Traveling Wave Solutions For Three Non-linear Equations By
\((G'/G)\)-expansion method

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Abstract: In this paper, we will try to obtain the new exact solutions of the DSSH equation, the KP-BBM equation and the (3+1) dimensional potential-YTSF equation. The three nonlinear equations are reduced to nonlinear ordinary differential equations (ODE) by using a simple transformation respectively. Then we construct the traveling wave solutions of the equations in terms of the hyperbolic functions, trigonometric functions and the rational functions by the \((G'/G)\)-expansion method.

Key–Words: \((G'/G)\)-expansion method, Traveling wave solutions, DSSH equation, KP-BBM equation, (3+1) dimensional potential-YTSF equation, exact solution, evolution equation, nonlinear equation

1 Introduction

Nonlinear evolution equations (NLEEs) have been the subject of study in various branches of mathematical-physical sciences such as physics, biology, chemistry, etc. The powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by many authors. Many efficient methods have been presented so far such as in [1-7].

In this paper, we pay attention to the analytical method for getting the exact travelling wave solutions of NLEES. Also there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the homogeneous balance method [8,9], the hyperbolic tangent expansion method [10,11], the trial function method [12], the tanh-method [13-15], the non-linear transform method [16], the inverse scattering transform [17], the Backlund transform [18,19], the Hirota bilinear method [20,21], the generalized Riccati equation [22,23], the Weierstrass elliptic function method [24], the theta function method [25-27], the sineCcosine method [28], the Jacobi elliptic function expansion [29,30], the complex hyperbolic function method [31-33], the truncated Painlevé expansion [34], the F-expansion method [35], the rank analysis method [36], the exp-function expansion method [37] and so on.

Recently, the \((G'/G)\)-expansion method, firstly introduced by Mingliang Wang [38], has become widely used to search for various exact solutions of NLEEs [39-42]. The value of the \((G'/G)\)-expansion method is that one can treat nonlinear problems by essentially linear methods.

Our aim in this paper is to present an application of the \((G'/G)\)-expansion method to some nonlinear problems. The rest of the paper is organized as follows. In Section 2, we give the main steps of the \((G'/G)\)-expansion method. In the subsequent sections, we will apply the method to the DSSH equation, the KP-BBM equation and the (3+1) dimensional potential-YTSF equation. The features of the \((G'/G)\)-expansion method are briefly summarized at the end of the paper.

2 Description of the \((G'/G)\)-expansion method

In this section we describe the \((G'/G)\)-expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables \(x, t,\) is given by

\[P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0, \quad (2.1)\]
or in three independent variables \( x, y \) and \( t \), is given by

\[
P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}, \ldots) = 0, \quad (2.2)
\]

where \( u = u(x, t) \) or \( u = u(x, y, t) \) is an unknown function, \( P \) is a polynomial in \( u, y, t \) or \( u(x, y, t) \) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following, we will give the main steps of the \((\frac{G'}{G'})\)-expansion method.

Step 1. Suppose that

\[
u(x, t) = u(\xi), \quad \xi = \xi(x, t) \quad (2.3)
\]

or

\[
u(x, y, t) = u(\xi), \quad \xi = \xi(x, y, t) \quad (2.4)
\]

The traveling wave variable (2.3) or (2.4) permits us reducing (2.1) or (2.2) to an ODE for \( u \),

\[
P(u, u', u'', \ldots) = 0, \quad (2.5)
\]

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in \((\frac{G'}{G'})\) as follows:

\[
u(\xi) = \alpha_m(\frac{G'}{G})^m + \ldots \quad (2.6)
\]

where \( G = G(\xi) \) satisfies the second order LODE in the form

\[
G'' + \lambda G' + \mu G = 0 \quad (2.7)
\]

\( \alpha_m, \ldots, \lambda \) and \( \mu \) are constants to be determined later, \( \alpha_m \neq 0 \). The unwritten part in (2.6) is also a polynomial in \((\frac{G'}{G'})\), the degree of which is generally equal to or less than \( m - 1 \). The positive integer \( m \) can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using second order LODE (2.7), collecting all terms with the same order of \((\frac{G'}{G'})\) together, the left-hand side of (2.5) is converted into another polynomial in \((\frac{G'}{G'})\). Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for \( \alpha_m, \ldots, \lambda \) and \( \mu \).

