Robust Stability Analysis for 2-D Continuous-Time Systems Via Parameter-Dependent Lyapunov Functions

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Abstract: New sufficient conditions to test stability of 2-D linear continuos-time systems described by Roesser model with polytopic uncertainty are presented in this paper. Robust stability is guaranteed by the existence of a parameter-dependent Lyapunov function obtained from the feasibility of a set of Linear Matrix Inequalities(LMIs), formulated at the vertices of the uncertainty polytope. Two conditions are presented, and the results are compared with the analysis based on quadratic stability (same Lyapunov function for the entire set of uncertainties). Several examples are presented to illustrate the results.

Keywords: 2-D continuous-time systems, Robust stability, parameter-dependent Lyapunov functions, Linear matrix inequalities (LMIs), polytopic uncertainty, Roesser model.

1 Introduction

Analysis of uncertain continuous-time linear systems is a major topic in automatic control [1], [2] and [21]. In these systems the uncertainties can be due to approximations when a linearized model is calculated, to neglected dynamics, or to parameter variations. In the application of Lyapunov function to robust control problem. The simplest approach consists in looking for a common quadratic Lyapunov function (Quadratic stability) that proves stability of the polytope of matrices (see e.g [17]).

Unfortunately, QS tests can lead to very conservative results in some cases. Recently, different techniques based on parameter-dependent [3], [4], [6-11] or piecewise Lyapunov functions [12-13] have appeared, providing less conservative results. Among this literature, some focus on continuous-time systems [5-7,10,11] and other on the discrete-time ones [3], [4], [10]. The key idea in the above paper is to introduce new variables and large the dimension of the LMIs to obtain sufficient conditions for the existence of a parameter dependent Lyapunov function.

In the study of distributed systems, partial differential equations will arise, these partial equations actually correspond to 2-D or n-D continuous-time systems [15], [16]. Therefore

the study of 2-D continuous-time systems is both of practical and theoretical importance.

In this paper, we deal with the problem of stability for uncertain 2-D continuous-time systems. The class of 2-D continuous-time systems under consideration is described by the Roesser model with polytopic-type uncertainty. New sufficient conditions for the robust stability are obtained from the feasibility of a set of linear matrix inequalities (LMIs) formulated at the vertices of uncertainty polytope. Several examples are presented to illustrate the results.

This note is organized as follows: Section 2 presents the problem formulation and some preliminaries results. In section 3 we give the main results. Section 4 uses numerical examples to illustrate the effectiveness of the proposed methods. Finally, some conclusions are presented.

Notation: The following notation will be used throughout this paper: R denote the set of real numbers, R^n denotes the n-dimensional Euclidean space and $R^{m \times n}$ denotes the set of $m \times n$ real matrices. The notation X > 0 and Y > 0, where X and Y are symmetric matrices, indicates that the matrix X - Y is positive definite.

2 Problem Formulation and Preliminaries

Let the 2-D continuous-time system described by Roesser model: [20].

$$(\Sigma): \begin{bmatrix} \frac{\partial}{\partial t_1} x^h(t_1, t_2) \\ \\ \frac{\partial}{\partial t_2} x^v(t_1, t_2) \end{bmatrix} = A \begin{bmatrix} x^h(t_1, t_2) \\ x^v(t_1, t_2) \end{bmatrix} \quad (1)$$

where $x^{h}(t_{1}, t_{2}) \in \mathbb{R}^{n_{1h}}$ and $x^{v}(t_{1}, t_{2}) \in \mathbb{R}^{n_{v}}$ aret he horizontal and vertical states, respectively; $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \in \mathbb{R}^{(n_{h}+n)_{v} \times (n_{h}+n_{v})}$ i

s the dynamic matrix.

We first introduce the notion of asymptotic stability of 2-D continuous-time systems.

Lemma 2.1[18],[19]. The 2-D linear continuous-time system (1) is asymptotically stable if there exist matrices $0 < P_h \in R^{n_h \times n_v}$ and $0 < P_v \in R^{n_h \times n_v}$ such that the following LMI holds

$$A^T P + P A < 0 \tag{2}$$

where $P = diag(P_h, P_v)$.

