Original Method for Stability Analysis of Mechatronic Systems with Asynchronous Machines

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Abstract: - This paper details an original method for the stability analysis of mechatronic systems driven by asynchronous machines. The mathematical basis of the method and a few characteristics obtained with an adequate Matlab program are presented. The windows of the program for monitoring of the experimental tests are also depicted. Finally a series of experimental tests and the conclusions confirming the validity of the proposed method are presented.

Key-Words: - stability, mechatronic system, asynchronous machine, variable frequency, parameters, software.

1 Introduction

Nowadays, the utilization on an ever larger scale of the asynchronous machine as an execution element in mechatronic systems, has imposed an ever ampler approach, in speciality papers, of problems concerning the dynamic regime of it and, implicitly, of stability problems.

The speciality literature [1], [12], [13], [16], [17] etc. presents a series of methods for the stability analysis of such systems.

Unfortunately, these methods either are very difficult to be implemented numerically or they have the drawback that they do not allow to study the inertia moment influence on stability, a very important thing especially in the case of the low power machines.

In order to eliminate these drawbacks, a new method for the stability study has been conceived, with the help of the equations with representative phasors written in per unit.

2 The mathematical model

The equations system that is used has the following form [8]:

$$
\begin{aligned}
&\omega_s^* = s_{ks}(\underline{\Psi}_s^* - k\underline{\Psi}_r^{/*}) + \frac{d\underline{\Psi}_s^*}{dt^*} + j\omega_s^*\underline{\Psi}_s^* \\
&0 = s_{kr}(\underline{\Psi}_r^{/*} - k\underline{\Psi}_s^*) + \frac{d\underline{\Psi}_r^{/*}}{dt^*} + j(\omega_s^* - \omega^*)\underline{\Psi}_r^{/*} \qquad (1)\\
&h\cdot\frac{d\omega^*}{dt^*} = -\frac{k}{x_{rt}^{/*}}\,\mathrm{Im}\left[(\underline{\Psi}_s^*)^*\,\underline{\Psi}_r^{/*}\right] - m_r^*
\end{aligned}
$$

These equations are linearized further on.

In order to do this thing, it is considered that the pulsation modifies in saltus with a very low value.

This variation will lead implicitly to a voltage modification, in saltus too, with the same value, so that the two quantities ratio to remain constant.

In this hypothesis the system (1) will modify as follows.

$$
\begin{aligned}
&\omega_s^* + \Delta\omega_s^* = s_{ks}\left[\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^* - k(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})\right] + \\
&+ \frac{d(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*)}{dt^*} + j(\omega_s^* + \Delta\omega_s^*)(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*) \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2)\\
&0 = s_{kr}\left[\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*} - k(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*)\right] + \\
&+ \frac{d(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})}{dt^*} + \\
&+ j(\omega_s^* + \Delta\omega_s^* - \omega^* - \Delta\omega^*)(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})
\end{aligned}
$$

$$
\begin{aligned}
&h\cdot\frac{d(\omega^* + \Delta\omega^*)}{dt^*} = -\frac{k}{x_{rt}^{/*}}\cdot \\
&.\mathrm{Im}\left\{\left[(\underline{\Psi}_s^*)^* + \Delta(\underline{\Psi}_s^*)\right]\cdot(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})\right\} - m_r^*
\end{aligned}
$$

The following relation is obtained by applying Laplace transformation to the first two equations of the systems (1) and (2), by subtracting member by member and by neglecting the products of the form $\Delta\cdot\Delta$:

$$\Delta\omega_s^* = (s_{ks} + j\omega_s^* + s)\cdot\Delta\underline{\Psi}_s^* - s_{ks}\cdot k\cdot\Delta\underline{\Psi}_r^{/*} + $$
$$+ j\cdot\underline{\Psi}_s^*\cdot\Delta\omega_s^*$$
$$0 = -s_{kr}\cdot k\cdot\Delta\underline{\Psi}_s^* + (s_{kr} + s)\Delta\underline{\Psi}_r^{/*} + j(\Delta\omega_s^* - \Delta\omega)\underline{\Psi}_r^{/*}$$
$$h\frac{d(\Delta\omega^*)}{dt} = -\frac{k}{x_{st}^*}\mathrm{Im}\left[\left(\underline{\Psi}_s^*\right)\cdot\Delta\underline{\Psi}_r^{/*} + \underline{\Psi}_r^{/*}\cdot\Delta\left(\underline{\Psi}_s^*\right)^*\right], \tag{3}$$

where s is the operational variable.

It must also be noticed that for simplifying the writing and for not producing confusions, both in the previous relation and in the following ones, it has been given up both to indicate the quantities depending on s ($\Delta\omega_s^*(s)$, $\Delta\omega^*(s)$ etc.) and to note them with capitals.

If it is considered that $\Delta\omega^*$ is not less than 0,1 in the previous relations, the following approximations may be made:

$$j\underline{\Psi}_s^* = 1 \quad \text{and} \quad j\underline{\Psi}_r^{/*} = k\,. \tag{4}$$

This way, the first two relations from (3) become:

$$0 = (s_{ks} + j\omega_s^* + s)\Delta\underline{\Psi}_s^* - s_{ks}\cdot k\cdot\Delta\underline{\Psi}_r^{/*} \tag{5}$$
$$k\left(\Delta\omega^* - \Delta\omega_s^*\right) = -s_{kr}\cdot k\cdot\Delta\underline{\Psi}_s^* + (s_{kr} + s)\Delta\underline{\Psi}_r^{/*}$$

The analysis of these relations can be simplified if it is considered that $R_s \cong 0$. But this simplifying hypothesis leads to satisfactory results only inside the interval $\omega_s^* \in (0{,}5 \div 1)$.

So it is imposed to analyze the situation when $R_s \neq 0$, but considering that the studied phenomenon is linearized.

In this purpose, it is considered that the motor operated without load before modifying the frequency. In this situation, owing to the low frequency of the rotor current, its active component may be neglected.

Thus, one can write:

$$\Delta\underline{i}_r^{/*} = \Delta i_{dr}^{/*} + j\Delta i_{qr}^{/*} \cong \Delta i_{dr}^{/*} = \frac{\Delta\underline{\Psi}_r^{/*} - k\Delta\underline{\Psi}_s^*}{dx_s^*}\,. \tag{6}$$

The following relation is obtained by computations, by solving the system (5) relatively to $\Delta\Psi_s^*$ and $\Delta\Psi_r^{/*}$, by replacing these relations in (6):

$$\Delta i_{dr}^{/*} = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)}\cdot \tag{7}$$
$$\cdot k(\Delta\omega^* - \Delta\omega_s^*)$$

where the following notation has been used:

$$\varepsilon = (1 - k^2)s_{ks} = \frac{r_s^*}{x_s^*} = \frac{r_s^*}{x_r^{/*}}\,. \tag{8}$$

When $\omega_s^* \geq 0{,}1$ it results that it can be considered (with approximation):

$$\left(\underline{\Psi}_s^*\right)^* = 1 \quad \text{and} \quad \underline{\Psi}_r^{/*} = -jk \tag{9}$$

In these conditions, the following relation is obtained y applying Laplace transformation to the relation (9):

$$hs\cdot\Delta\omega^* = -\frac{k}{x_{st}^*}\mathrm{Re}(\Delta\underline{\Psi}_r^{/*} - k\Delta\underline{\Psi}_s^*)\,, \tag{10}$$

or, equivalently

$$hs\cdot\Delta\omega^* = -\frac{k}{x_{st}^*}\mathrm{Re}(\Delta\underline{\Psi}_{dr}^{/*} - k\Delta\underline{\Psi}_{ds}^*)\,, \tag{11}$$

