w}_{C_1} + m_2\mathbf{w}_{C_2} = m\mathbf{w}_O + \boldsymbol{\varepsilon}_2 \times m\boldsymbol{\rho}_C + \boldsymbol{\omega}_2 \times \boldsymbol{\omega}_2 \times m\boldsymbol{\rho}_C + \\ &+ m_1(t)\left(\dot{\boldsymbol{\sigma}} \times \boldsymbol{\rho}_{C_1} + \boldsymbol{\sigma} \times \boldsymbol{\sigma} \times \boldsymbol{\rho}_{C_1} + 2\boldsymbol{\omega}_2 \times \left(\boldsymbol{\sigma} \times \boldsymbol{\rho}_{C_1} \right) \right); \\ &m = m_1 + m_2; \qquad m\boldsymbol{\rho}_C = m_1\boldsymbol{\rho}_{C_1} + m_2\boldsymbol{\rho}_{C_2}, \end{aligned} \tag{12}$$

where $\mathbf{F}^e$ is resultant of system of external active forces, $\mathbf{\Phi}_1^R$ is reactive (jet) force, $\mathbf{N}_{jk}^i$ are internal forces of interaction between bodies (*j*, *k*=1, 2). From (12) expression for acceleration $\mathbf{w}_O$ follow

$$\mathbf{w}_O = \frac{1}{m}\Big[\mathbf{F}^e + \mathbf{\Phi}_1^R - \mathbf{\varepsilon}_2 \times m\mathbf{\rho}_C - \mathbf{\omega}_2 \times \mathbf{\omega}_2 \times m\mathbf{\rho}_C - m_1(t)\left(\dot{\mathbf{\sigma}} \times \mathbf{\rho}_{C_1} + \mathbf{\sigma} \times \mathbf{\sigma} \times \mathbf{\rho}_{C_1} + 2\mathbf{\omega}_2 \times \left(\mathbf{\sigma} \times \mathbf{\rho}_{C_1}\right)\right)\Big]. \quad (13)$$

The $(-\mathbf{\rho}_C \times m\mathbf{w}_O)$ vector is represented using Eq. (13)

$$-\mathbf{\rho}_C \times m\mathbf{w}_O = -\mathbf{\rho}_C \times \Big[\mathbf{F}^e + \mathbf{\Phi}_1^R - \mathbf{\varepsilon}_2 \times m(t)\mathbf{\rho}_C - \mathbf{\omega}_2 \times \mathbf{\omega}_2 \times m(t)\mathbf{\rho}_C - m_1(t)\left(\dot{\mathbf{\sigma}} \times \mathbf{\rho}_{C_1} + \mathbf{\sigma} \times \mathbf{\sigma} \times \mathbf{\rho}_{C_1} + 2\mathbf{\omega}_2 \times \left(\mathbf{\sigma} \times \mathbf{\rho}_{C_1}\right)\right)\Big]. \quad (14)$$

If the change of mass of body 1, which has general tensors of inertia, is uniform along the whole volume, then tensors of inertia remain general tensors of inertia and the centre of mass of body 1 remains on a common axis of rotation Oz_2. Thus we will consider, that the body 2 also has general tensors of inertia. The following expressions in terms of projections onto the axes of $Ox_2y_2z_2$ will take place in this case

$$\begin{gathered}\mathbf{\sigma} = \left[0,0,\dot{\delta}\right]^T,\ \dot{\mathbf{\sigma}} = \left[0,0,\ddot{\delta}\right]^T,\ \mathbf{q}_C = \left[0,0,q_C\right]^T, \\ \mathbf{\rho}_{C_1} = \left[0,0,\rho_{C_1}(t)\right]^T, \quad \mathbf{\rho}_C = \left[0,0,\rho_C(t)\right]^T, \\ \left[-\mathbf{\rho}_C \times m\mathbf{w}_O\right]_{Ox_2y_2z_2} = \left[-\mathbf{\rho}_C \times \mathbf{F}^e - \mathbf{\rho}_C \times \mathbf{\Phi}_1^R\right]_{Ox_2y_2z_2} + \\ + m(t)\rho_C^2\left[\left(\dot{p}_2 - r_2q_2\right)\mathbf{i}_2 + \left(\dot{q}_2 - r_2p_2\right)\mathbf{j}_2 + 0\mathbf{k}_2\right].\end{gathered} \quad (15)$$

