FGN Based Telecommunication Traffic Models

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Abstract: - This paper addresses three models of traffic based on fractional Gaussian noise (fGn). The first is the standard fGn (fGn for short) that is characterized by a single Hurst parameter. The second is the generalized fGn (GfGn) indexed by two parameters. The third the local Hurst function. The limitation of fGn in traffic modeling is explained. We shall exhibit that the model of GfGn can be used to release that limitation. Finally, we discuss the local Hurst function to interpret that it is a simple model to express the multifractal property of traffic on a point-by-point basis.

Key-Words: - Internet traffic modeling; Fractional Gaussian noise; Fractal time series; Statistical computing.

1 Introduction

Teletraffic (traffic for short) modeling plays a role in telecommunications (Akimaru and Kawashima [1]). A modern telecommunication system is the Internet that is an infrastructure in modern societies. In principle, techniques of traffic modeling are application dependent. There are two categories of the traffic models, namely, stochastic modeling and deterministic modeling (Li and Borgnat [2]).

Let $x(t_i)$ be an arrival traffic function, implying the number of bytes in the ith packet arriving at t_i (i = 0, 1, 2, …), where t_i is the timestamp of the ith packet (Li et al. [3]). The function $x(t_i)$ may represent either aggregated traffic, consisting of arrival packets of all connections at the input of a server, or arrival packets of a specific connection or a specific class of connections. The former is called aggregated traffic while the later traffic at connection level. Network management concerns about stochastic modeling in the aggregated case while QoS relates to bounded modeling at connection level. Without confusions, we use $x(t)$ and $x(i)$ to represent a traffic trace in the continuous case and the discrete case, respectively.

The pioneering work of bounded modeling refers to Cruz [4] and that of TAMU (Raha et al. [5,6]). This type of models is developing towards stochastically bounded modeling, see e.g., Jiang [7], Jiang and Liu [8], Li et al. [9,10], Wang et al. [11,12], Starobinski and Sidi [13], Yaron and Sidi [14], Parekh and Gallager [15], Li and Zhao [16].

As far as the stochastic modeling of traffic was concerned, self-similar process may be the mostly used, see e.g., Partridge [17], Leland et al. [18], Crovella and Bestavros [19], Beran et al. [20], Paxson and Floyd [21], Tsybakov and Georganas [22], Willinger and Paxson [23], Adas [24], Michiel and Laevens [25], Stallings [26], Carmona et al. [27], Pitts and Schormans [28], MaDysan [29], Sheluhin et al. [30], Erramilli et al. [31], Karagiannis et al. [32], Chakraborty et al. [33], Song and Ng [34], Norros [35], Ma and Ji [36], Lee and Fapojuwo [37], He and Hou [38], Li [39,40], just citing a few.

Note that the fractional Gaussian noise (fGn) is the only stationary self-similar process while fractional Brownian motion (fBm) is the only nonstationary self-similar process. Therefore, we take fGn as the synonym of self-similar process if considering stationary processes or fBm when nonstationary ones in what follows.

The limitation of fGn in modeling traffic was noticed by Paxson and Floyd [21], Tsybakov and Georganas [22], and Beran [41]. Therefore, in addition to the fGn modeling, locally self-similar processes are paid attention to, such as the generalized Cauchy process (Li [42,43], Li and Lim [44-46]), alpha stable processes (Karasaridis and Hatzinakos [47], Shao and Nikias [48], Garroppo et al. [49]), Levy flights (Terdik and Gyires [50], Kogon and Manolakis [51], Li [52,53]). This paper focuses on the fGn based models of traffic.

By processing data of real traffic, it was reported that the fitting the data based on fGn is in the order of magnitude of 10^{-3} when the curve fitting is measured by mean square errors, see Li [54], Li et al. [55,56]. Those in [54-56] are quantitative results to describe the limitation stated in [21,22].

Recently, two models based on fGn were reported. One introduced by Li [57] is an fGn model with two parameters, which significantly improves the model accuracy. The other is the local Hurst function that is introduced in mathematics by Peltier and Levy-Vehel [58], see its application to traffic modelling in Li et al. [59,60]. We take the local Hurst function as a model that is fGn based.