respectively

$$hs\cdot\Delta\omega^* = -k\Delta i_{dr}^{/*}\,. \tag{12}$$

3 Simulations. Quantitative Results

Further on, for the study of the induction motor stability, the relations (7) and (12) established before are used. The first relation can be written in the form [8]:

$$\Delta\omega^* = -\frac{k}{hs}\cdot\Delta i_{dr}^{/*} \Leftrightarrow \Delta\omega^* = G_1(s)\cdot\Delta i_{dr}^{/*}\,, \tag{13}$$

with

$$G_1(s) = -\frac{k}{hs} \tag{14}$$

The second relation is processed analogously:

$$\Delta i_{dr}^{/*} = G_2(s)\cdot(\Delta\omega_s^* - \Delta\omega^*)\,, \tag{15}$$

where

$$G_2(s) = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)}\cdot k \tag{16}$$

The following configuration can be drawn by using (13) and (15).

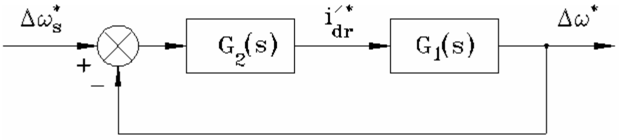
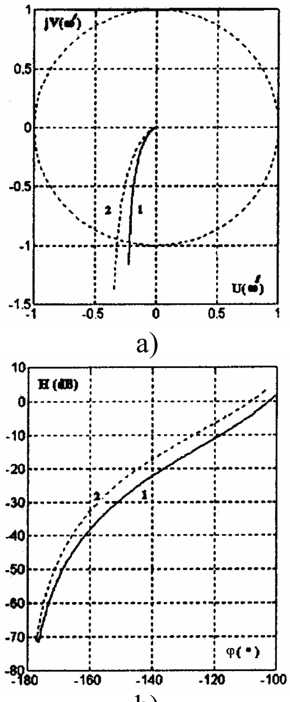


Fig.1. Machine block scheme in the mentioned situation.

Further on it is possible to pass to the stability study in our concrete case by using all these introductive notions. This analysis will be made with the help of a Matlab program conceived on the basis of the scheme depicted in figure 1 and of the relations (13), (14), (15) and (16).

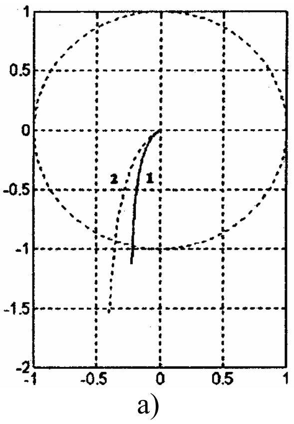
The following graphics have been obtained by running this program.



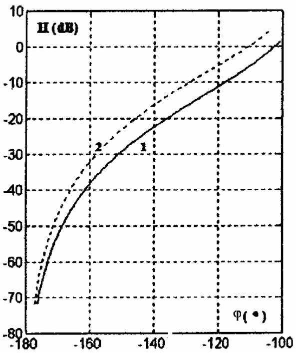
a)

b)

Fig.2. Transfer locus (a) and amplitude-phase characteristics (b) obtained in the case of the inductances modification: $L_s = 0{,}529$ H (1) and $L_s = 0{,}549$ H (2).

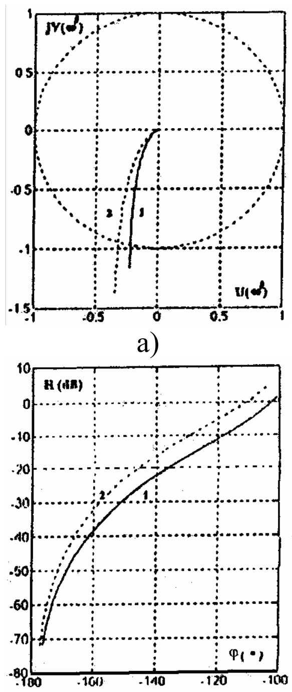


a)



b)

Fig.3. Transfer locus (a) and amplitude-phase characteristics (b) obtained in the case of the inductances modification: $L_r' = 0{,}528$ H (1) and $L_r' = 0{,}548$ H (2).



a)

b)

Fig.4. Transfer locus (a) and amplitude-phase characteristics (b) obtained in the case of the inductances modification: $L_{sh} = 0{,}498$ H (1) and $L_{sh} = 0{,}558$ H (2).

