Adaptive Fuzzy Tracking Control of Nonlinear Systems

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Abstract: -Adaptive linearization controllers have been shown to have nice control performance. However, two functions in the controllers are derived from the considered system. Thus, those controllers can only work for known systems. In this paper, we proposed a fuzzy modeling approach to model those two functions. The proposed approach is called the adaptive model reference fuzzy control. In this approach, the considered dynamic nonlinear model can be unknown. Different from previous adaptive fuzzy controllers, our approach does not need any auxiliary operations on input trajectories and on system states. The proposed controller and the weight update laws only need system states and the current desired output without using any their derivatives. The Lyapunov stability theorem is used to derive controller parameters update laws, which ensure that the system states be bounded and the plant output asymptotically tracks an arbitrary piecewise reference trajectory. The proposed method is successfully applied to an unstable nonlinear system and a chaotic system. The learning and control performance of our approach is nice and also superior to that of previous approaches.

Key-Words: - model reference, adaptive ,fuzzy control, Lyapunov stability, nonlinear system

1. Introduction

Adaptive control schemes for nonlinear systems via feedback linearization concept have been employed for decades. The idea of feedback linearization approaches is to transform a nonlinear dynamic system into a linear system through state feedback mechanisms. With such transformations, those well-explored linear control skills can then be applied to meet desired control specifications. Several nice results have been reported in [1-3]. The major deficiency of those approaches is that their good performance is largely relied on exact cancellation of nonlinear teams. If uncertainties exist in those nonlinear terms, the performance may be awful due to non-exact cancellation. Although several parameter adaptive control schemes have been proven to be asymptotically exact cancellation, they are valid only when the variations are on the coefficients of nonlinear terms [4]. In this study, we intended to apply fuzzy modeling techniques to cope with unknowns in systems while employing adaptive linearization control schemes.

Since Zadeh introduced the fuzzy set theorem in 1965 [5], it has received much attention from various fields and has also demonstrated nice performance in various applications. One of those successful fuzzy applications is to model unknown systems by a set of fuzzy rules. One important property of fuzzy modeling approaches is that they are universal approximators [6]. In other words, fuzzy systems can be used to model virtually any systems within a required accuracy provided that enough rules are given. It should be noticed that this universal approximation property only states the existence of such nice fuzzy systems and does not provide any mechanisms to obtain them. Various approaches have been proposed in the literature [7-10,21-24,26-27] to obtain fuzzy systems that can actually achieve nice modeling accuracy. In this paper, we reported our study in employing fuzzy systems to estimate functions that are required in constructing an adaptive linearization controller. In this approach, based on the Lyapunov stability theorem, a learning mechanism is proposed to guarantee asymptotical convergence of the estimation of those functions by fuzzy systems.

In fact, various fuzzy adaptive control schemes, which incorporate fuzzy systems into adaptive control schemes, have already been proposed in the literature [11-14]. Those approaches are based on the idea proposed by Wang [11]. The idea of this kind of adaptive control is to directly cancel nonlinear functions in dynamic systems and to form a response behavior model. However, due to the requirement of using up to the $(n\text{-}1)^{\text{th}}$ order derivatives of the output and the n^{th} order derivatives of the input trajectory, the approaches can only control unknown systems to track pre-defined continuous trajectories. As a matter of fact, those derivatives are generally not available while tracking trajectories that are not continuous or not previously defined. Another problem is that derivatives of real-world signals are very easily affected by noise. Thus, we proposed a novel adaptive fuzzy controller based on the linearization control ideas proposed in [4]. The proposed controller can avoid using those derivatives by including a reference model in the update law derivation. Since the proposed approach does not need any auxiliary operations on the system output and on the trajectory, it can be employed in various applications, especially used to trace piecewise continuous trajectories or input that is not previously defined.

A relative approach has also been reported in [18]. In that approach, the same form of controller is used and a radial basis function network is implemented to approximate two functions in the controller. A similar adaptive controller to our approach is obtained for systems with relative degree one. However, for higher relative degree systems, the update laws of that approach adopted the normalized gradient update law stated in [1]. In our opinion, there exist problems in such an approach. In our study, we have also implemented that update law in our simulation and the control performance is not acceptable as we expected. In fact, in [18], no higher degree systems are used for simulation. In this paper, an SPR-Lyapunov design method is employed to provide better controller parameter update laws for higher relative degree systems. It can be seen that our approach can have faster convergent behavior and better control performance when compared to the traditional adaptive fuzzy controllers or the approach proposed in [18].

The paper is organized as follows. Section 2 introduces the concept of feedback linearization controllers. Then, the proposed adaptive model reference fuzzy controller is shown in section 3. The updating algorithms and stability analysis for the proposed controller are derived in section 4. In section 5, simulation results are presented to confirm the feasibility and superiority of the proposed method. Finally, conclusion remarks are given in section 6.

