Resonance at motion of a body in the Mars's atmosphere under biharmonical moment

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Abstract: - An uncontrolled motion of a reentry vehicles in the rarefied Mars's atmosphere is investigated. Such reentry vehicle has small lengthening and a blunted form for effective braking [1]. This type of Lander can have three balancing positions of a spatial angle of attack: $\alpha^* = 0,\ \alpha^* \neq 0,\ \alpha^* = \pi$ depending on the position of the center mass. The change of a dynamic pressure the descent into the atmosphere can lead to resonance. It is shown, that the numerical integration of motion equations does not give reliable results due to the probabilistic character of transients with related to resonance. The conditions of movement stability are received identified for various areas of movement under resonance. The suggestion is to calculate the top and bottom parameters of movement using averaged equations. The research shows that the resonance could cause the accident the Beagle 2 Lander.

Key-Words: - Uncontrolled Reentry Vehicle, Resonance, Stability, Separatrice, Atmosphere, Mars

1 Introduction

One of the principal causes, resulting in abnormal behavior Reentry vehicle in an atmosphere, it is considered a parametrical resonance [2, 3] which arises at presence of small asymmetry when movement concerning the center of mass depends on two angular variables: a spatial angle of attack α and a angle of own rotation. If frequency of fluctuation of the angle of attack and average angular speed of own rotation under action of indignations become multiple to the relation of simple integers then the resonance arises. The resonance as the phenomenon of big change of amplitude fluctuations can to arise when small asymmetry does not have also movement depends on one angular variable: a spatial angle of attack, if coefficient of the aerodynamic static moment $m_\alpha(\alpha)$ addresses in a zero in three points on a interval $[0,\pi]$. In this case on a phase portrait $\dot{\alpha} = \dot{\alpha}(\alpha)$ three areas divided separatrices [4] can to take place. The dynamic pressure changes with height of flight and the phase portrait any more does not answer conservative system. In connection by it the evolution of phase trajectories takes place. As a result, these trajectories can to cross separatrices and fall into various phase portrait areas, which is followed by qualitative changes in the motion character. This is a resonance. In similar tasks frequently apply methods of chaotic dynamics [5-8], for example, Melnikov method [9]. However this method gives good result when phase trajectories are in a vicinity of unperturbed separatrices. In this paper the methods based on averaging of phase trajectories are used [10].

2 Problem Formulation

Uncontrolled reentry vehicle has small lengthening of the blunted form, which provides effective braking for descent in a rarefied atmosphere of Mars. This paper considers spatial motion around a re-entry's center of mass with the angle of attack dependence of the coefficient restoring moment having form of a biharmonical series

$m_\alpha(\alpha) = a\sin\alpha + b\sin 2\alpha$.

Such the angle of attack dependence of the coefficient restoring moment is typical for uncontrolled reentry vehicles of segmentally-conical, blunted conical, and other shapers (Soyuz, Mars, Apollo, Viking, Beagle 2 Lander). The presence of second harmonic in the moment characteristics causes the possibility of appearance of an additional equilibrium position of a reentry vehicle in the angle of attack, i.e., an additional singular point on a phase portrait $\alpha^* \in (0,\pi)$ of the system, which causes the transient mode – resonance. Fig. 1 shows a segmentally - conic body (analogue of the Beagle 2 Lander) and dependences of the coefficient static moment on the spatial angle of attack $m_\alpha(\alpha)$ at various positions of the center of the mass $\bar{x}_T = x_T / L$, counted from nose of a body (L - reference length), received on the shock theory of Newton.

For considered reentry vehicles position $\alpha = 0$ is stability. If the condition

$$|2b| > |a| \qquad (1)$$

take place, then there is an intermediate position of balance $\alpha^* \in (0,\pi)$.

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The purpose of the given report - to show an opportunity of occurrence of resonance, to find conditions of stability of the perturbed motion, to construct procedure of calculation of the top and bottom estimations of parameters of movement with use of the average equations [4].

