A New Model for Solving Portfolio Selections Based on Fuzzy Goals of Investors

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Abstract: - Since the 1960s, lots of scholars had begun to research in the portfolio selections based on the theory of mean-variance of Markowitz portfolio and relevant methods. All of these studies are under certainly of the assumption term, and then the researchers can get efficient set of portfolio selection. However, alone with the finance environment increasing of complexity, it is becoming more complex to build a model of portfolio selection, which requires further inquiries on its part based on the fact. In this paper, a new model of portfolio selection is proposed, and it is based on the fuzzy goals between risk and return which is made up of subjective measure factors. These factors can represent the investors' subjective thinking, and they are given before building the model of portfolio selection. Therefore, this goal will not change with the change of the portfolio strategy. The theory and empirical studies show that the new model is close to the practical finance environment and it simplify the process of solving and be valuable.

Key-Words: - Markowitz Portfolio Selection Model, fuzzy goals, subjective measure factors, subjective risk, subjective return

1 Introduction

Research on model of Portfolio investment is a branch of mathematical finance models. Mathematical finance model originated in 1900, when Louis Bachelier proposed his speculative theory. This new theory marked the first formal beginning of continuous time with random processes and the theory of continuous time with option pricing[1]. At a later time, investment analysis and mathematical finance to model had been improved significantly in the fifties. A new paper "Portfolio Selection" which is put first forward by American economist Harry Markowitz in March 1952 in the journal "Journal of Finance", which was included the theory of "Mean Variance Theory of Portfolio Selection". The real meaning of Markowitz's theory lies in quantization and balance analysis of risk and return. Maximize expected return when the level of risk is given, or minimize risk when the level of return is given, which are important ideas in Markowitz's theory. In mathematics, his ideas can be seen as an optimization problem with constraint conditions. At the same time, a new book with the same name "Portfolio Selection: Efficient Diversification of Investment" was published in 1959. Markowitz made a deep thought and research to the problems in his book, such as:

① how to study the optimal investments in the securities market with full of risk;

② to demonstrate the main reasons and related analysis methods between the risk and return of securities;

③to establish a basis framework with mean-variance model of portfolio investment;

All of his work has become the footstone of modern portfolio theories, and it symbolized the beginning of modern portfolio theories.

Based on the fact of above theory itself, it studied how to allocate the limited resource in the uncertain condition. This model first proposed the idea that used the variation of return rate of securities investment to measure the risk of investment, which initiated a precedent of measuring risk by the quantity index of evaluating. At the same time, this model proposed a method to solve how to allocate the investment capital between all kinds of investment object in optimal investment decision analysis. It looks all right in theory, however, this model was a nonlinear continuous programming problem, so it had some characteristics in common, for example, it contained too much data, solving hard and so on.

2 The traditional portfolio selection model

Harry Markowitz researched on inner relationship between the risk and return of investment portfolio by quantitative analysis in his theory, and he used quantitative method to describe his theory in order to build a mean-variance model for portfolio investment. At the same time, there are some hypotheses as follows: the returns of securities investment are normal distribution; and the expectation of returns can evaluate the future overall actual return; and the variance of return can evaluate the uncertainty of return. According to his theory, Harry Markowitz also thought that the investors were rational economic man, which meant that investors would have two investment modes when they were in the investment process:

① portfolio optimization based on principle of maximum returns within fixed risk;

② portfolio optimization based on principle of minimum risk within fixed return.

A good grasp of these two concepts enables get two single target models as follows:

Model (1)

$$\min \quad X^T V X$$

$$s.t.\begin{cases} X^T R = u \\ X^T I_n = 1(not\ short\ sale) \qquad x_i \geq 0 \\ or\ \ X^T I_n = 1(allowed\ short\ sale) \end{cases}$$

Model (2)

$$\max \quad X^T R$$

$$s.t.\begin{cases} X^T V X = \sigma \\ X^T I_n = 1(not\ short\ sale) \qquad x_i \geq 0 \\ or\ X^T I_n = 1(allowed\ short\ sale) \end{cases}$$

Where $R = (r_1, r_2, ..., r_n)^T$, and r_i mean i th expected return of securities, $i = 1, 2, ..., n$;

$X = (x_1, x_2, ..., x_n)^T$, and x_i mean the proportion of corresponding capital which is invested in i th securities, $i = 1, 2, ..., n$;

$V = (\sigma_{ij})_{n \times n}$

It is a variance-covariance matrix, which means n th kinds of portfolio of vector of return rate in the securities portfolio selection, and $\sigma_{ij} = \text{cov}(r_i r_j)$;

$I_n = (1,1,...,1)^T$ is a N dimension vector, and its value is 1.