Step 4. Assuming that the constants \( \alpha_m, \ldots, \lambda \) and \( \mu \) can be obtained by solving the algebraic equations in Step 3. Since the general solutions of the second order LODE (2.7) have been well known for us, then substituting \( \alpha_m, \ldots \) and the general solutions of (2.7) into (2.6) we have traveling wave solutions of the nonlinear evolution equation (2.1) or (2.2).

3 Application Of The \((\frac{G'}{G'})\)-Expansion Method For The DSSH Equation

In the subsequent sections, we will apply the \((\frac{G'}{G'})\)-expansion method to construct the traveling wave solutions for some nonlinear partial differential equations in mathematical physics as follows:

First we will consider the DSSH equation [43]:

\[
u_{xxxxxx} - 9\nu_x \nu_{xxxx} - 18\nu_{xx} \nu_{xxx} + 18\nu_{x}^{2} \nu_{xx} - \frac{1}{2} \nu_{tt} + \frac{1}{2} \nu_{xxtt} = 0 \quad (3.1)
\]

Similar to Section 3, we suppose that

\[
u(x, t) = u(\xi), \quad \xi = x - ct \quad (3.2)
\]

\( c \) is a constant that to be determined later.

Eq.(3.1) can be converted into an ODE

\[
\nu(6) - 9u'u^{(4)} - 18u''u^{m} + 18(u')^{2}u'' - \frac{1}{2} c^{2} u'' - \frac{1}{2} cu^{(4)} = 0 \quad (3.3)
\]

Integrating the ODE (3.3) with respect to \( \xi \) once, we obtain

\[
u(5) - 9u'u'' - \frac{9}{2} (u')^{2} + 6(u')^{3} - \frac{1}{2} c^{2} u' - \frac{1}{2} cu'' = g \quad (3.4)
\]
where $g$ is the integration constant that can be determined later.

Suppose that the solution of the ODE (3.4) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i \left(\frac{G'}{G}\right)^i \quad (3.5)$$

where $a_i$ are constants, $G = G(\xi)$ satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \quad (3.6)$$

where $\lambda$ and $\mu$ are constants.

Balancing the order of $u^{(5)}$ and $(u')^3$ in Eq. (3.4), we get that $m + 5 = 3m + 3 \Rightarrow m = 1$, so Eq. (3.5) can be rewritten as

$$u(\xi) = a_1 \left(\frac{G'}{G}\right) + a_0, \quad a_1 \neq 0 \quad (3.7)$$

$a_1, a_0$ are constants to be determined later.

Substituting (3.7) into (3.4) and collecting all the terms with the same power of $\left(\frac{G'}{G}\right)$ together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$\left(\frac{G'}{G}\right)^0: -22a_1^2 \lambda^2 \mu^2 + \frac{1}{2} c^2 a_1 \mu - \frac{27}{2} a_1^2 \lambda^2 \mu^2 + c a_1 \mu^2 - 16a_1 \mu^3 + \frac{1}{2} c a_1 \lambda^2 \mu = 0$$

$$-18a_1 \mu^3 - 6a_1^3 \mu^3 - a_1 \lambda^4 \mu - g = 0$$

$$\left(\frac{G'}{G}\right)^1: -108a_1^2 \lambda \mu^2 - 52a_1 \lambda^3 \mu - 27a_1^2 \lambda^3 \mu + 4ca_1 \lambda \mu + \frac{1}{2} c^2 a_1 \lambda + \frac{1}{2} c a_1 \lambda^3$$

$$-136a_1 \lambda \mu^2 - 18a_1^3 \lambda \mu^2 - a_1 \lambda^5 = 0$$

$$\left(\frac{G'}{G}\right)^2: \frac{1}{2} c^2 a_1 - \frac{27}{2} a_1^2 \lambda^4 - 18a_1^3 \mu^2 + \frac{7}{2} c a_1 \lambda^2 - 108a_1^2 \mu^2 + 4ca_1 \mu - 292a_1 \lambda^2 \mu$$

$$-189a_1^2 \lambda^2 \mu - 31a_1 \lambda^4 - 136a_1 \mu^2 - 18a_1^3 \lambda^2 \mu = 0$$

$$\left(\frac{G'}{G}\right)^3: -324a_1^2 \lambda \mu + 6ca_1 \lambda - 36a_1^3 \lambda \mu$$

