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Decentralized Adaptive Fuzzy Control for a class of Nonlinear Systems

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Abstract: - In this paper, stable direct and indirect decentralized adaptive fuzzy controls are proposed for a class of large-scale nonlinear systems with the strong interconnected. The feedback and adaptive mechanisms for each subsystem depend only upon local measurements to provide asymptotic tracking of a reference trajectory. In both approaches, the proposed controllers are used to approximate the unknown subsystems. In addition, each subsystem is able to adaptively compensate for interconnections without known bounds. Simulation results are given to illustrate the tracking performance of the proposed methods.

Keywords: Decentralized Control, Fuzzy Controller, Interconnected Nonlinear System.

1 Introduction

Decentralized adaptive control systems often arise from various complex situations where there exist physical limitations on information exchange among several systems for which there is insufficient capability to have a single control controller, and due to the physical configuration and high dimensionality of interconnected systems a centralized control is neither economically feasible nor even necessary. Therefore, the decentralized scheme is preferred in control design of large-scale interconnected system [1], [2], [9]. To control a large-scale system, one essential problem is how to handle the interactions among different systems. Intensive research has been devoted to the observer design for large-scale systems. Uncertainties in a large-scale system require the adaptive decentralized technique, for which many decentralized adaptive schemes have been developed, including the model reference adaptive control [1],[4], and nonlinear control with a special class of interconnections [7]. These approaches focus on stabilisation, where the dynamics of subsystems are assumed to be known or to be linear with a set of unknown parameters. However, in practice, large-scale systems may contain significant uncertainties, and/or with unknown parameters in nonlinear forms and unknown structures.

Fuzzy logic control as one of the most useful approaches for utilizing expert knowledge, has been an active field of research the past decade [8],[11]. Fuzzy logic control is generally applicable to plants that are mathematically poorly modelled and where experienced operators are available for providing qualitative guidance. The most important advantage of fuzzy-logic-control schemes lies in the fact that the developed controllers can deal with increasingly complex systems and controllers without precise knowledge of the model structure of the underlying system dynamic. Recently there have been significant research efforts on these issues in fuzzy control system [8],[11],[12] but these approaches work only for large-scale systems with a known or linear dynamics with a set of unknown parameters and bounds interconnections. In practice, however, not all states are usually available.

This paper presents two approaches which can easily tackle the output tracking control problem of a class of large-scale nonlinear system with unknown interconnections bounds. A direct adaptive approach approximates unknown control laws required to stabilize each subsystem, while an indirect adaptive is provided which identifies the isolated subsystem dynamics to produce a stabilizing controller. Both approaches ensure asymptotic tracking using only local measurement.

The organization of this paper is as follows: section 2 describes the problem under investigation; section 3, the direct adaptive decentralized control; while in section 4, we introduce the indirect approach. Experimental results are then used to demonstrate the effectiveness of the proposed approaches is presented in section 5, with a conclusion given in section 6.

2 Problem Formulation

Consider a class of nonlinear interconnected SISO subsystems S_i $(i=1,2,...,N)$ described as follows:

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \vdots \\ \dot{x}_{i,n_i} = f_i(x_i) + g_i(x_i)u_i + \Delta_i(\underline{x}) \\ y_i = x_{i,1} \end{cases} \quad (1)$$

where $\underline{x} = [x_1^T, x_2^T, ..., x_N^T]^T$, $x_i \in \Re^{n_i}$, is the global state vector, $u_i(t) \in \Re$ is the control signal input and $y_i \in \Re$ is the output of the plant for the subsystem S_i. The functions $f_i(.)$ and $g_i(.)$ are unknown and nonlinear, and $\Delta_i(\underline{x}) \in \Re$, are interconnection among subsystemsunknown ($i=1,2,...,N$).

The tracking error for S_i is defined by $e_{i0} = y_{ir} - y_i$. Our objective is to design an adaptive control for each subsystem which will cause the output y_i to track a desired output trajectory y_{ir} $(i,e., e_{i0} \to 0)$ in the presence of the strong interconnections using only local measurements.

Assumption 1: Let the scalars q_{ij} quantify the strength of the interconnections and the output vector for the i^{th} subsystem be defined by $e_i = [e_{i0}, \dot{e}_{i0}, ..., e_{i0}^{(d_i-1)}]^T$; it is assumed that the interconnections satisfy:

$$|\Delta_i(\underline{x})| \le \sum_{j=1}^{N} q_{ij} \|e_j\|_2 \quad (2)$$

where $\|.\|_2$ is the Euclidean vector norm. This assumption on the interconnections can be satisfied by a variety of decentralized nonlinear systems. For instance, in [10] it is shown to be satisfied for an intervehicle spacing regulation problem in a platoon of an automated highway system.

Subsystem (1) can be expressed as:

$$\dot{\varsigma}_i = \Lambda_{i0}\varsigma_i + b_i[f_i(\underline{x}_i) + g_i(\underline{x}_i)u_i + \Delta_i(\underline{x})] \quad (3)$$

where

$$\Lambda_{i0} = \begin{vmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{vmatrix} \quad b_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (4)$$

and

$\varsigma_i = [\varsigma_{i,1}, \varsigma_{i,2}, ..., \varsigma_{i,d_i}]^T$, $\varsigma_{i,1} = y_i$, $\varsigma_{i,2} = \dot{y}_i$, …, $\varsigma_{i,d_i} = y_i^{(d_i-1)}$

Assume that the given reference y_{ir} is bounded and have up to $d_i - 1$ bounded derivatives. The reference vector is denoted as $Y_{ir} = [y_{ir}, \dot{y}_{ir}, \ddot{y}_{ir}, ..., y_{\text{ir}}^{(d_i-1)}]^T$.