Suppose now that A is not precisely known, but belongs to a polytopic uncertain domain Ω . In this way, any matrix inside the domain Ω can be written as a convex combination of the vertices A_i of the uncertainty polytope: i.e.,

$$\Omega = \left\{ A(\alpha) : A(\alpha) = \sum_{i=1}^{N} \alpha_i A_i \quad ; \quad \sum_{i=1}^{N} \alpha_i = 1 \; ; \; \alpha_i \ge 0 \right\} (3)$$

where $A_i = \begin{bmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{bmatrix}$.

We begin our discussion by defining Robust stability with respect to system (1) and the structured model (3):

Definition 2.1. System (1) is robustly stable in the uncertainty domain (3) if there exists a parameter-dependent matrix $P(\alpha) = diag(P_h(\alpha), P_v(\alpha))$, such that

$$A^{T}(\alpha)P(\alpha) + P(\alpha)A(\alpha) < 0$$
 (4)

for all α such that $A(\alpha) \in \Omega$. where

To the authors' knowledge, there is no general and systematic way to formally

determine $P(\alpha)$ as a function of the uncertain parameter α . Such a matrix $P(\alpha)$ is called a parameter dependent Lyapunov matrix. The simplest method to solve this problem is to look for a single Lyapunov matrix $P(\alpha) = P$ which solves inequality (4). Unfortunately, this approach is known to provide quite conservative results, but it constitutes one of the first results in the quadratic approach. The test for this kind of stability, also known as quadratic stability (QS) test, is summarized in the following lemma:

Lemma 2.2 The uncertain system (1) is robustly stable in the uncertainty domain (3) if there exist matrices $0 < P_h \in \mathbb{R}^{n_h \times n_v}$ and $0 < P_v \in \mathbb{R}^{n_h \times n_v}$ such that

$$A_i^T P + P A_i < 0 \tag{6}$$

for all i = 1, ..., N.

3 Main Results

We begin this section by stating the following equivalence.

Theorem 3.1: The following conditions are equivalent:

i) there exist matrices $0 < P_h \in \mathbb{R}^{n_h \times n_h}$, and $0 < P_\nu \in \mathbb{R}^{n_\nu \times n_\nu}$ such that :

$$A^T P + PA < 0 \tag{7}$$

where $P = diag(P_h, P_v)$

ii) there exist matrices $0 < P_h \in \mathbb{R}^{n_h \times n_h}$, $0 < P_v \in \mathbb{R}^{n_v \times n_v}$ and matrices F and G such that

$$\begin{bmatrix} FA + A^T F^T & P - F + A^T G^T \\ * & -G - G^T \end{bmatrix} < 0 (8)$$

where $P = diag(P_h, P_v)$ and $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$.

Proof of Theorem 3.1: Before giving the proof of this theorem, a useful lemma is presented.

Lemma 3.1 [23] Let matrices $\widetilde{A} \in \mathbb{R}^{m \times n}$, $\widetilde{B} \in \mathbb{R}^{m \times l}$, $\widetilde{C} \in \mathbb{R}^{l \times n}$, $\widetilde{D} \in \mathbb{R}^{l \times l}$ and a matrix function $M : \mathbb{R}^{n \times m} \to \mathbb{R}^{n \times n}$ be given.

Then, the following two conditions are equivalent:

i)The matrix \widetilde{D} is invertible and there exist $\widetilde{P} \in \mathbb{R}^{n \times m}$ such that

$$M(\widetilde{P}) + He\left\{\widetilde{P}(\widetilde{A} + \widetilde{B}\widetilde{D}^{-1}\widetilde{C})\right\}$$
(9)

ii) There exist matrices $\widetilde{P} \in \mathbb{R}^{n \times m}$, $F_1 \in \mathbb{R}^{n \times l}$ and $F_2 \in \mathbb{R}^{l \times l}$ such that

$$\begin{bmatrix} M(\widetilde{P}) + He \left\{ \widetilde{P} \widetilde{A} \right\} & \widetilde{P} \widetilde{B} \\ \widetilde{B}^T \widetilde{P}^T & 0 \end{bmatrix} + He \left\{ \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \begin{bmatrix} \widetilde{C} & \widetilde{D} \end{bmatrix} \right\} < 0 \ (10)$$

Moreover, for every solution $\widetilde{P} = P$ of (9), then exists a sufficiently small $\varepsilon > 0$ such that $(\widetilde{P}, F_1, F_2) = (P, P\widetilde{B}\widetilde{D}^{-1}, \varepsilon\widetilde{D}^{-1})$ is a solution of (10). Conversely, every matrix \widetilde{P} such that (10) holds for some F_1 and F_2 also satisfies (9).

