New Exact Traveling Wave Solutions For Three Nonlinear Evolution Equations

Bin Zheng
Shandong University of Technology
School of Science
Zhangzhou Road 12, Zibo, 255049
China
zhengbin2601@126.com

Abstract: In this paper, we demonstrate the effectiveness of the $(\frac{G'}{G})$-expansion method by seeking more exact solutions of the SRLW equation, the (2+1) dimensional PKP equation and the (3+1) dimensional potential-YTSF equation. By the method, the two nonlinear evolution equations are separately reduced to non-linear ordinary differential equations (ODE) by using a simple transformation. As a result, the traveling wave solutions are obtained in three arbitrary functions including hyperbolic function solutions, trigonometric function solutions and rational solutions. When the parameters are taken as special values, we also obtain the soliton solutions of the fifth-order Kdv equation. The method appears to be easier and faster by means of a symbolic computation system.

Key–Words: $(\frac{G'}{G})$-expansion method, Traveling wave solutions, SRLW equation, (2+1) dimensional PKP equation, (3+1) dimensional potential-YTSF equation, exact solution, evolution equation, nonlinear equation

1 Introduction

The nonlinear phenomena exist in all the fields including either the scientific work or engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, and so on. It is well known that many non-linear evolution equations (NLEEs) are widely used to describe these complex phenomena. Research on solutions of NLEEs is popular. So, the powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far such as in [1-7].

In this paper, we pay attention to the analytical method for getting the exact solution of some NLEES. Among the possible exact solutions of NLEEs, certain solutions for special form may depend only on a single combination of variables such as traveling wave variables. In the literature, Also there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the homogeneous balance method [8,9], the hyperbolic tangent expansion method [10,11], the trial function method [12], the tanh-method [13-15], the non-linear transform method [16], the inverse scattering transform [17], the Backlund transform [18,19], the Hirotas bilinear method [20,21], the generalized Riccati equation [22,23], the Weierstrass elliptic function method [24], the theta function method [25-27], the sineCcosine method [28], the Jacobi elliptic function expansion [29,30], the complex hyperbolic function method [31-33], the truncated Painleve expansion [34], the F-expansion method [35], the rank analysis method [36], the exp-function expansion method [37] and so on.

In [38], Mingliang Wang proposed a new method called $(\frac{G'}{G})$-expansion method. Recently several authors have studied some nonlinear equations by this method [39-42]. The value of the $(\frac{G'}{G})$-expansion method is that one can treat nonlinear problems by essentially linear methods. The method is based on the explicit linearization of NLEEs for traveling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. Moreover, it transforms a nonlinear equation to a simple algebraic computation. The main merits of the $(\frac{G'}{G})$-expansion method over the other methods are that it gives more general solutions with some free parameters and it handles NLEEs in a direct manner with no requirement for initial/boundary condition or initial trial function at the outset.

Our aim in this paper is to present an application of the $(\frac{G'}{G})$-expansion method to some nonlinear problems to be solved by this method for the first time.

The rest of the paper is organized as follows. In Section 2, we describe the $(\frac{G'}{G})$-expansion method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the method to the SRLW equation, the (2+1) dimensional PKP equation and the (3+1) dimensional potential-YTSF equation. In section 5, the features of the $(\frac{G'}{G})$-expansion method are briefly summarized.

2 Description of the $(\frac{G'}{G})$-expansion method

In this section we describe the $(\frac{G'}{G})$-expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables $x,\ t$, is given by

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0, \tag{2.1}$$

or in three independent variables $x,\ y$ and t, is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}, ...) = 0, \tag{2.2}$$

where $u = u(x,t)$ or $u = u(x,y,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ or $u = u(x,y,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following, we will give the main steps of the $(\frac{G'}{G})$-expansion method.