This paper is organized as follows. We shall discuss fGn and its limitation in traffic modeling in Section 2. The limitation is further illustrated by using real-traffic traces in Section 3. In Section 4, we shall brief the two-parameter fGn and the local Hurst function modelling of traffic. Finally, Section 5 concludes the paper.

2. FGN and Its Limitation

Traffic on old telephony networks obeys the Poisson model. It has been successfully used in the design of infrastructure of old telephony networks for years (Gibson [61]). It is such a success on old telephony networks that it has almost been taken as an axiom for modelling traffic in communication systems. Due to unsatisfactory performances of the Internet, such as traffic congestions, people began doubting about the Poisson model. To re-evaluate the Internet traffic models, people began measuring the Internet at different sites during different periods of times (Paxson [62,63] and Traffic Archive at www.sigcomm.org/ITA/). Experimental processing real–traffic traces reveals that traffic has fractal properties. The early fractal model used for traffic modelling is fGn that is introduced in mathematics by Mandelbrot and van Ness [64].

In order to clarify the significance of fractal models, we shall first brief the basics of conventional time series in this section. Then, fBm and fGn are discussed.

Let $\{x_l(t)\}$ (l = 1, 2, …) be a 2-order stationary random process, where $x_l(t)$ is the lth sample function of the process. We use $x_l(t)$ to represent the process without confusion causing. Its mean is

$$\mu_x^s(t) = \lim_{N\to\infty} \frac{1}{N} \sum_{l=1}^{N} x_l(t) = \text{const}. \tag{1}$$

Its autocorrelation function (ACF) is given by

$$R_x^s(t, t+\tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{l=1}^{N} x_l(t) x_l(t+\tau) = R_x^s(\tau). \tag{2}$$

In (1) and (2), the superscript s implies that the mean and the ACF are computed by using spatial average. The mean and the ACF of a process expressed by time average are expressed by

$$\mu_x^t(t) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x_l(t) dt = \text{const}, \tag{3}$$

$$R_x^t(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x_l(t) x_l(t+\tau) d\tau, \tag{4}$$

where the superscript t indicates that the mean and the ACF are computed by time average.

The process $x_l(t)$ is said to be ergodic if (5) and (6) hold,

$$\mu_x^s(t) = \mu_x^t(t) = \mu_x = \text{const}, \tag{5}$$

$$R_x^s(\tau) = R_x^t(\tau) = R(\tau). \tag{6}$$

Note that a real-traffic trace is a series of single history. In what follows, consequently, we just use $x(t)$ to represent a traffic process.

Denote by $p(\xi)$ the probability density function (PDF) of traffic $x(t)$. Then, the probability is

$$P(x_2) - P(x_1) = \Pr\text{ob}[x_1 < \xi < x_2] = \int_{x_1}^{x_2} p(\xi) d\xi. \tag{7}$$

The mean and the ACF of $x(t)$ based on PDF are written by (8) and (9), respectively,

$$\mu_x = \int_{-\infty}^{\infty} x p(x) dx, \tag{8}$$

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t) x(t+\tau) p(x) dx. \tag{9}$$

Let σ_x^2 be the variance of x. Then, x is said to follow the Gaussian distribution if

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}. \tag{10}$$

The Poisson distribution is a discrete probability distribution that expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate and independently of the time since the last event. In communication networks, one is interested in the work focused on certain random variables N that count, among other things, a number of discrete occurrences (sometimes called "arrivals") that take place during a time-interval of given length. Denote the expected number of occurrences in this interval by a positive real number λ. Then, the probability that there are exactly k occurrences (k being a non-negative integer, k = 0, 1, 2, …) is given by the Poisson distribution below

$$p(x;\lambda)=\frac{\lambda^k e^{-\lambda}}{k!}. \tag{11}$$

One thing worth noting is that either (10) or (11) fast decays, more precisely, exponentially decays. Therefore, according to (8) and (9), μ_x and R_x are convergent, which is actually a defaulted assumption in the traditional theory of communication networks. However, actual traffic challenges such an assumption.