Let's transform the moments of external and reactive (jet) forces in expression (11)

$$\mathbf{M}_O^e = \mathbf{\rho}_C \times \mathbf{F}^e + \mathbf{M}_C^e; \quad \mathbf{M}_O^R = \mathbf{\rho}_C \times \mathbf{\Phi}_1^R + \mathbf{M}_C^R \quad (16)$$

Taking the expressions (15) and (16) into account, we will write Eqs (11) in the matrix form

$$[\mathbf{\delta}]\left\{\begin{bmatrix} A_1(t)\dot{p}_1 \\ B_1(t)\dot{q}_1 \\ C_1(t)\dot{r}_1 \end{bmatrix} + \begin{bmatrix} \left(C_1(t) - B_1(t)\right)q_1r_1 \\ \left(A_1(t) - C_1(t)\right)p_1r_1 \\ \left(B_1(t) - A_1(t)\right)q_1p_1 \end{bmatrix}\right\} + \left[\begin{bmatrix} A_2\dot{p}_2 \\ B_2\dot{q}_2 \\ C_2\dot{r}_2 \end{bmatrix} + \begin{bmatrix} \left(C_2 - B_2\right)q_2r_2 \\ \left(A_2 - C_2\right)p_2r_2 \\ \left(B_2 - A_2\right)q_2p_2 \end{bmatrix}\right] = \mathbf{M}_C^e + \mathbf{M}_C^R + m(t)\rho_C^2\begin{bmatrix} \dot{p}_2 - r_2q_2 \\ \dot{q}_2 - r_2p_2 \\ 0 \end{bmatrix} \quad (17)$$

Components of $\{p_1, q_1, r_1\}$ in Eq. (17) must be expressed via $\{p_2, q_2, r_2\}$ using (7).

We will add an equation describing the relative motion of the bodies to the Eq (11). A theorem on the change in the angular momentum projected onto the axis of rotation for the first body will have the following form

$$\left(\mathbf{L} + \mathbf{\omega}_1 \times \mathbf{K}_{1,O}\right)_{Oz_1} = M_{1,Oz}^e + M_z^R + M_\delta - \left[\mathbf{\rho}_C \times m\mathbf{w}_O\right]_{Oz_1} \quad (18)$$

where M_δ is the moment of the internal interaction of the bodies (e.g. action of internal engine or bearing friction), $M_{i,Oz}^e$ is the moment of external forces acting only on body *i*.

If tensors of inertia of body 1 remain the general ones for every moment of time, then Eq. (18) can be rewritten in the follow form

$$C_1(t)\dot{r}_1 + \left(B_1(t) - C_1(t)\right)q_1p_1 = M_{1,Oz}^e + M_z^R + M_\delta \quad (19)$$

We will supplement the dynamic equations (11) and (18) (or their simplified analogs (17) and (19)) by the following kinematical equations (Fig.3)

$$\begin{gathered}\dot{\delta} = \sigma; \qquad \dot{\gamma} = p_2\sin\varphi + q_2\cos\varphi \\ \dot{\psi} = \frac{1}{\cos\gamma}\left(p_2\cos\varphi - q_2\sin\varphi\right) \\ \dot{\varphi} = r_2 - \frac{\sin\gamma}{\cos\gamma}\left(p_2\cos\varphi - q_2\sin\varphi\right)\end{gathered} \quad (20)$$