2. Feedback Linearization Controllers

Consider a continuous time dynamic system of the form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ z &= h(x) \end{aligned} \qquad (1)$$

where $x = [x_1, x_2, \cdots, x_n]$ is an n-dimensional vector for the state variables, which are assumed measurable, $f(x)$, $g(x)$, and $h(x)$ are nonlinear smooth functions, u is the scalar manipulated input variable, and z is the output variable. The state feedback nonlinear control law [1] suggests that the control input be

$$u=\alpha+\beta v, \qquad (2)$$

where

$v = -a_\gamma L_f^{\gamma-1} h(x) - a_{\gamma-1} L_f^{\gamma-2} h(x) - \cdots - a_1 h(x) + a_1 z_{sp}$,

$\alpha = \dfrac{-L_f^\gamma h(x)}{L_g L_f^{\gamma-1} h(x)}$, and $\beta = \dfrac{1}{L_g L_f^{\gamma-1} h(x)}$. Here, a_1, …, a_γ are constants selected to define the response behavior, z_{sp} is a bounded reference input to be tracked for the current step, $L_f^\gamma h(x)$ and $L_g L_f^{\gamma-1} h(x)$ are *Lie* derivatives and are defined as

$L_f^\gamma h(x) = \dfrac{\partial (L_f^{\gamma-1} h(x))}{\partial x} f(x)$ with $L_f^0 h(x) = h(x)$

and $L_g L_f h(x) = \dfrac{\partial (L_f h)}{\partial x} g(x)$, and γ is the relative degree of the system. A system is said to have a strong relative degree γ [1] if $L_g h(x) = L_g L_f h(x) = \cdots = L_g L_f{}^{\gamma-2} h(x) = 0$ and $L_g L_f{}^{\gamma-1} h(x) \neq 0$ for all x. Usually, a_1, …, a_γ are chosen such that $s^\gamma + a_\gamma s^{\gamma-1} + \cdots + a_1$ is a Hurwitz polynomial [2].

Now, by defining

$n^*(x) = L_f^\gamma h(x) + a_\gamma L_f^{\gamma-1} h(x) + a_{\gamma-1} L_f^{\gamma-2} h(x) + \ldots + a_1 h(x)$

and $d^*(x) = L_g L_f^{\gamma-1} h(x)$, Eq. (2) can be rewritten as

$$u \equiv \frac{-n^*(x)+a_1 z_{sp}}{d^*(x)}. \tag{3}$$

It should be noted that the computation of $n^*(x)$ and $d^*(x)$ requires the knowledge of the system defined in Eq. (1). From [20], the above controller can yield the closed-loop dynamic response for Eq. (1) as:

$$z^{(\gamma)} + a_\gamma z^{(\gamma-1)} + a_{\gamma-1} z^{(\gamma-2)} + \ldots + a_1 z = a_1 z_{sp}.$$

The system can be made stable with a proper choice of those constants, $a_1, \ldots, a_\gamma$. Note that additional assumptions are required to ensure the internal stability of the closed-loop system due to the presence of an (n-γ)-dimensional nonlinear system called the "zero dynamics" [3]. A sufficient condition for bounded tracking is that the zero dynamics are exponentially stable and Lipschitz continuous [19]. Such a feedback linearization controller can have nice control performance as shown in [20].

In above, $n^*(x)$ and $d^*(x)$ are obtained from the system dynamics (Eq. (1)). However, when the system to be controlled is unknown, the above input-output linearization controller cannot work. This paper is to use a fuzzy system to approximate $n^*(x)$ and $d^*(x)$ based on the Lyapuonv theorem. With such a fuzzy system, the above controller can work well for unknown systems.

3. Adaptive Model Reference Fuzzy Controllers

In this study, we employed fuzzy systems to on-line estimate $n^*(x)$ and $d^*(x)$ from the measured state variables. Different from previous adaptive fuzzy controllers, the proposed approach does not need to use any auxiliary operations on input trajectories or on system outputs. This controller is called the adaptive model reference fuzzy controller. The idea is to include a reference model into the controller such that each step the controller only needs the current input and current output without using their derivatives.

The used fuzzy system is to perform a mapping from the current states to the estimated $n^*(x)$ and $d^*(x)$. The mapping consists of a set of fuzzy IF-THEN rules in which the l-th rule is of the form

$R^{(l)}$: **IF** x_1 is $F_1^{(l)}$ and ... and x_n is $F_n^{(l)}$ **THEN** $p=\theta^{(l)}$, (4)

where x_1, ..., x_n are the state variables defined in Eq. (1), $F_1^{(l)}$, …, $F_n^{(l)}$ are the corresponding fuzzy labels, p is the output variable for the fuzzy system, and $\theta^{(l)}$ is the corresponding output value for the l-th rule.

By using the product operations for the conjunction relations in the premise parts of fuzzy rules, the output of a fuzzy system consisting of N rules is obtained as:

$$p = \frac{\sum_{l=1}^{N} \theta^{(l)} (\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i))}{\sum_{l=1}^{N} (\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i))} = \theta^T \xi(x) \tag{5}$$

where $\mu_{F_i^{(l)}}(x_i)$ is the membership degree of x_i belonging to the fuzzy label $F_i^{(l)}$, $\theta = [\theta^{(1)}, \theta^{(2)}, \cdots, \theta^{(N)}]^T$, and $\xi = [\xi^{(1)}, \xi^{(2)}, \cdots, \xi^{(N)}]^T$ and is referred to as the regressive vector. Here the supscript T for a vector is the transpose of the vector, and $\xi^{(l)} = \frac{\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i)}{\sum_{l=1}^{N} (\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i))}$ is called the fuzzy basis function [6].