The expected return rate is:

$u = X^T R$

in this portfolio problem;

The covariance of return rate is:

$$\sigma = X^T V X = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}$$

in this portfolio problem;

Markowitz's ideas about portfolio investment can be widely accepted by investors; however, its theoretical model is based on lots of assumption conditions in his work: "Portfolio Selection":

① The whole securities business is fully effective market. That is to say, the price can be fully reflected by intact and effective information in this market, so this market is a fully competitive market, and it will never be hampered by any bad factors.

② All the investors are rational. They will hold a peaceable and orderly manner when they decide to invest in this market. They can evaluate the current market impersonally; have a high expecting level in obtaining more return; and they are risk aversion person.

③ The final result of this portfolio investment can be reflected by return rate of the securities. Investors can assume return rate as random variable which is a normal approximation distribution. Analysis of variance can be a measurement method for investment portfolio risk.

④ There is correlation between the returns of securities. By using the covariance matrices, investors can describe this correlation.

⑤ The securities assets which is used to invest can be divided, which means investors can buy any kind of securities according to their will. At the same time, the related fee in the course of a transaction such as commission, service charge, tax and so on will be considered negligible.

⑥ Sale behavior of investors will not impact the price of this market or the rate of return.

⑦ The purchase of securities by investors to close out a short sale is forbidden.

⑧ Investors hope to get a maximization investment of assets expected utility in the portfolio selection. So they will handle an investment in a single cycle of portfolio selection.

A view of above content can be obtained as follows: the portfolio theory of Markowitz has been limited by lots of conditions, which results in some limitation in the practice. Especially in china, because of its securities market has lots of rules, which make this model become not practical.

In recent years, some scholars begin to devote their selves to research and improve Markowitz's model and its related algorithms to solve this model in order to make it simple and practical.

Breen and Jackson [2] Firstly proposed to simplify large-scale portfolio investment model, Perold[3] used multi-factors method to decrease the rank of covariance matrix. This method can solve large-scale MV portfolio model which contains thousands of property. Konno[4] proposed a method called compact factorization of covariance matrices in order to solve large scale mean-variance models. At the same time, because the large scale MV models are very complex, the traditional methods which are used to solve them are not very effective. However, heuristic algorithm can solve these problems effectively, so the vast number of scholars use heuristic algorithm (such as genetic algorithm, simulated annealing algorithm, tabu search, neural network and so on) to solve the complex investment portfolio problem. Mansini and Speranza[5] pointed out that three different kinds of heuristic algorithm to solve the investment portfolio problems with minimum trading hand limit. Maria A. Osorio, Ana Ballinas and others [6] proposed the use of Mean/Variance multistage portfolio management for building efficient frontiers. The maximization of the returns yields the maximum and the variance minimization the minimum points in the efficient frontier. The efficient frontier is the graph describing all the optimal options to different percentages of the maximum utility expected. According to the investors' characteristics, a point in the graph, containing a complete set of investment strategies can be chosen. The stochastic quadratic and linear models use a scenario tree to represent the multistage discretization of the random returns. At a later time, chang-kyo suh[7] designed decision support systems (DSSs) which can help financial managers in evaluating proposals for strategic and long-range planning. Schaef [8] , Rolland [9] used heuristic algorithm to solve investment portfolio problems with mixed integer limitation. Crama, Schyns[10] and Gilli [11] used Heuristic algorithm to deal with the investment portfolio problems with complex constraint. Yu and Wang[12] used neural network to solve the mean-variance-skewness portfolio optimization model. Xia and Wang[13] used genetic algorithm to solve the portfolio model.

In china, our securities market has a lot of limitation, which mainly include the following aspects:

① The traditional MV model has a valid assumption: the securities market is fully effective. However, this assumption is not conforming strictly to actual situation;

② Rational securities investors are non-existent.