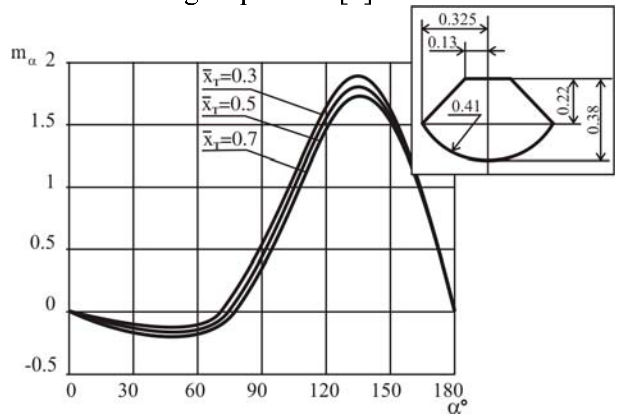

Fig. 1. The coefficient of the aerodynamic restoring moment at varicus positions of the mass center.

3 The disturbance equations of motion

The motion of an axial-symmetric body around the center of mass at descent in an atmosphere is described by the system with slowly varying parameters of type [3]

$$\ddot{\alpha} + F(\alpha, z) = -\varepsilon m_z(z)\dot{\alpha},$$

$$\dot{R} = -\varepsilon m_x(z) R = \varepsilon \Phi_R(z),$$

$$\dot{G} = -\varepsilon \left\{ m_y(z) G + \left[m_x(z) - m_y(z) \right] \cdot R \cos\alpha \right\}$$
$$= \varepsilon \Phi_G(\alpha, z),$$

$$\dot{V} = -c_{x\alpha}(\alpha) \cdot \frac{qS}{m} - g \sin\theta = \varepsilon \Phi_V(\alpha, z),$$

$$\dot{\theta} = -\frac{\cos\theta}{V}\left(g - \frac{V^2}{R_P + H}\right) = \varepsilon \Phi_\theta(\alpha, z),$$

$$\dot{H} = V \sin\theta = \varepsilon \Phi_H(\alpha, z),$$

$$F(\alpha) = \frac{(G - R\cos\alpha)(R - G\cos\alpha)}{\sin^3\alpha}$$
$$- A\sin\alpha - B\sin 2\alpha,$$

$$A = \frac{aSL}{I} q, \qquad B = \frac{bSL}{I} q, \tag{2}$$

where $z = (R, G, V, \theta, H)$ is the vector of slowly varying parameters; ε is the small parameter; *R* and *G* are, to an accuracy of a multiplier, the projections of the angular momentum vector onto the longitudinal axis and onto the velocity direction, respectively; *V* is the spacecraft motion velocity, θ is the trajectory inclination angle, *H* is the flight altitude, *g* is the acceleration of gravity, $c_{x\alpha}(\alpha)$ is the drag force coefficient, $q = \rho V^2 / 2$ is the dynamic pressure, ρ is the density of the atmosphere, *S* is the middle cross section area, *m* is the spacecraft mass, *I* is the transverse moment of inertia of the body, and *L* is its characteristic size, R_P is the planet's radius; $\varepsilon m_x(z)$, $\varepsilon m_y(z)$ and $\varepsilon m_z(z)$ are the projections of a small damping moment onto the axes of the right-handed coordinate system *Oxyz* chosen in such a manner that the *Ox* axis is directed along the spacecraft's axis of symmetry, the *Oy* axis lies in the plane formed by *Ox* and velocity vector *V*.

Evolution of motion occurs under action of disturbance at $\varepsilon \neq 0$.

4 The unperturbed solution

The disturbance system (2) is reduced to non-perturbed system with one degree of freedom at $\varepsilon = 0$

$$\ddot{\alpha} + F(\alpha) = 0. \tag{3}$$

Let's find the common decision of this equation. The energy integral of system (3) has the form of

$$\dot{\alpha}^2 / 2 + W(\alpha) = E, \tag{4}$$

where

$$W(\alpha) = \int F(\alpha) d\alpha = \frac{G^2 + R^2 - 2GR\cos\alpha}{2\sin^2\alpha}$$
$$+ A\cos\alpha + B\cos^2\alpha$$

is potential energy.