Let's analyze the motion of a system of two dynamically symmetrical bodies, equations (17) and (19) will be written in the following form

$$\begin{gathered}A(t)\dot{p}_2 + \left(C(t) - A(t)\right)q_2r_2 + C_1(t)q_2\sigma = M_{C,x}^e + M_x^R; \\ A(t)\dot{q}_2 - \left(C(t) - A(t)\right)p_2r_2 - C_1(t)p_2\sigma = M_{C,y}^e + M_y^R; \\ C(t)\dot{r}_2 + C_1(t)\dot{\sigma} = M_{C,z}^e + M_z^R; \\ C_r(t)\left(\dot{r} + \dot{\sigma}\right) = M_\delta + M_z^R + M_{1,Oz}^e\end{gathered} \quad (21)$$

where

$$A(t) = A_1(t) + A_2 - m\rho_C^2(t), \quad C(t) = C_1(t) + C_2.$$

Systems (21) and (20) together form a complete dynamic system for the research of attitude motion of dynamically symmetrical unbalanced gyrostat with variable mass.

4 Research of Attitude Motion of Variable Mass Coaxial Bodies and Unbalanced Gyrostat

Let's refer to a motion of coaxial bodies (unbalanced gyrostat) of variable mass under an action of dissipative and boosting external moments depending on components of angular velocities. Let the gyrostat consists of dynamically symmetrical main body (coaxial body 2) of a constant mass and a rotor (coaxial body 1) of the variable mass, which remains dynamically symmetrical during modification of a mass (fig. 3).

The fixed point O coincides with an initial geometrical position of a system's center of mass. The unbalanced gyrostat has a varying relative angular velocity of rotor rotation around the main body. It is possible in connection with the existence of internal moment M_δ acting between coaxial bodies. Let's assume there is a moment of jet forces only around a direct-axis Oz_1 $\left(M_x^R = M_y^R = 0\right)$.

Let's implement the new variables corresponding to the magnitude G of a vector of transversal angular velocity and angle F between this vector and axis Oy_2:

$$\begin{aligned} p_2 &= G(t)\sin F(t) \\ q_2 &= G(t)\cos F(t) \end{aligned} \qquad (22)$$

Equations (21) will be rewritten in new variables as follows:

$$\begin{cases} \dot{F} = -\dfrac{\left(C(t)-A(t)\right)r_2 + C_1(t)\sigma + f_F\left(G,F\right)}{A(t)}; \\ \dot{G} = f_G\left(G,F\right)\big/A(t); \quad \dot{r} = \left(M_{2,Oz}^e - M_\delta\right)\big/C_2; \\ \dot{\sigma} = \dfrac{C(t)M_\delta}{C_1(t)C_2} + \dfrac{M_z^R + M_{1,Oz}^e}{C_1(t)} - \dfrac{M_{2,Oz}^e}{C_2}. \end{cases} \qquad (23)$$

In equations (23) the following disturbing functions describing exposures take place:

$$f_G\left(G,F\right) = \left(M_{C,x}^e \sin F + M_{C,y}^e \cos F\right),$$

$$f_F\left(G,F\right) = \left(M_{C,x}^e \cos F - M_{C,y}^e \sin F\right)\big/G$$

We will consider a case when the module of a transversal angular velocity of main body is small in comparison to relative longitudinal rotation rate of the rotor:

$$\varepsilon = \sqrt{p_2^2 + q_2^2}\Big/\left|\sigma\right| << 1 \qquad (24)$$

From spherical geometry the formula for a nutation angel θ (an angle between axes OZ and Oz_2) follow

$$\cos\theta = \cos\psi\cos\gamma$$

We will assume angles γ and ψ to be small $\left(\gamma = O(\varepsilon),\ \psi = O(\varepsilon)\right)$. Then the nutation angle will be defined by the following approximated formula:

$$\theta^2 \cong \gamma^2 + \psi^2 \qquad (25)$$

Using the expressions (22) and kinematic equations (20) we can write (second order infinitesimal terms are omitted):

$$\begin{aligned} &\dot{\gamma} \cong G\cos\Phi(t), \quad \dot{\psi} \cong G\sin\Phi(t), \\ &\dot{\varphi} \cong r, \quad \dot{\delta} = \sigma, \quad \Phi(t) = F(t) - \varphi(t) \end{aligned} \qquad (26)$$

Function $\Phi(t)$ is a phase of spatial oscillations.

