Real-time Fuzzy Digital Filter by ARMA Systems

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Abstract:-This paper describes the Real-time Fuzzy Digital Filters (RTFDF) considering for it the fuzzy logic description (to see: [1], [3], [9], [11], [14], [15], [16], [20], [21]), adaptive digital filters concepts (to see: [2], [5], [11] and [19]) with Real-time properties (to see: [12], [13] and references in it).

The main characteristic operation of this filter is to use a region, which has a linguistic natural description and it is known as the knowledge base [21]. Illustratively, the classification of the output signals system respect to the desired answer in linguistic natural form (to see: [12] and [19]), requiring to establish the membership function set with respect to these for the characterization in computational sense as nature linguistic [3]. The RTFDF needs a group of rules inside of the knowledge base (in mathematical sense using logic connectors (IF and THEN) in agreement to [19]) for to select the best way that the filter need to take, respect to the natural evolution answer system. Novel RTFDF description considers quality answer into natural languages, and the interaction time restrictions with adaptive properties into the fuzzy structure filter.

The basic results described in this document are in formal sense, using the set definitions developed in the papers and referenced below without forgetting the Nyquist, Shannon, Zadeh and Passino criteria into convolution filter scheme ([7],[16],[18] and [21]). Experimentally, we propose a simulation scheme by this type of filters, using Matlab tools by basic ARMA models. The paper has seven basic sections compound as: 1. Introduction, 2. Membership Functions, 3. Fuzzy Digital Filters, 4. Real-time Fuzzy Digital Filters, 5. Simulations, 6.Conclusions and References.

Key-Words: Fuzzy logic, digital filter, adaptive filter, knowledge base, Real-time.

1 Introduction

In digital traditional concepts, the description techniques limit the interaction systems with natural description; but a traditional Fuzzy logic techniques used in artificial intelligent area helps to solve approximately this difficult respect to the system reference [20]: Because the natural description considers quality answer into natural languages sense, and the interaction time restrictions with adaptive properties in the real-time context.

A question around the interaction between the quantitative digital and traditional system and quantitative natural evolution reference system is: how do the interaction levels and variables get the corresponding real operation system conditions (to see: [19] and [21])?

A Fuzzy filter system [1] works in loop form adapting the parameters set dynamically ([2], [6], [7], and [16]). Commonly, the adaptation is a criterion based in the error signal $e(k) \in \mathbb{R}$ ([6] and [7]) optimizing the

velocity function estimation. This error e(k) is defined as the difference between the desired process responses (could be described as $y(k) \in \mathbb{R}$) and the actual signal $\hat{y}(k) \in \mathbb{R}$ ([6], [7], and [16]) generated by an identification * scheme.

The adaptive criterion (using the loop form concept [16]) previously selected, is the first material to establish the membership function, looking for the best approximation $\hat{y}(k)$ around to the desired signal y(k), adjusting the filter parameters dynamically, such that the error value e(k) converges near to the ball with radio $\gamma > 0$ previously defined [12].

Conceptually, each membership function is inside of the error distribution function in agreement to [21]. It means that the best approximation among the membership function respect to distribution description in Borel sense is the triangular form [6].

^{*} The difference between estimator and identifier is that the first describes the parameter evolution and the second describes the state velocity [12].

In general, the fuzzy filter has the following elements considering the concepts studied in [1], [2], [6], [7], [8], [11], [12], [14], and [15]:

- 1. Input Fuzzy Inference: In this stage, the natural process answer is an input filter but transforming it in metric sense.
- 2. *Rule base*: Dynamic range metrics respect to the input filter use the logical binary connector known as *IF*.
- 3. Inference Mechanism: The expert action respect to the rule base knowing as consequence, using logical binary connector *THEN*.
- 4. Output Fuzzy Inference: Finally, this is the last filter stage, which the digital result converted in natural answer process respect to the knowledge base, predefined.



Fig. 1 Fuzzy filter process: Description.