$$-99a_1^2 \lambda^3 - 480a_1 \lambda \mu - 6a_1^3 \lambda^3$$

$$-180a_1 \lambda^3 = 0$$

$$\left(\frac{G'}{G}\right)^4: 3ca_1 - 162a_1^2 \mu - 390a_1 \lambda^2$$

$$-18a_1^3 \lambda^2 - 240a_1 \mu - 18a_1^3 \mu$$

$$-\frac{459}{2} a_1^2 \lambda^2 = 0$$

$$\left(\frac{G'}{G}\right)^5: -18a_1^3 \lambda - 216a_1^2 \lambda - 360a_1 \lambda = 0$$

$$\left(\frac{G'}{G}\right)^6: -120a_1 - 72a_1^2 - 6a_1^3 = 0$$
Solving the algebraic equations above yields:

\[ a_1 = -2, \ a_0 = a_0, \ g = 0, \ c = \lambda^2 - 4\mu \]  \(\text{(3.8)}\)

where \(a_0, \lambda, \mu\) are arbitrary constants.

Substituting (3.8) into (3.7), we get that

\[ u(\xi) = -2\left(\frac{G'}{G}\right) + a_0, \ \xi = x - (\lambda^2 - 4\mu)t \]  \(\text{(3.9)}\)

where \(a_0, \lambda, \mu\) are arbitrary constants.

Substituting the general solutions of (3.6) into (3.9), we will have three types of travelling wave solutions of the DSSH equation (3.1) as follows:

Case (a): when \(\lambda^2 - 4\mu > 0\)

\[ u_1(\xi) = a_0 + \lambda - \sqrt{\lambda^2 - 4\mu} \]

\[ C_1 \sinh \left(\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi \right) + C_2 \cosh \left(\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi \right) \]

\[ = \left(\frac{C_1 \sinh \left(\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi \right) + C_2 \cosh \left(\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi \right)}{C_1 \cosh \left(\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi \right) + C_2 \sinh \left(\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi \right)}\right) \]

where

\[ \xi = x - (\lambda^2 - 4\mu)t, \]

\(C_1, C_2\) are arbitrary constants.

In particular, if \(C_1 = 1, \ C_2 = 0, \ \lambda = 2, \ \mu = 0\), then we have

\[ u(x, t) = 2 - 2\tanh(x + 4t) + a_0. \]

Case (b): when \(\lambda^2 - 4\mu < 0\)

\[ u_2(\xi) = a_0 + \lambda - \sqrt{4\mu - \lambda^2} \]

\[ -C_1 \sin \left(\frac{1}{2} \sqrt{4\mu - \lambda^2} \xi \right) + C_2 \cos \left(\frac{1}{2} \sqrt{4\mu - \lambda^2} \xi \right) \]

\[ = \left(\frac{-C_1 \sin \left(\frac{1}{2} \sqrt{4\mu - \lambda^2} \xi \right) + C_2 \cos \left(\frac{1}{2} \sqrt{4\mu - \lambda^2} \xi \right)}{C_1 \cos \left(\frac{1}{2} \sqrt{4\mu - \lambda^2} \xi \right) + C_2 \sin \left(\frac{1}{2} \sqrt{4\mu - \lambda^2} \xi \right)}\right) \]

where

\[ \xi = x - (\lambda^2 - 4\mu)t, \]

\(C_1, C_2\) are arbitrary constants.

In particular, if \(C_1 = 1, \ C_2 = 0, \ \lambda = 0, \ \mu = 1\), then

\[ u(x, t) = 2\tanh(x + 4t) + a_0. \]

Case (c): when \(\lambda^2 - 4\mu = 0\)

\[ u_3(\xi) = -2 \left[ \frac{2C_2 - C_1 \lambda - C_2 \lambda^2}{2(C_1 + C_2 \xi)} \right] + a_0 \]

where

\[ \xi = x - (\lambda^2 - 4\mu)t, \]

\(C_1, C_2\) are arbitrary constants.

In particular, if \(C_1 = C_2 = 1, \ \lambda = 2, \ \mu = 1\), then

\[ u(x, t) = 2 - \frac{1}{1 + x} + a_0. \]

4 Application Of The \(\left(\frac{G'}{G}\right)\)-Expansion Method For The KP-BBM Equation

we will consider the KP-BBM equation [44-45]:

\[ (u_t + uu_x - a(u^2)_x - bu_{xxt})_x + ku_{yy} = 0 \]  \(\text{(4.1)}\)

where \(a, b, k\) are constants.