Computer scientists claim that a traffic series is heavy-tailed, see e.g., [18-23], Resnick [65], Willinger et al. [66], Abry et al. [67], Cappe et al. [68], Li [69,70]. The tail of the PDF of traffic may be so heavy that its ACF decays hyperbolically. On the one hand, because of slowly decaying of the ACF, a random variable that represents a traffic series can be no longer considered to be independent. Hence, long-range dependence (LRD). On the other hand, the Fourier transform

$$S_x(\omega)=\int_{-\infty}^{\infty}R_x(\tau)e^{-j\omega\tau}d\tau, \tag{12}$$

of a slowly decayed ACF implies that the PSD of traffic with LRD obeys a power law. Hence, 1/*f* noise. These contents are actually in the domain of fractal time series.

Now we consider the fractional Brownian motion (fBm). Let $B(t)$ be a random process. Then, $B(t_{n+1}) - B(t_n)$ (n = 0, 1, 2, ...) is its increment process. If $B(t)$ has the following properties, it is called Brownian motion ([4], Hida [73]).

- The increments $B(t + t_0) - B(t_0)$ are Gaussian.
- $E[B(t + t_0) - B(t_0)] = 0$ and $\mathrm{Var}[B(t + t_0) - B(t_0)] = \sigma^2 t$, where $\sigma^2 = E\{[B(t + 1) - B(t)]^2\} = E\{[B(1) - B(0)]^2\} = E\{[B(1)]^2\}$.
- In non-overlapping intervals $[t_1, t_2]$ and $[t_3, t_4]$, the increments $B(t_4) - B(t_3)$ and $B(t_2) - B(t_1)$ are independent.
- $B(0) = 0$ and $B(t)$ is continuous at $t = 0$.

Let $B_H(t)$ be fBm with the Hurst parameter $H \in (0, 1)$. Let $\Gamma(\cdot)$ be the Gamma function. Then,

$$B_H(t)-B_H(0)=\frac{1}{\Gamma(H+1/2)}\left\{\begin{array}{l}\int_{-\infty}^{0}[(t-u)^{H-0.5}-(-u)^{H-0.5}]dB(u)\\ +\int_{0}^{t}(t-u)^{H-0.5}dB(u)\end{array}\right\}. \tag{13}$$

The function $B_H(t)$ has the following properties.

- $B_H(0) = 0$.
- The increments $B_H(t + t_0) - B_H(t_0)$ are Gaussian.
- $\mathrm{Var}[B_H(t + t_0) - B_H(t_0)] = \sigma^2 t^{2H}$, where $\sigma^2 = E\{[B_H(t + 1) - B_H(t)]^2\} = E\{[B_H(1) - B_H(0)]^2\} = E\{[B_H(1)]^2\}$. (14)

According to the properties of fBm, one has

$$\begin{aligned}&E\{[B_H(t_2) - B_H(t_1)]^2\}\\ &= E\{[B_H(t_2 - t_1) - B_H(0)]^2\}\\ &= E\{[B_H(t_2 - t_1)]^2\} = \sigma^2(t_2 - t_1)^{2H}.\end{aligned} \tag{15}$$

In addition,

$$\begin{aligned}&E\{[B_H(t_2) - B_H(t_1)]^2\}\\ &= E\{[B_H(t_2)]^2\} + E\{[B_H(t_1)]^2\}\\ &- 2E[B_H(t_2)B_H(t_1)]\\ &= \sigma^2(t_2)^{2H} + \sigma^2(t_1)^{2H} - 2r[B_H(t_2), B_H(t_1)].\end{aligned} \tag{16}$$

Thus, the ACF of $B_H(t)$, denoted by $r_{B_H,W}(t,s)$, is given by

$$r_{B_H,W}(t,s)=\frac{V_H}{(H+1/2)\Gamma(H+1/2)}\left[|t|^{2H}+|s|^{2H}-|t-s|^{2H}\right], \tag{17}$$

where

$$V_H=\mathrm{Var}[B_H(1)]=\Gamma(1-2H)\frac{\cos\pi H}{\pi H}. \tag{18}$$

Denote by $S_{B_H,W}(t,\omega)$ the PSD of $B_H(t)$. Then (Flandrin [71]),

$$S_{B_H,W}(t,\omega)=\frac{1}{|\omega|^{2H+1}}(1-2^{1-2H}\cos 2\omega t). \tag{19}$$

From the above, we see that either the ACF or the PDF of $B_H(t)$ is time varying. Therefore, $B_H(t)$ is nonstationary.