2 Membership Functions

A fuzzy filter uses an error signal functional (1) in distribution sense ([6] and [12]) for generate a basic membership function ([1], [3], [10], [11], [14], and [15]) where each n-cil interval required into adaptable algorithm ([2], [6], [9], and [11]) in agreement to objective function.

Although each member function is dynamically adjusted with gradualism respect to de functional error (1), consider in it the objective function, without forget the adaptation. In probability sense, for example, the objective function adjusts its parameters directly considering the n-cil intervals wide in each time respect to the density information input fuzzy inference.

The most used criterion into literature as objective function is the quadratic mean error (QME), described

it by the second probability moment (to see: [5], [6], and [12]) in a recursive form (1), as:

$$J(k) = \left[\frac{1}{k} \left((k-1)J(k-1)^2 + e(k)^2\right)\right]^{\frac{1}{2}}, \text{ with}$$

$$J(k) \in \mathbb{R}_+$$
(1)

it in metric sense [4].

In fuzzy filter concepts, each rule defines a membership function limited by a specific sequence with respect to correct operation system range in a distribution function desired answer sense [6].

The objective into the fuzzy filter is to give a specific natural response $\hat{y}(k)$ respect to the desired signal y(k), considering the black box idea, but each specific response limited by error criterion functional determining a fuzzy filter action response in agreement to knowledge base, generating a natural answers knowing as low, medium, or high in linguistic manner.

Basically, the knowledge base contains by pairs in metric sense [4], the product set $Y(k) \times \hat{Y}(k) \in \mathbb{R}^2$ defined symbolically as $T_N \in \mathbb{R}^2_{2[1,N]}$ where the desired set is $Y(k) = \{y(k)\}$ and the identification answer set is $\hat{Y}(k) = \{\hat{y}(k)\}$, respectively, labeling each of them from [1, N] to [1, N]:

$$T_N = \{ (y(k), \hat{y}(k)) \}_{k=1}^{2 \times N} \subseteq R_{2[1,N]}^2$$
(2)

Each membership function establishes the maximum correspondence value between the output $\hat{y}(k)$ and the desired signal y(k), where the best value (in ideal form) is the infimum cost for each sequence. In illustrative sense, looking inside of the control area (defined for T_N) having a subset convergence of ordered pairs.

In addition, mathematically the pair's correspondence expressed with respect to the second probability moment has the infimum value, described as:

$$J_{\min} = \min_{\mathcal{N}} \{ \inf J(y_0, \hat{y}) \}_{\mathcal{N}}$$
(3)

3 Fuzzy Digital Filters

The adaptation as a feedback law modifies the signal input filter, such that the answer fuzzy filter system $\hat{y}(k)$ tends to desired signal y(k), generating changes in its states.

The fuzzy filter classifies its different levels operation respect to the membership functions set, gives a specific response value in natural language in agreement to J_{min} (to see: (3)) limited by ball radius γ in a metric sense [4].

Classification system response realized by fuzzy filter bounding parameters gains in agreement to distribution function in agreement to (1), selecting the optimal criterion into the membership functions set considering that they are disjoint in a metric sense [13].

These regions permit to select into the knowledge base, the best values adjusting the input filter, considering the inference mechanisms. Inside of the Inference stage, the classification of the membership functions in agreement to [20] and [21] accomplishes the functional error respect to (1), in Borel sense [13].

Each membership function is inside the distribution function error with range interval criterion (to see: (1)) based in the Lebesgue sense [17] permitting express each of them in an iterate form describing in almost every point (to see: [6], [17], [10], and [12]).

In base of these properties about the RTFDF, we proceed to define how this kind of filters works in real-time manner.