Define the tracking error of the i^{th} subsystem as $e_{i0} = y_{ir} - y_i$. Then the error vector of the i^{th} subsystem is given by $e_i = [e_{i0}, \dot{e}_{i0}, ..., e_{i0}^{(d_i-1)}]^T$.

It is desired that the output error of the i^{th} subsystem follow $e_i^{(d_i)} + k_{i,d_i-1}e_{i0}^{(d_i-1)} + ... + k_{i,0}e_{i0} = 0$, here the coefficients are picked so that each:

$$\hat{L} = \varsigma_i^{d_i} + k_{i,d_i-1}\varsigma_i^{d_i-1} + ... + k_{i,0} \quad (5)$$

has its roots in the open left-half complex plane (in Hurwitz).

If the subsystem S_i is well known $(g_i(\underline{x}_i) \ne 0)$ and free of external disturbances; $(\Delta_i(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) = 0)$, then the primary control should be designed to have the following idealized control law:

$$u_i^* = \frac{1}{g_i(\underline{x}_i)}(-f_i(\underline{x}_i) + K_i^T e_i + y_{ir}^{(d_i)}) \quad (6)$$

In spite of the primary control (6) which mathematically cancels the given system and then places it in a stabilizing part, so as to guarantee $\lim_{t\to\infty} e_i = 0$, it is clear that, in practice, an exact cancellation of the given system nonlinearity is theoretically unrealizable and physically impossible. Thus, in this study, the direct adaptive approach implements an adaptive fuzzy system to approximate the idealized control action, and with the indirect approach we approximate the unknown dynamics for each subsystem ($f_i(\underline{x}_i)$ and $g_i(\underline{x}_i)$).

3 Direct adaptive fuzzy decentralized control

In this section, a direct adaptive output-feedback fuzzy decentralized controller is designed, with guaranteed stability of the integrated closed loop system.

Assume that in subsystem (1) $g_i(\underline{x}_i) \ne 0$. The direct adaptive controller is designed as:

$$u_i = u_i(\underline{x}_i, \underline{\theta}_i) + g_i^{-1}(\underline{x}_i)(a_i(t)e_i^T p_i b_i + u_{ih}) \quad (7)$$

where $u_i(\underline{x}_i, \underline{\theta}_i) = \underline{\theta}_i^T \varphi_i(\underline{x}_i)$, $\underline{\theta}_i = [\theta_i^1, \theta_i^2, ..., \theta_i^{m_i}]^T$ are parameter vectors and $\varphi_i(.) = [\varphi_{i,1}, \varphi_{i,2}, ..., \varphi_{i,m_i}]^T$ is a regressive vector with regressor φ_i^l $(1 \le l \le m_i$, where m_i is the number of rules), which is defined as a fuzzy basis function [5]. The term $a_i(t)e_i^T p_i b_i$ is used to compensate unknown effects from the interconnections ($p_i \in \Re^{d_i x d_i}$ is a positive definite matrix defined by a Lyapunov matrix equation and $b_i \in \Re^{d_i}$ is a vector), and u_{ih} .is auxiliary control compensation.

Substituting (7) to (1), and adding $g_i(\underline{x}_i)u_i^*$ and then subtracting $g_i(\underline{x}_i)u_i^*$ on the right-hand side of (1), we obtain,

$$\begin{aligned} y_i^{(d_i)} &= g_i(\underline{x}_i)(u_i(\underline{x}_i,\underline{\theta}_i) - u_i^*) + a_i(t)e_i^T p_i b_i \\ &+ u_{ih} + K_i^T e_i + y_{ir}^{(d_i)} + \Delta_i(\underline{x}) \end{aligned} \tag{8}$$

The error dynamics for the i^{th} subsystem may be expressed as:

$$\begin{aligned} e_i^{(d_i)} &= y_{ir}^{(d_i)} + g_i(\underline{x}_i)(u_i^* - u_i(\underline{x}_i,\underline{\theta}_i)) \\ &- a_i(t)e_i^T p_i b_i - u_{ih} - K_i^T e_i \\ &- y_{ir}^{(d_i)} - \Delta_i(\underline{x}) \end{aligned} \tag{9}$$

Define the optimal parameter vectors and fuzzy approximation error as:

$$\underline{\theta}_i^* = \arg\min_{\underline{\theta}_i \in \Omega_i} \left\{ \sup_{x_i \in \Re^{n_i}} \left| u_i^* - u_i(\underline{x}_i,\underline{\theta}_i) \right| \right\} \tag{10}$$

where $\Omega_i = \left\{ \underline{\theta}_i / \underline{\theta}_i^T \underline{\theta}_i \le M_i \right\}$ is the convex compact set which contains feasible parameter sets for $\underline{\theta}_i^*$.

Define the parameter error as $\Phi_i = \underline{\theta}_i^* - \underline{\theta}_i$.

