Traveling Wave Solutions For Three Non-linear Equations By $(\frac{G'}{G})$-expansion method

Qinghua Feng
Shandong University of Technology
School of Science
Zhangzhou Road 12, Zibo, 255049
China
fqhua@sina.com

Bin Zheng
Shandong University of Technology
School of Science
Zhangzhou Road 12, Zibo, 255049
China
zhengbin2601@126.com

Abstract: In this paper, we will try to obtain the new exact solutions of the DSSH equation, the KP-BBM equation and the (3+1) dimensional potential-YTSF equation. The three nonlinear equations are reduced to nonlinear ordinary differential equations (ODE) by using a simple transformation respectively. Then we construct the traveling wave solutions of the equations in terms of the hyperbolic functions, trigonometric functions and the rational functions by the $(\frac{G'}{G})$-expansion method.

Key–Words: $(\frac{G'}{G})$-expansion method, Traveling wave solutions, DSSH equation, KP-BBM equation, (3+1) dimensional potential-YTSF equation, exact solution, evolution equation, nonlinear equation

1 Introduction

Nonlinear evolution equations (NLEEs) have been the subject of study in various branches of mathematical-Cphysical sciences such as physics, biology, chemistry, etc. The powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by many authors. Many efficient methods have been presented so far such as in [1-7].

In this paper, we pay attention to the analytical method for getting the exact travelling wave solutions of NLEES. Also there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the homogeneous balance method [8,9], the hyperbolic tangent expansion method [10,11], the trial function method [12], the tanh-method [13-15], the non-linear transform method [16], the inverse scattering transform [17], the Backlund transform [18,19], the Hirotas bilinear method [20,21], the generalized Riccati equation [22,23], the Weierstrass elliptic function method [24], the theta function method [25-27], the sineCcosine method [28], the Jacobi elliptic function expansion [29,30], the complex hyperbolic function method [31-33], the truncated Painleve expansion [34], the F-expansion method [35], the rank analysis method [36], the exp-function expansion method [37] and so on.

Recently, the $(\frac{G'}{G})$-expansion method, firstly introduced by Mingliang Wang [38], has become widely used to search for various exact solutions of NLEEs [39-42]. The value of the $(\frac{G'}{G})$-expansion method is that one can treat nonlinear problems by essentially linear methods.

Our aim in this paper is to present an application of the $(\frac{G'}{G})$-expansion method to some nonlinear problems. The rest of the paper is organized as follows. In Section 2, we give the main steps of the $(\frac{G'}{G})$-expansion method. In the subsequent sections, we will apply the method to the DSSH equation, the KP-BBM equation and the (3+1) dimensional potential-YTSF equation. The features of the $(\frac{G'}{G})$-expansion method are briefly summarized at the end of the paper.

2 Description of the $(\frac{G'}{G})$-expansion method

In this section we describe the $(\frac{G'}{G})$-expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables x, t, is given by

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0, \tag{2.1}$$

or in three independent variables $x,\ y$ and t, is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}, ...) = 0, \quad (2.2)$$

where $u = u(x,t)$ or $u = u(x,y,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ or $u = u(x,y,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following, we will give the main steps of the $(\frac{G'}{G})$-expansion method.

Step 1. Suppose that

$$u(x,\ t) = u(\xi), \quad \xi = \xi(x,t) \quad (2.3)$$

or

$$u(x,\ y,\ t) = u(\xi), \quad \xi = \xi(x,y,t) \quad (2.4)$$

The traveling wave variable (2.3) or (2.4) permits us reducing (2.1) or (2.2) to an ODE for $u = u(\xi)$

$$P(u,\ u', u'',\ ...) = 0. \quad (2.5)$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \alpha_m (\frac{G'}{G})^m +\ ... \quad (2.6)$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \quad (2.7)$$

$\alpha_m, ..., \lambda$ and μ are constants to be determined later, $\alpha_m \neq 0$. The unwritten part in (2.6) is also a polynomial in $(\frac{G'}{G})$, the degree of which is generally equal to or less than $m - 1$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using second order LODE (2.7), collecting all terms with the same order of $(\frac{G'}{G})$ together, the left-hand side of (2.5) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, ..., \lambda$ and μ.

Step 4. Assuming that the constants $\alpha_m, ..., \lambda$ and μ can be obtained by solving the algebraic equations in Step 3. Since the general solutions of the second order LODE (2.7) have been well known for us, then substituting $\alpha_m, ...$ and the general solutions of (2.7) into (2.6) we have traveling wave solutions of the nonlinear evolution equation (2.1) or (2.2).

3 Application Of The $(\frac{G'}{G})$-Expansion Method For The DSSH Equation

In the subsequent sections, we will apply the $(\frac{G'}{G})$-expansion method to construct the traveling wave solutions for some nonlinear partial differential equations in mathematical physics as follows:

First we will consider the DSSH equation [43]:

$$u_{xxxxxx} - 9u_x u_{xxxx} - 18u_{xx}u_{xxx}$$
$$+18u_x^2 u_{xx} - \frac{1}{2}u_{tt} + \frac{1}{2}u_{xxxt} = 0 \quad (3.1)$$

Similar to Section 3, we suppose that

$$u(x,\ t) = u(\xi),\ \xi = x - ct \quad (3.2)$$

c is a constant that to be determined later.
Eq.(3.1) can be converted into an ODE

$$u^{(6)} - 9u'u^{(4)} - 18u''u''' + 18(u')^2 u'' - \frac{1}{2}c^2 u'' - \frac{1}{2}cu^{(4)} = 0 \quad (3.3)$$

Integrating the ODE (3.3) with respect to ξ once, we obtain

$$u^{(5)} - 9u'u''' - \frac{9}{2}(u'')^2 + 6(u')^3 - \frac{1}{2}c^2 u' - \frac{1}{2}cu''' = g \quad (3.4)$$

where g is the integration constant that can be determined later.

Suppose that the solution of the ODE (3.4) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i(\frac{G'}{G})^i \quad (3.5)$$

where a_i are constants, $G = G(\xi)$ satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \quad (3.6)$$

where λ and μ are constants.

Balancing the order of $u^{(5)}$ and $(u')^3$ in Eq.(3.4), we get that $m + 5 = 3m + 3 \Rightarrow m = 1$, so Eq.(3.5) can be rewritten as

$$u(\xi) = a_1(\frac{G'}{G}) + a_0,\ a_1 \neq 0 \quad (3.7)$$

$a_1,\ a_0$ are constants to be determined later.

Substituting (3.7) into (3.4) and collecting all the terms with the same power of $(\frac{G'}{G})$ together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$(\frac{G'}{G})^0:\ -22a_1\lambda^2\mu^2 + \frac{1}{2}c^2a_1\mu - \frac{27}{2}a_1^2\lambda^2\mu^2$$

$$+ca_1\mu^2 - 16a_1\mu^3 + \frac{1}{2}ca_1\lambda^2\mu$$

$$-18a_1^2\mu^3 - 6a_1^3\mu^3 - a_1\lambda^4\mu - g = 0$$

$$(\frac{G'}{G})^1:\ -108a_1^2\lambda\mu^2 - 52a_1\lambda^3\mu - 27a_1^2\lambda^3\mu$$

$$+4ca_1\lambda\mu + \frac{1}{2}c^2a_1\lambda + \frac{1}{2}ca_1\lambda^3$$

$$-136a_1\lambda\mu^2 - 18a_1^3\lambda\mu^2 - a_1\lambda^5 = 0$$

$$(\frac{G'}{G})^2:\ \frac{1}{2}c^2a_1 - \frac{27}{2}a_1^2\lambda^4 - 18a_1^3\mu^2$$

$$+\frac{7}{2}ca_1\lambda^2 - 108a_1^2\mu^2 + 4ca_1\mu - 292a_1\lambda^2\mu$$

$$-189a_1^2\lambda^2\mu - 31a_1\lambda^4 - 136a_1\mu^2 - 18a_1^3\lambda^2\mu = 0$$

$$(\frac{G'}{G})^3:\ -324a_1^2\lambda\mu + 6ca_1\lambda - 36a_1^3\lambda\mu$$

$$-99a_1^2\lambda^3 - 480a_1\lambda\mu - 6a_1^3\lambda^3$$

$$-180a_1\lambda^3 = 0$$

$$(\frac{G'}{G})^4:\ 3ca_1 - 162a_1^2\mu - 390a_1\lambda^2$$

$$-18a_1^3\lambda^2 - 240a_1\mu - 18a_1^3\mu$$

$$-\frac{459}{2}a_1^2\lambda^2 = 0$$

$$(\frac{G'}{G})^5:\ -18a_1^3\lambda - 216a_1^2\lambda - 360a_1\lambda = 0$$

$$(\frac{G'}{G})^6:\ -120a_1 - 72a_1^2 - 6a_1^3 = 0$$

Solving the algebraic equations above yields:

$$a_1 = -2, \;\; a_0 = a_0, \;\; g = 0, \;\; c = \lambda^2 - 4\mu \quad (3.8)$$

where a_0, λ, μ are arbitrary constants.

Substituting (3.8) into (3.7), we get that

$$u(\xi) = -2(\frac{G'}{G}) + a_0, \quad \xi = x - (\lambda^2 - 4\mu)t \quad (3.9)$$

where a_0, λ, μ are arbitrary constants.

Substituting the general solutions of (3.6) into (3.9), we will have three types of travelling wave solutions of the DSSH equation (3.1) as follows:

Case (a): when $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = a_0 + \lambda - \sqrt{\lambda^2 - 4\mu}$$
$$\cdot(\frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi})$$

where
$$\xi = x - (\lambda^2 - 4\mu)t,$$
C_1, C_2 are arbitrary constants.

In particular, if $C_1 = 1, \; C_2 = 0, \; \lambda = 2, \; \mu = 0,$ then we have

$$u(x, t) = 2 - 2tanh(x - 4t) + a_0.$$

Case (b): when $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = a_0 + \lambda - \sqrt{4\mu - \lambda^2}$$
$$\cdot(\frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi})$$

where
$$\xi = x - (\lambda^2 - 4\mu)t,$$
C_1, C_2 are arbitrary constants.

In particular, if $C_1 = 1, \; C_2 = 0, \; \lambda = 0, \; \mu = 1,$ then
$$u(x, t) = 2tanh(x + 4t) + a_0.$$

Case (c): when $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = -2\left[\frac{2C_2 - C_1\lambda - C_2\lambda\xi}{2(C_1 + C_2\xi)}\right] + a_0$$

where
$$\xi = x - (\lambda^2 - 4\mu)t,$$
C_1, C_2 are arbitrary constants.

In particular, if $C_1 = C_2 = 1, \; \lambda = 2, \; \mu = 1,$ then
$$u(x, t) = 2\frac{1}{1 + x} + a_0.$$

4 Application Of The $(\frac{G'}{G})$-Expansion Method For The KP-BBM Equation

we will consider the KP-BBM equation [44-45]:

$$(u_t + u_x - a(u^2)_x - bu_{xxt})_x + ku_{yy} = 0 \quad (4.1)$$

where a, b, k are constants.

Suppose that

$$u(x, \; y, \; t) = u(\xi), \; \xi = x + y - ct \quad (4.2)$$

c is a constant that to be determined later.

By using the wave variable (4.2), Eq.(4.1) is converted into an ODE

$$-cu'' + u'' - 2a(u')^2 - 2auu'' + bcu^{(4)} + ku'' = 0 \quad (4.3)$$

Integrating (4.3) once, we obtain

$$(-c + 1 + k)u' + bcu^{(3)} - 2auu' = g \quad (4.4)$$

where g is the integration constant that can be determined later.

Suppose that the solution of the ODE (4.4) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi)=\sum_{i=0}^{m}a_i(\frac{G'}{G})^i \qquad (4.5)$$

where a_i are constants, $G=G(\xi)$ satisfies the second order LODE in the form:

$$G''+\lambda G'+\mu G=0 \qquad (4.6)$$

where λ and μ are constants.

Balancing the order of uu' and u''' in Eq.(4.4), we get that $m+m+1=m+3 \;\Rightarrow\; m=2$. So Eq.(4.5) can be rewritten as

$$u(\xi)=a_2(\frac{G'}{G})^2+a_1(\frac{G'}{G})+a_0,\ a_2\neq 0 \qquad (4.7)$$

$a_2,\ a_1,\ a_0$ are constants to be determined later. Then it follows:

$$u'(\xi)=-2a_2(\frac{G'}{G})^3+(-a_1-2a_2\lambda)(\frac{G'}{G})^2$$
$$+(-a_1\lambda-2a_2\mu)(\frac{G'}{G})-a_1\mu$$

$$u''(\xi)=6a_2(\frac{G'}{G})^4+(2a_1+10a_2\lambda)(\frac{G'}{G})^3$$
$$+(8a_2\mu+3a_1\lambda+4a_2\lambda^2)(\frac{G'}{G})^2$$
$$+(6a_2\lambda\mu+2a_1\mu+a_1\lambda^2)(\frac{G'}{G})$$
$$+2a_2\mu^2+a_1\lambda\mu$$

$$u'''(\xi)=-24a_2(\frac{G'}{G})^5+(-54a_2\lambda-6a_1)(\frac{G'}{G})^4$$
$$+(-12a_1\lambda-38a_2\lambda^2-40a_2\mu)(\frac{G'}{G})^3$$
$$+(-52a_2\lambda\mu-7a_1\lambda^2-8a_2\lambda^3-8a_1\mu)(\frac{G'}{G})^2$$
$$+(-14a_2\lambda^2\mu-a_1\lambda^3-16a_2\mu^2-8a_1\lambda\mu)(\frac{G'}{G})$$
$$-a_1\lambda^2\mu-2a_1\mu^2-6a_2\lambda\mu^2$$

Substituting(4.7) into (4.4) and collecting all the terms with the same power of $(\frac{G'}{G})$ together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$(\frac{G'}{G})^0:\ (c-k-1)a_1\mu-g-2bca_1\mu^2$$
$$+2aa_0a_1\mu-6bca_2\lambda\mu^2-bca_1\lambda^2\mu=0$$

$$(\frac{G'}{G})^1:\ -a_1\lambda-16bca_2\mu^2+2ca_2\mu-bca_1\lambda^3$$
$$+ca_1\lambda-2a_2\mu+2aa_1^2\mu-8bca_1\lambda\mu$$
$$-2ka_2\mu+2aa_0a_1\lambda-14bca_2\lambda^2\mu$$
$$-ka_1\lambda+4aa_0a_2\mu=0$$

$$(\frac{G'}{G})^2:\ -52bca_2\lambda\mu+(2c-2k-2)a_2\lambda$$
$$+2aa_1^2\lambda-7bca_1\lambda^2-a_1+ca_1$$
$$-8bca_1\mu+4aa_0a_2\lambda-ka_1$$
$$+6aa_1a_2\mu-8bca_2\lambda^3+2aa_1a_0=0$$

$$(\frac{G'}{G})^3:\ (2c-2k-2)a_2-40bca_2\mu+4aa_2^2\mu$$

$$+6aa_1a_2\lambda - 12bca_1\lambda + 4aa_0a_2$$
$$-38bca_2\lambda^2 + 2aa_1^2 = 0$$

$$(\frac{G'}{G})^4 : \ -6bca_1 - 54bca_2\lambda + 4aa_2^2\lambda + 6aa_1a_2 = 0$$

$$(\frac{G'}{G})^5 : \ -24bca_2 + 4aa_2^2 = 0$$

Solving the algebraic equations above yields:

$$a_2 = a_2, \ a_1 = a_2\lambda, \ a_0 = a_0, \ c = \frac{1}{6}\frac{aa_2}{b}, \ g = 0 \tag{4.8}$$

where a_0, a_2, λ are arbitrary constants.

Substituting (4.8) into (4.7), we have

$$u(\xi) = a_2(\frac{G'}{G})^2 + a_2\lambda(\frac{G'}{G}) + a_0$$
$$\xi = x + y - \frac{1}{6}\frac{aa_2}{b}t \tag{4.9}$$

Substituting the general solutions of (4.6) into (4.9), we will have three types of travelling wave solutions of the Kadomtsev-Petviashvili equation (4.1) as follows:

Case (a): When $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = a_0 - \frac{a_2}{4}\lambda^2 + \frac{a_2}{4}(\lambda^2 - 4\mu)$$
$$.(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi})^2$$

where
$$\xi = x + y - \frac{1}{6}\frac{aa_2}{b}t,$$
C_1, C_2 are arbitrary constants.

Case (b): When $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = a_0 - \frac{a_2}{4}\lambda^2 + \frac{a_2}{4}(4\mu - \lambda^2)$$
$$.(\frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi})^2$$

where
$$\xi = x + y - \frac{1}{6}\frac{aa_2}{b}t,$$
C_1, C_2 are arbitrary constants.

Case (c): When $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = -\frac{1}{4}\lambda^2 a_2 + \frac{a_2 C_2^2}{(C_1 + C_2\xi)^2} + a_0$$

where
$$\xi = x + y - \frac{1}{6}\frac{aa_2}{b}t,$$
C_1, C_2 are arbitrary constants.

5 Application Of The $(\frac{G'}{G})$-Expansion Method For The (3+1) dimensional potential-YTSF equation

At last, we consider the (3+1) dimensional potential-YTSF equation [46]:

$$-4u_{xt} + u_{xxxz} + 4u_x u_{xz} + 2u_{xx}u_z + 3u_{yy} = 0 \tag{5.1}$$

Suppose that

$$u(x,\ y,\ z,\ t) = u(\xi),\ \xi = kx + ly + mz + \omega t \tag{5.2}$$

k, l, m, ω are constants that to be determined later.
By using (5.2), (5.1) is converted into:

$$k^3 m u^{(4)} + 6mk^2 u' u'' + (3l^2 - 4k\omega)u'' = 0 \tag{5.3}$$

Integrating (5.3) once, we obtain

$$k^3 m u''' + 3mk^2 (u')^2 + (3l^2 - 4k\omega)u' = g \tag{5.4}$$

where g is the integration constant that can be determined later.

Similar to the last example, suppose:

$$u(\xi)=\sum_{i=0}^{m}a_i(\frac{G'}{G})^i \tag{5.5}$$

where a_i are constants, $G=G(\xi)$ satisfies the second order LODE in the form:

$$G''+\lambda G'+\mu G=0 \tag{5.6}$$

where λ and μ are constants.

Balancing the order of u''' and $(u')^2$ in Eq.(5.24), we have $m+3=2+2m \Rightarrow m=1$. So

$$u(\xi)=a_1(\frac{G'}{G})+a_0,\ a_1\neq 0 \tag{5.7}$$

$a_1,\ a_0$ are constants to be determined later.

Substituting (5.7) into (5.4) and collecting all the terms with the same power of $(\frac{G'}{G})$ together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$(\frac{G'}{G})^0:\ -g+4a_1k\omega\mu-3a_1l^2\mu-2k^3ma_1\mu^2$$

$$-k^3ma_1\lambda^2\mu+3k^2ma_1^2\mu^2=0$$

$$(\frac{G'}{G})^1:\ 4a_1k\omega\lambda-8k^3ma_1\lambda\mu-k^3ma_1\lambda^3$$

$$+6k^2ma_1^2\lambda\mu-3a_1l^2\lambda=0$$

$$(\frac{G'}{G})^2:\ -7k^3ma_1\lambda^2+3k^2ma_1^2\lambda^2-8k^3ma_1\mu$$

$$+4a_1k\omega+6k^2ma_1^2\mu-3a_1l^2=0$$

$$(\frac{G'}{G})^3:\ 6k^2ma_1^2\lambda-12k^3ma_1\lambda=0$$

$$(\frac{G'}{G})^4:\ 3k^2ma_1^2-6k^3ma_1=0$$

Solving the algebraic equations above, yields

$$\begin{aligned}a_1&=2k,\\ a_0&=a_0,\\ k&=k,\\ l&=l,\\ m&=m,\\ \omega&=\frac{k^3m\lambda^2-4k^3m\mu+3l^2}{4k},\\ g&=0\end{aligned} \tag{5.8}$$

Then

$$u(\xi)=2k(\frac{G'}{G})+a_0$$

$$\xi=kx+ly+mz+\frac{k^3m\lambda^2-4k^3m\mu+3l^2}{4k}t \tag{5.9}$$

where k,l,m,a_0 are arbitrary constants.

Substituting the general solutions of Eq.(5.6) into (5.9), we have:

When $\lambda^2-4\mu>0$

$$u_1(\xi)=-k\lambda+k\sqrt{\lambda^2-4\mu}$$
$$\left(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)+a_0$$

where

$$\xi=kx+ly+mz+\frac{k^3m\lambda^2-4k^3m\mu+3l^2}{4k}t,$$

C_1, C_2, k, l, m, a_0 are arbitrary constants.

When $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = -k\lambda + k\sqrt{4\mu - \lambda^2}$$

$$\left(\frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi} \right) + a_0$$

where

$$\xi = kx + ly + mz + \frac{k^3 m\lambda^2 - 4k^3 m\mu + 3l^2}{4k} t,$$

C_1, C_2, k, l, m, a_0 are arbitrary constants.

When $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{k(2C_2 - C_1\lambda - C_2\lambda\xi)}{(C_1 + C_2\xi)} + a_0$$

where

$$\xi = kx + ly + mz + \frac{k^3 m\lambda^2 - 4k^3 m\mu + 3l^2}{4k} t,$$

C_1, C_2, k, l, m, a_0 are arbitrary constants.

6 Conclusions

In this paper we have seen that the traveling wave solutions of the DSSH equation, the KP-BBM equation and the (3+1) dimensional potential-YTSF equation are successfully found by using the $(\frac{G'}{G})$-expansion method.

This study shows that the $(\frac{G'}{G})$-expansion method is quite efficient and practically well suited for use in finding exact solutions for the problems considered here.

Being concise and effective, the method can also be used to many other nonlinear equations.

7 Acknowledgements

I would like to thank the anonymous referees for their useful and valuable suggestions.

References:

[1] Damelys Zabala, Aura L. Lopez De Ramos, Effect of the Finite Difference Solution Scheme in a Free Boundary Convective Mass Transfer Model, WSEAS Transactions on Mathematics, Vol. 6, No. 6, 2007, pp. 693-701

[2] Raimonds Vilums, Andris Buikis, Conservative Averaging and Finite Difference Methods for Transient Heat Conduction in 3D Fuse, WSEAS Transactions on Heat and Mass Transfer, Vol 3, No. 1, 2008

[3] Mastorakis N E., An Extended Crank-Nicholson Method and its Applications in the Solution of Partial Differential Equations: 1-D and 3-D Conduction Equations, WSEAS Transactions on Mathematics, Vol. 6, No. 1, 2007, pp 215-225

[4] Nikos E. Mastorakis, Numerical Solution of Non-Linear Ordinary Differential Equations via Collocation Method (Finite Elements) and Genetic Algorithm, WSEAS Transactions on Information Science and Applications, Vol. 2, No. 5, 2005, pp. 467-473

[5] Z. Huiqun, Commun. Nonlinear Sci. Numer. Simul. 12 (5) (2007) 627-635.

[6] Wazwa Abdul-Majid. New solitary wave and periodic wave solutions to the (2+1)-dimensional Nizhnik-Nivikov-veselov system. Appl. Math. Comput. 187 (2007) 1584-1591.

[7] Senthil kumar C, Radha R, lakshmanan M. Trilinearization and localized coherent structures and periodic solutions for the (2+1) dimensional K-dv and NNV equations. Chaos, Solitons and Fractals. 39 (2009) 942-955.

[8] M. Wang, Solitary wave solutions for variant Boussinesq equations, Phys. Lett. A 199 (1995) 169-172.

[9] E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, On the solitary wave solutions for nonlinear Hirota-Satsuma coupled KdV equations, Chaos, Solitons and Fractals 22 (2004) 285-303.

[10] L. Yang, J. Liu, K. Yang, Exact solutions of nonlinear PDE nonlinear transformations and reduction of nonlinear PDE to a quadrature, Phys. Lett. A 278 (2001) 267-270.

[11] E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, Group analysis and modified tanh-function to find the invariant solutions and soliton solution

for nonlinear Euler equations, Int. J. Nonlinear Sci. Numer. Simul. 5 (2004) 221-234.

[12] M. Inc, D.J. Evans, On traveling wave solutions of some nonlinear evolution equations, Int. J. Comput. Math. 81 (2004) 191-202.

[13] M.A. Abdou, The extended tanh-method and its applications for solving nonlinear physical models, Appl. Math. Comput. 190 (2007) 988-996

[14] E.G. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (2000) 212-218.

[15] W. Malfliet, Solitary wave solutions of nonlinear wave equations, Am. J. Phys. 60 (1992) 650-654.

[16] J.L. Hu, A new method of exact traveling wave solution for coupled nonlinear differential equations, Phys. Lett. A 322 (2004) 211-216.

[17] M.J. Ablowitz, P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform, Cambridge University Press, Cambridge, 1991.

[18] M.R. Miura, Backlund Transformation, Springer-Verlag, Berlin, 1978.

[19] C. Rogers, W.F. Shadwick, Backlund Transformations, Academic Press, New York, 1982.

[20] R. Hirota, Exact envelope soliton solutions of a nonlinear wave equation, J. Math. Phys. 14 (1973) 805-810.

[21] R. Hirota, J. Satsuma, Soliton solution of a coupled KdV equation, Phys. Lett. A 85 (1981) 407-408.

[22] Z.Y. Yan, H.Q. Zhang, New explicit solitary wave solutions and periodic wave solutions for WhithamCBroerCKaup equation in shallow water, Phys. Lett. A 285 (2001) 355-362.

[23] A.V. Porubov, Periodical solution to the nonlinear dissipative equation for surface waves in a convecting liquid layer, Phys. Lett. A 221 (1996) 391-394.

[24] K.W. Chow, A class of exact periodic solutions of nonlinear envelope equation, J. Math. Phys. 36 (1995) 4125-4137.

[25] E.G. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (2000) 212-218.

[26] Engui Fan, Multiple traveling wave solutions of nonlinear evolution equations using a unifiex algebraic method, J. Phys. A, Math. Gen. 35 (2002) 6853-6872.

[27] Z.Y. Yan, H.Q. Zhang, New explicit and exact traveling wave solutions for a system of variant Boussinesq equations in mathematical physics, Phys. Lett. A 252 (1999) 291-296.

[28] S.K. Liu, Z.T. Fu, S.D. Liu, Q. Zhao, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Phys. Lett. A 289 (2001) 69-74.

[29] Z. Yan, Abundant families of Jacobi elliptic functions of the (2 + 1)-dimensional integrable DaveyCStawartson-type equation via a new method, Chaos, Solitons and Fractals 18 (2003) 299-309.

[30] C. Bai, H. Zhao, Complex hyperbolic-function method and its applications to nonlinear equations, Phys. Lett. A 355 (2006) 22-30.

[31] E.M.E. Zayed, A.M. Abourabia, K.A. Gepreel, M.M. Horbaty, On the rational solitary wave solutions for the nonlinear HirotaCSatsuma coupled KdV system, Appl. Anal. 85 (2006) 751-768.

[32] K.W. Chow, A class of exact periodic solutions of nonlinear envelope equation, J. Math. Phys. 36 (1995) 4125-4137.

[33] M.L. Wang, Y.B. Zhou, The periodic wave equations for the KleinCGordonCSchordinger equations, Phys. Lett. A 318 (2003) 84-92.

[34] M.L. Wang, X.Z. Li, Extended F-expansion and periodic wave solutions for the generalized Zakharov equations, Phys. Lett. A 343 (2005) 48-54.

[35] M.L. Wang, X.Z. Li, Applications of F-expansion to periodic wave solutions for a new Hamiltonian amplitude equation, Chaos, Solitons and Fractals 24 (2005) 1257-1268.

[36] X. Feng, Exploratory approach to explicit solution of nonlinear evolutions equations, Int. J. Theo. Phys. 39 (2000) 207-222.

[37] J.H. He, X.H. Wu, Exp-function method for nonlinear wave equations, Chaos, Solitons and Fractals 30 (2006) 700-708.

[38] Mingliang Wang, Xiangzheng Li, Jinliang Zhang, The $(\frac{G'}{G})$-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. Physics Letters A, 372 (2008) 417-423.

[39] Mingliang Wang, Jinliang Zhang, Xiangzheng Li, Application of the $(\frac{G'}{G})$-expansion to travelling wave solutions of the Broer-Kaup and the approximate long water wave equations. Appl. Math. Comput. , 206 (2008) 321-326.

[40] Ismail Aslan, Exact and explicit solutions to some nonlinear evolution equations by utilizing the $(\frac{G'}{G})$-expansion method. Appl. Math. Comput. In press, (2009).

[41] Xun Liu, Lixin Tian, Yuhai Wu, Application of $(\frac{G'}{G})$-expansion method to two nonlinear evolution equations. Appl. Math. Comput. , in press, (2009).

[42] Ismail Aslan, Turgut $\ddot{O}$zis, Analytic study on two nonlinear evolution equations by using the $(\frac{G'}{G})$-expansion method. Appl. Math. Comput. 209 (2009) 425-429.

[43] Abdul-Majid Wazwaz, The ColeCHopf transformation and multiple soliton solutions for the integrable sixth-order DrinfeldCSokolovC-SatsumaCHirota equation, Appl. Math. Comput. 207 (2009) 248-255.

[44] A.M. Wazwaz, Exact solutions of compact and noncompact structures for the KP-BBM equation. Appl. Math. Comput. ,169 (1) (2005) 700-712.

[45] A.M. Wazwaz, The extended tanh method for new compact and noncompact solutions for the KP-BBM and the ZK-BBM equations. Chaos Solitons Fract. ,38 (5) (2008) 1505-1516.

[46] Z. Yan, New families of nontravelling wave solutions to a new (3 + 1)-dimensional potential-YTSF equation, Phys. Lett. A 318 (2003) 78-83.