Precession motion of the gyrostat with small nutation angles is obviously described by a phase space of variables $\{\gamma,\psi\}$. The phase trajectory in this space completely characterizes motion of the direct-axis Oz_2 (an apex of the direct-axis). Therefore our further researches will be connected to the analysis of this phase space and chances of behaviors of phase curves in this space.

We can develop a special qualitative method of the analysis of a phase space. Main idea of the method is the evaluation of a phase trajectory curvature in the phase plane $\{\gamma,\psi\}$.

On the indicated plane the phase point will have following components of a velocity and acceleration:

$$V_\gamma = \dot{\gamma},\ V_\psi = \dot{\psi},\ W_\gamma = \ddot{\gamma},\ W_\psi = \ddot{\psi}$$

With the help of expressions (26) the curvature of a phase trajectory (k) is evaluated as follows:

$$k^2 = \left(\dot{\gamma}\ddot{\psi} - \dot{\psi}\ddot{\gamma}\right)^2\Big/\left(\dot{\gamma}^2 + \dot{\psi}^2\right)^3 = \dot{\Phi}^2\big/G^2 \qquad (27)$$

If curvature magnitude increase, there will be a motion on a twisted spiral trajectory similar to a steady focal point (fig. 4, case "a") and if decreases - on untwisted. On twisted spiral trajectory motion condition can be noted as:

$$|k|\uparrow \Rightarrow k\dot{k} > 0 \ \Rightarrow\ \dot{\Phi}\ddot{\Phi}G - \dot{G}\dot{\Phi}^2 > 0 \qquad (28)$$

For the analysis of the condition realization it is necessary to study a disposition of zero point of a following function:

$$P(t) = \dot{\Phi}\ddot{\Phi}G - \dot{G}\dot{\Phi}^2 \qquad (29)$$

Function (29) will be defined as a function of phase trajectory evolutions.

Different qualitative cases of phase trajectory behaviors are possible depending on a zero point of function $P(t)$. Fig.4 illustrate three main *qualitative type* of phase trajectory, which will be described further on examples-based. So, in the first case (fig. 4, case "a") the function is positive and has no zero on a considered slice of time $t \in [0,T]$, thus the phase trajectory is spirally twisting. In the second case (fig. 4, case “b”) there exists one zero point and there is one modification in a monotonicity of the trajectory curvature. The Cornu spiral, also known as clothoid, take place in case “b”. The third case (fig. 4, case “c”) represents a number of zero points and the trajectory has alternation of untwisted and twisted segments of motion; also there are some points of self-intersection.

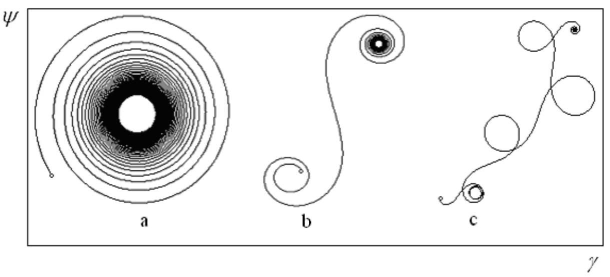

Fig. 4

4.1 Example 1

As an first example we will refer to a motion of coaxial bodies of variable mass under the influence of constant internal moment ($M_\delta = const$) and constant moment of jet forces ($M_z^R = const$). The analysis of a phase space is conducted using a developed method of curvature evaluation.