The fuzzy system is used to estimate both $n^*(x)$ and $d^*(x)$ in the controller (Eq. (3)). Thus, the fuzzy system has two output variables for $n^*(x)$ and $d^*(x)$, respectively. $n^*(x)$ and $d^*(x)$ then can be approximated as

$$\begin{aligned} \hat{n}^*(x) = p_n &= \frac{\sum_{l=1}^{N} \theta_n{}^{(l)} (\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i))}{\sum_{l=1}^{N} (\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i))} \\ &= \theta_n{}^T \xi(x) \end{aligned} \tag{6}$$

$$\begin{aligned} \hat{d}^*(x) = p_d &= \frac{\sum_{l=1}^{N} \theta_d{}^{(l)} (\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i))}{\sum_{l=1}^{N} (\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i))} \\ &= \theta_d{}^T \xi(x). \end{aligned} \tag{7}$$

By using the above estimations of $n^*(x)$ and $d^*(x)$, Eq. (3) can be rewritten as

$$u \cong \frac{-\theta_n^T \xi(x) + a_1 z_{sp}}{\theta_d^T \xi(x)}. \quad (8)$$

It should be noted that in Eq. (8), θ_n and θ_d are parameters to be determined from the identification process and those fuzzy labels are pre-defined and fixed in our implementation. In the next section, we shall present the algorithms for obtaining θ_n and θ_d such that the stability of the system is guaranteed.

4. The Update Algorithms and Stability Analysis

The update algorithm is to recursively update the controller parameters (i.e., θ_n and θ_d) such that the plant output asymptotically tracks the output of a reference model. Two assumptions are made to facilitate the Lyapunov design of the parameter update laws. The first one is $\theta_d^T \xi(x) \neq 0$, which ensures that the nonlinear control law Eq. (8) remains well defined. The second assumption is the existence of "true" controller parameters $\theta_n^*(x)$ and $\theta_d^*(x)$ such that it is possible to achieve

$$\theta_n^{*T} \xi(x) = L_f^\gamma h(x) + a_\gamma L_f^{\gamma-1} h(x) + a_{\gamma-1} L_f^{\gamma-2} h(x) + \ldots + a_1 h(x) \quad (9)$$

and $\theta_d^{*T} \xi(x) = L_g L_f^{\gamma-1} h(x)$. (10)

Now, suppose that the considered system is of relative degree γ; i.e., $L_g h = L_g L_f h = \cdots = L_g L_f^{\gamma-2} h = 0$ and $L_g L_f^{\gamma-1} h \neq 0$. Then, an appropriate reference model is

$$z_m^{(\gamma)} = -a_\gamma z_m^{(\gamma-1)} - a_{\gamma-1} z_m^{(\gamma-2)} - \cdots - a_1 z_m + a_1 z_{sp}, \quad (11)$$

where a_γ, $a_{\gamma-1}$, … and , a_1 are chosen such that the reference model is stable. The derivative of the output with respect to time along the system trajectories can be written from Eq. (1) as

$$\dot{z} = L_f h(x) + L_g h(x) u \ . \quad (12)$$

Since the system is of relative degree γ, then $L_g h = L_g L_f h = \cdots = L_g L_f^{\gamma-2} h = 0$ and $L_g L_f^{\gamma-1} h \neq 0$. The control input vanishes in Eq. (12); i.e.,

$$\dot{z} = L_f h(x). \quad (13)$$

Again, take derivative of the above equation with respect to time along trajectories and yield:

$$\ddot{z} = L_f L_f h(x) + L_g L_f h(x) u \ . \quad (14)$$

The control input still vanishes. By repeating the above process, we finally have

$$z^{(\gamma)} = L_f^\gamma h(x) + L_g L_f^{\gamma-1} h(x) u \ . \quad (15)$$

Since $L_g L_f^{\gamma-1} h(x) \neq 0$, the control input appears in the output of the system.

Substituting Eqs. (9) and (10) into Eq. (15), we have

$$\begin{aligned} z^{(\gamma)} &= -a_\gamma L_f^{\gamma-1} h - a_{\gamma-1} L_f^{\gamma-2} h - a_{\gamma-2} L_f^{\gamma-3} h - \ldots - a_1 h + \theta_n^{*T} \xi(x) + \theta_d^{*T} \xi(x) u \\ &= -a_\gamma L_f^{\gamma-1} h - a_{\gamma-1} L_f^{\gamma-2} h - a_{\gamma-2} L_f^{\gamma-3} h - \ldots - a_1 h + \theta_n^{*T} \xi(x) \\ &\quad + \theta_d^{*T} \xi(x) u + \theta_d^T \xi(x) u - \theta_d^T \xi(x) u \end{aligned} \quad .(16)$$

Now, by employing Eq. (8) as the control input, we have

$$\begin{aligned} z^{(\gamma)} &= -a_\gamma L_f^{\gamma-1} h - a_{\gamma-1} L_f^{\gamma-2} h - a_{\gamma-2} L_f^{\gamma-3} h - \ldots - a_1 h + \theta_n^{*T} \xi(x) \\ &\quad - \theta_n^T \xi(x) + a_1 z_{sp} + \theta_d^{*T} \xi(x) u - \theta_d^T \xi(x) u \\ &= -a_\gamma L_f^{\gamma-1} h - a_{\gamma-1} L_f^{\gamma-2} h - a_{\gamma-2} L_f^{\gamma-3} h - \ldots - a_1 h + a_1 z_{sp} \\ &\quad - \left(\theta_n - \theta_n^*\right)^T \xi(x) - \left(\theta_d - \theta_d^*\right)^T \xi(x) u \end{aligned} \quad (17)$$