Note that $B_H(t)$ is self-similar because it satisfies the definition of self-similarity. In fact,

$$B_H(at)\equiv a^H B_H(t),\quad a>0, \tag{20}$$

where $\equiv$ denotes equality in the sense of probability distribution.

From (19), one sees that the PSD of fBm is divergent at $\omega = 0$, exhibiting a case of $1/f^{\alpha}$ noise, see Csabai [72] for the early work of 1/*f* noise of traffic. The relationship between the fractal dimension of fBm, denoted by D_{fBm}, and its Hurst parameter denoted by H_{fBm} is given by

$$D_{\mathrm{fBm}}=2-H_{\mathrm{fBm}}. \tag{21}$$

Note that the increment series, $B_H(t+s) - B_H(t)$, is fGn. Thus, one has

$$\begin{aligned} &\mathrm{E}\{[B_H(t_4) - B_H(t_3)][B_H(t_2) - B_H(t_1)]\} \\ &= r\{[B_H(t_4) - B_H(t_3)], [B_H(t_2) - B_H(t_1)]\} \\ &= \mathrm{E}\{[B_H(t_4)B_H(t_2) - B_H(t_4)B_H(t_1) \\ &\quad - B_H(t_3)B_H(t_2)] + B_H(t_3)B_H(t_1)\} \\ &= \mathrm{E}[B_H(t_4)B_H(t_2)] - \mathrm{E}[B_H(t_4)B_H(t_1)] \\ &\quad - \mathrm{E}[B_H(t_3)B_H(t_2)] + \mathrm{E}[B_H(t_3)B_H(t_1)] \\ &= r[B_H(t_4), B_H(t_2)] - r[B_H(t_4), B_H(t_1)] \\ &\quad - r[B_H(t_3), B_H(t_2)] + r[B_H(t_3), B_H(t_1)]. \quad (22) \end{aligned}$$

According to (17), therefore, one has

$$r[B_H(t_4), B_H(t_2)] = \frac{\sigma^2}{2}[(t_4)^{2H} + (t_2)^{2H} - (t_4 - t_2)^{2H}], \quad (23)$$

$$r[B_H(t_4), B_H(t_1)] = \frac{\sigma^2}{2}[(t_4)^{2H} + (t_1)^{2H} - (t_4 - t_1)^{2H}], \quad (24)$$

$$r[B_H(t_3), B_H(t_2)] = \frac{\sigma^2}{2}[(t_3)^{2H} + (t_2)^{2H} - (t_3 - t_2)^{2H}], \quad (25)$$

$$r[B_H(t_3), B_H(t_1)] = \frac{\sigma^2}{2}[(t_3)^{2H} + (t_1)^{2H} - (t_3 - t_1)^{2H}]. \quad (26)$$

Replacing the right hand of (22) by (23) ~ (26) yields

$$\begin{aligned} &\mathrm{E}\{[B_H(t_4) - B_H(t_3)][B_H(t_2) - B_H(t_1)]\} \\ &= r\{[B_H(t_4) - B_H(t_3)], [B_H(t_2) - B_H(t_1)]\} \\ &= \frac{\sigma^2}{2}[(t_4 - t_2)^{2H} + (t_3 - t_2)^{2H} - (t_4 - t_2)^{2H} - (t_3 - t_1)^{2H}]. \quad (27) \end{aligned}$$

In the discrete case, we let $t_1 = n$, $t_2 = n + 1$, $t_3 = n + k$, $t_4 = n + k + 1$. Then,

$$\begin{aligned} &r\{[B_H(t_4) - B_H(t_3)], [B_H(t_2) - B_H(t_1)]\} \\ &= \frac{\sigma^2}{2}[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}]. \quad (28) \end{aligned}$$