We will suppose that the moments of inertia are linear functions of time $\left(A_1(t) = A_r - at;\right.$ $C_1(t) = C_r - ct;$ $A_2 = A_m;$ $\left.C_2 = C_m\right)$ and magnitudes $m\rho_C^2$ are omitted. In a considered case equations (23) will obtain the following form:

$$\dot{F} = -\frac{\left(C_m + C_r - ct - A_m - A_r + at\right) r_2 + (C_r - ct)\sigma}{A_m + A_r - at}$$

$$\dot{G} = 0, \qquad \dot{\sigma} = \frac{\left(C_m + C_r - ct\right) M_\delta}{(C_r - ct) C_m} + \frac{M_z^R}{(C_r - ct)} \tag{30}$$

$$\dot{r}_2 = -M_\delta / C_m$$

Analytical solutions for angular velocity $r_2(t)$ and $\sigma(t)$ are derived from equations (30):

$$r_2 = r_0 - \frac{M_\delta}{C_m} t, \quad \sigma = \sigma_0 + s_1 t + s_2 \ln\left(1 - c_1 t\right)$$
$$s_1 = M_\delta / C_m, \;\; s_2 = -\frac{1}{c}\left(M_\delta + M_z^R\right), \;\; c_1 = c / C_r \tag{31}$$

Solutions (31) make it possible to receive an expansion in a series of a right part of an equation for phase $F(t)$ (30):

$$\dot{F} = F_0 + \sum_{i=1}^{\infty} F_i t^i \tag{32}$$

Following values appear in expression (32):

$$F_0 = -\frac{D_1}{A_m + A_r}; \qquad F_1 = -\frac{D_1 a_1 + D_2}{A_m + A_r}$$

$$\underset{(i\ge 2)}{F_i} = \frac{-1}{A_m + A_r}\left(\sum_{k=1}^{3} D_k a_1^{i-k+1} + \sum_{j=1}^{i-2} E_{j+1,j+2} a_1^{i-j-2}\right)$$

$$D_1 = B r_0 + C_r \sigma_0; \qquad B = C_r + C_m - A_r - A_m$$

$$D_2 = C_r\left(s_1 - s_2 c_1\right) - c\sigma_0 - b r_0 - B\frac{M_\delta}{C_m}$$

$$D_3 = b\frac{M_\delta}{C_m} - c\left(s_1 - s_2 c_1\right) - C_r c_1^2 s_2 / 2$$

$$E_{k,l} = c s_2 c_1^k / k - C_r s_2 c_1^l / l; \;\; a_1 = \frac{a}{A_m + A_r}; \;\; b = c - a;$$

The obtained expression (32) converges uniformly on the interval $t \in \left[0, 1/a_1\right) \bigcap \left[0, 1/c_1\right)$. Kinematic equations can be used to receive a solution for the angle φ:

$$\varphi = \varphi_0 + r_0 t - \frac{M_\delta}{2C_m} t^2 \tag{33}$$

Expression for a time derivative of a spatial oscillations phase $\dot{\Phi}$ can be obtained using (33):

$$\dot{\Phi} = \dot{F} - \dot{\varphi} = f_0 + \sum_{i=1}^{\infty} f_i t^i; \quad f_0 = F_0 - r_0$$
$$f_1 = F_1 + M_\delta / C_m; \quad f_j = F_j \quad (j = 2..\infty) \tag{34}$$

Further we will investigate the simplest case when the expansion for $\dot{\Phi}$ has only linear part (other terms of expansion are not taken into account). On the basis of expressions (34) we can get a polynomial of the first degree for the phase trajectory evolutions function (29):

$$P(t) = f_1\left(f_0 + f_1 t\right) = f_1^2 t + f_1 f_0 \tag{35}$$

There is a unique zero point of this function: $t_1 = -f_0 / f_1$. For implementation of a condition (28) of twisted spiral motion it is necessary for the polynomial to be stable ($t_1 < 0$) and positive for all $t \ge 0$. It is possible only in case the following condition fulfills:

$$f_1 f_0 > 0 \tag{36}$$

We will consider a case when following contingencies are correct:

$$r_0 = 0; \;\; \sigma_0 < 0$$

In this case value f_0 will be positive:

$$f_0 = -C_r \sigma_0 / \left(A_m + A_r\right) > 0$$

In order f_1 to be also positive the following condition must be satisfied:

$$f_1 = \frac{\sigma_0\left[c\left(A_m + A_r\right) - C_r a\right]}{\left(A_m + A_r\right)^2} - \frac{M_z^R}{A_m + A_r} > 0 \quad (37)$$