Observation 1

In order to establish the characteristics depicted in the previous figures it has been considered that the induction motor has the following parameters [6]: r_s^*=0,0989; $r_r'^*$=0,0725; x_s^*=2,1907; $x_r'^*$=2,1865; x_{1m}^*=2,0623; x_{st}^*=0,2456; x_{rt}^*=0,2451; s_{ks}=0,4026; s_{kr}=0,2958; k=0,9414; h=32,4; ε=0,0458.

Observation 2

With the help of a specially conceived Matlab program and of the characteristics corresponding to the cases when a parameter from the ones depicted in the second column of the table 1 is successively modified (over the initial case), the margins of phase depicted in the third column of the same table are obtained.

Table 1

Par.	Abs. value [Ω], [H], [kgm^2]	Per unit par.	Per unit value	Phase margin [degree]
R_s	7,5	r_s^*	0,0988	75,54
	2,5		0,0330	74,20
$R_r^/$	5,5	$r_r^{/*}$	0,0725	75,54
	4,5		0,0593	53,71
L_s	0,529	x_s^*	2,1907	75,54
	0,549		2,2735	69,13
$L_r^/$	0,528	$x_r^{/*}$	2,1865	75,54
	0,548		2,2694	67,31
L_{sh}	0,498	x_{1m}^*	2,0623	75,54
	0,438		1,8138	75,76
J	0,004	h	32,4	75,54
	0,003		24,3	47,65

The following **conclusions** can be emphasized, by analyzing the previous results:

- the decrease of the stator winding resistance leads to the stability decrease;
- the rotor resistance decrease has also as an effect, the decrease of the machine stability and conversely;
- the increase of the stator winding inductivity leads to the stability decrease;
- at the same time with the rotor inductivity increase, the system stability decreases;
- the main inductivity increase has a non-stabilizing effect;
- the inertia moment increase contributes to the stability increase.

In order to catch quantitatively these interdependences, the following table can be filled.

Table 2

Parameter	Per cent variation of the parameter	Per cent variation of the phase margin
R_s	66,6	2,04
$R_r^/$	18,2	28,89
L_s	3,64	8,48
$L_r^/$	3,93	10,89
L_{sh}	12,04	0,29
J	25	36,92

4 Program of experimental verification

In order to verify the conclusions emphasized before, the experimental circuit detailed in [10] has been carried out.

An original program has been conceived in Visual Basic for acquiring and processing the obtained data.

This program can be run by a double click applied on the pictogram placed on the desktop (figure 5).

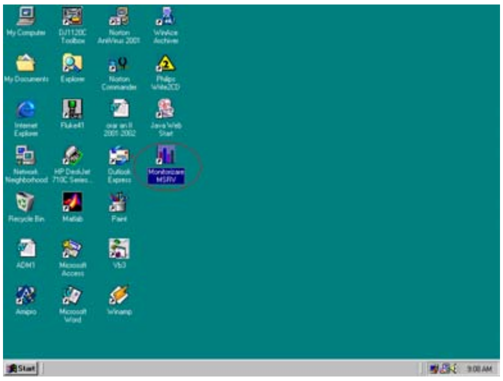


Fig.5. Desktop.

When running the program, the presentation cover of the program appears on the display, figure 6 (the texts corresponding to the following windows are written in the Romanian language).

Fig.6. Window „Coperta”.

The main windows of the program are detailed in figures 7-15.