We introduce a new variable $u = \cos\alpha$, and then the energy integral (4) takes the form

$$\frac{\dot{u}^2}{2(1-u^2)} + \frac{G^2 + R^2 - 2GRu}{2(1-u^2)} + Au + Bu^2 = E$$

or

$$\dot{u}^2 = f(u), \tag{5}$$

$$f(u) = 2(1-u^2)(E - Au - Bu^2)$$
$$+ 2GRu - G^2 - R^2. \tag{6}$$

The equation (5) may be resulted in a quadrature

$$t - t_0 = \int_{u_0}^{u} \left[f(u) \right]^{-1/2} du. \tag{7}$$

The polynomial $f(u)$ has the fourth degree be relative *u*. It means, that the integral (7) is elliptic [12]. Four roots u_1, u_2, u_3, u_4 of the polynomial (6) depend on coefficients A, B, R, G, E. There is a limited number of characteristic variants of an arrangement of roots. We research function $f(u)$. This function at $u = \pm\infty$, $u = \pm 1$ is equal

$$f(\pm\infty) = sign(b) \cdot \infty,$$

$$f(\pm 1) = -(G \mp R)^2 \leq 0. \tag{8}$$

For real motion $u = \cos\alpha$ belongs to an interval $[-1, +1]$ and function $f(u) \geq 0$ by virtue of the equation (5).

From (8) follows, that at $u = \pm 1$ function $f(u) \leq 0$ then on the specified interval the polynomial (6) should have even amount of the real roots.

First of all let us consider the case $b > 0$. All four roots are real and two of them are in an interval $[-1,+1]$ according to conditions (8) (see Fig. 2 a). We choose number of the root so that for real motion $[u_2, u_1] \subset [-1,+1]$. Two others of the root satisfy to the conditions: $u_3 \le -1$, $u_4 \ge 1$.

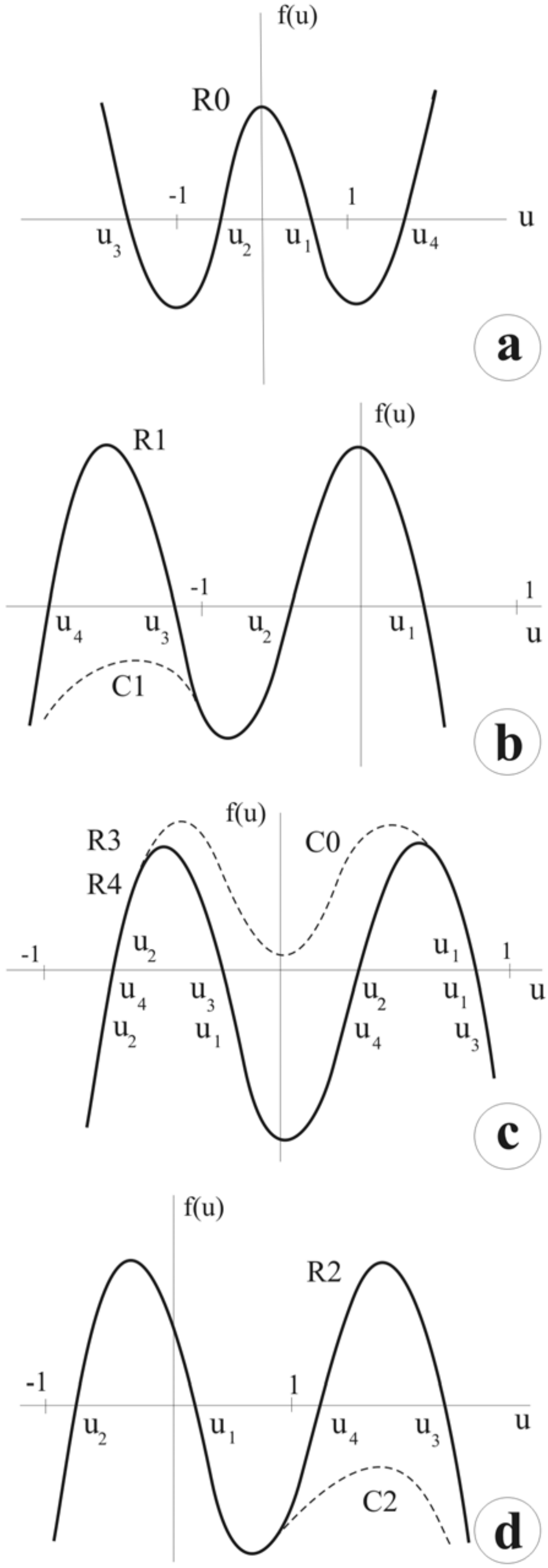

Fig. 2. Roots of the polynomial $f(u)$.