Suppose that

\[ u(x, y, t) = u(\xi), \ \xi = x + y - ct \]  \(\text{(4.2)}\)

\(c\) is a constant that to be determined later.

By using the wave variable (4.2), Eq.(4.1) is converted into an ODE

\[ -cu'' + u'' - 2a(u')^2 - 2auu'' + bcu^{(4)} + ku'' = 0 \]  \(\text{(4.3)}\)

Integrating (4.3) once, we obtain

\[ (-c + 1 + k)u' + bcu^{(3)} - 2auu' = g \]  \(\text{(4.4)}\)
where \( g \) is the integration constant that can be determined later.

Suppose that the solution of the ODE (4.4) can be expressed by a polynomial in \((G')/G\) as follows:

\[
u(\xi) = \sum_{i=0}^{m} a_i (G')^i
\]

where \( a_i \) are constants, \( G = G(\xi) \) satisfies the second order LODE in the form:

\[
G'' + \lambda G' + \mu G = 0
\]

where \( \lambda \) and \( \mu \) are constants.

Balancing the order of \( uu' \) and \( u'' \) in Eq.(4.4), we get that \( m + m + 1 = m + 3 \implies m = 2 \). So Eq.(4.5) can be rewritten as

\[
u(\xi) = a_2 (G')^2 + a_1 (G') + a_0, \ a_2 \neq 0
\]

\( a_2, \ a_1, \ a_0 \) are constants to be determined later. Then it follows:

\[
u'(\xi) = -2a_2 (G')^3 + (-a_1 - 2a_2 \lambda)(G')^2
\]

\[
+(-a_1 \lambda - 2a_2 \mu)(G') - a_1 \mu
\]

\[
u''(\xi) = 6a_2 (G')^4 + (2a_1 + 10a_2 \lambda)(G')^3
\]

\[
+(8a_2 \mu + 3a_1 \lambda + 4a_2 \lambda^2)(G')^2
\]

\[
+(6a_2 \lambda \mu + 2a_1 \mu + a_1 \lambda^2)(G')
\]

\[
+2a_2 \mu^2 + a_1 \lambda \mu
\]

\[
u'''(\xi) = -24a_2 (G')^5 + (-54a_2 \lambda - 6a_1)(G')^4
\]

\[
+(-12a_1 \lambda - 38a_2 \lambda^2 - 40a_2 \mu)(G')^3
\]

\[
+(-52a_2 \lambda \mu - 7a_1 \lambda^2 - 8a_2 \lambda^3 - 8a_1 \mu)(G')^2
\]

\[
+(-14a_2 \lambda^2 \mu - a_1 \lambda^3 - 16a_2 \mu^2 - 8a_1 \lambda \mu)(G')
\]

\[
-a_1 \lambda^2 \mu - 2a_1 \mu^2 - 6a_2 \lambda \mu^2
\]

Substituting(4.7) into (4.4) and collecting all the terms with the same power of \((G')/G\) together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

\[
(G')^0: \quad (c - k - 1)a_1 \mu - g - 2 bca_1 \mu^2
\]

\[
+2a_0 a_1 \mu - 6 bca_2 \lambda \mu^2 - bca_1 \lambda^2 \mu = 0
\]

\[
(G')^1: \quad -a_1 \lambda - 16 bca_2 \mu^2 + 2 bca_2 \mu - bca_1 \lambda^2
\]

\[
+ca_1 \lambda - 2a_2 \mu + 2a_1^2 \mu - 8 bca_2 \lambda \mu
\]

\[
-2ka_2 \mu + 2a_0 a_1 \lambda - 14 bca_2 \lambda^2 \mu
\]

\[
-ka_1 \lambda + 4a_0 a_2 \mu = 0
\]

\[
(G')^2: \quad -52 bca_2 \lambda \mu + (2c - 2k - 2)a_2 \lambda
\]

\[
+2a_0^2 \lambda - 7 bca_1 \lambda^2 - a_1 + ca_1
\]

\[
-8 bca_1 \mu + 4a_0 a_2 \lambda - ka_1
\]

\[
+6a_1 a_2 \mu - 8 bca_2 \lambda^3 + 2 a_0 a_1 a_0 = 0
\]

\[
(G')^3: \quad (2c - 2k - 2)a_2 - 40 bca_2 \mu + 4 a_0^2 \mu
\]
\[ +6aa_1a_2\lambda - 12bca_1\lambda + 4aa_0a_2 -38bca_2\lambda^2 + 2aa_1^2 = 0 \]

\[(G')^4 : -6bca_1 - 54bca_2\lambda + 4aa_0^2\lambda + 6aa_1a_2 = 0 \]

\[(G')^5 : -24bca_2 + 4aa_2^2 = 0 \]