Conditions (37) provide the following expressions:

$$\frac{c}{C_r} < \frac{a}{A_m + A_r}; \quad M_z^R < 0 \qquad (38)$$

or two other similar groups of conditions:

$$\left\{\frac{c}{C_r} < \frac{a}{A_m + A_r};\ M_z^R > 0;\ \frac{\sigma_0\left[c\left(A_m + A_r\right) - C_r a\right]}{\left(A_m + A_r\right)} > M_z^R\right\};$$

$$\left\{\frac{c}{C_r} > \frac{a}{A_m + A_r};\ M_z^R < 0;\ \left|\frac{\sigma_0\left[c\left(A_m + A_r\right) - C_r a\right]}{\left(A_m + A_r\right)}\right| < \left|M_z^R\right|\right\} \qquad (39)$$

In figure (fig. 5) illustrates the results of phase trajectory numeric calculations.

Figures "a" and "b" (fig. 5) demonstrate the situation when (38) and (39) are satisfied, "c" and "d" (fig. 5) show the opposite case. System parameters and initial conditions for obtained solutions are listed in table 1.

Conditions (38) (or (39)) can be used for the synthesis of dual-spin spacecraft parameters. In order to enhance the accuracy of SC's longitudinal axis positioning it is necessary to realize precession motion with a decreasing nutation angle. This motion is realized when the conditions (38) or (39) are satisfied.

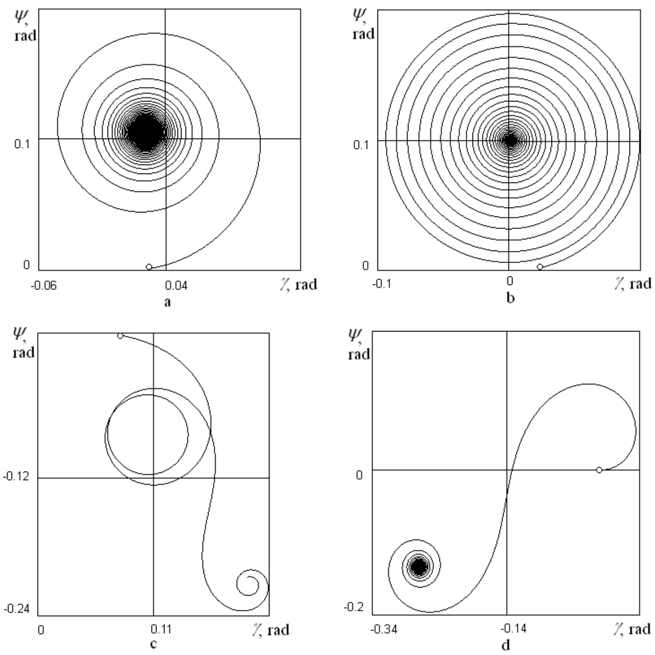

Fig. 5

An effort of system to find a local stability point in phase domain (fig.5) is connected to fact that in considered cases absolute magnitude of phase trajectory curvature increase at completion phase of motion. If to assume, that the magnitude of curvature becomes a constant, then the phase trajectory form will pass to a circle of constant radius. If the magnitude of curvature will have undamped oscillations, then the form also will be a variable and complex.

Table 1

Case (fig. 4)	a	b	c	d
A_m, kg·m^2	2.5	2.5	2	2.5
A_r, kg·m^2	2.5	2.5	2	2.5
C_m, kg·m^2	1	1	1	1
C_r, kg·m^2	1	1	1	1
a, kg·m^2/sec	0.2	0.2	0.2	0.2
c, kg·m^2/sec	0.01	0.01	0.1	0.01
M_δ, N·m	1	1	3	1
M^R_z, N·m	-3	0.1	4	2.3
σ_0, radian/sec	-10	-10	-1	-10
G_0, radian/sec	0.2	0.2	0.1	0.1

For realization of more accurate researches, certainly, it is necessary to take higher degrees polynomials $P(t)$ (29) into account. However it was shown that an implemented linear analysis (using a first degree polynomial (35)) provides an adequate description of the angular motion evolutions of variable mass coaxial bodies.