Step 1. Suppose that

$$u(x,\ t) = u(\xi), \quad \xi = \xi(x,t) \tag{2.3}$$

or

$$u(x,\ y,\ t) = u(\xi), \quad \xi = \xi(x,y,t) \tag{2.4}$$

The traveling wave variable (2.3) or (2.4) permits us reducing (2.1) or (2.2) to an ODE for $u = u(\xi)$

$$P(u,\ u', u'',\ ...) = 0. \tag{2.5}$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \alpha_m(\frac{G'}{G})^m +\ ... \tag{2.6}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \tag{2.7}$$

$\alpha_m, ..., \lambda$ and μ are constants to be determined later, $\alpha_m \neq 0$. The unwritten part in (2.6) is also a polynomial in $(\frac{G'}{G})$, the degree of which is generally equal to or less than $m-1$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using second order LODE (2.7), collecting all terms with the same order of $(\frac{G'}{G})$ together, the left-hand side of (2.5) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, ..., \lambda$ and μ.

Step 4. Assuming that the constants $\alpha_m, ..., \lambda$ and μ can be obtained by solving the algebraic equations in Step 3. Since the general solutions of the second order LODE (2.7) have been well known for us, then substituting $\alpha_m, ...$ and the general solutions of (2.7) into (2.6) we have traveling wave solutions of the nonlinear evolution equation (2.1) or (2.2).

3 Application Of The $(\frac{G'}{G})$-Expansion Method For The SRLW Equation

In the following three sections, we will apply the $(\frac{G'}{G})$-expansion method for getting the exact solutions of some nonlinear equations. First We consider the SRLW equation [43]:

$$u_{xxtt} + u_{tt} + u_{xx} + uu_{xt} + u_xu_t = 0 \tag{3.1}$$

Suppose that

$$u(x,\ t) = u(\xi),\ \xi = kx + \omega t \tag{3.2}$$

k, ω are constants that to be determined later.

By using (3.2), Eq.(3.1) is converted into an ODE

$$(\omega^2 + k^2)u'' + k\omega uu'' + k\omega(u')^2 + k^2\omega^2 u^{(4)} = 0 \tag{3.3}$$

Integrating (3.3) once, we obtain

$$(\omega^2 + k^2)u' + k\omega uu' + k^2\omega^2 u''' = g \tag{3.4}$$

where g is the integration constant that can be determined later.

Suppose that the solution of the ODE (3.4) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i (\frac{G'}{G})^i \tag{3.5}$$

where a_i are constants, $G = G(\xi)$ satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \tag{3.6}$$

where λ and μ are constants.

Balancing the order of $u^{(3)}$ and uu' in Eq.(3.4), we get that

$$m + 3 = m + m + 1 \ \Rightarrow\ m = 2$$

So Eq.(3.5) can be rewritten as

$$u(\xi) = a_2(\frac{G'}{G})^2 + a_1(\frac{G'}{G}) + a_0,\ a_2 \neq 0 \tag{3.7}$$

$a_2,\ a_1,\ a_0$ are constants to be determined later.

Substituting Eq.(3.7) into the ODE (3.4) and collecting all the terms with the same power of $(\frac{G'}{G})$ together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$\begin{gathered}(\frac{G'}{G})^0:\ -k^2a_1\mu - \omega^2 a_1\mu - g \\ -2k^2\omega^2 a_1\mu^2 - k\omega a_0 a_1\mu - k^2\omega^2 a_1\lambda^2\mu \\ -6k^2\omega^2 a_2\lambda\mu^2 = 0\end{gathered}$$

$$\begin{gathered}(\frac{G'}{G})^1:\ -k\omega a_1^2\mu - k\omega a_0 a_1\lambda - k^2\omega^2 a_1\lambda^3 \\ -14k^2\omega^2 a_2\lambda^2\mu - 2k\omega a_0 a_2\mu - \omega^2 a_1\lambda \\ -16k^2\omega^2 a_2\mu^2 - k^2 a_1\lambda - 8k^2\omega^2 a_1\lambda\mu \\ -2\omega^2 a_2\mu - 2k^2 a_2\mu = 0\end{gathered}$$

$$\begin{gathered}(\frac{G'}{G})^2:\ -8k^2\omega^2 a_2\lambda^3 - 2k\omega a_0 a_2\lambda - k^2 a_1 \\ -k\omega a_1^2\lambda - 8k^2\omega^2 a_1\mu - k\omega a_0 a_1 \\ -3k\omega a_1 a_2\mu - 52k^2\omega^2 a_2\lambda\mu - \omega^2 a_1 \\ -2k^2 a_2\lambda - 7k^2\omega^2 a_1\lambda^2 - 2\omega^2 a_2\lambda = 0\end{gathered}$$