Remark 3.1: In Lemma 3.1, the matrix inequality (9) can be regarded as an LMI for the analysis and synthesis frequently used in the previous studies (Boyd et al. 1994, skelton et al. 1997), while (10) is a dilated LMI corresponding to (9). This Lemma is simple but crucial generalization of the results in the oliveira and skelton (2001) and Peaucelle and al. (2000).

Proof of Theorem 3.1: Let us take M = 0, $\tilde{A} = 0$, $\tilde{B} = I$, $\tilde{C} = A$, $\tilde{D} = I$ and $\tilde{P} = P > 0$. Then, the matrix inequality (9) reduces to the Lyapunov inequality (7) with respect to the Lyapunov variable *P*. On the other hand, the matrix inequality (10)

reduces to (8) with $F_1 = F$ and $F_2 = G$.

Remark 3.2: Condition *ii*) appears as a direct expansion of condition *i*) via its "Schur complement" formulation, where introducing the new additional matrices F and G, we obtain a linear matrix inequality in which the Lyapunov matrix P is not involved in any product with the dynamic matrix. This feature enables one to write a new robust asymptotic stability conditions which, although sufficient, are assumed not too conservative due to presence of the extra degree of freedom provides by the introduction of matrices F and G (see the Numerical examples).

The next two Lemmas state sufficient conditions for the existence of a parameter-

dependent Lyapunov function
$$P(\alpha) = P^{T}(\alpha) > 0$$
, given by:

$$P(\alpha) = \sum_{i=1}^{N} \alpha_i \begin{bmatrix} P_{hi} & 0\\ 0 & P_{vi} \end{bmatrix}, \ \alpha_i \ge 0, \ \sum_{i=1}^{N} \alpha_i = 1 \quad (11)$$

such that

$$M(\alpha) = A^{T}(\alpha)P(\alpha) + P(\alpha)A(\alpha) < 0 \quad 12)$$

Holds

Lemma 3.2 The uncertain system (1) is robustly stable in the uncertainty domain (3) if there exist matrices $0 < P_{hi} \in \mathbb{R}^{n_h \times n_v}$ and $0 < P_{vi} \in \mathbb{R}^{n_h \times n_v}$ such that

$$M_i = A_i^T P_i A_i - P_i < -I, \quad i = 1,.., N$$
(11)

$$M_{jk} = A_j^T P_k + A_k^T P_j + P_j A_k + P_k A_j < \frac{2}{N-1}I,$$

$$j = 1, ..., N-1, , k = j+1, ..., N$$
(14)

Proof:

Note that $M(\alpha)$ in (12) can be written as

$$M(\alpha)\sum_{i=1}^{N}\alpha_{i}^{2}M_{i} + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_{i}\alpha_{j}M_{ij} \quad (15)$$

Imposing (13) and (14) and taking into account that $\alpha_i \ge 0$, with $\sum_{i=1}^{N} \alpha_i = 1$, it is possible to apply the results of [21]. This results imply that (12) holds for all admissible α .

Lemma 3.3 2-D Uncertain system (1) is robustly stable in the uncertainty domain (3) if there exist matrix $P_i = diag(P_{hi}, P_{vi})$ F_i and G_i such that

$$S_{i} = \begin{bmatrix} F_{i}A_{i} + A_{i}^{T}F_{i}^{T} & -F_{i} + P_{i} + A_{i}^{T}G^{T} \\ * & -G_{i} - G_{i}^{T} \end{bmatrix} < -I$$

$$i = I, \dots, N \qquad (16)$$

$$S_{jk} \equiv \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ * & \Gamma_{22} \end{bmatrix} < \frac{2}{N-1}I, \qquad (17)$$

$$i = 1, \dots, N-1 \qquad k = j+1, \dots, N$$

where

$$\begin{split} \Gamma_{11} &= F_k A_j + F_j A_k + A_j^T F_k^T + A_k^T F_j^T \\ \Gamma_{12} &= -F_j - F_k + P_j + P_k + A_j^T G_k^T + A_k^T G_j^T \\ \Gamma_{22} &= -G_j - G_k - G_j^T - G_k^T \end{split}$$