Using Eqs. (11) and (17), the dynamics of the tracking error becomes

$$e^{(\gamma)} = -a_\gamma e^{(\gamma-1)} - a_{\gamma-1} e^{(\gamma-2)} - \ldots - a_1 e - \phi_n^T \xi(x) - \phi_d^T \xi(x) u, (18)$$

where $\phi_n = \theta_n - \theta_n^*$ and $\phi_d = \theta_d - \theta_d^*$. Equation (18) can be rewritten as

$$e = M(s)\left[-\phi_n^T \xi(x) - \phi_d^T \xi(x) u\right], \quad (19)$$

where $M(s) = \dfrac{1}{s^\gamma + a_\gamma s^{\gamma-1} + \cdots + a_1}$. It has been shown in the literature [1] that when an error dynamic system has the property of strictly positive real (SPR), the parameter update law for adaptive controllers can easily be obtained. Here a dynamic system is said to be SPR if it is strictly minimum-phase and its transfer function has relative degree 0 or 1 [3]. But, when $\gamma > 1$, by properly selecting $a_\gamma, a_{\gamma-1}, \cdots, a_1$, $M(s)$ can be a stable transfer function but not strictly positive real [1]. In order to use the SPR-Lyapunov design method [25], Eq. (19) is further rewritten as

$$e = M(s) N(s)\left[-\phi_n^T \varsigma(x) - \phi_d^T \varsigma(x) u\right], \quad (20)$$

where $\varsigma(x) = N^{-1}(s)\xi(x)$ and $N(s)$ is chosen so that $N^{-1}(s)$ is a proper stable transfer function and $M(s)N(s)$ is a proper SPR transfer function. Define k= $\gamma - 1$ and let $N(s)$= $s^k + b_1 s^{k-1} + b_2 s^{k-2} + \cdots + b_k$ such that

$M(s)N(s)$ is a proper SPR transfer function. Then the dynamic equation for Eq. (20) becomes:

$$\dot{\bar{e}}_s = \bar{A}_s \bar{e}_s + \bar{B}_s(-\phi_n^T \varsigma(x) - \phi_d^T \varsigma(x)u)$$
$$e = \bar{C}_s^T \bar{e}_s \tag{21}$$

where $\bar{e}_s = \begin{bmatrix} e & \dot{e} & \ddot{e} & \dots & e^{(\gamma-1)} \end{bmatrix}^T$,

$$\bar{A}_s = \begin{bmatrix} -a_\gamma & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & 0 & \cdots \\ -a_3 & \vdots & \vdots & \vdots & \vdots \\ -a_2 & 0 & 0 & \cdots & 1 \\ -a_1 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{\gamma\times\gamma},$$

$\bar{B}_s^T = \begin{bmatrix} 1 & b_1 & b_2 & b_3 & \cdots & b_k \end{bmatrix} \in R^\gamma$, and $\bar{C}_s^T = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \in R^\gamma$. From the above equations, we then can directly define the parameter update laws as:

$$\dot{\theta}_n = \dot{\phi}_n = \lambda_1 e \varsigma(x) \tag{22}$$

$$\dot{\theta}_d = \dot{\phi}_d = \lambda_2 e \varsigma(x) u \,. \tag{23}$$

where λ_1 and λ_2 are two constants used to define the importance of the errors in $n^*(x)$ and $d^*(x)$, respectively.

Theorem 1: Consider a minimum-phase nonlinear unknown system as Eq. (1). Suppose that it satisfies the above two mentioned assumptions and the used control input is Eq. (8). Then if the adaptive laws are Eqs. (22) and (23), $e(t)$ converges to zero as $t \to \infty$ and all signals in the closed system are also bounded.

Proof: Define a Lyapunov function as

$$V(t) = \frac{1}{2}\bar{e}_s^T P \bar{e}_s + \frac{1}{2\lambda_1}\phi_n^T \phi_n + \frac{1}{2\lambda_2}\phi_d^T \phi_d \tag{24}$$

where P is a symmetric positive definite matrix. Take derivative of Eq. (24) with respect to time, we have

$$\dot{V} = \frac{1}{2}\dot{\bar{e}}_s^T P \bar{e}_s + \frac{1}{2}\bar{e}_s^T P \dot{\bar{e}}_s + \frac{1}{\lambda_1}\phi_n^T \dot{\phi}_n + \frac{1}{\lambda_2}\phi_d^T \dot{\phi}_d \tag{25}$$

Substituting Eq. (21) into Eq. (25) and with some manipulations, we have

$$\dot{V} = \frac{1}{2}\bar{e}_s^T(\bar{A}_s^T P + P\bar{A}_s)\bar{e}_s + \bar{e}_s^T P\bar{B}_s(-\phi_n^T \varsigma(x) - \phi_d^T \varsigma(x)u) + \frac{1}{\lambda_1}\phi_n^T\dot{\phi}_n + \frac{1}{\lambda_2}\phi_d^T\dot{\phi}_d \tag{26}$$