4.2 Example 2

Let's refer to the other mode of motion with the following external and internal dissipative force moments:

$$M^e_{C,x} = -\nu p, \quad M^e_{C,y} = -\nu q$$
$$M^e_{2,Oz} = -\lambda r, \quad M^e_{1,Oz} = -\mu\left(r + \sigma\right) \qquad (40)$$

In this case the evolution function can be rewritten as:

$$P(t) = G\left(\frac{1}{2}\frac{d\dot{\Phi}^2}{dt} + \frac{\nu\,\dot{\Phi}^2}{A(t)}\right) \qquad (41)$$

Omitting the solution details we receive the following numeric simulation results for the evolution function and phase trajectory (Fig. 6). Parameters of system and initial conditions for calculations of fig.6 are listed in table 2.

It can be noticed (Fig. 6) that curvature function $P(t)$ (41) has 5 zero points and a phase trajectory has 6 evolutions.

The phase trajectory in the considered example corresponds to the form of the trajectory of the figure 4-c.

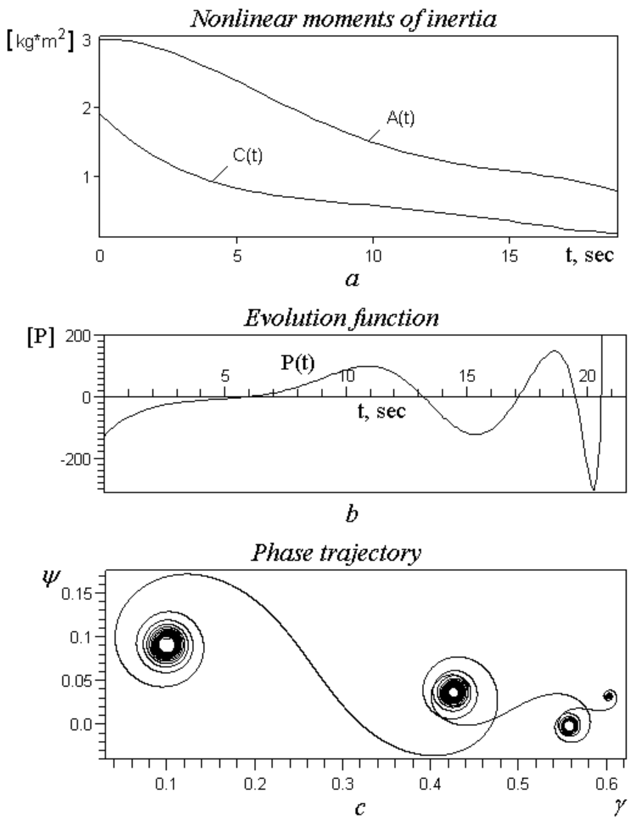

Fig.6

Table 2

$A(t)$, kg·m^2	Fig.6-a
A_2, kg·m^2	1.5
$C(t)$, kg·m^2	Fig.6-a
C_2, kg·m^2	1.6
μ, kg·m^2/sec	0.01
ν, kg·m^2/sec	0.02
λ, kg·m^2/sec	0.03
M_δ, N·m	-10
M^R_z, N·m	-20
r_0, radian/sec	30
σ_0, radian/sec	30
G_0, radian/sec	0.32

4.3 Example 3

Let's consider third mode of motion without external forces moments but in presence of constant internal moment ($M_\delta = const$) and piecewise continuous moment of jet forces ($M_z^R(t)$) with piecewise smooth nonlinear time-dependences inertia moments (Fig.7-a).

We receive the following numeric simulation results for the evolution function (Fig.7-b) and phase trajectory (Fig. 7-c).