4 Real-time Fuzzy Digital Filters

DEFINITION 1. (Real-time Fuzzy Digital Filter - RTFDF). A Real-time Fuzzy Digital Filter is an adaptive filter performed in according to ([5], [7], [15] and [20]):

- a) Extraction and emission of the fuzzy information through limited intervals respect to the process response within limited intervals according to the stability criteria as Lyapunov, Root-Hurwitz, and others (see: [5], [7]).
- b) Extract and emit information through semi-open time intervals [4], synchronized with evolution time of the process [6] described in a relative way by semi-open time intervals, considering the criterion expressed by Kotel'nikov (1933), Nyquist (1928), Whittaker, (1915), Shannon (1948), and Ecker (2000) (to see references included in [15], [16], [17] and [18]).
- c) A membership functions group forming a discourse linguistic universe, according to the

properties considered in a) and b) points, respectively.

- *A* set of fuzzy rules builds the knowledge base depending of the fuzzy desired signal ŷ(k) respect to reference model y(k) without objective function.
- e) The adaptation algorithm actualizing the filter coefficients according to the membership function corresponds with the established error criterion symbolically expressed by γ_*

In fuzzy filter, the knowledge base has all information that the filter requires for to adjust its gains in optimal form¹ and give a good enough answer accomplishing the convergence range, inside of the time interval (indexed with $k \in \mathbb{Z}_+$) in agreement to Nyquist sense, without lose the stability properties:

a) y(k) knew as measurable value classified in ranges in linguistic sense (describing all of them into space state variable bounded symbolically in a linguistic natural expressions as high, medium or low values),

b) T(k) is the control area described by pairs formed with $\hat{y}(k)$ and y(k), limited with time interval (it has a velocity change bounded in the sense exposed by [15]),

c) e(k) is the fuzzy value defined by the difference among $\hat{y}(k)$ and y(k), where this linguistic value is bounded by the set $\{\gamma_i : \gamma_i > 0, \forall i \in Z_+\}$, $\inf\{\gamma_i\} \rightarrow |\lambda_*|$, such that $|\lambda_*| > 0$, $\sup\{\gamma_i\} \rightarrow |\lambda^*|$, $|\lambda^*| < 1$, means that $\hat{y}(k)$ is approximately equal to y(k)metrically speaking, but in linguistic sense both correspond to the same natural value.

THEOREM 1: The change velocity of the input and output signals is described by semi open intervals [4], respect to the evolution time frequency of the reference system [12 and 13].

PROOF: Each semi open time interval is temporally limited respect to its inputs and outputs [6], because in another hand, the entropy would increase [18], the information could grow up and the system will collapse, having a process saturations that not complain with the conditions established respect to

¹ In natural language, the convergence is around of the point known as optimal, but in metric description (inside of the filter); the convergence is robust because it is defined by intervals.

the reference system evolution.

THEOREM 2: In a temporal sense, the control area T_N has a limited rank operation according to the established criterion [18] respect to the reference system.

PROOF: According to the Nyquist and Shannon criterion [18], the system limited into time intervals the properties of the control area T_N , instead of this, the answer time would be outside of context [9]; therefore, the answer would be after of the required time by the reference system [12 and 13].

DEFINITION 2 (Local and global description) *An RTFDF in local and global temporal sense, has quality response according to the convergence criterion (1) (to see: Theorem (1) and [6 and 8]) respect to real time conditions [18].*

Global characteristics: The convergence intervals defined by $[0, \varepsilon \pm \alpha)$ with a measure up to zero through error functional J(k) and considering [6] and (1), temporally parameterized to the membership function respect to the linguistic variables values ([1 and 14]), without lose that /e(k)/<1 in agreement to [6].