③ The statistical data shows that the return rate of securities actually do not follow Gaussian distributions. So if it does not change according to Gaussian distribution, MV model will has different from the actual.

④ The model of Markowitz in investment portfolio measures the credit risk which is not conforming strictly to actual situation. For example, variance means the degree of scatter of indexes' value compared with their average, which shows two biases: negative or positive, however, it is not acceptable if we think the two biases are fair. When investors begin to invest portfolio, their measurement for investment portfolio risk has a greater difference. Most scholars admit this opinion: the risk will appear when the return rate of investment portfolio is below expected return rate, which means this risk is in the non-existence when the return rate of investment portfolio is above expected return rate.

⑤ Furthermore, To ignore all kinds of transaction cost will induce non-effective investment portfolio[14] . That is to say, the transaction cost has been not neglected in investment portfolio. At the same time, Markowitz's model of investment portfolio has another assumption. He thought that the investors just had a specific number of capital and they did not hold any security when they begun to invest portfolio. However, there is another instance for investors to make a investment portfolio decision: the investors have held a share of securities when he first make his selection, and they will reinvest the securities which are aiming for adjusting investment proportion again in order to get the best return and minimize investment portfolio risk..

So our scholars begun to research this market according to our actual situation, and analyze the theory of MV model. Some scholars proposed some new measurement method for investment portfolio risk, for example, sensitivity analysis, VAR, and to avoid the disadvantages of variance in measuring market risk, the semi-variance and LPM are introduced. Sensitivity analysis assumes that there has linear relationship between the change of portfolio value and all kinds of market factors. However, the changes of portfolio value and market factors are often non-linear or slowly time varying. Semi-variance shows a deviation of returns of portfolio, but it does not possess good statistical aspects, and it can not reflect how large the risk of portfolio returns. VAR is a good method to measure the portfolio risk, the prominent advantage of VAR is that it can measure risk with synthesis. It can integrate all kinds of market factors, market risk to a number, so it can measure potential loss which is produced by different source of risk and their interactions accurately. At the same time, some description is given such as pressure tested and extremum method as a complement of VAR.

Generally speaking, the research on the portfolio problems is mainly focuses on the financial risk measurement. That is to say, the scholars analyze the level of loss which is induced by financial risk. However, as the economic activities become more and more frequent and complex, how to deal with the financial risk in portfolio selection become more and more difficult. These problems require improved tools and technology. There are four kinds of commonly used methods; according to reference (15), they are as follows:

① simple arithmetic functions

Here both deviation rate method and price discrepancy (PD) method are involved, and generally, price discrepancy method is one of the earliest and simplest methods. Its computation formula is as follows:

$$PD = 2 \times \frac{\max imum\ price - \min imum\ price}{\max imum\ price + \min imum\ price} \times 100\%$$

Where $\max imum\ price$ and $\min imum\ price$ mean the maximum price and minimum price of selected assets in a certain period of time respectively. This formula makes the possible fluctuating range as an index to measure risk, so more larger PD is, more bigger the possible fluctuating range of price of assets is, and more bigger risk is. This is a very simple method to measure the selection risk. But its drawback is very obviously, and there is no other information outside the maximum price and minimum price of price of assets in this method, so this method can not measure the risk in portfolio selection accurately.

② mean-variation model

This model is usually used to measure the balance between expected return and risk in portfolio selection. The best known model is Markowitz's model. However, mean-variation model has a lot of defects; it can measure the discrete level of expected return of portfolio itself. This expected return includes the lower part of expectation and the upper part of expectation. However, when investors decide to invest in the financial market, they like the return beyond the expected part. That is to say, investors will not think the upper part of expectation as risk. So many scholars still improve this model based on this reason.

③ Value at Risk

Deliberate on the defect of Markowitz's model, the new methods with measurement of downside-risk such as Value at Risk (VaR) are proposed. VAR focuses on the return rate of portfolio which is lower the given return rate. When VaR is first proposed by J.P.Morgan in 1994, more and more financial institution begun to use this method to measure and manage the financial risk.