In the analysis to follow, we will use the fact that $u_i^* - u_i(\underline{x}_i,\underline{\theta}_i) = \Phi_i^T \varphi_i(\underline{x}_i) - w_i$ where $\Phi_i = [\tilde{\Phi}_{i,1},\tilde{\Phi}_{i,2},...,\tilde{\Phi}_{i,m_i}]^T$ are parameter vectors, $\varphi_i(\underline{x}_i) = [\varphi_{i,1},\varphi_{i,2},...,\varphi_{i,m_i}]^T$ are defined above which is defined as a fuzzy basis function [5].

The dynamics equations of the i^{th} subsystem can be written as:

$$\begin{aligned} \dot{e}_i &= \Lambda_i e_i + b_i[g_i(\underline{x}_i)\Phi_i^T \varphi_i(\underline{x}_i) + \beta_i(\underline{x}_i)w_i \\ &- a_i(t)e_i^T p_i b_i - u_{ih} - \Delta_i(\underline{x})] \end{aligned} \tag{11}$$

and w_i represent the fuzzy approximation error of the i^{th} subsystem.

Assumption 2: We assume that, there exists a function $\mathrm{T}_{w_i}(\underline{x}_i) > 0$ such that:

$$\left| \beta_i(\underline{x}_i)w_i \right| < \mathrm{T}_{w_i}(\underline{x}_i) \ \ \forall 1 \le i \le N \tag{12}$$

The direct adaptive fuzzy decentralized control that we have proposed in (7) can be classified as:

$$\dot{\theta}_i = \eta_i \Pr oj[.] \tag{13}$$

where $\Pr oj[.]$ is the projection operator [5]

$$u_{ih} = T_{w_i}(\underline{x}_i) sign(e_i^T p_i b_i) \tag{14}$$

$$\dot{a}_i = \eta_{a_i}(e_i^T p_i b_i)^2 \tag{15}$$

where $\eta_i > 0$, and $\eta_{a_i} > 0$ are fixed adaptive gains.

Theorem 1:
Consider the nonlinear subsystem (1), suppose that assumptions 1-2 are satisfied. If there exists a matrix $p_i = p_i^T > 0$ *satisfying the Lyapunov equation:* $\Lambda_i^T p_i + p_i \Lambda_i + Q_i = 0$ *where* $Q_i = Q_i^T > 0$. *The adaptive fuzzy decentralized controller law is chosen as (7) with parameter adaptation law (13)-(15). Then the proposed fuzzy decentralized control scheme can guarantee that (i) all the variables of the closed-loop system are bounded and (ii) performance tracking is achieved.*

Proof: consider the following Lyapunov function for the i^{th} subsystem:

$$v_i = e_i^T p_i e_i + \frac{1}{\eta_i}\Phi_i^T \Phi_i + \frac{1}{2\eta_{a_i}}\Phi_{a_i}^T \Phi_{a_i} \tag{16}$$

where $\Phi_i = \underline{\theta}_i^* - \underline{\theta}_i$, $\Phi_{a_i} = a_i - \tau_i^*$, τ_i^* well be defined shortly , and each $p_i \in \Re_i^{d_i x d_i}$ is a positive definite and symmetric matrix.

Taking the time derivative of v_i, yields

$$\dot{v}_i = (\dot{e}_i^T p_i e_i + e_i^T p_i \dot{e}_i) + \frac{2}{\eta_i}\Phi_i^T \dot{\Phi}_i + \frac{1}{\eta_{a_i}}\Phi_{a_i}^T \dot{\Phi}_{a_i} \tag{17}$$

Substituting (11) into (17), applying (12)-(14) and choosing $\zeta_i(t) = e_i^T p_i b_i$ yields

$$\begin{aligned} \dot{v}_i &= -e_i^T Q_i e_i - \tau_i^*[(e_i^T p_i b_i)^2 \\ &+ 2e_i^T p_i b_i \frac{\Delta_i(\underline{x})}{\tau_i^*} + (\frac{\Delta_i(\underline{x})}{\tau_i^*})^2 \\ &- (\frac{\Delta_i(\underline{x})}{\tau_i^*})^2] \\ &= -e_i^T Q_i e_i - \tau_i^*(e_i^T p_i b_i + \frac{\Delta_i(\underline{x})}{\tau_i^*})^2 \\ &+ \frac{1}{\tau_i^*}(\Delta_i(\underline{x}))^2 \end{aligned} \tag{18}$$

so that if each $\tau_i^* > 0$, we simply obtain

$$\dot{v}_i \le -e_i^T Q_i e_i + \frac{1}{\tau_i^*}(\Delta_i(\underline{x}))^2 \tag{19}$$

Now consider the composite system Lyapunov candidate $V = \sum_{i=1}^{N} \varepsilon_i v_i$ where each $\varepsilon_i > 0$, yields

$$\dot{V} \le \sum_{i=1}^{N} \varepsilon_i [-e_i^T Q_i e_i + \frac{1}{\tau_i^*}(\sum_{j=1}^{N} q_{ij} \left\| e_j \right\|_2)^2] \tag{20}$$

Since $\sum_{j=1}^{N} q_{ij} \left\| e_j \right\|_2 = \psi^T \chi_i$, where $\psi = [\|e_1\|_2, \|e_2\|_2, ..., \|e_N\|_2]^T$ and

$\chi_i = [q_{i1}, q_{i2}, ..., q_{iN}]^T$, let λ_i the real part of the eigenvalue of Q_i, (20) may be written as

$$\dot{V} \le \sum_{i=1}^{N} \varepsilon_i [-\lambda_i \|e_i\|_2^2 + \frac{1}{\tau_i^*} \psi^T \chi_i \chi_i^T \psi] \quad (21)$$