According with the fuzzy concepts, the global characteristics specified in a stochastic sense according with [15], where $J(\tau_m) = \inf\{\min\{\mathcal{J}_k\}\} \le \varepsilon$ (to see: (3)), with $\{\mathcal{J}_k\} \subseteq \{J_k\}$ and $P(\mathcal{J}_k \le \varepsilon \pm \sigma) = 1, \ \sigma << \varepsilon$ without lose its natural evolution described by [18]:

$$\tau_{\min} = 0.5 f_{\max}^{-1}.$$
 (4)

Local characteristics: Implies the process convergence through the digital filter adjusting its parameters $\{a(k)_i\} \subseteq T_N$; selecting each of its through the fuzzy variables, without lose the finalized times $\{f(k)_i\}$ establishing for the filter delimited by its corresponding limits $[ld(k)_{i_min}, LD(k)_i)$ in Nyquist and Shannon sense [18] according to [12 and 13] thus $\mu [ld(k)_{i_min}, LD(k)_i] < \tau_{min}$:

$$\lim_{lk\to m} \left\{ (\hat{y}(k)) \right\}_i \to [0, \gamma_* \pm \alpha).$$
(5)

Fig. 2, shows the execution of the fuzzy filter task in global form through time intervals



The filtering process has a temporal limit at the arrival time (l_i) , start time (s_i) , execution time (c_i) , finalized time (f_i) and maximal time limit (d_i) , in (5) considering τ_{min} .

5 Simulations

The simulation of a fuzzy filter composed in this case by the Kalman filter [6] with a transition matrix described by the knowledge base according to the error functional criterion ([4] and [6]). The evolution times inside of a soft kind system into PC AMD Sempron processor 3100+, with *k* intervals having an mean evolution time as 0.004 sec \pm 0.0002 sec.

The basic system is in discrete space estates expressed as:

$$x(k+1) = a(k)x(k) + w(k).$$
 (6)

And the output:

$$y(k) = x(k) + v(k) : \quad x(k), \ w(k), \ v(k) \in R$$
(7)

Where: x(k) is the internal estate, $\{a(k)\}$ is a parameters sequence, $\{w(k)\}$ is perturbation noise set, y(k) is the output answer system, v(k) is the output noises.

Different operation levels were built in probability sense in agreement to the functional error (1) respect to output answer system y(k) and Kalman answer, $\hat{y}(k)$ respectively; bounded all them by second probability moment (5).

In agreement to error answer levels, we obtain the transition parameter as depicted into the Fig. 3 as:





Fig. 4, shows the response levels respect to Kalman filter:

The evolution time is less than the reference process proposed as 0.09 sec., complying with the condition described in (4).

Fig. 5, shows the functional described in (3) respect to the filter:



Thus, the global convergence time is 0.08 sec, which is less than the evolution condition of the system knowing as *LDmax*, oscillating around of 0.09 sec accordingly whit its properties previously described into definitions.

6 Conclusions

This paper described in section four, the Real-time Fuzzy Digital Filters (RTFDF) concepts considering the fuzzy logic description (to see: [1], [3], [11], [14], [15], [16]), adaptive digital filters concepts (to see: [2], [4], [8] and [12]) with Real-time properties (to see: [12], [13] and references in it). The basic characteristic operation of these kind filters expressed as operation regions knowing it as linguistic natural descriptions and bounded as T_N .

Illustratively, it showed the output signals system classification respect to desired answer in linguistic

natural form (to see: [12] and [13]), requiring to establish the membership function set respect to the linguistic description [3].

Then RTFDF needed a group rules, to consult and to discriminate inside of the knowledge base built in a probabilistic manner [19].

A novel RTFDF description evaluated in two senses, in quality answer (considering the natural languages), and interaction time restrictions, considering in both cases, the adaptive properties.

The basic results described in this document in formal sense, used the set definitions developed in the papers referenced below without forgetting the Nyquist and Shannon criterion into the convolution filter scheme. Also, the paper shows a simulation of the basic concepts of RTFDF.

The RTFDF has many applications, such that: navigation systems, biological signals with interaction levels answer, and hybrid systems with natural language and discrete operation with Fuzzy logic adaptation requirements. Nevertheless, the interactions between any biological system and its environment will be better considering the artificial intelligences tools as genetic algorithms, neural networks and other topics interacting with these filters description.

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