Proof: Following the same steps of the proof of Lemma 3.2, note that (12) can be written as

$$M(\alpha) = T(\alpha) \left[\sum_{i=1}^{N} \alpha_i^2 S_i + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \alpha_j \alpha_j S_{ij} \right] T^T(\alpha) < 0$$

with

 $T(\alpha) = [I \quad A^T(\alpha)]$

3.1 Maximum stability domain

An important problem in uncertain system is to compute the maximum robust stability domain. In fact, as the techniques proposed in the literature and in this paper are sufficient, the best method to compare them is to estimate the maximum robust stability domain obtained with the corresponding technique: the biggest the domain is, the least conservative the proposed technique.

It is now briefly discussed how using the results of this paper it is possible to estimate such domain of stability. For this, consider the system described by the following state-space equation

$$:\left[\frac{\frac{\partial}{\partial t_1}x^h(t_1,t_2)}{\frac{\partial}{\partial t_2}x^\nu(t_1,t_2)}\right] = (A_0 + \sum_{i=1}^N \alpha_i E_i) \begin{bmatrix} x^h(t_1,t_2) \\ x^\nu(t_1,t_2) \end{bmatrix} (19)$$

where $x^{h}(t_{1}, t_{2}) \in R^{n_{1h}}$ and $x^{v}(t_{1}, t_{2}) \in R^{n_{v}}$ are horizontal and vertical states, the respectively; $A_0 \in R^{(n_h+n)_v \times (n_h+n_v)}$ is the nominal matrix of the system, $E_i \in \mathbb{R}^{(n_h+n)_v \times (n_h+n_v)}, i = 1,..,N$, are given matrices representing the directions of perturbation and $\alpha_i \in R, i = 1, \dots, N$ are constant values defining the amount of perturbations allowed for $\alpha_i \in [-\alpha_i^m, \alpha_i^M]$, i = 1, ..., N. The system (19) can be viewed as an uncertain dynamic system belonging to the convex compact set defined by its 2^N vertices.

Supposing that A_0 is asymptotically stable and using the results of Lemmas 3.2 and 3.3, and also applying the algorithm given in [10], it is possible to compute an estimate of the maximum robust stability domain, for details concerning the algorithm; the reader is referred to [11]

4 Examples

Some Examples have been generated in order to provide a numerical evaluation of the

conditions for robust stability presented in the paper. They show that the conditions stated by Lemma 3.3 are less conservative than the ones stated by Lemma 3.2 and Lemma 2.2.

4.1 First Example

In the first example the stability domain radius (i.e., symmetric domain) is investigated.

		Ine	e sy	ster	n 15	gıv	/en	ı by	(19),		
					W	ith						
	- 0.	057	1	- 0.	009	6	:	0.0	873		0.0	300 -
	- 0.	075	5	- 0.	035	5	:	0.0	245		0.0	574
$A_{0} =$:					••
	- 0.	045	2	- 0.	015	5	:	- 0.0)954	4	- 0.0	0306
	0.	064	7	- 0.	099	3	:	- 0.0)03′	7	- 0.0	0390
		0	0	0	0			[0	0	0	0	
E	7 _	a	0	0	0	F	_	0	а	0	0	
	$z_1 =$	0	0	0	0	<i>L</i> ₂	=	0	0	0	a	
		a	0	0	0			0	0	0	0	

where *a* is a free parameter.

.33
.9
4
.9
6
64
)

Table 1: Symmetric Stability Domain

Nsy	Lemma 2.2			
а	$\alpha_{_1}$	α_{2}		
0.1	[-0.1240,0.1240]	[-0.1240,0.1240]		
0.2	[-0.0620,0.0620]	[-0.0620,0.0620]		
0.5	[-0.0248,0.0248]	[-0.0248,0.0248]		
0.8	[-0.0155,0.0155]	[-0.0155,0.0155]		
1	[-0.0124,0.0124]	[-0.0124,0.0124]		