Define $K^* = [\tau_1^*, \tau_2^*, ..., \tau_N^*]$. Let $\tau_i^* = \tau^* . i = 1,2,...,N$ for some $0 < \tau^*$. Define $D = diag\{\varepsilon_1 \lambda_1, \varepsilon_2 \lambda_2, ..., \varepsilon_N \lambda_N\}$ and $M = \sum_{i=1}^{N} \varepsilon_i \chi_i \chi_i^T$ so that $\dot{V} \le -\psi^T A \psi$, where $A = D - \frac{1}{\tau^*} M$. Then for some sufficiently large $\tau^* > 0$ the matrix A is positive definite. The diagonal dominance property may be established using Gershgorin's Theorem [3]. Now define $K^* = [\tau^*, \tau^*, ..., \tau^*]^T \in \Re^N$ as

$$K^* = \arg \min_{\substack{K^* \in \Re^N \\ 0 < \tau^*}} \left\{ \begin{array}{l} K^{*T} K^* : A = D - \frac{1}{\tau^*} M \; is \\ \qquad positive\ definite \end{array} \right\} \quad (22)$$

There exists sufficiently large τ^* such that A, defined by (22), is positive definite, which implies that $V \in \ell_\infty$, and thus $\|\psi\|_2 \in \ell_\infty$. Also

$$\int_0^\infty \psi^T A \psi dt \le -\int_0^\infty \dot{V} dt + const \quad (23)$$

so that $\|\psi\|_2 \in \ell_2$. Since all of the signals are well defined, we also have $\dot{e}_i \in \ell_\infty^{d_i}$ so that $d/dt \|e_i\|_2 = e_i^T \dot{e}_i / \|e_i\|_2 \le \|e_i\|_2 \in \ell_\infty$. Using Barbalat's Lemma, we thus establish that $\lim_{t \to \infty} \|\psi\|_2 = 0$, thus we are guaranteed asymptotically stable tracking for each of the subsystems.

4 Indirect adaptive fuzzy decentralized control

In this section, it is assumed that the function $f_i(\underline{x}_i)$ and $g_i(\underline{x}_i)$ are unknown. Take a universal fuzzy system $\hat{f}_i(\underline{x}_i / \underline{\theta}_i)$ with $\underline{x}_i \in U_{x_i}$ for some compact set U_{x_i} to approximate the uncertain term $f_i(\underline{x}_i)$ where θ_i contains the tunable parameters. Here the linearly parameterized fuzzy model [8] is employed in the approximation procedure. Then we replace $f_i(\underline{x}_i)$ and $g_i(\underline{x}_i)$ by the fuzzy system $\hat{f}_i(\underline{x}_i / \underline{\theta}_{i1})$ and $\hat{g}_i(\underline{x}_i / \underline{\theta}_{i2})$ respectively, with singleton fuzzifier, center average defuzzifier, and product inference. The fuzzy system $\hat{f}_i(\underline{x}_i / \underline{\theta}_{i1})$ and $\hat{g}_i(\underline{x}_i / \underline{\theta}_{i2})$ can be expressed as:

$\hat{f}_i(\underline{x}_i / \underline{\theta}_{i1}) = \underline{\theta}_{i1}^T \varphi_i(\underline{x}_i)$ and

$$\hat{g}_i(\underline{x}_i / \underline{\theta}_{i2}) = \underline{\theta}_{i2}^T \varphi_i(\underline{x}_i) \text{ for } i = 1,2,...,N \quad (24)$$

where $\underline{\theta}_{ik} = [\theta_{ik}^1, \theta_{ik}^2, ..., \theta_{ik}^{m_i}] \in \Re^{m_i}$ is a parameter vector $(k = 1,2)$ and $\varphi_i(\underline{x}_i) = [\varphi_i^1(\underline{x}_i), \varphi_i^2(\underline{x}_i), ..., \varphi_i^{m_i}(\underline{x}_i)] \in \Re^{m_i}$ is a regressive vector with the regressor $\varphi_i^l(\underline{x}_i)$ defined as,

$$\varphi_i^l(\underline{x}_i) = \frac{\prod_{j=1}^{n_i} \mu_{F_{i,j}^l}(x_i^j)}{\sum_l^{m_i} \prod_{j=1}^{n_i} \mu_{F_{i,j}^l}(x_i^j)} \quad (25)$$

where $\mu_{F_{i,j}^l}(.)$ is the membership functions for $1 \le l \le m_i$ (m_i is the number of rules) and $1 \le j \le n_i$ in this paper, we present the decentralized adaptive fuzzy controller defined as

$$u_i = u_{if} + \frac{1}{\hat{g}_i(\underline{x}_i / \theta_{i2})} (-\hat{f}_i(\underline{x}_i / \theta_{i1}) + K_i^T e_i + y_{ir}^{(d_i)} + a_i(t) e_i^T p_i b_i / 2 + u_{ih}) \quad (26)$$

where u_{fi} is the fuzzy controller, introduced to perform the main control action, which is given by synthesizing fuzzy control rules from human experts and/or by trial and error designing tools.