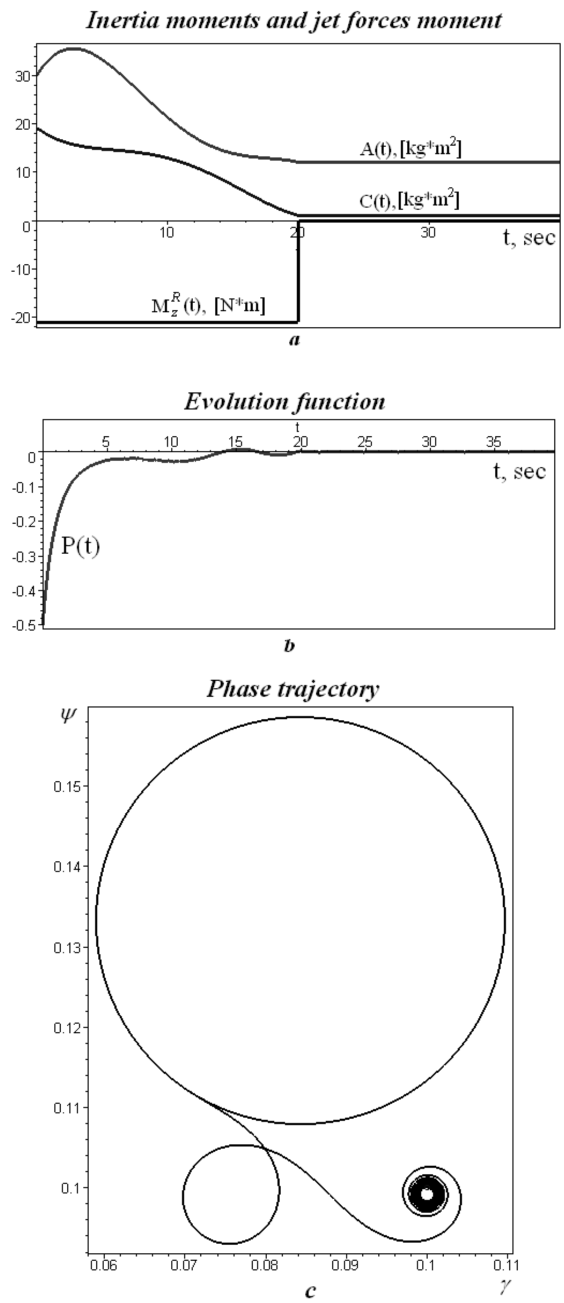

Fig.7

In considered case the magnitude of curvature becomes a constant after "shutdown" of jet moment and variability of the inertia moments. The phase trajectory form will pass to a circle of constant radius.

Parameters of system and initial conditions for calculations of fig.7 are listed in table 3.

Certainly, other interesting researches of motion modes are possible. So it is necessary to note the motion of gyrostat with various nonlinear time-dependences of the moments of inertia in presence of various fields of forces, and also at action of vibrations effect and effects of the nutation, precision disturbing and wobbling. It is a quite plan for further investigation.

Table 3

$A(t)$, $C(t)$, kg·m^2	Fig.7-a
C_2, kg·m^2	1.6
M_δ, N·m	-3
M^R_z, N·m	Fig.7-a
r_0, radian/sec	-2
σ_0, radian/sec	22
G_0, radian/sec	0.01

5 Conclusion

The article described a research of the phase space of a non-autonomous dynamical system of coaxial bodies and unbalanced gyrostats of variable mass using a new method of analysis for the behavior of non-autonomous dynamical system. Results of the research have an important applied value for the problems of space flight mechanics.

Appreciation:

The following grants and programs supported this work:

- The Program of the Russian Federation President on supporting the Russian scientists and leading science schools of the Russian Federation: projects # MK-4707.2006.8, MK-2263.2004.8;
- The Russian Foundation for Basic Research (http://www.rfbr.ru): # 06-08-00325, 06-01-00151.

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