Thus, the ACF of the discrete fGn (dfGn) is

$$r(k) = \frac{\sigma^2}{2}[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}]. \quad (29)$$

Since the ACF is an even function, we have

$$r(k) = \frac{\sigma^2}{2}\left[(|k|+1)^{2H} + \left||k|-1\right|^{2H} - 2|k|^{2H}\right], \quad (30)$$

where $k \in \mathbb{Z}$. Denote by $C_H(\tau;\ \varepsilon)$ the ACF of fGn in the continuous case. Then,

$$C_H(\tau;\ \varepsilon) = \frac{V_H \varepsilon^{2H-2}}{2}\left[\left(\frac{|\tau|}{\varepsilon}+1\right)^{2H} + \left|\frac{|\tau|}{\varepsilon}-1\right|^{2H} - 2\left|\frac{\tau}{\varepsilon}\right|^{2H}\right], \quad (31)$$

where $\varepsilon > 0$ is used by smoothing fBm so that the smoothed fBm is differentiable.

The PSD of dfGn was derived out quite early by Sinai [74]. It is given by

$$S_{\mathrm{dfGn}}(\omega) = 2C_f(1-\cos\omega)\sum_{n=-\infty}^{\infty}|2\pi n + \omega|^{-2H-1}, \quad (32)$$

where $C_f = V_H^2(2\pi)^{-1}\sin(\pi H)\Gamma(2H+1)$ and $\omega \in [-\pi, \pi]$. The PSD of fGn is (Li and Lim [75])

$$S_{\mathrm{fGn}}(\omega) = V_H^2 \sin(H\pi)\Gamma(2H+1)|\omega|^{1-2H}, \quad (33)$$

which exhibits that fGn is a type of $1/f$ noises.

We say that $f(t)$ is asymptotically equivalent to $g(t)$ under the limit $x \to c$ if $f(t)$ and $g(t)$ are such that $\lim_{x\to c}\frac{f(t)}{g(t)} = 1$ (Murray [76]), i.e.,

$$f(t) \sim g(t)\ (t \to c) \text{ if } \lim_{x\to c}\frac{f(t)}{g(t)} = 1, \quad (34)$$

where c can be infinity. It has the property expressed by

$$f(t) \sim g(t) \sim h(t)\ (t \to c). \quad (35)$$

In this sense, $f(t)$ is called slowly varying function if $\lim_{u\to\infty}\frac{f(ut)}{f(u)} = 1$ for all t.

A random series $x(i)$ is said to be of LRD if

$$r(k) \sim ck^{-\beta}\ (k \to \infty) \text{ for } c > 0,\ \beta \in (0, 1), \quad (36)$$

where c can also be a slowly varying function.

Eq. (36) implies that the ACF of a series with LRD is non-summable. That is,

$$\sum_k r(k) = \infty. \quad (37)$$

Replacing β by the Hurst parameter H yields

$$\beta = 2 - 2H. \quad (38)$$

Thus, another expression of (36) is written by

$$r(k) \sim ck^{2H-2}\ (k \to \infty) \text{ for } c > 0,\ H \in (0.5, 1). \quad (39)$$

On the other side, if $\beta > 1$ or $H \in (0, 0.5)$, $r(k)$ is summable, corresponding to the case of short-range dependence (SRD).

Note that $0.5[(\tau+1)^{2H} - 2\tau^{2H} + (\tau-1)^{2H}]$ can be approximated by $H(2H-1)(\tau)^{2H-2}$. In fact, that is the finite 2-order difference of $0.5(\tau)^{2H}$ (Mandelbrot [77]). Approximating it with the 2-order differential of $0.5(\tau)^{2H}$ yields

$$0.5[(\tau+1)^{2H} - 2\tau^{2H} + (\tau-1)^{2H}] \approx H(2H-1)(\tau)^{2H-2}. \quad (40)$$

From the above, one immediately sees that fGn contains three subclasses of time series. In the case of $H \in (0.5, 1)$, the ACF is non-summable and the corresponding series is of LRD. For $H \in (0, 0.5)$, the ACF is summable and fGn in this case is of SRD. FGn reduces to white noise when $H = 0.5$.