The decentralized fuzzy controller u_{if} is constructed from the following $\prod_{k=1}^{d_i} n_{i,k}$ rules:

$$R_i^{l_1,...,l_{d_i}} : \; if \; e_{i,1} is \; A_{i,1}^{l_1} \; and \; e_{i,2} is \; A_{i,2}^{l_2} \; and \; ... \; and \; e_{i,d_i} is \; A_{i,d_i}^{l_{d_i}} \; then \; L_i^{l_1,...,l_{d_i}} \; is \; C_i^{l_1,...,l_{d_i}} \quad (27)$$

where $n_{i,k}$ define the number of fuzzy sets $A_{i,k}^{l_k}$ in $U_{i,k}$ ($1 \le l_k \le n_{i,k}$ and $1 \le k \le d_i$) such that for any $e_{i,k} \in U_{i,k}$, there exists a fuzzy set $A_{i,j}^{l_j}$ so that the memberships function $\mu_{A_{i,j}^{l_j}}(e_{i,k}) \ne 0$. The centres of these fuzzy sets are adapted by the proposed low which will be defined below.

According to the universal approximation theorem [6], there exist optimal approximation parameters θ_{i1}^* and θ_{i2}^* such that $\hat{f}_i(\underline{x}_i / \theta_{i1}^*)$ and $\hat{g}_i(\underline{x}_i / \theta_{i2}^*)$ can, respectively, approximate $f_i(\underline{x}_i)$ and $g_i(\underline{x}_i)$ as best as possible. Define the optimal parameter vectors and fuzzy approximation errors

$$\theta_{i1}^* = \arg\min_{\theta_{i1}\in\Omega_{i1}} \left\{ \sup_{x_i\in\Re^{n_i}} \left| f_i(\underline{x}_i) - \hat{f}_i(\underline{x}_i/\theta_{i1}) \right| \right\}$$
$$\theta_{i2}^* = \arg\min_{\theta_{i2}\in\Omega_{i2}} \left\{ \sup_{x_i\in\Re^{n_i}} \left| g_i(\underline{x}_i) - \hat{g}_i(\underline{x}_i/\theta_{i2}) \right| \right\} \quad (28)$$

where Ω_{i1} and Ω_{i2} are the convex compact sets, which contain feasible parameter sets for θ_{i1}^* and θ_{i2}^* respectively, and

$$\Omega_{i1} = \left\{ \theta_{i1} : \theta_{i1}\theta_{i1}^T \le M_{i1} \right\}$$
$$\Omega_{i2} = \left\{ \theta_{i2} : tr(\theta_{i2}\theta_{i2}^T) \le M_{i2} \right\} \quad (29)$$

define

$$\Delta f_i(\underline{x}_i) = f_i(\underline{x}_i) - \hat{f}_i(\underline{x}_i/\theta_{i1}^*)$$

and

$$\Delta g_i(\underline{x}_i) = g_i(\underline{x}_i) - \hat{g}_i(\underline{x}_i/\theta_{i2}^*) \quad (30)$$

which denote the minimum approximation errors. Throughout this section we need the following assumption:

Assumption 3: there exists a positive function $0 < M_{w_i}(\underline{x}_i)$ such that

$$\left| \Delta f_i(\underline{x}_i) + \Delta g_i(\underline{x}_i) u_i \right| \le M_{w_i}(\underline{x}_i) \quad \forall\ 1 \le i \le N \quad (31)$$

Substituting (26.) to (1), the tracking error dynamic equation can be written as

$$\begin{aligned} \dot{e}_i = \Lambda_i e_i + b_i[& f_i(\underline{x}) - \hat{f}_i(\underline{x}_i/\theta_{i1}) \\ & + (g_i(\underline{x}_i) - \hat{g}_i(\underline{x}_i,\theta_{i2}))u_i \\ & - a_i(t) e_i^T p_i b_i / 2 - u_{ih} - \Delta_i(\underline{x})] \end{aligned} \quad (32)$$

where

$$\Lambda_i = \begin{vmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -K_{i0} & K_{i1} & & \cdots & K_{id_i} \end{vmatrix}, \; b_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (33)$$

from (24) and (30), (31) can be written as

$$\begin{aligned} \dot{e}_i = \Lambda_i e_i + b_i[& \Phi_{i1}^T \varphi_i(\underline{x}_i) + \\ & \Phi_{i2}^T \varphi_i(\underline{x}_i) u_i + \Delta f_i(\underline{x}) + \Delta g_i(\underline{x}_i) \\ & - a_i(t) e_i^T p_i b_i / 2 - u_{ih} - \Delta_i(\underline{x})] \end{aligned} \quad (34)$$

where the parameter error vectors are defined as $\Phi_{i1} = \theta_{i1}^* - \theta_{i1}, \Phi_{i2} = \theta_{i2}^* - \theta_{i2}$