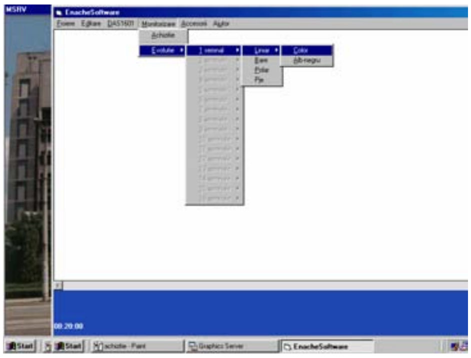


Fig.7. Window „Meniu”.

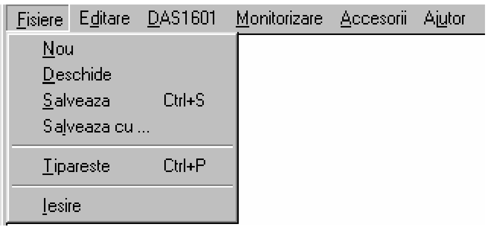


Fig.8. Details of menu „Fisiere”.

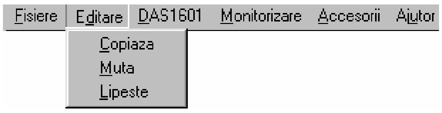


Fig.9. Details of menu „Editare”.

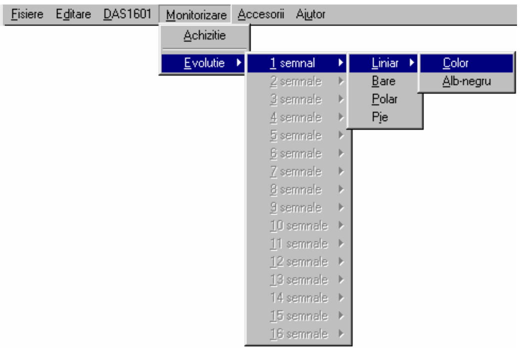
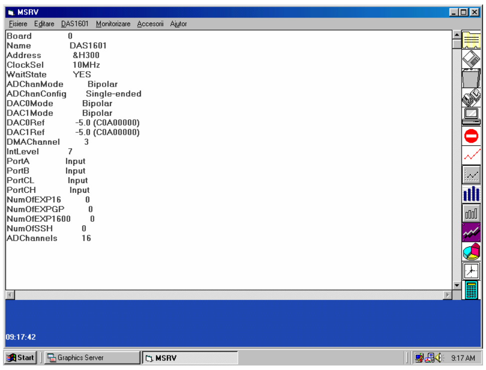
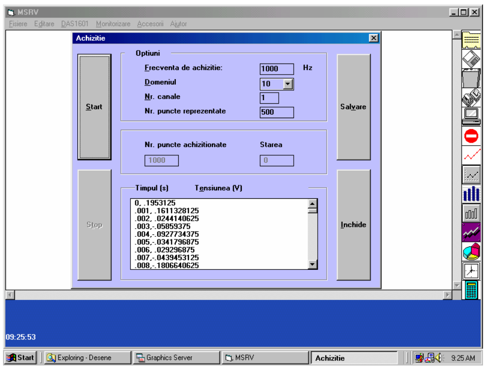


Fig.10. Details of menu Monitorizare”.



```
Board            0
Name             DAS1601
Address           &H300
ClockSel          10MHz
WaitState         YES
ADChanMode          Bipolar
ADChanConfig        Single-ended
DAC0Mode           Bipolar
DAC1Mode           Bipolar
DAC0Ref            -5.0 (C0A00000)
DAC1Ref            -5.0 (C0A00000)
DMAChannel          3
IntLevel         7
PortA           Input
PortB           Input
PortCL           Input
PortCH           Input
NumOfEXP16         0
NumOfEXPGP          0
NumOfEXP1600        0
NumOfSSH          0
ADChannels         16
```

Fig.11. Window „Configurare”.