Among LRD processes, fGn has its advantage in traffic modeling. For example, it can be used to easily represent two types of traffic series, namely, self-similar process and processes with LRD.

Note that LRD is a global property of traffic. However, in principle, self-similarity is a local property of traffic. It is measured by fractal dimension D, see e.g., Hall and Roy [78], Chan et al.

[79], Adler [80], Kent and Wood [81]. In fact, if $R(\tau)$ of $X(t)$ is sufficiently smooth on $(0, \infty)$ and if

$$R(0) - R(\tau) \sim c|\tau|^{\alpha} \text{ for } |\tau| \to 0, \tag{41}$$

where c is a constant, then one has the fractal dimension of $X(t)$ as

$$D = 2 - \frac{\alpha}{2}. \tag{42}$$

Denote D_{fGn} the fractal dimension of fGn. Then, according to the asymptotic expression (40), one has

$$r_{\text{fGn}}(0) - r_{\text{fGn}}(\tau) \sim c|\tau|^{2H} \text{ for } |\tau| \to 0. \tag{43}$$

According to (39) and (42), therefore, one immediately gets

$$D_{\text{fGn}} = 2 - H. \tag{44}$$

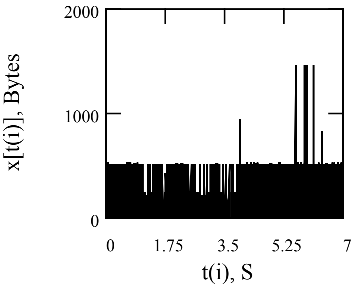
Hence, for fGn, the local properties happen to be reflected in the global ones as noticed by Mandelbrot [82, p. 27].

The above discussions exhibit that fGn has its limitation in traffic modeling because it uses a single parameter H to characterize two different phenomena, that is, local property and global one. Recently, Li [57] introduced a generalized fGn indexed by two parameters, releasing the limitation of fGn in traffic modeling.

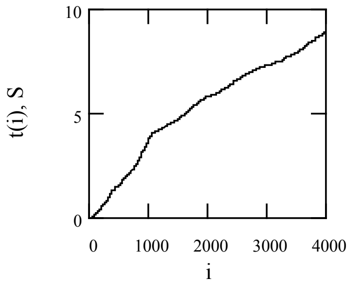
3 Demonstrations

Real data used in this paper consist of two traces. One is DEC-pkt-1.TCP and the other DEC-pkt-1.UDP, where DEC implies that data were measured at Digital Equipment Corporation. Denote $R(k)$ by $R(k; H)$ for the illustrations below.

The series $x[t(i)]$ of DEC-pkt-1.TCP is indicated in Fig. 1 (a) and timestamp series $t(i)$ is in Fig. 1 (b). The interarrival series $s(i)$ is in Fig. 2. Denote by $M^2(R)$ the minimum mean square error for the data fitting. Then, $M^2(R) = 2.264\times10^{-3}$ for $s(i)$ of DEC-pkt-1.TCP. The measured ACF of $s(i)$ is plotted in Fig. 3 (a). The modeled ACF $R(k)$ of $s(i)$ of DEC-pkt-1.TCP using fGn is indicated in Fig. 3 (b). Fig. 3(c) shows the fitting the data. By eye, one sees that fGn does not satisfactorily fits the ACF of $s(i)$ of DEC-pkt-1.TCP for short-term lags.



(a)



(b)

Fig. 1. Traffic DEC-pkt-1.TCP. (a). $x[t(i)]$. (b). $t(i)$.