The following update laws are now defined for the decentralized indirect adaptive controller:

$$\dot{\theta}_{i1} = \eta_{i1} e_i^T p_i b_i \varphi_i(\underline{x}_i) \quad (35)$$

$$\dot{\theta}_{i2} = \eta_{i2} e_i^T p_i b_i \varphi_i(\underline{x}_i) u_i \quad (36)$$

$$u_{ih} = -M_{w_i}(\underline{x}_i) sign(e_i^T p_i b_i) \quad (37)$$

$$\dot{a}_i = \eta_{a_i} (e_i^T p_i b_i)^2 \quad (38)$$

$$u_{if} = \sigma_i \Gamma_i^T \varphi_i(\underline{x}_i) \quad (39)$$

where η_{i1}, η_{i2} and η_{a_i} are fixed adaptive gains. The parameters update low for the isolated system identifier (35)-(36) are used to estimate the dynamics of the subsystem under control. The update law (37) is designed to compensate for the effects of the representation error, where (38) is used to stabilize the subsystem by estimating the effects of the interconnections. The vector $\Gamma_i^{l_1,\ldots,l_{d_i}} = (\Gamma_1^1, \Gamma_1^2, \ldots, \Gamma_1^{\prod_{j=1}^{d_i} n_{i,j}})$ is the centre average of $C_i^{l_1,\ldots,l_{d_i}}$. We define $\Gamma_i = (\Gamma_1^1, \Gamma_1^2, \ldots, \Gamma_1^{\prod_{j=1}^{d_i} n_{i,j}})$ as the collection of $\Gamma_i^{l_1,\ldots,l_{d_i}}$'s for $l_1 = 1,2,\ldots,M_{i,1};\ldots;l_{d_i} = 1,2,\ldots,n_{i,d_i}$. The centre of the l^{ih} set C_i^l ($l = (l_i, l_2, \ldots, l_{d_i})$) is given by the proposed equation

$$\Gamma_i^{l_1,\ldots,l_{d_i}} = \begin{cases} \Gamma_i^{l_1,\ldots,l_{d_i}} & \text{if } \max_{j=1,2,\ldots,n_i} \mu_{A_{ij}^{l_j}}(e_{ij}) = 0 \\ \dfrac{\sum_{j=1}^{d_i} e_{ij} (\mu_{A_{i,j}^{l_j}}(e_{ij}))}{\max_{j=1,2,\ldots,d_i} (\mu_{A_{ij}^{l_j}}(e_{ij}))} & otherwise \end{cases} \quad (40)$$

σ_i is positive constant.

Theorem 2:

Consider the nonlinear subsystem (1), with the assumptions 1 and 3 are satisfied. If there exists a matrix $p_i = p_i^T > 0$ satisfying the Lyapunov equation: $\Lambda_i^T p_i + p_i \Lambda_i + Q_i = 0$ where $Q_i = Q_i^T > 0$. Then the proposed control(26) with adaptation laws (35-38) will ensure that, for $i = 1,2,\ldots,N$. (i) all the variables of the closed-loop system are bounded and (ii) performance tracking is achieved.

Proof: take the error dynamic equation (34), and consider the Lyapunov function candidate

$$\begin{aligned} v_i = & e_i^T p_i e_i + \frac{1}{\eta_{i1}} \Phi_{i1}^T \Phi_{i1} \\ & + \frac{1}{\eta_{i2}} \Phi_{i2}^T \Phi_{i2} + \frac{1}{2\eta_{a_i}} \Phi_{a_i}^T \Phi_{a_i} \end{aligned} \quad (41)$$

where $\Phi_{i1} = \theta_{i1}^* - \theta_{i1}, \Phi_{i2} = \theta_{i2}^* - \theta_{i2}$, $\Phi_{a_i} = a_i - \tau_i^*$, τ_i^* well be defined shortly , and each $P_i \in \Re_i^{d_i x d_i}$ is a positive definite and symmetric matrix.

The time derivative of v_i along the error trajectory (41)

$$\dot{v}_i = \dot{e}_i^T p_i e_i + e_i^T p_i \dot{e}_i + \frac{2}{\eta_{i1}} \Phi_{i1}^T \dot{\Phi}_{i1} + \frac{2}{\eta_{i2}} \Phi_{i2}^T \dot{\Phi}_{i2} + \frac{1}{\eta_{a_i}} \Phi_{a_i}^T \dot{\Phi}_{a_i} \quad (42)$$

Substituting (34) into (42) and choosing $\zeta_i(t) = e_i^T p_i b_i$ we obtain:

$$\begin{aligned}\dot{v}_i &= e_i^T (\Lambda_i^T p_i + p_i \Lambda_i) e_i \\ &+ 2 e_i^T p_i b_i [\Phi_{i1}^T \varphi_i(\underline{x}_i) + \Phi_{i2}^T \varphi_i(\underline{x}_i) u_i \\ &+ \Delta f_i(\underline{x}) + \Delta g_i(\underline{x}_i) u_i - a_i(t) e_i^T p_i b_i \\ &- u_{ih} - \Delta_i(\underline{x}_i)] + \frac{2}{\eta_{i1}} \Phi_{i1}^T \dot{\Phi}_{i1} \\ &+ \frac{2}{\eta_{i2}} \Phi_{i2}^T \dot{\Phi}_{i2} + \frac{1}{\eta_{a_i}} \Phi_{a_i}^T \dot{\Phi}_{a_i}\end{aligned} \quad (43)$$

Applying parameters adaptation laws (35)-(38), yields

$$\begin{aligned}\dot{v}_i &= -\frac{1}{2} e_i^T Q_i e_i - 2 e_i^T p_i b_i \Delta_i(\underline{x}_i) \\ &- \tau_i^* (e_i^T p_i b_i)^2 \\ &\le -\frac{1}{2} e_i^T Q_i e_i + \frac{1}{\tau_i^*} (\Delta_i(\underline{x}_i))^2\end{aligned} \quad (44)$$

now consider the composite system Lyapunov candidate $V = \sum_{i=1}^{N} \varepsilon_i v_i$,where each $\varepsilon_i > 0$. Taking the derivate of V gives

$$\dot{V} \le \sum_{i=1}^{N} \varepsilon_i [-\lambda_i \|e_i\|_2^2 + \frac{1}{\tau_i^*} \psi^T \chi_i \chi_i^T \psi] \quad (45)$$

where λ_i is the real part of the eigenvalue of Q_i. Define $K^* = [\tau_1^*, \tau_2^*, ..., \tau_N^*] \in \Re^N$.