Since $M(s)N(s)$ is SPR, there exists $P = P^T > 0$ [1] such that

$$\bar{A}_s^T P + P\bar{A}_s = -Q$$
$$P\bar{B}_s = \bar{C}_s \tag{27}$$

where $Q = Q^T > 0$. By using Eqs. (27) and (21), Eq. (26) becomes

$$\dot{V} = -\frac{1}{2}\bar{e}_s^T Q \bar{e}_s + e(-\phi_n^T \varsigma(x) - \phi_d^T \varsigma(x)u) + \frac{1}{\lambda_1}\phi_n^T\dot{\phi}_n + \frac{1}{\lambda_2}\phi_d^T\dot{\phi}_d .$$

Then, it is easy to see that when the update laws are Eqs. (22) and (23), $\dot{V} = -\frac{1}{2}\bar{e}_s^T Q\bar{e}_s \le 0$. When $V>0$ and $\dot{V} \le 0$, it can easily be found that e, $\dot{e}$, θ_n, and θ_d are all bounded from Eq. (24). It follows from Barbalat's lemma [19] that $\lim_{t\to\infty} e(t) = 0$. Moreover, since the chosen reference model is stable, then z and z_m are also bounded. The exponentially stable and Lipschitz continuous assumptions imposed on the zero dynamics ensure that x be bounded. □

Note that when the relative degree is 1, $N(s)$ can be selected to be 1. Then the update laws become

$$\dot{\theta}_n = \dot{\phi}_n = \lambda_1 e \xi(x) \tag{28}$$

$$\dot{\theta}_d = \dot{\phi}_d = \lambda_2 e \xi(x) u \,, \tag{29}$$

In fact, it can be found that the obtained update law for relative degree one systems is exactly the same as that derived in [18].

5. Simulation Results

In this paper, two examples are used to verify the performance of the proposed controller. One is an unstable nonlinear system and the other is a Duffing forced oscillation system. Both examples are used in [11].

Example 1: The used system is defined as

$$\dot{x}(t) = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + u(t), \quad z = x, \tag{30}$$

where x is the system state, and u and z are the system input and output variables, respectively. The used reference model is

$$\dot{z}_m = -10 z_m + 10 z_{sp}, \tag{31}$$

where z_m and z_{sp} are the reference model output and the set-point, respectively.

In the implementation, six fuzzy sets are defined over the interval [-3,3], with labels *N3*, *N2*, *N1*, *P1*, *P2*, and *P3*, and their membership functions are $u_{N3}(x)=1/(1+\exp(5(x+2)))$, $u_{N2}(x)=\exp(-(x+1.5)^2)$, $u_{N1}(x)=\exp(-(x+0.5)^2)$, $u_{P1}(x)=\exp(-(x-0.5)^2)$, $u_{P2}(x)=\exp(-(x-1.5)^2)$, and $u_{P3}(x)=1/(1+\exp(-5(x-2)))$. Those membership functions are also shown in Figure 1. The initial values of $\theta_n(0)$ and $\theta_d(0)$ are randomly chosen from interval [-2, 2]. In this example, $\lambda_1 = 300$, $\lambda_2 = 25$, $\alpha_1 = 10$, $N(s)$=1 (because γ=1), the simulation time period is 20 seconds, and the initial state $x(0)$ is zero.

The simulation is conducted in a *Matlab* environment with a step size 0.01. Several input functions are used in our simulation. Square wave functions with the amplitudes ± 0.5 and ± 0.75 and the periods 6.25 seconds and 8.5 seconds, respectively, are considered as examples of non-differentiable signals. A sinusoidal function and a constant function are used as examples for differentiable signals. Figure 2 and 3 shows the tracking performance of our proposed scheme when the reference signal is a square wave with different amplitudes and different periods. Figure 4 shows the tracking performance when the reference input is a sinusoidal wave. Figure 5 shows the tracking performance when the reference input is a constant signal. All examples demonstrated good tracking capability after a couple cycles of learning. It is evident that our controllers can indeed have nice adaptive capability.

In order to show the superiority of our approach, we also implemented the adaptive fuzzy controller proposed in [11]. Figure 6 shows the capability of regulation by considering zero input with initial state $x(0)$ being 1. From the simulation, it can be found that our proposed approach has the better control performance. Figure 7 shows the tracking performances for a sinusoidal function. Table 1 shows the sums of the absolute errors in the first three cycles. Again, our proposed approach can have the better control performance. Note that since the approach in [11] needs the derivative of the trajectory, square waves cannot be directly used as the reference trajectory for [11]. By considering the reference model as the desired output, the approach of [11] still can be applied. However, the simulation revealed that the tracking performance of the approach in [11] is inadequate as shown in Figure 8. Table 2 shows the sums of the absolute errors in the first three cycles.