Solving the algebraic equations above yields:

\[ a_2 = a_2, \ a_1 = a_2\lambda, \ a_0 = a_0, \ c = \frac{1}{6}aa_2 \ b, \ g = 0 \quad (4.8) \]

where \( a_0, a_2, \lambda \) are arbitrary constants.

Substituting (4.8) into (4.7), we have

\[ u(\xi) = a_2(G')^2 + a_2\lambda(G') + a_0 \]

\[ \xi = x + y - \frac{1}{6}aa_2 \ b t \quad (4.9) \]

Substituting the general solutions of (4.6) into (4.9), we will have three types of travelling wave solutions of the Kadomtsev-Petviashvili equation (4.1) as follows:

Case (a): When \( \lambda^2 - 4\mu > 0 \)

\[ u_1(\xi) = a_0 - \frac{a_2}{4}\lambda^2 + \frac{a_2}{4}(\lambda^2 - 4\mu) \]

\[ C_1 \sinh \sqrt{\frac{1}{2}(\lambda^2 - 4\mu)}\xi + C_2 \cosh \sqrt{\frac{1}{2}(\lambda^2 - 4\mu)}\xi \]

where

\[ \xi = x + y - \frac{1}{6}aa_2 t, \]

\[ C_1, C_2 \] are arbitrary constants.

Case (b): When \( \lambda^2 - 4\mu < 0 \)

\[ u_2(\xi) = a_0 - \frac{a_2}{4}\lambda^2 + \frac{a_2}{4}(4\mu - \lambda^2) \]

\[ \frac{C_1 \sin \sqrt{\frac{1}{2}(4\mu - \lambda^2)}\xi + C_2 \cos \sqrt{\frac{1}{2}(4\mu - \lambda^2)}\xi}{C_1 \cos \sqrt{\frac{1}{2}(4\mu - \lambda^2)}\xi + C_2 \sin \sqrt{\frac{1}{2}(4\mu - \lambda^2)}\xi} \]

where

\[ \xi = x + y - \frac{1}{6}aa_2 t, \]

\[ C_1, C_2 \] are arbitrary constants.

Case (c): When \( \lambda^2 - 4\mu = 0 \)

\[ u_3(\xi) = -\frac{1}{4}\lambda^2 a_2 + \frac{a_2^2}{(C_1 + C_2\xi)} + a_0 \]

where

\[ \xi = x + y - \frac{1}{6}aa_2 t, \]

\[ C_1, C_2 \] are arbitrary constants.

5 Application Of The \((G')\)-Expansion Method For The (3+1) dimensional potential-YTSF equation

At last, we consider the (3+1) dimensional potential-YTSF equation [46]:

\[ -4u_{xt} + u_{xxxx} + 4u_xu_{xx} + 2u_xu_{xz} + 3u_{yy} = 0 \quad (5.1) \]

Suppose that

\[ u(x, y, z, t) = u(\xi), \ \xi = kx + ly + mz + \omega t \quad (5.2) \]

\( k, l, m, \omega \) are constants that to be determined later.

By using (5.2), (5.1) is converted into:

\[ k^3mu^{(4)} + 6mk^2u''u'' + (3l^2 - 4k\omega)u'' = 0 \quad (5.3) \]

Integrating (5.3) once, we obtain

\[ k^3mu''' + 3mk^2(u')^2 + (3l^2 - 4k\omega)u' = g \quad (5.4) \]
where \( g \) is the integration constant that can be determined later.

Similar to the last example, suppose:

\[
u(\xi) = \sum_{i=0}^{m} a_i \left( \frac{G'}{G} \right)^i \tag{5.5}\]

where \( a_i \) are constants, \( G = G(\xi) \) satisfies the second order LODE in the form:

\[
G'' + \lambda G' + \mu G = 0 \tag{5.6}
\]

where \( \lambda \) and \( \mu \) are constants.