Let $D = diag\{\varepsilon_1 \lambda_1(Q_1), ..., \varepsilon_N \lambda_{N1}(Q_N)\}$ and $M = \sum_{i=1}^{N} \varepsilon_i \chi_i \chi_i^T$, so that $\dot{V} \le -\psi^T A \psi$, where $A = D - \frac{1}{\tau_i^*} M$. Then for some sufficiently large $\tau^* > 0$, the matrix A is positive define. The remainder of the theorem (2) follows as for the direct adaptive case.

5 Simulation results

a double-inverted pendulum connected by a spring can be considered as the simplified example of the large-scale system. Each pendulum may be positioned by a torque input u_i applied by a servomotor at its base. It is assumed that both ϕ_i and $\dot{\phi}_i$ (angular position and rate) are available to the i^{th} controller for $i = 1,2$ .Fig.1.

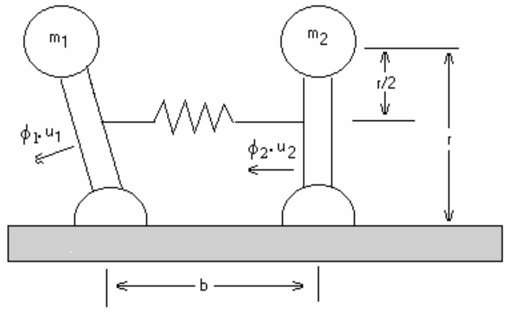

Fig.1. Two inverted pendulums connected by a spring.

Consider a double-inverted pendulum model [10]. The equations which describe the motion of the pendulums are defined by

$$\begin{aligned}\dot{x}_{11} &= x_{12} \\ \dot{x}_{12} &= (\frac{m_1 g r}{j_1} - \frac{k r^2}{4 j_1}) \sin(x_{11}) + \frac{k r}{2 j_1}(l - b) \\ &+ \frac{u_1}{j_1} + \frac{k r^2}{4 j_1} \sin(x_{21})\end{aligned} \quad (46)$$

$$\begin{aligned}\dot{x}_{21} &= x_{22} \\ \dot{x}_{22} &= (\frac{m_1 g r}{j_2} - \frac{k r^2}{4 j_2}) \sin(x_{21}) + \frac{k r}{2 j_2}(l - b) \\ &+ \frac{u_2}{j_2} + \frac{k r^2}{4 j_2} \sin(x_{12})\end{aligned} \quad (47)$$

where $x_{11} = \phi_1$ and $x_{21} = \phi_2$ are the angular displacements of the pendulums from vertical. The parameters $m_1 = 2kg$ and $m_2 = 2.5kg$ are the pendulum end masses, $j_1 = 0.5kg$ and $j_2 = 0.625kg$ are the moments of inertia, the constant of connecting spring is $k = 100N/m$, the pendulum height is r=0.5m, the natural length of the spring is $l = 0.5m$ and the gravitational acceleration is $g = 9.81m/s^2$. The distance between the pendulum hinges is defined as $b = 0.4m$ (with $b < l$ in this example, so that the pendulum links repels each other when both are in the upright position (Fig 1).

In (45) and (47)

$f_1(x_1) = (\frac{m_1 g r}{j_1} - \frac{k r^2}{4 j_1}) \sin(x_{11})$, $g_1(x_1) = 1/j_1$;

$f_2(x_2) = (\frac{m_1 g r}{j_2} - \frac{k r^2}{4 j_2}) \sin(x_{21})$, $g_2(x_2) = 1/j_2$

$$\Delta_1(x) = \frac{kr}{2j_1}(l-b) + \frac{kr^2}{4j_1}\sin(x_{21}),$$

$$\Delta_2(x) = \frac{kr}{2j_2}(l-b) + \frac{kr^2}{4j_2}\sin(x_{12}),$$

the motion equations fit the format of system (1). Here we will attempt to drive the angular positions to zero, so that $e_i = -\phi_i$ (i.e., $y_{1r} = y_{2r} = 0$) for $i = 1,2$

To construct the fuzzy approximators $u_i(\underline{x}_i, \underline{\theta}_i)$ in (9) and $\hat{\alpha}_{ij}(\underline{x}_i/\theta_{i1})$, $\hat{\beta}_{ij}(\underline{x}_i/\theta_{i2})$ in (24), we define three fuzzy sets for component of each $\underline{x}_1 = (x_{11}, x_{12})$ and $\underline{x}_2 = (x_{21}, x_{22})$ with labels $A^1_{x_{ij}}$, $A^2_{x_{ij}}$, $A^3_{x_{ij}}$, $A^4_{x_{ij}}$ and $A^5_{x_{ij}}$ characterized by:

$$\mu_{A^1_{x_{ij}}}(x_{ij}) = \exp(-(x_{ij}+0.8)^2)$$

$$\mu_{A^2_{x_{ij}}}(x_{ij}) = \exp(-(x_{ij}+0.4)^2)$$

$$\mu_{A^3_{x_{ij}}}(x_{ij}) = \exp(-(x_{ij})^2)$$

$$\mu_{A^4_{x_{ij}}}(x_{ij}) = \exp(-(x_{ij}-0.4)^2)$$

$$\mu_{A^5_{x_{ij}}}(x_{ij}) = \exp(-(x_{ij}-0.8)^2)$$

with $n_{i,j} = 5$, $j = 1,2$ and $i = 1,2$.