If $b < 0$, there are some variants of an arrangement of roots. Let roots u_3, u_4 are real then exists two variants when these roots are located outside of the interval $[-1,+1]$: $u_4 < u_3 < -1$ and $u_3 > u_4 > 1$ (see Fig. 2.16 c, d), and two variants - when inside the interval $[-1,+1]$: $-1 < u_4 < u_3 < u_2 < u_1 < 1$ and $-1 < u_2 < u_1 < u_4 < u_3 < 1$ (Fig. 2 b). When roots u_3, u_4 are in a conjugate complex, the real part is at the left, on the right or inside the interval $[u_2, u_1]$ (see Fig. 2 b, c, d). All variants of position of the roots are in Tab. 1.

Table 1.
All variants of position of the roots polynomial $f(u)$

Var.	b	u_1, u_2	u_3, u_4	Type of roots
R0	$b > 0$		$u_3 < -1$, $u_4 > 1$	
R1			$u_4 < u_3 < -1$	
R2			$u_3 > u_4 > 1$	
R3			$-1 < u_4$, $u_4 < u_3$, $u_3 < u_2$, $u_2 < u_1$, $u_1 < 1$	All roots u_1, u_2, u_3, u_4 are real roots
R4	$b < 0$	$-1 < u_2$, $u_2 < u$, $u_1 < 1$	$-1 < u_2$, $u_2 < u_1$, $u_1 < u_4$, $u_4 < u_3$, $u_3 < 1$	
C0			$u_2 < u_{34} < u_1$	Roots u_1, u_2 are real roots.
C1			$u_{34} < u_2$	Conjugate complex roots
C2			$u_{34} > u_1$	$u_{3,4} = u_{34} + iv$

The integral (7) may be reduced to normal elliptic integrals with the change of variables $u = u(\gamma)$ [12]. For variants R0..R4 the change is

$$u = \frac{u_1(u_2 - u_3) + u_3(u_1 - u_2)\cos^2\gamma}{(u_2 - u_3) + (u_1 - u_2)\cos^2\gamma}, \tag{9}$$

for variants C0..C2 -

$$u = \frac{(u_2 + u_1\xi) - (u_2 - u_1\xi)\cos\gamma}{(1+\xi) - (1-\xi)\cos\gamma}, \tag{10}$$

Where $\xi = \cos\chi_1 / \cos\chi_2$, $tg\chi_1 = (u_1 - u_{34})/v$, $tg\chi_2 = (u_2 - u_{34})/v$.

The changes (9) and (10) give the following expressions for (7)

$$\beta t+\tau_0=\int_0^{\gamma}\frac{d\gamma}{\left[1-k^2\sin^2\gamma\right]^{1/2}}=F(\gamma,k), \qquad (11)$$

where $\beta=2|b|/\mu$, $F(\gamma,k)$ is incomplete elliptic integral of the first kind [12], μ, k - parameters which are expressed through roots $u_1 ... u_4$, τ_0 - a constant of integration.