Optiuni

Frecventa de achizitie: 1000 Hz

Domeniul 10

Nr. canale 1

Nr. puncte reprezentate 500

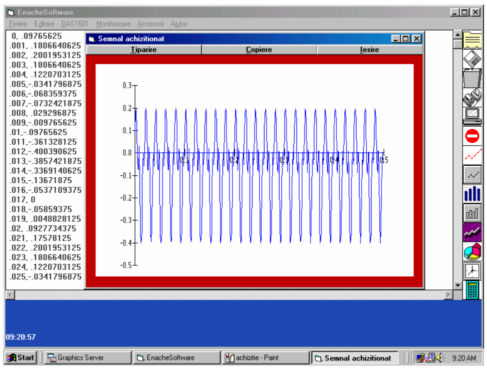
Nr. puncte achizitionate 1000

Starea 0

Timpul (s) Tensiunea (V)

```
0, .1953125
.001, .1611328125
.002, .0244140625
.003,-.05859375
.004,-.0927734375
.005,-.0341796875
.006, .029296875
.007,-.0439453125
.008,-.1806640625
```

Fig.12. Window „Achizitie”.



```
 0, .09765625
.001, .1806640625
.002, .2001953125
.003, .1806640625
.004, .1220703125
.005,-.0341796875
.006,-.068359375
.007,-.0732421875
.008, .029296875
.009,-.009765625
.01,-.09765625
.011,-.361328125
.012,-.400390625
.013,-.3857421875
.014,-.3369140625
.015,-.13671875
.016,-.0537109375
.017, 0
.018,-.05859375
.019, .0048828125
.02, .0927734375
.021, .17578125
.022, .2001953125
.023, .1806640625
.024, .1220703125
.025,-.0341796875
```

Fig.13. Window „Semnal”.

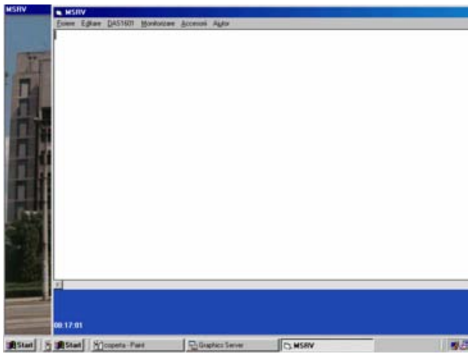
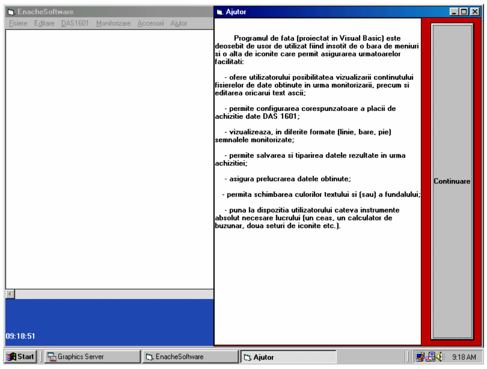


Fig.14. Window „Editare”.



Programul de fata (proiectat in Visual Basic) este deosebit de usor de utilizat fiind insotit de o bara de meniuri si o alta de iconite care permit asigurarea urmatoarelor facilitati:

- ofere utilizatorului posibilitatea vizualizarii continutului fisierelor de date obtinute in urma monitorizarii, precum si editarea oricarui text ascii;

- permite configurarea corespunzatoare a placii de achizitie date DAS 1601;

- vizualizeaza, in diferite formate (linie, bare, pie) semnalele monitorizate;

- permite salvarea si tiparirea datele rezultate in urma achizitiei;

- asigura prelucrarea datele obtinute;

- permita schimbarea culorilor textului si (sau) a fundalului;

- puna la dispozitia utilizatorului cateva instrumente absolut necesare lucrului (un ceas, un calculator de buzunar, doua seturi de iconite etc.).

Fig.15. Window „Ajutor”.