VaR is usually used to evaluate risk based on statistics. And the VaR method has very wide range of applications, such as portfolio selection, risk control, information disclosure, financial supervision and so on. Under normal market conditions, VaR means the greatest loss of particular portfolio selection based on given confidence level $1-\alpha$, given period of time. the related formula is as follows:

$$P(\Delta p > VaR) = \alpha$$

Where Δp is the loss of particular portfolio selection based on given period of time, VaR is the value of assets based on the confidence level $1-\alpha$.

Generally, confidence level α is set by investors themselves. It can reflect how allergic to risk investors are. More higher confidence level is, more allergic investors to risk.

In order to understand the basic concept of VaR, its mathematical expression is as follows:

Suppose the initial value of portfolio is I, the expected return at the end of investment term is $E-R$, and the minimum expected return at the end of investment term is $E-R_{\min}$. At the same time, the expectations of $E-R$ is μ; and the standard deviation is σ, a known confidence level is α.

Then we can get the following result:

The minimum value of portfolio at the end of investment term:

$$I_{\min} = I \times (1 + E - R_{\min})$$

Then

$$\begin{aligned} Value\ at\ Risk &= E(I) - I_{\min} \\ &= \mu I - I(1 + E - R_{\min}) \qquad (1) \\ &= I \times (\mu - E - R_{\min}) \end{aligned}$$

According to the concept of VaR, it can be described as follows:

If we know the probability distribution of portfolio, then

$$\alpha = \int_{E-R_{\min}}^{+\infty} f(I)dI \qquad (2)$$

Where $f(I)$ is the density function of probability distribution, and this probability distribution follows Gaussian distribution.

Formula (1) is equivalent to following formula:

$$1-\alpha = \int_{-\infty}^{E-R_{\min}} f(I)dI \qquad (3)$$

Suppose a is the quantile of Gaussian distribution, then

$$1-\alpha = \int_{-\infty}^{a} f(I)dI = \int_{-\infty}^{a} \varphi(\varepsilon)d\varepsilon$$

And $\varphi(\varepsilon)$ is the density function of probability distribution, too. Since

$$P(E-R < E-R_{\min}) = 1-\alpha$$

That is to say

$$\begin{aligned} 1-\alpha &= P(E-R < E-R_{\min}) \\ &= P\left(\frac{E-R-\mu}{\sigma} < \frac{E-R_{\min}-\mu}{\sigma}\right) \end{aligned}$$

$$\frac{E-R-\mu}{\sigma} = 1-a \qquad (4)$$

$$\frac{E-R_{\min}-\mu}{\sigma} = a \qquad (5)$$

So according to formula (5)

$$E-R_{\min} = \mu + a\sigma \qquad (6)$$

Put the formula (5) into formula (1), then we can get

$$\begin{aligned} Value\ at\ Risk &= I \times (\mu - E - R_{\min}) \\ &= I \times (-a\sigma) \qquad (7) \\ &= -a\sigma I \end{aligned}$$

Formula (7) represent the best known computation method of VaR, which is variance-covariance method, which is proposed by J.P. Morgan in 1994. This method has a hypothesis that the return rate actually does follow Gaussian distributions.

This method based on the hypothesis of Gaussian distributions is very simple, and it can evaluate the risk overnight or the risk during a typical workday efficiently. However, the distribution of return rate is with fat tails in the long term. Then it will grossly underestimate the value of VaR for extreme precipitation financial event under the hypothesis of Gaussian distributions. So when researchers use this method, they will follow other distribution such as normal mixture mode, t distribution and so on.

④ beta coefficient method

Beta coefficient is a very important concept which is applied for measuring the financial risk. Generally, investors can predict the risk in stock market after estimating the beta coefficient. That is to say, beta coefficient needs estimate the former data about return rate of stock. However, this beta coefficient only express the past beta coefficient.

If we want to reflect the current or future risk in stock market, then we should make sure the beta coefficient has stability. So when beta coefficient is used, how to check its stability is very important.

In order to discuss the stability of beta coefficient, firstly, suppose:

σ_i^2 is the variance of return rate for risk assets portfolio i. σ^2 is the variance of return rate for the whole risk assets portfolio. ρ_i is the covariance between risk assets portfolio i and the whole risk assets portfolio. r_i is the return rate of risk assets i; and R is the return rate of the whole risk assets portfolio.