Making use of inversion of integral (11)

$\gamma=am[\beta t+\tau_0,k]$

and formulas (9) or (10), the general solution of equation (3) can be written as

$$\cos\alpha=u=L+\frac{M}{1+Ncn^{\delta}(\beta t+\tau_0,k)}. \qquad (12)$$

The values of coefficients L, M, N, β,δ and k are determined depending on the type of roots:

- four roots are real: change (9) ($\delta=2$)

$$L=u_3,\ M=u_1-u_3,$$
$$N=\frac{u_1-u_2}{u_2-u_3},$$
$$k^2=\frac{(u_1-u_2)(u_3-u_4)}{(u_1-u_3)(u_2-u_4)},$$
$$\beta=\left[-0.5b(u_1-u_3)(u_2-u_4)\right]^{1/2}; \qquad (13)$$

- two roots are real and two ones are complex-conjugate: change (10) ($\delta=1$)

$$L=\frac{u_1\xi-u_2}{\xi-1},\qquad M=\frac{2\xi(u_2-u_1)}{\xi^2-1},$$
$$N=\frac{\xi-1}{\xi+1},\qquad k^2=\frac{1}{2}\left(1-\frac{\zeta}{\eta}\right),$$
$$\beta=(-2b\eta)^{1/2},\quad \xi=\sqrt{\frac{(u_2-u_{34})^2+v^2}{(u_1-u_{34})^2+v^2}},$$
$$\eta=\sqrt{(u_1-u_{34})^2+v^2}\sqrt{(u_2-u_{34})^2+v^2},$$
$$\zeta=(u_1-u_{34})(u_2-u_{34})+v^2. \qquad (14)$$

The size τ_0 is defined from entry conditions

$$\tau_0=F(\gamma_0,k),$$
$$\gamma_0=-sign(\dot\alpha_0)\arccos\left[\frac{L+M-\cos\alpha_0}{N(\cos\alpha_0-L)}\right]^{1/\delta}. \quad (15)$$

Taking into account, that the period elliptic cosine $cn(x,k)$ is equal $4K(k)$, where $K(k)$ is complete elliptic integral of the first kind, the period T_α and frequency ω_α of fluctuations of the angle of attack can be found from the general solution (12).

$$T_\alpha=4K(k)/\delta\beta,\qquad \omega_\alpha=\delta\pi\beta/2K(k).$$

Let's consider transient cases:

R3-C0, R4-C0, when $u_2=u_3, v=0$ or $u_1=u_4$, $v=0$; R1-C1, R2-C2, R3-C1, R4-C2, when $u_3=u_4$, $v=0$ (Fig.2.).

Cases R1-C1 and R3-C1 or R2-C2 and R4-C2 also can coincide among themselves, forming accordingly cases R1-R3-C1, when $u_3=u_4=-1$, or R2-R4-C2, when $u_3=u_4=1$.

From (13), (14) follows, that in variants R1-C1, R2-C2, R3-C1, R4-C2 the module of elliptic integrals $k=0$. It is possible to show, that in this case formulas (13), (14) at substitution in (9) and (10) give the same common decision in which elliptic functions are replaced trigonometrical. Variants R3-C0, R4-C0 correspond to motion on a separatrice, $k=1$, $\omega_\alpha\to 0$ and $T_\alpha\to\infty$.

5 The phase portrait

Let us consider the energy integral system (3). The energy potential can be expressed as function of the variable $u=\cos\alpha$ in the form of two components

$$W(u)=W_g(u)+W_r(u), \qquad (16)$$

where $W_g(u)=\dfrac{G^2+R^2-2GRu}{2(1-u^2)}$,

$W_r(u)=Au+Bu^2$.

We research behavior of the function (16) at various combinations of the parameters: R, G, A, B.

Let's find a derivative of the function $W_g(u)$ on the variable u

$$W_g'(u)=\frac{(R^2+G^2)u-RG(1+u^2)}{(1-u^2)^2}.$$

The numerator of this expression has the valid mutually return roots R/G and G/R from which only one belongs to interval $[-1,+1]$. Thus, there is a unique extremum of the function $W_g(u)$, and it is a minimum equal $0.5\max(R^2,G^2)\geq 0$.

The second derivative

$$W_g''(u)=\left((R^2+G^2)(1+3u^2)-2RGu(3+u^2)\right)(1-u^2)^{-3} \qquad (17)$$

shows, that it and the function $W_g(u)$, everywhere on the interval $[-1,+1]$ are nonnegative. Really, the numerator has extremums in already known points R/G and G/R, equal $(G^2-R^2)^2/R^2\geq 0$ and $(G^2-R^2)^2/G^2\geq 0$, and it on the ends of the interval $u=\pm 1$ equals $4(G\mp R)^2\geq 0$. From here follows, that function $W_g(u)$ has no points of inflexion, and the derivative $W_g'(u)$ is monotonically increasing quantity on the interval $[-1,+1]$.