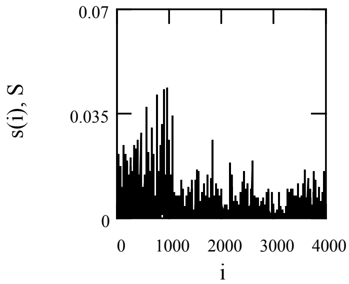
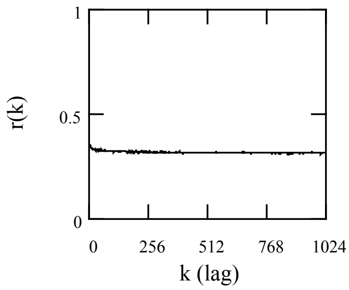
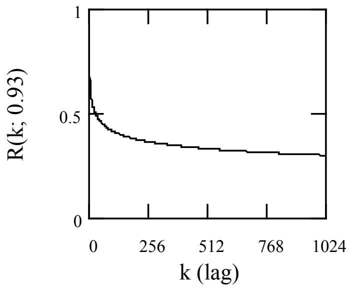


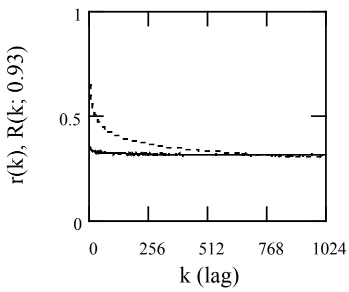
Fig. 2. Interarrival series $s(i)$ of DEC-pkt-1.TCP.



(a)



(b)



(c) —— $r(k)$, ······ $R(k)$.

Fig. 3. Modeling procedure. (a). $r(k)$: Measured ACF of $s(i)$ of DEC-pkt-1.TCP. (b). $R(k)$: Modeled ACF based on fGn. (c). Fitting the data.

Real series $t(i)$ for DEC-pkt-1.UDP is shown in Fig. 4 and $s(i)$ in Fig. 5, respectively. The measured ACF of $s(i)$ is shown in Fig. 6 (a). Fig. 6 (b) indicates the modeled ACF using fGn and Fig. 6 (c) shows the fitting the data with $M^2(R) = 6.09\times10^{-3}$.

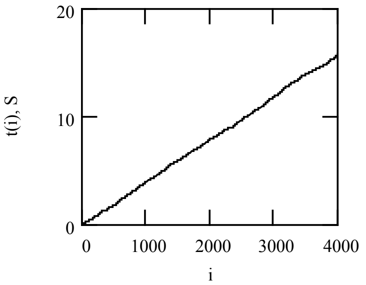


Fig. 4. Real series $t(i)$ for DEC-pkt-1.UDP.

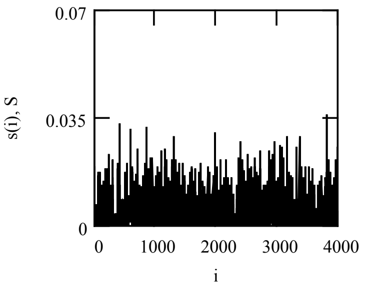
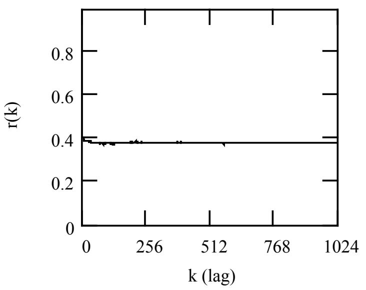
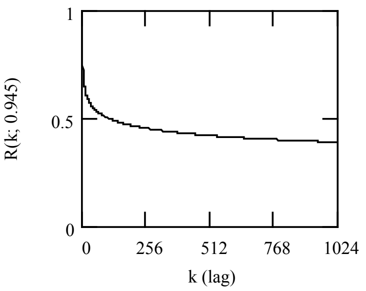


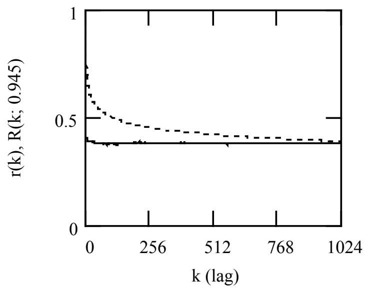
Fig. 5. Real series $s(i)$ for DEC-pkt-1.UDP.



(a)



(b)



(c) —— $r(k)$, ······ $R(k)$.

Fig. 6. Modeling procedure. (a). Measured ACF of $s(i)$ of DEC-pkt-1.UDP. (b). $R(k)$: Modeled ACF based on fGn. (c). Fitting the data.