$$\begin{gathered}(\frac{G'}{G})^3:\ -2\omega^2 a_2 - 38k^2\omega^2 a_2\lambda^2 - 3k\omega\lambda a_1 a_2 \\ -40k^2\omega^2 a_2\mu - 12k^2\omega^2 a_1\lambda - k\omega a_1^2 \\ -2k\omega a_2^2\mu - 2k^2 a_2 - 2k\omega a_0 a_2 = 0\end{gathered}$$

$$\begin{gathered}(\frac{G'}{G})^4:\ -2k\omega a_2^2\lambda - 6k^2\omega^2 a_1 - 54k^2\omega^2 a_2\lambda \\ -3k\omega a_1 a_2 = 0\end{gathered}$$

$$(\frac{G'}{G})^5: \ -24k^2\omega^2 a_2 - 2k\omega a_2^2 = 0$$

Solving the algebraic equations above, yields:

$$k = k$$
$$\omega = \omega$$
$$g = 0$$
$$a_2 = -12k\omega$$
$$a_1 = -12k\omega\lambda$$
$$a_0 = -\frac{\omega^2 + k^2\omega^2\lambda^2 + 8k^2\omega^2\mu + k^2}{k\omega} \qquad (3.8)$$

where λ, μ are arbitrary constants.

Substituting (3.8) into (3.7), we get that

$$u(\xi) = -12k\omega(\frac{G'}{G})^2 - 12k\omega\lambda(\frac{G'}{G}) - \frac{\omega^2 + k^2\omega^2\lambda^2 + 8k^2\omega^2\mu + k^2}{k\omega}$$

$$\xi = kx + \omega t \qquad (3.9)$$

where k, ω are arbitrary constants.

Substituting the general solutions of Eq.(3.6) into (3.9), we can obtain:

When $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = 3k\omega\lambda^2 - 3k\omega(\lambda^2 - 4\mu) \cdot \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right)^2 - \frac{\omega^2 + k^2\omega^2\lambda^2 + 8k^2\omega^2\mu + k^2}{k\omega}$$

where

$$\xi = kx + \omega t,$$

k, ω, C_1, C_2 are arbitrary constants.

When $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = 3k\omega\lambda^2 - 3k\omega(4\mu - \lambda^2) \cdot \left(\frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}\right)^2 - \frac{\omega^2 + k^2\omega^2\lambda^2 + 8k^2\omega^2\mu + k^2}{k\omega}$$

where

$$\xi = kx + \omega t,$$

k, ω, C_1, C_2 are arbitrary constants.

When $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = 3k\omega\lambda^2 - \frac{12k\omega C_2^2}{(C_1 + C_2\xi)^2} - \frac{\omega^2 + k^2\omega^2\lambda^2 + 8k^2\omega^2\mu + k^2}{k\omega}$$

where

$$\xi = kx + \omega t,$$

k, ω, C_1, C_2 are arbitrary constants.

4 Application Of The $(\frac{G'}{G})$-Expansion Method For The (2+1) Dimensional PKP Equation

In this section we consider the (2+1) dimensional PKP equation [44]:

$$\frac{1}{4}u_{xxxx} + \frac{3}{2}u_x u_{xx} + \frac{3}{4}u_{yy} + u_{xt} = 0 \qquad (4.1)$$

Suppose that

$$u(x,\ y,\ t) = u(\xi),\ \xi = kx + ly + \omega t \qquad (4.2)$$

k, l, ω are constants that to be determined later.

By using (4.2), (4.1) is converted into:

$$\frac{1}{4}k^4u^{(4)} + \frac{3}{2}k^3u'u'' + (\frac{3}{4}l^2 + k\omega)u'' = 0 \quad (4.3)$$

Integrating (4.3) once, we obtain

$$\frac{1}{4}k^4u''' + \frac{3}{4}k^3(u')^2 + (\frac{3}{4}l^2 + k\omega)u' = g \quad (4.4)$$

where g is the integration constant that can be determined later.

Suppose that the solution of (4.4) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i(\frac{G'}{G})^i \quad (4.5)$$

where a_i are constants, $G = G(\xi)$ satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \quad (4.6)$$

where λ and μ are constants.