Then theoretical basis of beta coefficient is the capital asset pricing model (CAPM) which is proposed by Lintner(1965), Mossin(1966), and Sharpe(1964). It can be defined as follows:

$$\beta_i = \frac{\rho_i}{\sigma_i^2} = \frac{COV(r_i, R)}{VAR(R)}$$

And in CAPM, beta coefficient can be described:

$$E(r_i) = r_f + \beta_i [E(R) - r_f]$$

Where r_f is the return rate with riskless.

All above methods are base on this assumption: risk is the uncertainty of loss. However, the definition of risk is a loss in value of property which is induced by some uncertain factors. That is to say, the nature of risk is an uncertainty.

In this paper, the theories of fuzzy mathematics can be used to research investment portfolio, and it can measure the risk in portfolio selection more objectively.

3 A New Model for Solving Portfolio Selections Based on Fuzzy Goals of Investors

As mentioned above, there are two different kinds of investment portfolio studies. One is traditional portfolio theory, which focuses on all kinds of constraints. This kind of theory thinks investors request return of investment portfolio, so scholars research how to invest from current revenues and capital appreciation in order to meet investors' requirement. Their researches are usually qualitative analysis. The other theory is called modern portfolio theory, which researches how to reduce the risk of investment in order to maximize revenue. This theory is based on relationship between expected return of securities and its related risk, and a number of methods of quantitative analysis are used, which further verifying the correctness and practicality of this theory.

At the same time, the current study most focused on the research or improve MV model, and to obtain a series of available solution as the goal. However, this does not mean we can get the real optimal solution, that is to say, even if we can prove one solution is optimal by some methods, however, we can not use a practical method to get the final optimal combination, which means it was not effective if we used traditional computational method to solve complex investment portfolio selection. So a large number of experts and scholars use heuristic algorithm to solve complex portfolio problems.

In this paper, a new model of investment portfolio based on zeng jianhua's theory [16] is proposed, and the risk measure model on the basis of MV model is improved in this paper. Markowitz portfolio model has a lot of above mentioned limitations, and its risk measurement is very simple, this is far from enough to meet the practical demand. Therefore, this paper proposed new risks of investment and portfolio model from the risks perspective in order to make portfolio model more accurately.

Generally, the financial system itself has complexities, investment market itself is unpredictable, and there exist a lot of speculative transactions in this market. All these distinctive features make these investors can not obtain the accurate expect return when they face two usually contradictory goals: maximize the return of the securities portfolio and minimum its system risk. These characters of investment decide that risk and return are two fuzzy goals for these investors. Therefore, investors will consider these two prerequisite before they decide to invest:

① the return from investments will be in an affordable scope as a. Then the bigger is return from investment, the better is this portfolio selection based on a;

② the risks from investment will be controlled in an affordable scope as b. Then the smaller is risk from investment, the better is this portfolio selection based on b.

As mentioned above, a and b can be seen as two fuzzy goals which can measure the risk in portfolio selection, then two membership function can be built based on these two fuzzy goals.

There is a hypothesis firstly as follows:

These investors want to achieve above mentioned fuzzy goals: ① or ②, so they will make a decision for portfolio selection.

These investors will invest their capital in n securities.

x_i means investment portion in securities i. $i = 1, 2, \ldots\ldots, n$.

R_i means return rate of investment in securities i, this parameter is a random variable, $r_i = E(R_i)$, r_i is expected value of R_i.

A definition: k_i is a very useful parameter, which means the transaction cost of risk securities per unit change, then: for each investment portfolio update, the transaction cost can be described as follows:

$$f_i = k_i \left| X^1 - X^0 \right| \ , \ i = 1, 2, ..., n$$

Where X^0 is a given investment portfolio, and $X^0 = (x_1^0, x_2^0, ..., x_n^0)$

X^1 is a new investment portfolio, and $X^1 = (x_1^1, x_2^1, ..., x_n^1)$

Then the total transaction cost is:

$$\sum_{i=1}^{m} f_i = \sum_{i=1}^{m} k_i \left| X^1 - X^0 \right|$$

And the total return of this portfolio is:

$$R(X) =$$

$$E(\sum_{i=1}^{n} R_i x_i) - \sum_{i=1}^{m} f_i$$

$$= \sum_{i=1}^{n} r_i x_i - \sum_{i=1}^{m} k_i \left| X^1 - X^0 \right|$$

At the same time, the risk of this investment portfolio is:

$$V(X) = \sum_{i=1}^{n} E \left| (R_i - E(R_i)) \right| x_i$$

$$= \sum_{i=1}^{n} d_i x_i$$

To analysis investment portfolio X^0 and investment portfolio X^1 respectively:

In this paper, we think X^0 and X^1 are two groups of correlative data and make a statistical design, then define return average of each group.

$$\overline{x_i} = \frac{\sum_{j=1}^{n} x_{ij} r_j}{n} \ , \ i = 0,1$$

Then an overall average is:

$$\overline{x} = \frac{\overline{x_0} + \overline{x_1}}{2}$$

A variance is defined in this paper in order to measure dispersion degree between investment portfolio X^0 and X^1.

$$\delta^2 = \frac{\sum_{i=0}^{1} (\overline{x_i} - \overline{x})}{2}$$

In this paper, this indicator can measure and reflect the practical expected value of investors.

Assume this investment environment to be fuzzy, then two membership functions are defined based on fuzzy goals ① and ② according to the subjective judge of investors. With the help of these two membership functions a new investment portfolio model can be built.

Membership functions J are described below:

① the subjective return of investment is in an affordable range: $[I_l, I_u]$

I_l, I_u are the subjective measure factors, which can be chosen by investors.

(1) $J = 0$,
$if \quad R(X) < I_l$

(2) $J = 1$,
$if \quad R(X) > I_u$

(3) $J = \frac{R(X) - I_l}{I_u - I_l}$,
$if \quad I_l < R(X) < I_u$

② the subjective risk of investment is controlled in an affordable range: $[R_l, R_u]$

R_l, R_u are the subjective measure factors, which can be chosen by investors.

(1) $J = 1$,
$if \quad \delta^2 < R_l$

(2) $J = 0$,
$if \quad \delta^2 > R_u$

(3) $J = \frac{\delta^2}{R_u - R_l}$,
$if \quad R_l < \delta^2 < R_u$

Membership functions J as subjective measure function can portray a fuzzy investment portfolio environment more accurately. Then a new investment portfolio model based on fuzzy decision making and indistinct programming theory[17] can be described as follows:

$$\max \quad J$$

$$\begin{aligned} & S.t. \\ & J \le R_u \\ & J \ge I_l \qquad (3) \\ & \sum_{i=1}^{n} x_i = 1 \\ & x_i \ge 0 \ , \ i = 1, 2, ..., n \end{aligned}$$

According to the literature[16] : A Model for Portfolio Selection Based on Fuzzy Decision-making Theory, we can get the following theory:

Theorem 1: x^* is an optimal solution for the investment portfolio model (3), if and only if there is y^*, which makes (x^*, y^*) be the optimal solution for the following programming problem:

$$\max \quad J$$

$$\begin{aligned} & S.t. \\ & J \le R_u \\ & J \ge I_l \qquad (4) \\ & y_i + x_i - x_i^0 \ge 0 \\ & y_i - x_i + x_i^0 \ge 0 \\ & \sum_{i=1}^{n} x_i = 1 \\ & x_i \ge 0 \ , \ i = 1, 2, ..., n \end{aligned}$$

The mathematical proof is omitted.

Then even this new model (3) is used to deal with large scale complex portfolio selection problem, we can transform this problem into model (4), which can make the calculation of model become simple greatly.

4 Results and Discussion

In this section , model validation is then carried out with experimental data. According to literature[18], some stock return data from stock market of China in 2006 is used in this paper in table .

According to the investors' subjective opinion, $[I_l, I_u]$ and $[R_l, R_u]$ can be given; then we can get different kinds of investment strategies according to investment portfolio model (4). The concrete results are as follows:

1. if $I_l = 0.0678$, $I_u = 0.1043$, $R_l = 0.1892$, $R_u = 0.4370$

Then:

$(x \ , x_2, ..., x_{20}) =$
$(0, 0, 0, 0, 0.2113, \quad 0, 0, 0, 0, 0.0027,$
$0.1124, 0, 0, 0, 0.2011, \quad 0, 0, 0.4725, 0, 0)$

And we can have the value: $J = 0.9827$

2. if $I_l = 0.0973$, $I_u = 0.3215$, $R_l = 0.1170$, $R_u = 0.5021$

Then:

$(x \ , x_2, ..., x_{20}) =$
$(0, 0.7435, 0, 0, 0, \quad 0.1012, 0, 0, 0, 0,$
$0, 0, 0, 0.0013, 0.0092, \quad 0, 0, 0, 0.1448, 0)$

And we can have the value: $J = 0.8999$

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1#	0.070	0.080	0.180	0.007	0.031	0.070	-0.037	0.080	-0.010	0.008	0.017	-0.037
2#	0.263	0.373	0.501	0.073	0.155	0.263	0.254	0.358	0.486	0.073	0.154	0.254
3#	0.006	-0.043	-0.145	0.005	0.005	0.006	-0.027	-0.045	-0.150	-0.011	-0.011	-0.027
4#	0.117	0.122	0.132	0.035	0.066	0.117	0.082	0.107	0.067	0.031	0.054	0.082
5#	0.504	0.152	0.299	0.379	0.406	0.504	0.422	0.075	0.191	0.379	0.406	0.422

6#	0.050	0.060	0.012	0.008	0.011	0.050	0.060	0.110	-0.003	0.010	0.021	0.052
7#	0.001	-0.208	0.017	0.010	0.011	0.001	-0.031	-0.106	0.022	-0.010	-0.004	-0.031
8#	0.002	-0.164	-0.620	0.000	0.001	0.002	-0.045	-0.165	-0.620	-0.019	-0.024	-0.045
9#	0.155	0.017	0.009	-0.013	0.053	0.155	-0.041	-0.045	-0.184	-0.013	-0.032	-0.041
10#	0.240	0.188	0.249	0.080	0.156	0.240	0.240	0.185	0.236	0.080	0.153	0.240
11#	-0.024	0.057	0.102	-0.073	-0.055	-0002	-0.093	0.006	-0.076	-0.083	-0.070	-0.093
12#	-0.063	-0.129	-0.250	-0.012	-0.062	-0.063	-0.069	-0.133	-0.260	-0.360	-0..69	-0.069
13#	0.000	0.012	0.014	0.003	-0.005	0.000	0.000	0.005	-0.102	0.003	-0.005	0.000
14#	0.150	0.160	0.110	0.050	0.110	0.150	0.130	0.130	0.010	0.031	0.100	0.130
15#	0.002	0.009	0.032	-0.001	-0.022	0.002	0.003	0.032	0.032	0.011	-0.022	0.003
16#	0.031	0.060	0.050	-0.003	0.030	0.031	0.023	0.059	0.030	-0.003	0.030	0.023
17#	0.363	0.039	0.020	-0.320	0.035	0.036	0.036	0.040	0.020	-0.031	0.037	0.037
18#	0.061	0.013	-0.050	0.004	0.015	0.016	0.008	0.014	-0.050	0.005	0.004	0.008
19#	0.056	0.089	0.120	0.030	0.037	0.056	0.056	0.088	0.120	0.030	0.033	0.056
20#	0.050	0.030	0.023	0.008	0.030	0.050	0.050	0.030	0.025	0.009	0.030	0.050

Table1. Stocks return data from stock market of China in 2006 in literature[17]

5 Conclusion

In this study, we propose a new model for solving portfolio selections by using subjective measure factors. This make the new model be more suitable for practical investment environment. At the same time, the empirical analysis supports the model (4) in the fourth chapter. After quantitative analysis and theoretical exploration, some results and conclusions can be got as follows:

(1) The new model introduces membership function based on the fuzzy goals, which is coupled to the investors' judge. So this new model can reflect better subjective expectancy of investors in order to construct a new portfolio model, which is more accordant with the actual conditions. This new model can make investment analysis is more flexible by defining four subjective measure factors: I_l, I_u, R_l, R_u. So we can allocate the investable assets with return and risk acceptable to different investors when the investment environment is uncertain.

(2) According to Theorem 1, this new model can predigest the process of calculation, and make us get the optimal solution of investment portfolio problem. At the same time, to solve large-scale portfolio selection problem become possible because of this.

(3) The results in chapter 4 show that the result of portfolio is more decentralization based on the investment portfolio model (4), which can prove the thinking that the risk of portfolio should be dispersive.

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