Example 2: The used system is

$$\dot{x}_1 = x_2 \qquad (32)$$

$$\dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos(t) + u(t) \qquad (33)$$

$$z = x_1 \qquad (34)$$

This is a Duffing forced oscillation system. Note that when $u(t)$=0, the system is chaotic. The reference model is defined as

$$\ddot{z}_m + 10\dot{z}_m + 25z_m = 25z_{sp} \qquad (35)$$

In this example, the system has relative degree two and the function $L_gL_fh(x)$=1 is known. Only $n^*(x)$ in Eq. (3) needs to be estimated. The six fuzzy labels shown in Figure 1 are also used for both x_1 and x_2. The following parameters are used in our implementation; λ_1=100, $\theta_n(0)$=1, the simulation time period=50 sec, $N(s)$=s+2, a_2=10, a_1=25 and $[x_1(0) \;\; x_2(0)]$=[2 0]. The simulation is conducted with a step size 0.02. Figure 9 shows the tracking results when sin(t) is the input function. Figure 10 show the tracking results in the phase plane. Another reference input, a triangular wave, is also used and the results are shown in Figure 11. For comparison, Figure 12 shows the tracking results for a sinusoidal function for our approach and for the approach in [11]. Table 3 shows the sums of the absolute errors in the first seven cycles in Figure 12 for those two approaches. Figure 13 shows the tracking results for a triangular wave in [11]. Table 4 shows the sums of the absolute errors in the first seven cycles in Figures 11 and 13, respectively, for those two approaches. Finally, the normalized gradient update law proposed in [18] is used in this example. It can be found that that approach leads to failure in tracking control as shown in Figure 14. In fact, we have tried various parameters, and the results are all similar. In other words, the update law proposed in [18] cannot converge for this example.

6. Conclusions

In this paper, a novel adaptive fuzzy controller is proposed. It is called the adaptive model reference fuzzy controller in our research. The control law of our approach is based on a traditional adaptive linearization controller, which has been shown to have nice control performance in the literature. Since the considered dynamic nonlinear model is unknown in our problem, two functions required in the adaptive linearization controller are on-line tuned under a fuzzy rule structure. Based on the Lyapunov stability theorem, the corresponding adaptive update laws are also derived. With the use these update laws, the stability of the closed systems is guaranteed. Different from previous adaptive fuzzy controllers, our approach does not need any auxiliary operations on input trajectories and on system states. With this property, our approach can be used to track any trajectory, even a trajectory not previously known. This method has been applied to control an unstable nonlinear system to track a piecewise reference trajectory and to control a chaotic system to track a sinusoidal reference trajectory and a triangular reference trajectory. The computer simulation results showed that the proposed scheme could perform a successful control. When compared to previous approaches, our approach can have better learning and control performance in those examples for all used trajectories.

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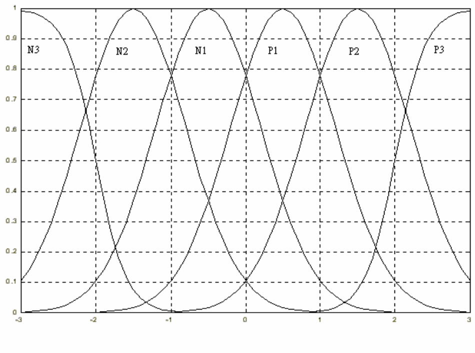

Fig. 1 The used fuzzy membership functions.

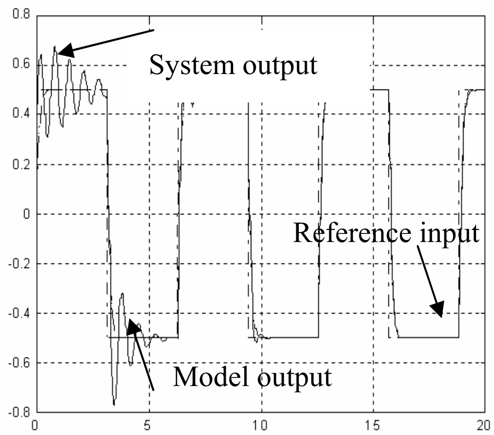

Fig. 2 The tracking results when the input is a square wave with the amplitude ±0.5 and the time period 6.25 seconds.

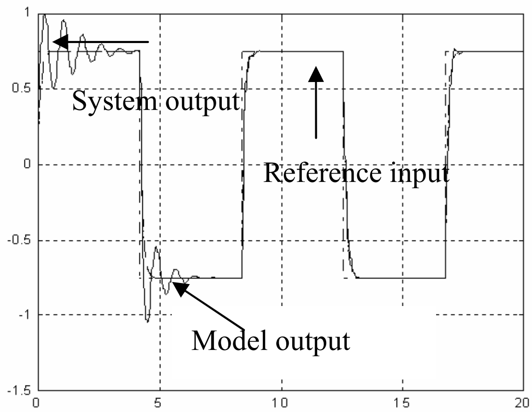

Fig.3 The tracking results when the input is a square wave with the amplitude ±0.75 and the time period 8.5 seconds.

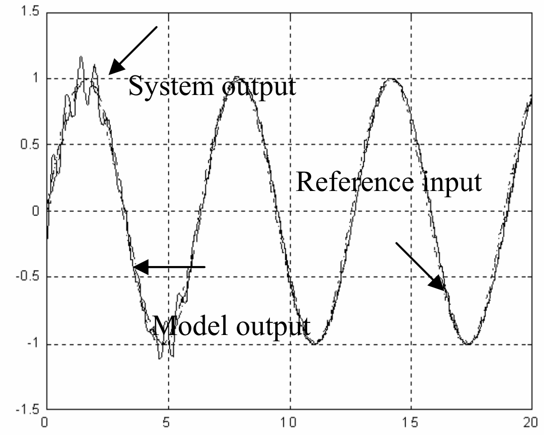

Fig. 4 The tracking results when the input is a sinusoid function.