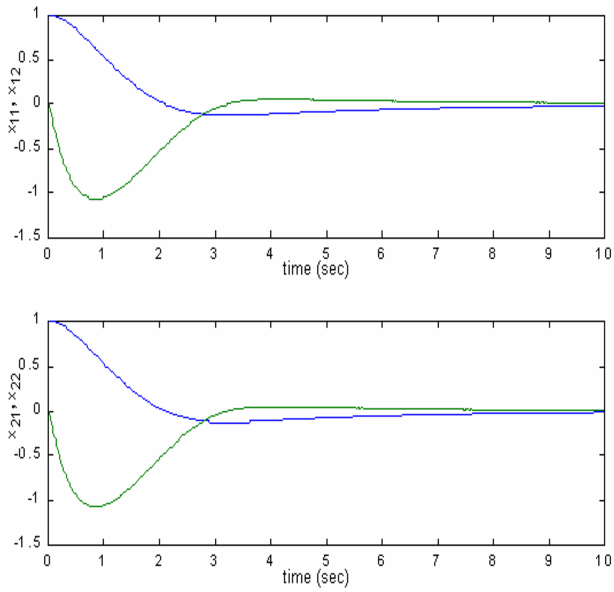

Fig. 2, Control of the pendulums using the proposed direct adaptive decentralized technique.

Defining 25 fuzzy rules, in the following linguistic description:

$R_i^{(l)}$: *if* x_{i1} *is* $A^{j_1}_{x_{i1}}$ *and* x_{i2} *is* $A^{j_2}_{x_{i2}}$ *then* y_i^l *is* C_i^l

Denoting $D_i = \sum_{l=1}^{25}\prod_{k=1}^{2}\mu_{A^l_{x_{ik}}}(x_{ik})$,

$\varphi_i(x_i) = [\varphi_{i,1}(x_i)/D_i, \varphi_{i,2}(x_i)/D_i, \ldots, \varphi_{i,25}(x_i)/D_i]^T$

we can construct the fuzzy system (7) and (24) respectively, as follows: we choose $\eta_i = 0.01$, $\eta_{a_i} = 0.001$ and $\eta_{i1} = 0.001$, $\eta_{i2} = 0.001$, $\eta_{a_i} = 0.0001$, $\sigma_i = 0.15$, $Q_i = diag(10,10)$, and each Λ_i so that $\hat{L}(\varsigma) = \varsigma^2 + 4\varsigma + 4$ has roots at $(-2, -2)$.

Choose the initial conditions to be the same for both direct and indirect approaches in the simulations: $(x_{11}, x_{12}, x_{21}, x_{22})^T = (1,1,1,1)^T$), $\theta_{i1} = \theta_{i2} = 0_{25x1}$ and $\Gamma_i = [-1\ -0.75\ -0.5\ -0.25\ 0\ 0.25\ 0.5\ 0.75\ 1]$. For the direct approach, the simulation results are shown in Fig 2. The simulation results are given in Fig.3

Both direct and indirect fuzzy decentralized controllers achieve good performance, as can be seen from the simulation results.

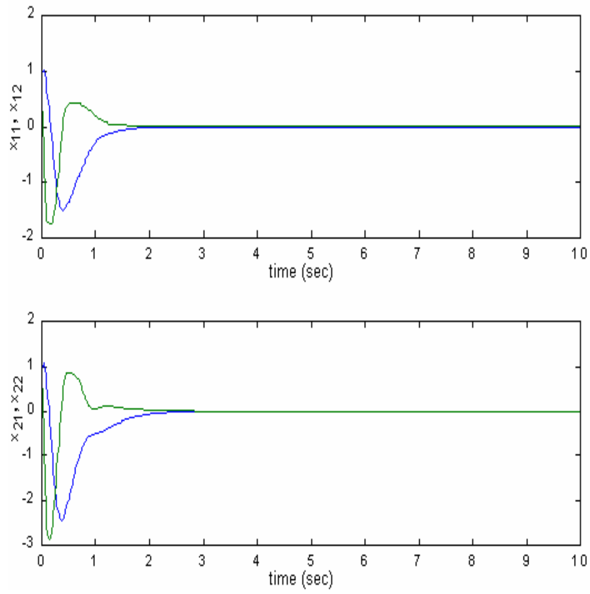

Fig. 3, Control of the pendulums using the proposed indirect adaptive decentralized technique.

Conclusion

In the course of this paper, we have presented an adaptive output-feedback fuzzy decentralized control for a class of large-scale nonlinear systems. In both, direct and indirect adaptive proposed design methods, fuzzy logic systems are used to estimate the part the decentralized adaptive fuzzy controller and unknown nonlinear dynamics without knowing bounds of interconnections. Furthermore, the stability of nonlinear interconnected systems is also guaranteed and ensures asymptotic tracking using only local measurement. The proposed approaches are simple without complex algorithms. Simulations have shown that the proposed controls methodology is effective, with guaranteed stability and satisfying performance.

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