Balancing the order of u''' and $(u')^2$ in Eq.(4.4), we have

$$m + 3 = 2 + 2m \ \Rightarrow \ m = 1$$

So Eq.(4.5) can be rewritten as

$$u(\xi) = a_1(\frac{G'}{G}) + a_0, \ a_1 \neq 0 \quad (4.7)$$

$a_1, \ a_0$ are constants to be determined later.

Substituting (4.7) into (4.4) and collecting all the terms with the same power of $(\frac{G'}{G})$ together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$(\frac{G'}{G})^0: \ -g - k\omega a_1\mu - \frac{3}{4}l^2a_1\mu - \frac{1}{2}k^4a_1\mu^2$$

$$-\frac{1}{4}k^4a_1\lambda^2\mu + \frac{3}{4}k^3a_1^2\mu^2 = 0$$

$$(\frac{G'}{G})^1: \ -2k^4a_1\lambda\mu + \frac{3}{2}k^3a_1^2\lambda\mu - \frac{1}{4}k^4a_1\lambda^3$$

$$-k\omega a_1\lambda - \frac{3}{4}a_1l^2\lambda = 0$$

$$(\frac{G'}{G})^2: \ -\frac{7}{4}k^4a_1\lambda^2 + \frac{3}{4}k^3a_1^2\lambda^2 - 2k^4a_1\mu$$

$$+\frac{3}{2}k^3a_1^2\mu - k\omega a_1 - \frac{3}{4}a_1l^2 = 0$$

$$(\frac{G'}{G})^3: \ \frac{3}{2}k^3a_1^2\lambda - 3k^4la_1\lambda = 0$$

$$(\frac{G'}{G})^4: \ \frac{3}{4}k^3a_1^2 - \frac{3}{2}k^4a_1 = 0$$

Solving the algebraic equations above, yields

$$a_1 = 2k, \ \ a_0 = a_0, \ \ k = k, \ \ l = l,$$

$$\omega = -\frac{1}{4}\frac{k^4\lambda^2 - 4k^4\mu + 3l^2}{k}, \ \ g = 0 \quad (4.8)$$

Substituting (4.8) into (4.7), we have

$$u(\xi) = 2k(\frac{G'}{G}) + a_0$$

$$\xi = kx + ly - \frac{1}{4}\frac{k^4\lambda^2 - 4k^4\mu + 3l^2}{k}t \quad (4.9)$$

where k, l, a_0 are arbitrary constants.

Substituting the general solutions of Eq.(4.6) into (4.9), we have:

When $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = -k\lambda + k\sqrt{\lambda^2 - 4\mu} \cdot \left(\frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi} \right) + a_0$$

where $\xi = kx + ly - \frac{1}{4}\frac{k^4\lambda^2 - 4k^4\mu + 3l^2}{k}t$, C_1, C_2, k, l, a_0 are arbitrary constants.

When $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = -k\lambda + k\sqrt{4\mu - \lambda^2} \cdot \left(\frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi} \right) + a_0$$

where $\xi = kx + ly - \frac{1}{4}\frac{k^4\lambda^2 - 4k^4\mu + 3l^2}{k}t$, C_1, C_2, k, l, a_0 are arbitrary constants.

When $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{k(2C_2 - C_1\lambda - C_2\lambda\xi)}{(C_1 + C_2\xi)} + a_0$$

where $\xi = kx + ly - \frac{1}{4}\frac{k^4\lambda^2 - 4k^4\mu + 3l^2}{k}t$, C_1, C_2, k, l, a_0 are arbitrary constants.

5 Application Of The $(\frac{G'}{G})$-Expansion Method For The (3+1) Dimensional Potential-YTSF Equation

We consider the (3+1) dimensional potential-YTSF equation [45]:

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0 \quad (5.1)$$

Suppose that

$$u(x,\ y,\ z,\ t) = u(\xi),\ \xi = kx + my + rz + \omega t \quad (5.2)$$

k, r, m, ω are constants that to be determined later.