Table 2: Nonsymmetric Stability Domain

Nsy	Lemma 3.2			
a	$\alpha_{_1}$	α_{2}		
0.1	[-0.1651,0.1649]	[-0.7572,0.1649]		
0.2	[-0.0827,0.0824]	[-0.3784,0.0824]		
0.5	[-0.0332,0.0329]	[-0.1532,0.0329]		
0.8	[-0.0206,0.0206]	[-0.0967,0.0206]		
1	[-0.0167,0.0164]	[-0.0775,0.0164]		
Table	2. Nongruman strig St	ability Domain		

 Table 3: Nonsymmetric Stability Domain

Nsy	Lemma 3.3			
а	$\alpha_{_1}$	α_{2}		
0.1	[-0.1651,0.1649]	[-1.3720,0.1649]		
0.2	[-0.0827,0.0824]	[-0.6857,0.0824]		

0.5	[-0.0332,0.0329]	[-0.2743,0.0329]
0.8	[-0.0206,0.0206]	[-0.1717,0.0206]
1	[-0.0167,0.0164]	[-0.1371,0.0164]

Table 4: Nonsymmetric Stability Domain

Table 1 exhibits the maximum values of α with the conditions of Lemmas 2.2, 3.2 and 3.3 (symmetric domains). On the other hand, tables 2, 3 and 4 show the results for maximum non symmetric robust stability domains, the overall volume of the domains achieved with the results of Lemma 3.3 is always greater than the corresponding ones obtained with Lemma 3.2 and Lemma 2.2.

4.2 Second Example

The second example is given by the stable polytope that corresponds to the following vertices (randomly generated):

	-0.0571	- 0.0096	÷	0.0873	0.0300	
	-0.0595	-0.0195	÷	0.0245	0.0574	
$A_1 =$			÷			
	-0.0452	-0.0155	÷	-0.0954	- 0.0146	
	-0.0487	- 0.0993	÷	-0.0037	- 0.0390	
	- 0.0571	- 0.0096	÷	0.0873	0.0300	
	- 0.0595	-0.1355	÷	0.0245	0.0574	
$A_2 =$			÷			
	-0.0452	-0.0155	÷	- 0.0954	- 0.1306	
	- 0.0487	- 0.0993	÷	-0.0037	- 0.0390	
	-0.0571	-0.0096	÷	0.0873	0.0300	
	-0.0915	- 0.0195	÷	0.0245	0.0574	
$A_3 =$			÷			
	-0.0452	-0.0155	÷	-0.0954	- 0.0146	
	-0.0807	- 0.0993	÷	-0.0037	- 0.0390	
	-0.0571	-0.0096	÷	0.0873	0.0300	
	-0.0915	-0.1355	÷	0.0245	0.0574	1
$A_{4} =$			÷			1
	-0.0452	-0.0155	÷	-0.0954	-0.1306	
	-0.0807	-0.0993	÷	-0.0037	-0.0390	

n this case, sufficient conditions presented in Lemmas 2.2 and 3.2 fail. However, using the proposed Lemma 3.3, one gets a solution, corresponding to the vertices of the parameter dependent Lyapunov matrices.

$$P_{1} = 10^{5} \times \begin{bmatrix} 8.3578 & -0.9567 & 0 & 0 \\ -0.9567 & 3.8710 & 0 & 0 \\ 0 & 0 & 8.7898 & -0.3901 \\ 0 & 0 & -0.3901 & 1.9270 \end{bmatrix},$$
$$P_{2} = 10^{6} \times \begin{bmatrix} 1.0029 & 0.5782 & 0 & 0 \\ 0.5782 & 1.4293 & 0 & 0 \\ 0 & 0 & 0.5916 & -0.0760 \\ 0 & 0 & -0.0760 & 1.3072 \end{bmatrix},$$
$$P_{3} = 10^{6} \times \begin{bmatrix} 0.4064 & -0.2106 & 0 & 0 \\ -0.2106 & 0.1705 & 0 & 0 \\ 0 & 0 & 1.1822 & -0.1179 \\ 0 & 0 & -0.1179 & 0.0507 \end{bmatrix},$$

	1.0199	0.7813	0	0]
$p = 10^{6}$ v	07813	1.3399	0	0
$P_4 = 10 \times$	0	0	0.4660	-0.0970
	0	0	-0.0970	1.1031