This program has many facilities:

- allows the configuration of the data acquisition board;
- ensures the work aided by a help window.
- ensures the acquisition corresponding to the dynamic signal we want;
- allows the visualization in different forms (line, bars, pie) for the acquired signal;
- allows to edit the files ASCII of the obtained data;
- allows to save and to type data;
- ensures the access to a series of accessories useful during work (pocket computer, clock);
- allows the configuration corresponding to the work interface (background and text colour, text dimensions, icons on the desktop).

5 Experimental results

A series of graphic results have been obtained with the help of the acquisition program detailed before; the following figures are presented further on.

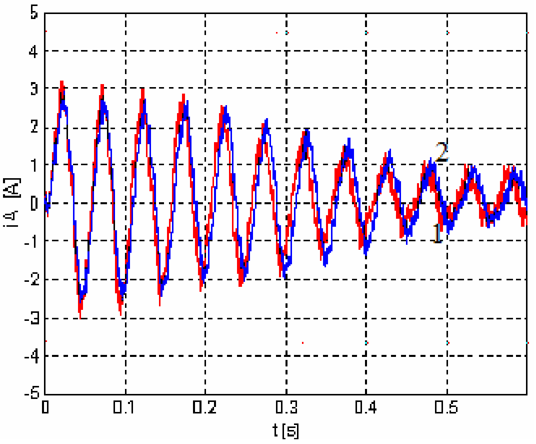


Fig.16. Graphics dependences corresponding to the cases L_s=0,529 H (1) and L_s=0,549 H (2).

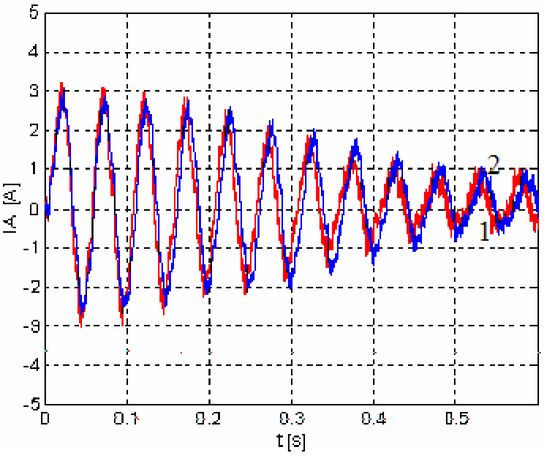


Fig.17. Graphics dependences corresponding to the cases $L^{/}$ =0,528 H (1) and $L^{/}$ =0,548 H (2).

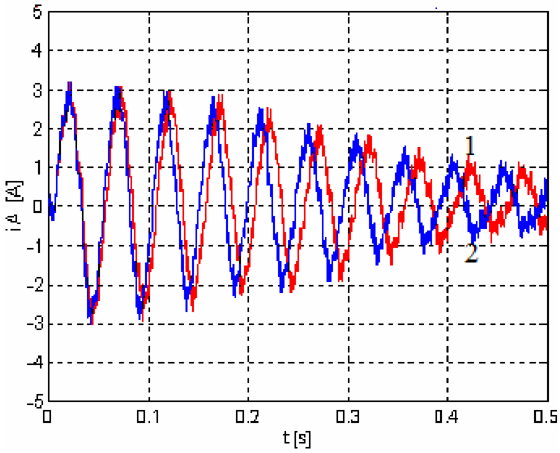


Fig.18. Graphics dependences corresponding to the cases L_{sh}=0,498 H (1) and L_{sh}=0,438 H (2).

6 Conclusion

These graphics lead to the following **conclusions**:

- when the value of the stator inductance increases the transient process duration increases (the stability decrease);
- the increase of the rotor inductance also involves the increase of the transient process duration (the stability decreases);
- the decrease of the main inductance value determines a faster stabilization of the process (stability increase);

These conclusions confirm the theoretical analysis performed before.

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