By using (5.2), (5.1) is converted into:

$$k^3 m u^{(4)} + 6mk^2 u' u'' + (2m\omega - 3kr)u'' = 0 \quad (5.3)$$

Integrating (5.3) once, we obtain

$$k^3 m u''' + 3mk^2 (u')^2 + (2m\omega - 3kr)u' = g \quad (5.4)$$

where g is the integration constant that can be determined later.

Similar to the last example, suppose:

$$u(\xi) = \sum_{i=0}^{m} a_i (\frac{G'}{G})^i \quad (5.5)$$

where a_i are constants, $G = G(\xi)$ satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \quad (5.6)$$

where λ and μ are constants.

Balancing the order of u''' and $(u')^2$ in Eq.(5.4), we have

$$m + 3 = 2 + 2m \ \Rightarrow\ m = 1$$

So

$$u(\xi) = a_1(\frac{G'}{G}) + a_0,\ a_1 \neq 0 \quad (5.7)$$

$a_1,\ a_0$ are constants to be determined later.

Substituting (5.7) into (5.4) and collecting all the terms with the same power of $(\frac{G'}{G})$ together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$(\frac{G'}{G})^0: \ -g + 3a_1kr\mu - 2a_1m\omega\mu - 2k^3ma_1\mu^2$$
$$-k^3ma_1\lambda^2\mu + 3k^2ma_1^2\mu^2 = 0$$

$$(\frac{G'}{G})^1: \ 3a_1kr\lambda - 8k^3ma_1\lambda\mu - k^3ma_1\lambda^3$$
$$+6k^2ma_1^2\lambda\mu - 2a_1m\omega\lambda = 0$$

$$(\frac{G'}{G})^2: \ -7k^3ma_1\lambda^2 + 3k^2ma_1^2\lambda^2 - 8k^3ma_1\mu$$
$$+3a_1kr + 6k^2ma_1^2\mu - 2a_1m\omega = 0$$

$$(\frac{G'}{G})^3: \ 6k^2ma_1^2\lambda - 12k^3ma_1\lambda = 0$$

$$(\frac{G'}{G})^4: \ 3k^2ma_1^2 - 6k^3ma_1 = 0$$

Solving the algebraic equations above, yields

$$a_1 = 2k, \ a_0 = a_0, \ k = k$$
$$r = r, \ m = m, \ g = 0$$
$$\omega = \frac{-k^3m\lambda^2 + 4k^3m\mu + 3kr}{2m} \tag{5.8}$$

Then

$$u(\xi) = 2k(\frac{G'}{G}) + a_0$$

$$\xi = kx + my + rz + \frac{-k^3m\lambda^2 + 4k^3m\mu + 3kr}{2m}t \tag{5.9}$$

where k, r, m, a_0 are arbitrary constants.

Substituting the general solutions of Eq.(5.6) into (5.9), we have:

When $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = -k\lambda + k\sqrt{\lambda^2 - 4\mu}$$
$$\cdot \left(\frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi} \right) + a_0$$

where

$$\xi = kx + my + rz + \frac{-k^3m\lambda^2 + 4k^3m\mu + 3kr}{2m}t,$$

C_1, C_2, k, r, m, a_0 are arbitrary constants.

When $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = -k\lambda + k\sqrt{4\mu - \lambda^2}$$
$$\cdot \left(\frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi} \right) + a_0$$

where

$$\xi = kx + my + rz + \frac{-k^3m\lambda^2 + 4k^3m\mu + 3kr}{2m}t$$

C_1, C_2, k, r, m, a_0 are arbitrary constants.

When $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{k(2C_2 - C_1\lambda - C_2\lambda\xi)}{(C_1 + C_2\xi)} + a_0$$

where

$$\xi = kx + my + rz + \frac{-k^3m\lambda^2 + 4k^3m\mu + 3kr}{2m}t,$$

C_1, C_2, k, r, m, a_0 are arbitrary constants.

6 Conclusions

In this paper we have seen that the traveling wave solutions of the SRLW equation, the (2+1) dimensional PKP equation and the (3+1) dimensional potential-YTSF equation are successfully found by using the

$(\frac{G'}{G})$-expansion method. Now we briefly summarize the method in the following.

The main points of the method are that assuming the solution of the ODE reduced by using the traveling wave variable as well as integrating can be expressed by an m-th degree polynomial in $(\frac{G'}{G})$, where $G = G(\xi)$ is the general solutions of a second order LODE. The positive integer m is determined by the homogeneous balance between the highest order derivatives and nonlinear terms appearing in the reduced ODE, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations resulted from the process of using the method.