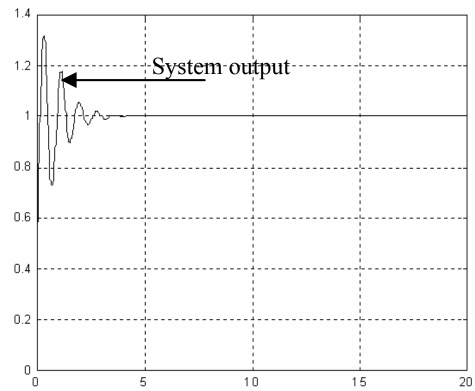

Fig. 5 The tracking results when the input is a constant signal.

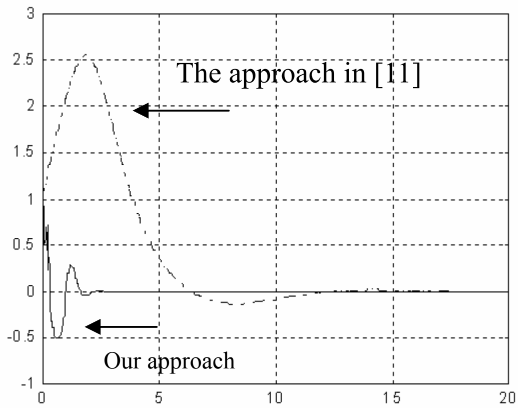

Fig. 6. Simulation results for the proposed method and the approach in [11].

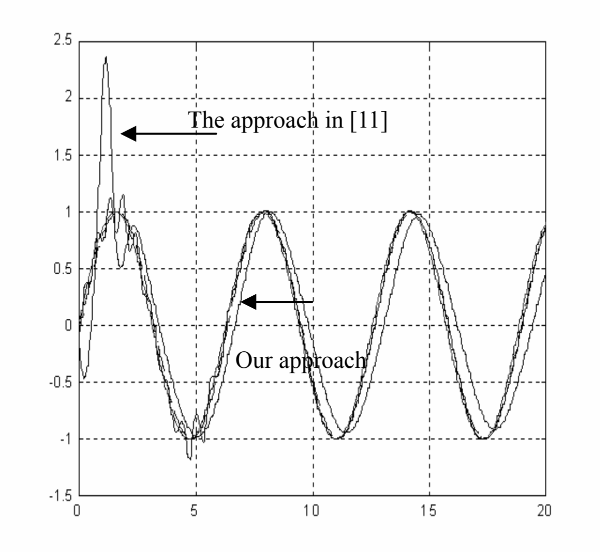

Fig.7. Simulation results for the proposed method and the approach in [11].

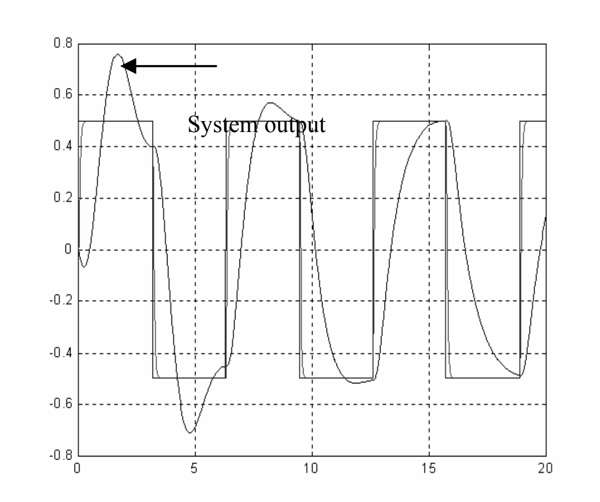

Fig. 8 The tracking results when the input is a square wave for the approach in [11].

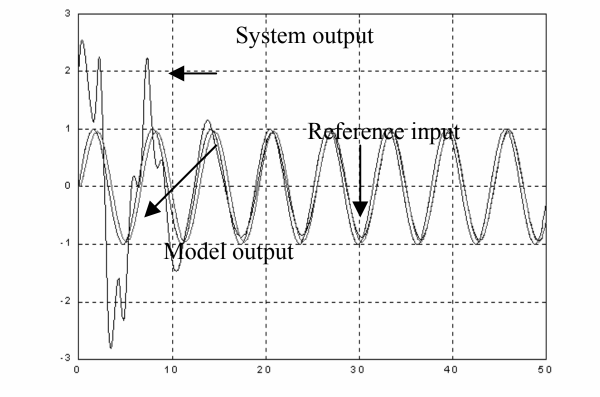

Fig. 9. The output trajectory using the proposed controller for a sinusoid function.

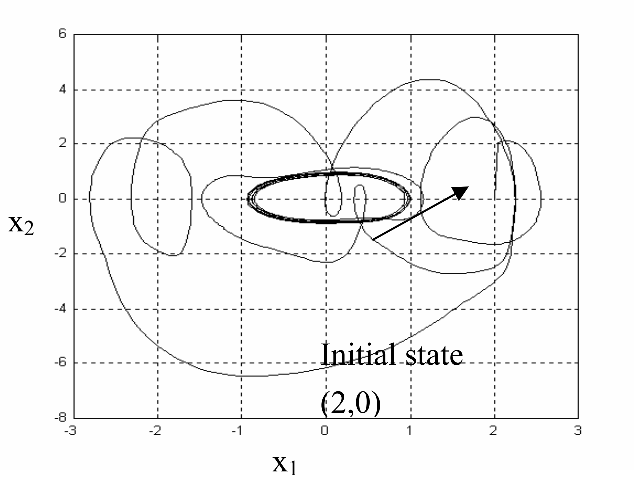

Fig. 10. The closed-loop system trajectory $(x_1(t), x_2(t))$ using the proposed controller for a sinusoid function.