Let's consider now the quadratic function $W_r(u)$. It has an extremum in a point $(-A/2B)$ where its derivative $W_r'(u)=A+2Bu$ go to zero. The second derivative $W_r''(u)=2B$ is a constant. Therefore at performance of the condition

$$B \ge -\left[\min_{-1\le u\le 1}\left(0.5W_g''(u)\right)\right] \equiv B^*, \qquad (18)$$

the second derivative $W''(u)=W_g''(u)+W_r''(u)$ is positive or equal to zero. Function $W(u)$ has no points of inflexion on the interval $[-1,+1]$. It means, that on a phase portrait of the system there is a unique steady position of balance, and saddle point is absent. From (18) follows, that b^* is positive or equal to zero always. The function $W_g''(u)$ degenerates at $R=G=0$, therefore, $b^*=0$ and the condition (18) can be written $b\ge 0$. From (17) follows, that the size b^* decreases at increase in absolute values of parameters R and G, i.e. the condition (18) is weakened. Obviously, the saddle point will be absent also at enough small absolute size of the coefficient b. Really, if

$$|b| \le 0.5|a|, \qquad (19)$$

that function $W_r''(u)$ on all the interval has the same sign and the derivative $W'(u)=W_g'(u)+W_r'(u)$ is zero in a unique point, and function $W(u)$ has a unique extremum - a minimum.

If any of conditions (18), (19) is not satisfied, presence of two minima and one maximum of function $W(u)$ on the interval $[-1,+1]$ that corresponds to presence on a phase portrait of a unstable point such as a saddle (fig. 3) is possible. The specified situation will take place at performance of a condition

$$W'(u_{*1})\cdot W'(u_{*2})<0, \qquad (20)$$

where u_{*1}, u_{*2} are roots of the equation:

$$W''(u)=\frac{d^2W(u)}{du^2}=0.$$

From the condition (20) follows, that if inside the interval $[-1,+1]$ the saddle point is absent in a plane case ($R=G=0$) it will be absent and in case of spatial fluctuations irrespective of size of the parameters R and G. On the other hand, if at $R=G=0$ the saddle point takes place to provide its absence for any values of energy E according to (17), (18) it is possible a choice enough big on the module R and G.

The phase plane divided the separatrice into three areas: external A_0 and two internal A_1 and A_2, if the condition (20) satisfies.

It is possible to find connection between three balancing positions $\alpha^*=0, \alpha^*\ne 0, \alpha^*=\pi$ and positions of balance of the system (3) on a phase portrait at performance of conditions (1).

If $E>W_*$, where W_* is value $W(u)$ in saddle point $u=u_*$, then motion occurs in external area A_0, as can been from fig. 3. Otherwise ($E<W_*$) motion can to take place in any of internal areas A_1 or A_2 depending on entry conditions. Equality $E=W_*$ satisfies to motion on the separatrice.

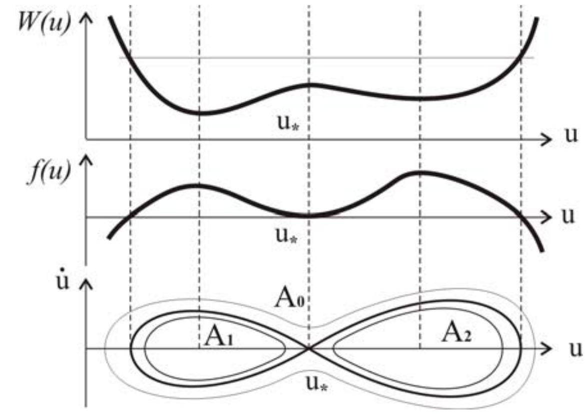

Fig. 3. Phase portrait.