4 Other fGn Based Models

As mentioned previously, fGn has its limitation in modeling small lags of traffic. To release that limitation, Li [57] introduced the generalized fGn (GfGn). Its ACF in the discrete case is given by

$$r_{\mathrm{GfGn}}(k;H,a)=\frac{V_H^2}{2}\left(\left||k|^a+1\right|^{2H}-2\left||k|^a\right|^{2H}+\left||k|^a-1\right|^{2H}\right), \tag{45}$$

where $0 < a \le 1$. It can be easily seen that the above $r_{\mathrm{GfGn}}(k;H,a)$ becomes the ACF of the standard fGn if $a = 1$.

Traffic has multifractal properties, see e.g., Abrey and Veitch [83], Taqqu et al. [84], Feldmann et al. [85]. Due to this, we need considering processes that are locally self-similar. One of possible processes is to generalize fBm by replacing the Hurst parameter H by a continuously deterministic function $H(t)$ (Lim and Muniandy [86]). The function $H(t)$ satisfies H: $[0, \infty] \to (0, 1)$. Denote the generalized fBm by $X(t)$, instead of $B_H(t)$, so as to distinguish it from the standard one. Then,

$$X(t)=\frac{1}{\Gamma(H(t)+1/2)}\left\{\begin{array}{l}\int_{-\infty}^{0}[(t-u)^{H(t)-0.5}-(-u)^{H(t)-0.5}]dB(u)\\+\int_{0}^{t}(t-u)^{H(t)-0.5}dB(u)\end{array}\right\}. \tag{46}$$

The following ACF holds for $\tau \to 0$

$$\mathrm{E}[X(t)X(t+\tau)]=\frac{V_{H(t)}}{(H(t)+1/2)\Gamma(H(t)+1/2)}\left[|t|^{2H(t)}+|t+\tau|^{2H(t)}-|\tau|^{2H(t)}\right]. \tag{47}$$

In fact, $H(t)$ can be regarded as a tool to characterize local properties of fBm. This can be seen when the self-similarity is expressed by

$$X(at) \equiv a^{H(t)} X(t), \quad a > 0. \tag{48}$$

Based on the local growth of the increment process, one may write a sequence expressed by

$$S_k(j) = \frac{m}{N-1} \sum_{j=0}^{j+k} |X(i+1) - X(i)|, \ 1 < k < N, \tag{49}$$

where m is the largest integer not exceeding N/k. Then, $H(t)$ at point $t = j/(N-1)$ is given by (Peltier and Levy-Vehel [58])

$$H(t) = -\frac{\log(\sqrt{\pi/2} S_k(j))}{\log(N-1)}. \tag{50}$$

Fig. 7 plots a real-traffic trace. Fig. 8 shows its $H(t)$, easily giving the evidence of the multifractal property of traffic on a point-by-point basis.

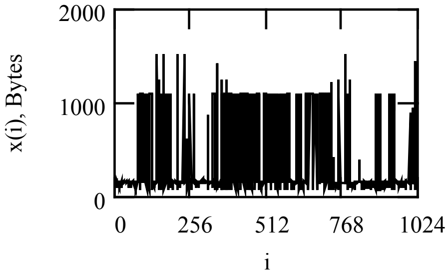


Fig. 7. Traffic of BC-pAug89.

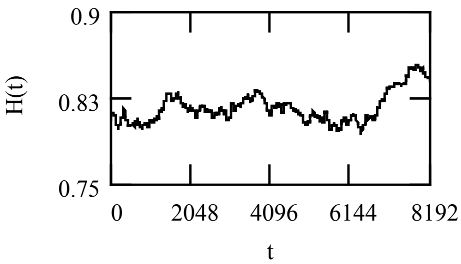


Fig. 8. Local Hurst function of $X(i)$.

Note that traffic theory relates to computational techniques, such as wavelet, fractals, time series and statistical computing in short [87-99], which we shall discuss in future.

5 Conclusion

We have explained 3 models of traffic based on fGn. The limitation of fGn has been addressed. The generalized fGn with two parameters is discussed and the local Hurst function to easily show the multifractal property of traffic is illustrated.

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