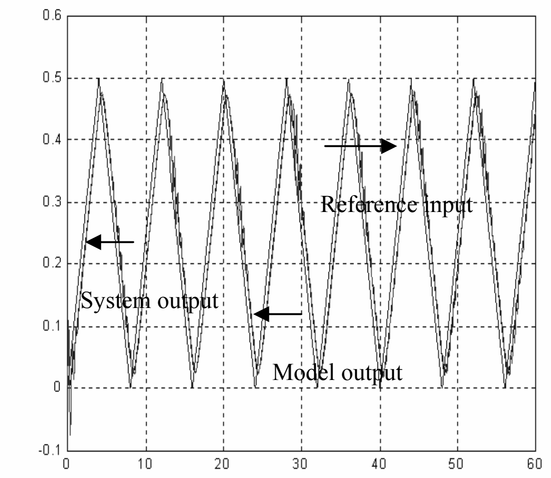

Fig. 11. The output trajectory using the proposed controller for a triangular wave input.

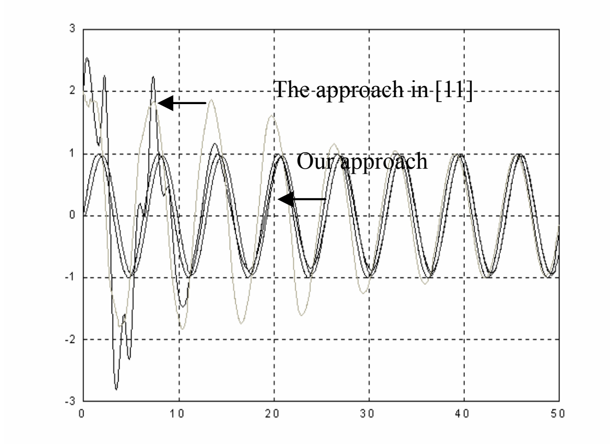

Fig. 12. Simulation results for the proposed method and the approach in [11].

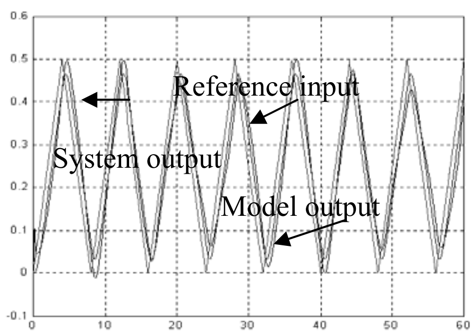

Fig. 13. The output trajectory using the approach in [11] for a triangular wave input

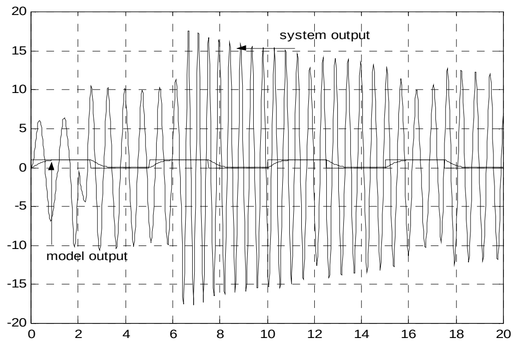

Fig. 14. Simulation results for the adaptive fuzzy control with the normalized gradient update law [18]

Cycle # / Approach	The total errors in the first cycle	The total errors in the second cycle	The total errors in the third cycle
Our approach	54.3185	17.0945	9.3585
The approach in [11]	210.0848	135.5928	168.4118

Table 1. The comparison of the sums of the absolute errors by cycles for Example 1 with the input as a sinusoid function.

Cycle # / Approach	The total errors in the first cycle	The total errors in the second cycle	The total errors in the third cycle
Our approach	44.0042	9.3253	1.3316
The approach in [11]	113.7631	147.2520	187.0868

Table 2. The comparison of the sums of the absolute errors by cycles for Example 1 with the input as a square wave.

Cycle # / Approach	The 1st cycle	The 2nd cycle	The 3rd cycle
Our approach	404.7307	208.0629	98.6355
The approach in [11]	311.6347	264.1880	217.7920
The 4th cycle	The 5th cycle	The 6th cycle	The 7th cycle
32.0753	13.5955	8.4675	7.2423
158.6114	83.9198	55.3928	39.8578

Table 3. The comparison of the sums of the absolute errors by cycles for Example 2 with the input as a sinusoid.

Cycle # / Approach	The 1st cycle	The 2nd cycle	The 3rd cycle
Our approach	4.6887	2.9081	2.9129
The approach in [11]	10.6872	9.9598	10.0013
The 4th cycle	The 5th cycle	The 6th cycle	The 7th cycle
3.8077	2.9315	2.7694	3.7884
11.4130	10.2748	6.6289	9.7996

Table 4. The comparison of the sums of the absolute errors by cycles for Example 2 with the input as a triangular wave.