Compared to the methods used before, one can see that this method is direct, concise and effective. As we can use the MATHEMATICA or MAPLE to find out a useful solution of the algebraic equations resulted, so we can also avoids tedious calculations. This method can also be used to many other nonlinear equations.

7 Acknowledgements

I would like to thank the anonymous referees for their useful and valuable suggestions.

References:

[1] Damelys Zabala, Aura L. Lopez De Ramos, Effect of the Finite Difference Solution Scheme in a Free Boundary Convective Mass Transfer Model, WSEAS Transactions on Mathematics, Vol. 6, No. 6, 2007, pp. 693-701

[2] Raimonds Vilums, Andris Buikis, Conservative Averaging and Finite Difference Methods for Transient Heat Conduction in 3D Fuse, WSEAS Transactions on Heat and Mass Transfer, Vol 3, No. 1, 2008

[3] Mastorakis N E., An Extended Crank-Nicholson Method and its Applications in the Solution of Partial Differential Equations: 1-D and 3-D Conduction Equations, WSEAS Transactions on Mathematics, Vol. 6, No. 1, 2007, pp 215-225

[4] Nikos E. Mastorakis, Numerical Solution of Non-Linear Ordinary Differential Equations via Collocation Method (Finite Elements) and Genetic Algorithm, WSEAS Transactions on Information Science and Applications, Vol. 2, No. 5, 2005, pp. 467-473

[5] Inc M, New exact solutions for the ZK-MEW equation by using symbolic computation. Appl. Math. Comput. 189 (2007) 508.

[6] Zhang Huiqun, Extended Jacobi elliptic function expansion method and its applications. Communications in Nonlinear Science and Numerical Simulation 12 (2007) 627-635.

[7] A.M. Wazwaz, Multiple-front solutions for the burgers equation and the coupled burgers equations, Appl. Math. Comput. 190 (2007) 1198-1206.

[8] M. Wang, Solitary wave solutions for variant Boussinesq equations, Phys. Lett. A 199 (1995) 169-172.

[9] E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, On the solitary wave solutions for nonlinear Hirota-Satsuma coupled KdV equations, Chaos, Solitons and Fractals 22 (2004) 285-303.

[10] L. Yang, J. Liu, K. Yang, Exact solutions of nonlinear PDE nonlinear transformations and reduction of nonlinear PDE to a quadrature, Phys. Lett. A 278 (2001) 267-270.

[11] E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, Group analysis and modified tanh-function to find the invariant solutions and soliton solution for nonlinear Euler equations, Int. J. Nonlinear Sci. Numer. Simul. 5 (2004) 221-234.

[12] M. Inc, D.J. Evans, On traveling wave solutions of some nonlinear evolution equations, Int. J. Comput. Math. 81 (2004) 191-202.

[13] M.A. Abdou, The extended tanh-method and its applications for solving nonlinear physical models, Appl. Math. Comput. 190 (2007) 988-996

[14] E.G. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (2000) 212-218.

[15] W. Malfliet, Solitary wave solutions of nonlinear wave equations, Am. J. Phys. 60 (1992) 650-654.

[16] J.L. Hu, A new method of exact traveling wave solution for coupled nonlinear differential equations, Phys. Lett. A 322 (2004) 211-216.

[17] M.J. Ablowitz, P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform, Cambridge University Press, Cambridge, 1991.

[18] M.R. Miura, Backlund Transformation, Springer-Verlag, Berlin, 1978.

[19] C. Rogers, W.F. Shadwick, Backlund Transformations, Academic Press, New York, 1982.

[20] R. Hirota, Exact envelope soliton solutions of a nonlinear wave equation, J. Math. Phys. 14 (1973) 805-810.

[21] R. Hirota, J. Satsuma, Soliton solution of a coupled KdV equation, Phys. Lett. A 85 (1981) 407-408.

[22] Z.Y. Yan, H.Q. Zhang, New explicit solitary wave solutions and periodic wave solutions for WhithamCBroerCKaup equation in shallow water, Phys. Lett. A 285 (2001) 355-362.