Balancing the order of \( u''' \) and \( (u')^2 \) in Eq.\((5.24)\), we have \( m + 3 = 2 + 2m \Rightarrow m = 1 \). So

\[
u(\xi) = a_1 \left( \frac{G'}{G} \right) + a_0, \quad a_1 \neq 0 \tag{5.7}\]

\( a_1, a_0 \) are constants to be determined later.

Substituting \((5.7)\) into \((5.4)\) and collecting all the terms with the same power of \((G'G)\) together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

\[
\left( \frac{G'}{G} \right)^0 : -g + 4a_1 k \omega \mu - 3a_1 t^2 \mu - 2k^3 m a_1 \mu^2 \]

\[-k^3 m a_1 \lambda^2 \mu + 3k^2 m a_1^2 \mu^2 = 0 \]

\[
\left( \frac{G'}{G} \right)^1 : 4a_1 k \omega \lambda - 8k^3 m a_1 \lambda \mu - k^3 m a_1 \lambda^3 \]

\[+ 6k^2 m a_1^2 \lambda \mu - 3a_1 t^2 \lambda = 0 \]

\[
\left( \frac{G'}{G} \right)^2 : -7k^3 m a_1 \lambda^2 + 3k^2 m a_1^2 \lambda^2 - 8k^3 m a_1 \mu \]

\[+ 4a_1 k \omega + 6k^2 m a_1^2 \mu - 3a_1 t^2 = 0 \]

Solving the algebraic equations above, yields

\[
a_1 = 2k, \quad a_0 = a_0, \quad k = k, \quad l = l, \quad m = m, \quad \omega = \frac{k^3 m \lambda^2 - 4k^3 m \mu + 3l^2}{4k}, \quad g = 0 \tag{5.8}\]

Then

\[
u(\xi) = 2k \left( \frac{G'}{G} \right) + a_0 \]

\[
\xi = kx + ly + mz + \frac{k^3 m \lambda^2 - 4k^3 m \mu + 3l^2}{4k} t \tag{5.9}\]

where \( k, l, m, a_0 \) are arbitrary constants.

Substituting the general solutions of Eq.\((5.6)\) into \((5.9)\), we have:

When \( \lambda^2 - 4\mu > 0 \)

\[
u_1(\xi) = -k \lambda + k \sqrt{\lambda^2 - 4\mu} \]

\[
\left( \frac{C_1 \sinh \left( \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi \right) + C_2 \cosh \left( \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi \right)}{C_1 \cosh \left( \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi \right) + C_2 \sinh \left( \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi \right)} \right) + a_0 \]

where

\[
\xi = kx + ly + mz + \frac{k^3 m \lambda^2 - 4k^3 m \mu + 3l^2}{4k} t, \]

\[
\frac{G'}{G} : 6k^2 m a_1^2 \lambda - 12k^3 m a_1 \lambda = 0 \]

\[
\frac{G'}{G} : 3k^2 m a_1^2 - 6k^3 m a_1 = 0 \]
$C_1, C_2, k, l, m, a_0$ are arbitrary constants.

When $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = -k\lambda + k\sqrt{4\mu - \lambda^2}$$

$$\left(\frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2 \xi} + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2 \xi}}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2 \xi} + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2 \xi}}\right) + a_0$$

where

$$\xi = kx + ly + mz + \frac{k^3m\lambda^2 - 4k^3m\mu + 3l^2}{4k}t,$$

$C_1, C_2, k, l, m, a_0$ are arbitrary constants.

When $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{k(2C_2 - C_1\lambda - C_2\lambda\xi)}{(C_1 + C_2\xi)} + a_0$$

where

$$\xi = kx + ly + mz + \frac{k^3m\lambda^2 - 4k^3m\mu + 3l^2}{4k}t,$$

$C_1, C_2, k, l, m, a_0$ are arbitrary constants.

### 6 Conclusions

In this paper we have seen that the traveling wave solutions of the DSSH equation, the KP-BBM equation and the (3+1) dimensional potential-YTSF equation are successfully found by using the $(G'/G)$-expansion method.

This study shows that the $(G'/G)$-expansion method is quite efficient and practically well suited for use in finding exact solutions for the problems considered here.

Being concise and effective, the method can also be used to many other nonlinear equations.

### 7 Acknowledgements

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### References:


