

# Traveling Wave Solutions For The Generalized Burgers Equation And The (2+1) Dimensional Dispersive Equation By $(\frac{G'}{G})$ -expansion method

Qinghua Feng  
Shandong University of Technology  
School of Science  
Zhangzhou Road 12, Zibo, 255049  
China  
fqhua@sina.com

Bin Zheng  
Shandong University of Technology  
School of Science  
Zhangzhou Road 12, Zibo, 255049  
China  
zhengbin2601@126.com

**Abstract:** In this paper, we test the validity and reliability of the  $(\frac{G'}{G})$ -expansion method by applying it to get the exact travelling wave solutions of the generalized Burgers equation and the (2+1) dimensional dispersive equation. The traveling wave solutions are obtained in three forms. Being concise and less restrictive, the method can also be applied to many other nonlinear partial differential equations.

**Key-Words:**  $(\frac{G'}{G})$ -expansion method, Traveling wave solutions, generalized Burgers equation, (2+1) dimensional dispersive equation, exact solution, evolution equation, nonlinear equation

## 1 Introduction

The nonlinear phenomena exist in all the fields including either the scientific work or engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, and so on. It is well known that many non-linear evolution equations (NLEEs) are widely used to describe these complex phenomena. Research on solutions of NLEEs is popular. So, the powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far such as in [1-7].

In this paper, we pay attention to the analytical method for getting the exact solution of NLEEs. Among the possible exact solutions of NLEEs, certain solutions for special form may depend only on a single combination of variables such as traveling wave variables. In the literature, Also there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the homogeneous balance method [8,9], the hyperbolic tangent expansion method [10,11], the trial function method [12], the tanh-method [13-15], the non-linear transform method [16], the inverse scattering transform [17], the Backlund transform [18,19], the Hirota's bilinear method [20,21], the generalized Riccati equation [22,23], the Weierstrass elliptic func-

tion method [24], the theta function method [25-27], the sineCcosine method [28], the Jacobi elliptic function expansion [29,30], the complex hyperbolic function method [31-33], the truncated Painleve expansion [34], the F-expansion method [35], the rank analysis method [36], the exp-function expansion method [37] and so on.

In [38], Mingliang Wang proposed a new method called  $(\frac{G'}{G})$ -expansion method. Recently several authors have studied some nonlinear equations by this method [39-42]. The value of the  $(\frac{G'}{G})$ -expansion method is that one can treat nonlinear problems by essentially linear methods. The method is based on the explicit linearization of NLEEs for traveling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. Moreover, it transforms a nonlinear equation to a simple algebraic computation. The main merits of the  $(\frac{G'}{G})$ -expansion method over the other methods are that it gives more general solutions with some free parameters and it handles NLEEs in a direct manner with no requirement for initial/boundary condition or initial trial function at the outset.

Our aim in this paper is to present an application of the  $(\frac{G'}{G})$ -expansion method to some nonlinear problems to be solved by this method for the first time. The rest of the paper is organized as follows. In

Section 2, we describe the  $(\frac{G'}{G})$ -expansion method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the method to the generalized Burgers equation and the (2+1) dimensional dispersive equation. In section 5, the features of the  $(\frac{G'}{G})$ -expansion method are briefly summarized.

## 2 Description of the $(\frac{G'}{G})$ -expansion method

In this section we describe the  $(\frac{G'}{G})$ -expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables  $x, t$ , is given by

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \dots) = 0, \quad (2.1)$$

or in three independent variables  $x, y$  and  $t$ , is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}, \dots) = 0, \quad (2.2)$$

where  $u = u(x, t)$  or  $u = u(x, y, t)$  is an unknown function,  $P$  is a polynomial in  $u = u(x, t)$  or  $u = u(x, y, t)$  and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following, we will give the main steps of the  $(\frac{G'}{G})$ -expansion method.

Step 1. Suppose that

$$u(x, t) = u(\xi), \quad \xi = \xi(x, t) \quad (2.3)$$

or

$$u(x, y, t) = u(\xi), \quad \xi = \xi(x, y, t) \quad (2.4)$$

The traveling wave variable (2.3) or (2.4) permits us reducing (2.1) or (2.2) to an ODE for  $u = u(\xi)$

$$P(u, u', u'', \dots) = 0. \quad (2.5)$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in  $(\frac{G'}{G})$  as follows:

$$u(\xi) = \alpha_m \left(\frac{G'}{G}\right)^m + \dots \quad (2.6)$$

where  $G = G(\xi)$  satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \quad (2.7)$$

$\alpha_m, \dots, \lambda$  and  $\mu$  are constants to be determined later,  $\alpha_m \neq 0$ . The unwritten part in (2.6) is also a polynomial in  $(\frac{G'}{G})$ , the degree of which is generally equal to or less than  $m - 1$ . The positive integer  $m$  can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using second order LODE (2.7), collecting all terms with the same order of  $(\frac{G'}{G})$  together, the left-hand side of (2.5) is converted into another polynomial in  $(\frac{G'}{G})$ . Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for  $\alpha_m, \dots, \lambda$  and  $\mu$ .

Step 4. Assuming that the constants  $\alpha_m, \dots, \lambda$  and  $\mu$  can be obtained by solving the algebraic equations in Step 3. Since the general solutions of the second order LODE (2.7) have been well known for us, then substituting  $\alpha_m, \dots$  and the general solutions of (2.7) into (2.6) we have traveling wave solutions of the nonlinear evolution equation (2.1) or (2.2).

## 3 Application Of The $(\frac{G'}{G})$ -Expansion Method For The Generalized Burgers Equation

In this section, we apply the  $(\frac{G'}{G})$ -expansion method to construct the traveling wave solutions for some nonlinear partial differential equations in mathematical physics as follows:

We begin with the generalized Burgers equation [43]:

$$u_t + \alpha u_{xx} + u^m u_x = 0, \quad \alpha \neq 0 \quad (3.1)$$

In order to obtain the travelling wave solutions of Eq.(3.1), we suppose that

$$u(x, t) = u(\xi), \quad \xi = x - ct \quad (3.2)$$

$c$  is a constant that to be determined later.

By using (3.2), (3.1) is converted into an ODE

$$-cu' + \alpha u'' + u^m u' = 0 \quad (3.3)$$

Suppose that the solution of (3.3) can be expressed by a polynomial in  $(\frac{G'}{G})$  as follows:

$$u(\xi) = \sum_{i=0}^n a_i \left(\frac{G'}{G}\right)^i \quad (3.4)$$

where  $a_i$  are constants.

Balancing the order of  $u^m u'$  and  $u''$  in Eq.(3.3), we have  $mn + n + 1 = n + 2 \Rightarrow n = \frac{1}{m}$ . So we make a variable  $u = v^{\frac{1}{m}}$ , then (3.3) is converted into

$$-cmv v' + \alpha(1-m)(v')^2 + \alpha m v v'' + m v^2 v' = 0 \quad (3.5)$$

Suppose that the solution of (3.5) can be expressed by a polynomial in  $(\frac{G'}{G})$  as follows:

$$v(\xi) = \sum_{i=0}^l b_i \left(\frac{G'}{G}\right)^i \quad (3.6)$$

where  $b_i$  are constants,  $G = G(\xi)$  satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \quad (3.7)$$

where  $\lambda$  and  $\mu$  are constants.

Balancing the order of  $v^2 v'$  and  $v v''$  in Eq.(3.5), we have  $2l + l + 1 = l + l + 1 \Rightarrow l = 1$ . So Eq.(3.6) can be rewritten as

$$v(\xi) = b_1 \left(\frac{G'}{G}\right) + b_0, \quad b_1 \neq 0 \quad (3.8)$$

$b_1, b_0$  are constants to be determined later. Then we can obtain

$$v'(\xi) = b_1 \left[ -\lambda \left(\frac{G'}{G}\right) - \mu - \left(\frac{G'}{G}\right)^2 \right]$$

$$v''(\xi) = 2b_1 \left(\frac{G'}{G}\right)^3 + 3\lambda b_1 \left(\frac{G'}{G}\right)^2 + (\lambda^2 b_1 + 2b_1 \mu) \left(\frac{G'}{G}\right) + \lambda \mu b_1$$

Substituting (3.8) into (3.5) and collecting all the terms with the same power of  $(\frac{G'}{G})$  together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$\left(\frac{G'}{G}\right)^0 : -mb_0^2 b_1 \mu + \alpha b_1^2 \mu^2 - \alpha b_1^2 m \mu^2$$

$$+ \alpha m b_0 b_1 \lambda \mu + c m b_0 b_1 \mu = 0$$

$$\left(\frac{G'}{G}\right)^1 : c m b_1^2 \mu - 2m b_1^2 b_0 \mu - m b_1 b_0^2 \lambda$$

$$+ 2\alpha m b_0 b_1 b_2 \mu + \alpha m b_0 b_1 \lambda^2$$

$$- \alpha b_1^2 m \lambda \mu + c m b_0 b_1 \lambda + 2\alpha b_1^2 \lambda \mu = 0$$

$$\left(\frac{G'}{G}\right)^2 : -m b_1^3 \mu + 2\alpha b_1^2 \mu^3 + c m b_1^2 \lambda$$

$$- 2m b_0 b_1^2 \lambda + \alpha b_1^2 \lambda^2 - m b_0^2 b_1$$

$$+ 3\alpha m b_0 b_1 \lambda + c m b_0 b_1 = 0$$

$$\left(\frac{G'}{G}\right)^3 : \alpha b_1^2 m \lambda + c m b_1^2 - 2m b_1^2 b_0$$

$$+2\alpha mb_0b_1 - mb_1^3\lambda + 2\alpha b_1^2\lambda = 0$$

$$\left(\frac{G'}{G}\right)^4 : \alpha mb_1^2 - mb_1^3 + \alpha b_1^2 = 0$$

Solving the algebraic equations above, yields:

Case 1: when  $\lambda^2 - 4\mu > 0$

$$\begin{aligned} b_1 &= \frac{\alpha(m+1)}{m}, \\ b_0 &= \frac{\alpha(m+1)}{m} \left[ \frac{1}{2}\lambda \pm \frac{1}{2}\sqrt{\lambda^2 - 4\mu} \right], \\ c &= \pm \frac{\alpha}{m} \sqrt{\lambda^2 - 4\mu} \end{aligned} \quad (3.9)$$

Substituting (3.9) into (3.8), we have

$$\begin{aligned} v(\xi) &= \frac{\alpha(m+1)}{m} \left( \frac{G'}{G} \right) \\ &+ \frac{\alpha(m+1)}{m} \left[ \frac{1}{2}\lambda \pm \frac{1}{2}\sqrt{\lambda^2 - 4\mu} \right] \\ \xi &= x \mp \frac{\alpha}{m} \sqrt{\lambda^2 - 4\mu} t \end{aligned} \quad (3.10)$$

Substituting the general solutions of (3.7) into (3.10), we have:

$$\begin{aligned} v_1(\xi) &= -\frac{\alpha\lambda(m+1)}{2m} + \frac{\alpha(m+1)}{2m} \sqrt{\lambda^2 - 4\mu} \\ &\left( \frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi} \right) \\ &+ \frac{\alpha(m+1)}{m} \left[ \frac{1}{2}\lambda \pm \frac{1}{2}\sqrt{\lambda^2 - 4\mu} \right] \end{aligned}$$

Then

$$u_1(\xi) = (v_1(\xi))^{\frac{1}{m}}.$$

Where

$$\xi = x \mp \frac{\alpha}{m} \sqrt{\lambda^2 - 4\mu} t,$$

$C_1$  and  $C_2$  are two arbitrary constants.

Case 2: when  $\lambda^2 - 4\mu < 0$

$$\begin{aligned} b_1 &= \frac{\alpha(m+1)}{m}, \\ b_0 &= \frac{\alpha(m+1)}{m} \\ \left[ \frac{1}{2}\lambda \pm \frac{1}{2}\sqrt{4\mu - \lambda^2}i \right], \quad c &= \pm \frac{\alpha}{m} \sqrt{4\mu - \lambda^2}i \end{aligned} \quad (3.11)$$

Substituting (3.11) into (3.8), we have

$$\begin{aligned} v(\xi) &= \frac{\alpha(m+1)}{m} \left( \frac{G'}{G} \right) \\ &+ \frac{\alpha(m+1)}{m} \left[ \frac{1}{2}\lambda \pm \frac{1}{2}\sqrt{4\mu - \lambda^2}i \right] \\ \xi &= x \mp \frac{\alpha}{m} \sqrt{4\mu - \lambda^2}it \end{aligned} \quad (3.12)$$

Substituting the general solutions of (3.7) into (3.12), we have:

$$\begin{aligned} v_2(\xi) &= -\frac{\alpha\lambda(m+1)}{2m} + \frac{\alpha(m+1)}{2m} \sqrt{4\mu - \lambda^2}i \\ &\left( \frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi} \right) \\ &+ \frac{\alpha(m+1)}{m} \left[ \frac{1}{2}\lambda \pm \frac{1}{2}\sqrt{4\mu - \lambda^2}i \right] \end{aligned}$$

Then

$$u_2(\xi) = (v_2(\xi))^{\frac{1}{m}}.$$

Where

$$\xi = x \mp \frac{\alpha}{m} \sqrt{4\mu - \lambda^2}it,$$

$C_1$  and  $C_2$  are two arbitrary constants.

Case 3: when  $\lambda^2 - 4\mu = 0$

$$b_1 = \frac{\alpha(m+1)}{m},$$

$$\begin{aligned} b_0 &= \frac{\alpha\lambda(m+1)}{2m}, \\ c &= 0 \end{aligned} \quad (3.13)$$

Substituting (3.13) into (3.8), we have

$$v(\xi) = \frac{\alpha(m+1)}{m} \left(\frac{G'}{G}\right) + \frac{\alpha\lambda(m+1)}{2m}, \quad \xi = x \quad (3.14)$$

Substituting the general solutions of (3.7) into (3.14), we have:

$$\begin{aligned} v_3(\xi) &= \frac{\alpha(m+1)(2C_2 - C_1\lambda - C_2\lambda\xi)}{2m(C_1 + C_2\xi)} \\ &\quad + \frac{\alpha\lambda(m+1)}{2m} \end{aligned}$$

Then

$$u_3(\xi) = (v_3(\xi))^{\frac{1}{m}}.$$

Where  $\xi = x$ ,  $C_1$  and  $C_2$  are two arbitrary constants.

#### 4 Application Of The $(\frac{G'}{G})$ -Expansion Method For The (2+1) Dimensional Dispersive Equation

In this section, we will consider the (2+1) dimensional dispersive equation [44]:

$$u_{yt} + v_{xx} + (uu_x)_y = 0 \quad (4.1)$$

$$v_t + u_x + (uv)_x + u_{xxy} = 0 \quad (4.2)$$

Supposing that

$$\xi = k(x + ly + st) \quad (4.3)$$

By (4.3), (4.1) and (4.2) are converted into ODEs

$$slu'' + v'' + l(u')^2 + luu'' = 0 \quad (4.4)$$

$$sv' + u' + (uv)' + k^2lu''' = 0 \quad (4.5)$$

Suppose that the solution of (4.4) and (4.5) can be expressed by a polynomial in  $(\frac{G'}{G})$  as follows:

$$u(\xi) = \sum_{i=0}^m a_i \left(\frac{G'}{G}\right)^i \quad (4.6)$$

$$v(\xi) = \sum_{i=0}^n b_i \left(\frac{G'}{G}\right)^i \quad (4.7)$$

where  $a_i, b_i$  are constants,  $G = G(\xi)$  satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \quad (4.8)$$

where  $\lambda$  and  $\mu$  are constants.

Balancing the order of  $(u')^2$  and  $v''$  in Eq.(4.4), the order of  $u'''$  and  $(uv)'$  in Eq.(4.5), then we can obtain  $2m+2 = n+2$ ,  $m+3 = m+n+1 \Rightarrow m=1$ ,  $n=2$ , so Eq.(4.6) and (4.7) can be rewritten as

$$u(\xi) = a_1 \left(\frac{G'}{G}\right) + a_0, \quad a_1 \neq 0 \quad (4.9)$$

$$v(\xi) = b_2 \left(\frac{G'}{G}\right)^2 + b_1 \left(\frac{G'}{G}\right) + b_0, \quad b_1 \neq 0 \quad (4.10)$$

$a_1, a_0, b_2, b_1, b_0$  are constants to be determined later.

Substituting (4.9) and (4.10) into (4.4) and (4.5) and collecting all the terms with the same power of  $(\frac{G'}{G})$  together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

For Eq.(4.4):

$$\left(\frac{G'}{G}\right)^0 : b_1\lambda\mu + la_1^2\mu^2 + 2b_2\mu^2$$

$$+sla_1\lambda\mu + la_1a_0\lambda\mu = 0$$

$$\left(\frac{G'}{G}\right)^1 : b_1\lambda^2 + la_1a_0\lambda^2 + (6b_2 + 3la_1^2)\lambda\mu$$

$$+sla_1\lambda^2 + 2sla_1\mu + 2b_1\mu + 2la_1a_0\mu = 0$$

$$\left(\frac{G'}{G}\right)^2 : 4la_1^2\mu + 3la_1a_0\lambda + 3b_1\lambda$$

$$+4b_2\lambda^2 + 3sla_1\lambda + 8b_2\mu + 2la_1^2\lambda^2 = 0$$

$$\left(\frac{G'}{G}\right)^3 : 2b_1 + 2sla_1 + 10b_2\lambda + 2la_1a_0 + 5la_1^2\lambda = 0$$

$$\left(\frac{G'}{G}\right)^4 : 6b_2 + 3la_1^2 = 0$$

For Eq.(4.5):

$$\left(\frac{G'}{G}\right)^0 : -sb_1\mu - k^2la_1\lambda^2\mu - 2k^2la_1\mu^2$$

$$-b_1a_0\mu - a_1\mu - b_0a_1\mu = 0$$

$$\left(\frac{G'}{G}\right)^1 : -k^2la_1\lambda^3 - sb_1\lambda - a_1\lambda -$$

$$2sb_2\mu - 8k^2la_1\lambda\mu - 2b_2a_0\mu$$

$$-2b_1a_0\lambda - b_0a_1\lambda - 2b_1a_1\mu = 0$$

$$\left(\frac{G'}{G}\right)^2 : -a_0b_1 - 8k^2la_1\mu - 2sb_2\lambda$$

$$-a_1 - 2b_2a_0\lambda - 2b_1a_1\lambda$$

$$-3b_2a_1\mu - b_0a_1 - 7k^2la_1\lambda^2 - sb_1 = 0$$

$$\left(\frac{G'}{G}\right)^3 : -3b_2a_1\lambda - 12k^2la_1\lambda - 2a_0b_2$$

$$-2sb_2 - 2a_1b_1 = 0$$

$$\left(\frac{G'}{G}\right)^4 : -6k^2la_1 - 3a_1b_2 = 0$$

Solving the algebraic equations above yields:

$$a_1 = a_1,$$

$$a_0 = a_0,$$

$$b_2 = -\frac{1}{2}a_1^2l,$$

$$b_1 = -\frac{1}{2}a_1^2l\lambda,$$

$$b_0 = -1 - \frac{1}{2}a_1^2l\mu$$

$$k = \pm\frac{1}{2}a_1,$$

$$l = l,$$

$$s = -a_0 + \frac{1}{2}a_1\lambda \quad (4.11)$$

where  $a_1, a_0, l$  are arbitrary constants.

Substituting (4.11) into (4.9) and (4.10), yields:

$$u(\xi) = a_1\left(\frac{G'}{G}\right) + a_0 \quad (4.12)$$

$$v(\xi) = -\frac{1}{2}a_1^2l\left(\frac{G'}{G}\right)^2 - \frac{1}{2}a_1^2l\lambda\left(\frac{G'}{G}\right) - 1 - \frac{1}{2}a_1^2l\mu \quad (4.13)$$

where

$$\xi = \pm \frac{1}{2}a_1[x + ly + (-a_0 + \frac{1}{2}a_1\lambda)t].$$

Substituting the general solutions of (4.8) into (4.12) and (4.13), we have:

When  $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = -\frac{a_1\lambda}{2} + \frac{a_1\sqrt{\lambda^2 - 4\mu}}{2} \left( \frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi} \right) + a_0$$

$$v_1(\xi) = \frac{a_1^2\lambda^2 l}{8} - \frac{a_1^2 l}{8}(\lambda^2 - 4\mu) \left( \frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi} \right)^2 - 1 - \frac{1}{2}a_1^2 l\mu$$

Where

$$\xi = \pm \frac{1}{2}a_1[x + ly + (-a_0 + \frac{1}{2}a_1\lambda)t],$$

$a_1, a_0, l$  are arbitrary constants.

When  $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = -\frac{a_1\lambda}{2} + \frac{a_1\sqrt{4\mu - \lambda^2}}{2} \left( \frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi} \right) + a_0$$

$$v_2(\xi) = \frac{a_1^2\lambda^2 l}{8} - \frac{a_1^2 l}{8}(4\mu - \lambda^2)$$

$$\left( \frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi} \right)^2 - 1 - \frac{1}{2}a_1^2 l\mu$$

Where

$$\xi = \pm \frac{1}{2}a_1[x + ly + (-a_0 + \frac{1}{2}a_1\lambda)t],$$

$a_1, a_0, l$  are arbitrary constants.

When  $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{a_1(2C_2 - C_1\lambda - C_2\lambda\xi)}{2(C_1 + C_2\xi)} + a_0$$

$$v_3(\xi) = \frac{a_1^2\lambda^2 l}{8} - \frac{a_1^2 l C_2^2}{2(C_1 + C_2\xi)^2} - 1 - \frac{1}{2}a_1^2 l\mu$$

Where

$$\xi = \pm \frac{1}{2}a_1[x + ly + (-a_0 + \frac{1}{2}a_1\lambda)t],$$

$a_1, a_0, l$  are arbitrary constants.

## 5 Conclusions

In this paper we have seen that the traveling wave solutions of the generalized Burgers equation and the (2+1) dimensional dispersive equation are successfully found by using the  $(\frac{G'}{G})$ -expansion method. Now we briefly summarize the method in the following.

The main points of the method are that assuming the solution of the ODE reduced by using the traveling wave variable as well as integrating can be expressed by an  $m$ -th degree polynomial in  $(\frac{G'}{G})$ , where  $G = G(\xi)$  is the general solutions of a second order LODE. The positive integer  $m$  is determined by the homogeneous balance between the highest order derivatives and nonlinear terms appearing in the reduced ODE, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations resulted from the process

of using the method. The main merits of the  $(\frac{G'}{G})$ -expansion method over the other methods are that it gives more general solutions with some free parameters and it handles NLEEs in a direct manner with no requirement for initial/boundary condition or initial trial function at the outset.

Compared to the methods used before, one can see that this method is direct, concise and effective. As we can use the MATHEMATICA or MAPLE to find out a useful solution of the algebraic equations resulted, so we can also avoid tedious calculations. This method can also be used to many other nonlinear equations.

## 6 Acknowledgements

I would like to thank the anonymous referees for their useful and valuable suggestions.

### References:

- [1] Damelys Zabala, Aura L. Lopez De Ramos, Effect of the Finite Difference Solution Scheme in a Free Boundary Convective Mass Transfer Model, WSEAS Transactions on Mathematics, Vol. 6, No. 6, 2007, pp. 693-701
- [2] Raimonds Vilums, Andris Buikis, Conservative Averaging and Finite Difference Methods for Transient Heat Conduction in 3D Fuse, WSEAS Transactions on Heat and Mass Transfer, Vol 3, No. 1, 2008
- [3] Mastorakis N E., An Extended Crank-Nicholson Method and its Applications in the Solution of Partial Differential Equations: 1-D and 3-D Conduction Equations, WSEAS Transactions on Mathematics, Vol. 6, No. 1, 2007, pp 215-225
- [4] Nikos E. Mastorakis, Numerical Solution of Non-Linear Ordinary Differential Equations via Collocation Method (Finite Elements) and Genetic Algorithm, WSEAS Transactions on Information Science and Applications, Vol. 2, No. 5, 2005, pp. 467-473
- [5] Z. Huiqun, Commun. Nonlinear Sci. Numer. Simul. 12 (5) (2007) 627-635.
- [6] Wazwa Abdul-Majid. New solitary wave and periodic wave solutions to the (2+1)-dimensional Nizhnik-Nivikov-veselov system. Appl. Math. Comput. 187 (2007) 1584-1591.
- [7] Senthil kumar C, Radha R, lakshmanan M. Trilinearization and localized coherent structures and periodic solutions for the (2+1) dimensional K-dv and NNV equations. Chaos, Solitons and Fractals. 39 (2009) 942-955.
- [8] M. Wang, Solitary wave solutions for variant Boussinesq equations, Phys. Lett. A 199 (1995) 169-172.
- [9] E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, On the solitary wave solutions for nonlinear Hirota-Satsuma coupled KdV equations, Chaos, Solitons and Fractals 22 (2004) 285-303.
- [10] L. Yang, J. Liu, K. Yang, Exact solutions of nonlinear PDE nonlinear transformations and reduction of nonlinear PDE to a quadrature, Phys. Lett. A 278 (2001) 267-270.
- [11] E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, Group analysis and modified tanh-function to find the invariant solutions and soliton solution for nonlinear Euler equations, Int. J. Nonlinear Sci. Numer. Simul. 5 (2004) 221-234.
- [12] M. Inc, D.J. Evans, On traveling wave solutions of some nonlinear evolution equations, Int. J. Comput. Math. 81 (2004) 191-202.
- [13] M.A. Abdou, The extended tanh-method and its applications for solving nonlinear physical models, Appl. Math. Comput. 190 (2007) 988-996
- [14] E.G. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (2000) 212-218.
- [15] W. Malfliet, Solitary wave solutions of nonlinear wave equations, Am. J. Phys. 60 (1992) 650-654.
- [16] J.L. Hu, A new method of exact traveling wave solution for coupled nonlinear differential equations, Phys. Lett. A 322 (2004) 211-216.
- [17] M.J. Ablowitz, P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform, Cambridge University Press, Cambridge, 1991.
- [18] M.R. Miura, Backlund Transformation, Springer-Verlag, Berlin, 1978.
- [19] C. Rogers, W.F. Shadwick, Backlund Transformations, Academic Press, New York, 1982.
- [20] R. Hirota, Exact envelope soliton solutions of a nonlinear wave equation, J. Math. Phys. 14 (1973) 805-810.



- [21] R. Hirota, J. Satsuma, Soliton solution of a coupled KdV equation, *Phys. Lett. A* 85 (1981) 407-408.
- [22] Z.Y. Yan, H.Q. Zhang, New explicit solitary wave solutions and periodic wave solutions for Whitham-CBroer-Kaup equation in shallow water, *Phys. Lett. A* 285 (2001) 355-362.
- [23] A.V. Porubov, Periodical solution to the nonlinear dissipative equation for surface waves in a convecting liquid layer, *Phys. Lett. A* 221 (1996) 391-394.
- [24] K.W. Chow, A class of exact periodic solutions of nonlinear envelope equation, *J. Math. Phys.* 36 (1995) 4125-4137.
- [25] E.G. Fan, Extended tanh-function method and its applications to nonlinear equations, *Phys. Lett. A* 277 (2000) 212-218.
- [26] Engui Fan, Multiple traveling wave solutions of nonlinear evolution equations using a unified algebraic method, *J. Phys. A, Math. Gen.* 35 (2002) 6853-6872.
- [27] Z.Y. Yan, H.Q. Zhang, New explicit and exact traveling wave solutions for a system of variant Boussinesq equations in mathematical physics, *Phys. Lett. A* 252 (1999) 291-296.
- [28] S.K. Liu, Z.T. Fu, S.D. Liu, Q. Zhao, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, *Phys. Lett. A* 289 (2001) 69-74.
- [29] Z. Yan, Abundant families of Jacobi elliptic functions of the  $(2 + 1)$ -dimensional integrable Davey-Stewartson-type equation via a new method, *Chaos, Solitons and Fractals* 18 (2003) 299-309.
- [30] C. Bai, H. Zhao, Complex hyperbolic-function method and its applications to nonlinear equations, *Phys. Lett. A* 355 (2006) 22-30.
- [31] E.M.E. Zayed, A.M. Abourabia, K.A. Gepreel, M.M. Horbaty, On the rational solitary wave solutions for the nonlinear Hirota-Satsuma coupled KdV system, *Appl. Anal.* 85 (2006) 751-768.
- [32] K.W. Chow, A class of exact periodic solutions of nonlinear envelope equation, *J. Math. Phys.* 36 (1995) 4125-4137.
- [33] M.L. Wang, Y.B. Zhou, The periodic wave equations for the Klein-Gordon-Schrodinger equations, *Phys. Lett. A* 318 (2003) 84-92.
- [34] M.L. Wang, X.Z. Li, Extended F-expansion and periodic wave solutions for the generalized Zakharov equations, *Phys. Lett. A* 343 (2005) 48-54.
- [35] M.L. Wang, X.Z. Li, Applications of F-expansion to periodic wave solutions for a new Hamiltonian amplitude equation, *Chaos, Solitons and Fractals* 24 (2005) 1257-1268.
- [36] X. Feng, Exploratory approach to explicit solution of nonlinear evolution equations, *Int. J. Theo. Phys.* 39 (2000) 207-222.
- [37] J.H. He, X.H. Wu, Exp-function method for nonlinear wave equations, *Chaos, Solitons and Fractals* 30 (2006) 700-708.
- [38] Mingliang Wang, Xiangzheng Li, Jinliang Zhang, The  $(\frac{G'}{G})$ -expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics. *Physics Letters A*, 372 (2008) 417-423.
- [39] Mingliang Wang, Jinliang Zhang, Xiangzheng Li, Application of the  $(\frac{G'}{G})$ -expansion to travelling wave solutions of the Broer-Kaup and the approximate long water wave equations. *Appl. Math. Comput.*, 206 (2008) 321-326.
- [40] Ismail Aslan, Exact and explicit solutions to some nonlinear evolution equations by utilizing the  $(\frac{G'}{G})$ -expansion method. *Appl. Math. Comput.* In press, (2009).
- [41] Xun Liu, Lixin Tian, Yuhai Wu, Application of  $(\frac{G'}{G})$ -expansion method to two nonlinear evolution equations. *Appl. Math. Comput.*, in press, (2009).
- [42] Ismail Aslan, Turgut Özis, Analytic study on two nonlinear evolution equations by using the  $(\frac{G'}{G})$ -expansion method. *Appl. Math. Comput.* 209 (2009) 425-429.
- [43] Nikolai A. Kudryashov, Maria V. Demina, Traveling wave solutions of the generalized nonlinear evolution equations, *Appl. Math. Comput.* 210 (2009) 551-557.
- [44] Cuicui Kong, Dan Wang, Lina Song, Hongqing Zhang, New exact solutions to MKDV-Burgers

equation and  $(2 + 1)$ -dimensional dispersive long wave equation via extended Riccati equation method, Chaos, Solitons and Fractals 39 (2009) 697-706.