6 Disturbed motion stability

We investigate the disturbed motion research in areas A_0, A_1, A_2. Movement can begin both in external area A_0, and in any of internal areas A_1 and A_2. If the area in which began movement, is unstable, the phase trajectory will cross through separatrice some limited time by virtue action of disturbances. It is obvious, that at the moment of crossing separatrice two situations take place: two areas are unstable, one is stable and, on the contrary, one is unstable, and two are stable. At action of small disturbances the average of full energy $\bar{E}$ and of potential energy W_* in saddle point $u=u_*$ slowly changes. For definition of stability it is enough to calculate derivatives on time from these functions [3]. The internal area (A_1 or A_2) will be stable, if in neighborhood separatrice the following condition satisfies

$$\dot{\bar{E}}(z)<\dot{W}(u_*,z). \qquad (21)$$

For external area A_0 the condition of stability looks like:

$$\dot{\bar{E}}(z)>\dot{W}(u_*,z). \qquad (22)$$

Function (21) in saddle point $u=u_*$ is equal

$$f_* \equiv f(u_*, z) = 2(1-u_*^2)[\bar{E}(z) - W(u_*, z)]\,. \qquad (23)$$

In neighborhood separatrice take place

$\bar{E}(z) - W(u_*, z) = O(\varepsilon)\,, \quad \dot{u}_*(z) = O(\varepsilon)\,.$

Differentiation of function (23) on time gives the following result to within of the order infinitesimal ε^2

$$\dot{f}_* = 2(1-u_*^2)[\dot{\bar{E}}(z) - \dot{W}(u_*, z)]\,. \qquad (24)$$

From (24) follows, that conditions (21) and (22) are answered with the following conditions, accordingly (see Fig. 4, 5)

$\dot{f}_* < 0\,, \quad \dot{f}_* > 0\,.$

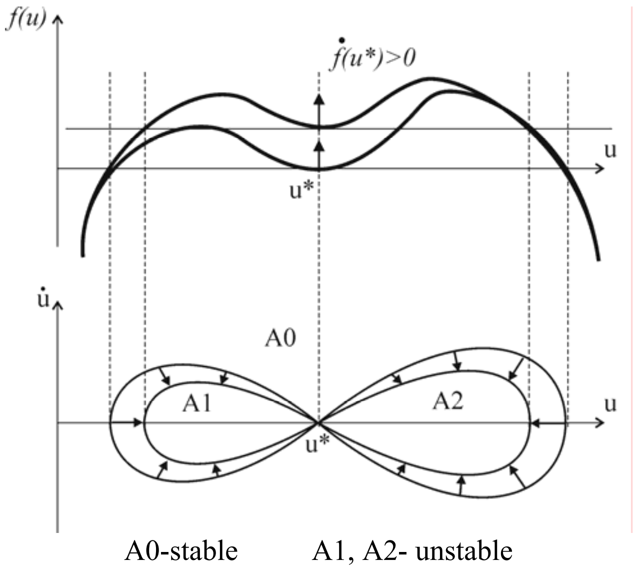

Fig. 4. Evolution of the phase portrait.

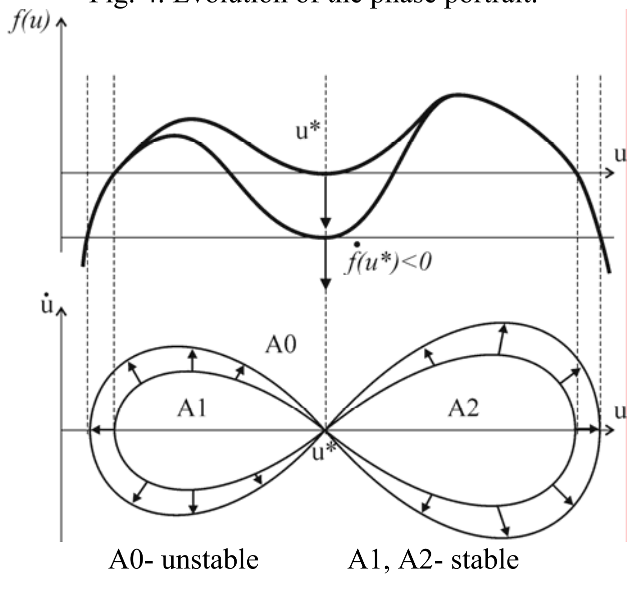

Fig. 5. Evolution of the phase portrait.

From integral of energy (4) follows, that at $\dot{\alpha} = 0$

$\bar{E}(z) = W(\alpha_m, z)\,,$

where $\alpha = \alpha_m$ is amplitude of attack angle.

For system (2) average equations of motion, are received in [4, 11]. We calculate derivatives $\dot{\bar{E}}(z)$ and $\dot{W}(\alpha_*, z)$ by virtue of the average equations:

$$\dot{\bar{E}}(z) = \left.\frac{\partial W}{\partial \alpha}\right|_{\alpha=\alpha_m} \cdot \dot{\alpha}_m + \left.\frac{\partial W}{\partial z}\right|_{\alpha=\alpha_m} \cdot \dot{z}$$

$$= F(\alpha_m, z) \cdot \dot{\alpha}_m + \left.\frac{\partial W}{\partial z}\right|_{\alpha=\alpha_m} \cdot \dot{z}\,,$$

$$\dot{W}(\alpha_*, z) = \left.\frac{\partial W}{\partial z}\right|_{\alpha=\alpha_*} \cdot \dot{z}.$$

For definition of stability of the disturbed motion in neighborhood separatrice we introduce new criterion

$$\Lambda \equiv F(\alpha_m, z) \cdot \dot{\alpha}_m + \left.\frac{\partial W}{\partial z}\right|_{\alpha_*}^{\alpha_m} \cdot \dot{z}\,, \qquad (25)$$

Then conditions of stability (21) for internal area (A_1 or A_2) and (22) for external area A_0 will become, accordingly

$$\Lambda < 0\,,\ \Lambda > 0\,. \qquad (26)$$

On the basis of the carried out analysis it is possible to offer the following procedure of calculation of the top and bottom estimations of motion parameters with use of the average equations [4]. Numerical integration of the average equations is carried out from an initial point belonging to one of areas till the moment of crossing separatrice. Then it is calculated criterion (25) for each of areas A_0, A_1, A_2, and with the help of conditions (26) stability of disturbed motion is defined. The area from which there is an exit on separatrice, always is unstable, therefore there can be or one, or two stable areas. In the first case, numerical integration proceeds in stable area. In the second case, numerical integration for each stable area is carried out, in result is received the top and bottom estimations of the decision.

As an example the uncontrolled motion of analogue the Beagle 2 Lander is considered in the rarefied Mars's atmosphere. On fig. 6 two branches of decisions for angle attack are shown: $A_0 \rightarrow A_1$ and $A_0 \rightarrow A_2$.

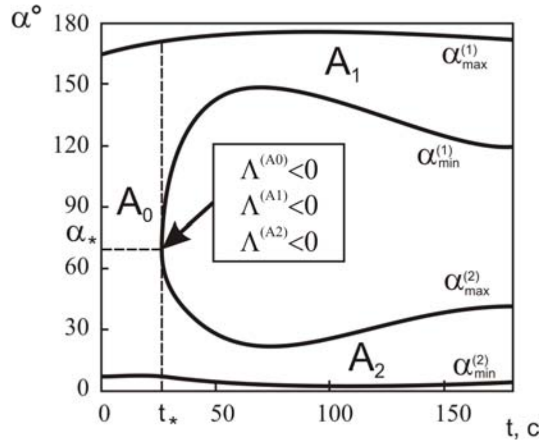

Fig. 6. Amplitudes of oscillations of the angle of attack.

7 Conclusion

Thus, we have shown, that exist transitive modes (resonance) at which parameters of motion considerably change at descent in the atmosphere of Mars for an axial-symmetric bodies having the biharmonic restoring moment. Criteria of stability of transitive modes are found and procedure is offered for the analysis of motion uncontrolled reentry vehicles of blunted conical shaper. It is shown, that if not to carry out the similar analysis of stability it is possible to overlook one of branches possible decisions, hence, to receive not genuine result.

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