The corresponding slack matrices F_i , G_i are given by:

$$F_{1} = 10^{5} \times \begin{bmatrix} 8.2638 & -1.0275 & 0.2537 & 0.0579 \\ -1.0272 & 3.9205 & 0.0298 & 0.1026 \\ -0.0222 & 0.3015 & 8.3247 & -0.4820 \\ -0.0182 & -0.1833 & -0.3517 & 1.9831 \end{bmatrix},$$

$$F_{2} = 10^{6} \times \begin{bmatrix} 0.9899 & 0.5440 & 0.0637 & -0.0119 \\ 0.5546 & 1.3465 & 0.1013 & -0.0135 \\ -0.0010 & 0.0367 & 0.5256 & -0.0777 \\ -0.0020 & 0.0158 & -0.1176 & 1.2891 \end{bmatrix},$$

$$F_{3} = 10^{5} \times \begin{bmatrix} 4.0266 & -2.0742 & 0.1882 & 0.0885 \\ -2.1520 & 1.8697 & -0.1310 & 0.1254 \\ -0.0214 & 1.7345 & 9.8856 & -1.3720 \\ -0.0378 & -0.1729 & -1.3247 & -0.5841 \end{bmatrix}$$

$$F_{4} = 10^{6} \times \begin{bmatrix} 0.9997 & 0.7242 & 0.0367 & -0.0137 \\ 0.7519 & 1.2280 & 0.1122 & -0.0125 \\ -0.0011 & 0.0552 & 0.3810 & -0.0967 \\ -0.4042 & 0.0293 & -0.1626 & 1.0869 \end{bmatrix}$$

$$G_{1} = 10^{5} \times \begin{bmatrix} 1.5059 & 0.7396 & 0.7098 & 0.5088 \\ -0.7058 & 1.6299 & 1.0853 & -0.3346 \\ -0.7380 & -1.3160 & 1.5168 & -0.2601 \\ -0.5183 & 0.2984 & 0.2777 & 1.4978 \end{bmatrix},$$

	1.5132	0.2722	0.3435	0.1488
$C = 10^5 v$	-0.2425	1.5800	-2.1255	1.4475
$G_2 = 10^{\circ} \times$	-0.4399	1.5540	2.3298	-0.8291
	-0.1315	-1.3832	0.6882	1.5245
	1.5062	0.7816	0.5755	0.4925]
$C = 10^5 v$	-0.6300	2.7461	2.5486	-0.3892
$G_3 = 10 \times$	-0.7065	-4.2665	2.0711	0.3943
	-0.5177	0.2391	-0.2199	1.5011
	[1.5133	0.0851	0.0470	0.5803
$C = 10^5 \text{ v}$	-0.0351	1.7246	-2.7039	1.8735
$G_4 = 10^{\circ} \times$	-0.1509	1.5531	3.0168	-0.6309
	-0.5544	-1.7707	0.4676	1.5289

4.3 Third ExampleNumerical Evaluation

A numerical evaluation procedure is now considered, to check the improvements obtained with the proposed methods. Recall that the Kind of systems we are dealing with is characterized by its order $(n = n_h + n_y)$, and the number of vertices (N). For each fixed $(n,N) \in [2,4], 1000$ systems were randomly generated of the form [22]. Thus, a total of 9000 stable polytopes was generated. Each polytope of stable plants was evaluated by the different lemmas, and the results are summarized in Table 5, that gives the number of success of the different methods, which gives а measure of their performance and conservativeness. It can be seen that the tests given by Lemma 3.2 and Lemma 3.3 prove to be less conservative than test given by Lemma 2.2. However, the test using Lemma 3.3 is more demanding computationally when compared to other tests.

n	Ν	Lemma2.3	Lemma3.2	Lemma3.3
	2	958	1000	1000
2	3	880	1000	1000
	4	808	994	1000
	2	833	935	950
3	3	628	825	840
	4	445	710	745
	2	452	588	630
4	3	157	325	352
	4	44	155	221

Table I. Number of stable polytopes identified by the proposed Lemmas for $2 \le n \le 4$, and $2 \le N \le 4$.

5 Conclusion

New robust sufficient stability conditions for 2-D linear continuous-time systems described by Roesser model have been given. The conditions are formulated in terms of a set of LMIs described only in terms of the vertices of the uncertainty domain. Several examples have been presented that illustrate the results, showing the feasibility of the proposed approaches.

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