[23] A.V. Porubov, Periodical solution to the nonlinear dissipative equation for surface waves in a convecting liquid layer, Phys. Lett. A 221 (1996) 391-394.

[24] K.W. Chow, A class of exact periodic solutions of nonlinear envelope equation, J. Math. Phys. 36 (1995) 4125-4137.

[25] E.G. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (2000) 212-218.

[26] Engui Fan, Multiple traveling wave solutions of nonlinear evolution equations using a unifiex algebraic method, J. Phys. A, Math. Gen. 35 (2002) 6853-6872.

[27] Z.Y. Yan, H.Q. Zhang, New explicit and exact traveling wave solutions for a system of variant Boussinesq equations in mathematical physics, Phys. Lett. A 252 (1999) 291-296.

[28] S.K. Liu, Z.T. Fu, S.D. Liu, Q. Zhao, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Phys. Lett. A 289 (2001) 69-74.

[29] Z. Yan, Abundant families of Jacobi elliptic functions of the (2 + 1)-dimensional integrable DaveyCStawartson-type equation via a new method, Chaos, Solitons and Fractals 18 (2003) 299-309.

[30] C. Bai, H. Zhao, Complex hyperbolic-function method and its applications to nonlinear equations, Phys. Lett. A 355 (2006) 22-30.

[31] E.M.E. Zayed, A.M. Abourabia, K.A. Gepreel, M.M. Horbaty, On the rational solitary wave solutions for the nonlinear HirotaCSatsuma coupled KdV system, Appl. Anal. 85 (2006) 751-768.

[32] K.W. Chow, A class of exact periodic solutions of nonlinear envelope equation, J. Math. Phys. 36 (1995) 4125-4137.

[33] M.L. Wang, Y.B. Zhou, The periodic wave equations for the KleinCGordonCSchordinger equations, Phys. Lett. A 318 (2003) 84-92.

[34] M.L. Wang, X.Z. Li, Extended F-expansion and periodic wave solutions for the generalized Zakharov equations, Phys. Lett. A 343 (2005) 48-54.

[35] M.L. Wang, X.Z. Li, Applications of F-expansion to periodic wave solutions for a new Hamiltonian amplitude equation, Chaos, Solitons and Fractals 24 (2005) 1257-1268.

[36] X. Feng, Exploratory approach to explicit solution of nonlinear evolutions equations, Int. J. Theo. Phys. 39 (2000) 207-222.

[37] J.H. He, X.H. Wu, Exp-function method for nonlinear wave equations, Chaos, Solitons and Fractals 30 (2006) 700-708.

[38] Mingliang Wang, Xiangzheng Li, Jinliang Zhang, The $(\frac{G'}{G})$-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. Physics Letters A, 372 (2008) 417-423.

[39] Mingliang Wang, Jinliang Zhang, Xiangzheng Li, Application of the $(\frac{G'}{G})$-expansion to travelling wave solutions of the Broer-Kaup and the approximate long water wave equations. Appl. Math. Comput. , 206 (2008) 321-326.

[40] Ismail Aslan, Exact and explicit solutions to some nonlinear evolution equations by utilizing the $(\frac{G'}{G})$-expansion method. Appl. Math. Comput. In press, (2009).

[41] Xun Liu, Lixin Tian, Yuhai Wu, Application of $(\frac{G'}{G})$-expansion method to two nonlinear evolution equations. Appl. Math. Comput. , in press, (2009).

[42] Ismail Aslan, Turgut Özis, Analytic study on two nonlinear evolution equations by using the

$(\frac{G'}{G})$-expansion method. Appl. Math. Comput. 209 (2009) 425-429.

[43] Fei Xu, Application of Exp-function method to Symmetric Regularized Long Wave (SRLW) equation. Physics Letters A 372 (2008) 252C257.

[44] A. Borhanifar, M.M. Kabir, New periodic and soliton solutions by application of Exp-function method for nonlinear evolution equations. Journal of Computational and Applied Mathematics. 229 (2009) 158-167.

[45] Ahmet Boz, Ahmet Bekir, Application of Exp-function method for (3 + 1)-dimensional nonlinear evolution equations. Computers and Mathematics with Applications. 56 (2008) 1451C1456.