Sensitivity Analysis of Hopfield Neural Network in Classifying Natural RGB Color Space

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Abstract: - This paper presents a study of the sensitivity analysis of the artificial Hopfield Neural Network (HNN) when segmenting natural color images. The color distinction or vision system relies on two step process which, first classifies the different regions in the scene into a given number of clusters, and then assigns to each cluster a color that is likely to one of its corresponding region in the raw image. The classification process is performed using the minimization of an energy function typically the Sum of Squared Errors (SSE). The optimization process is found sensitive to the step taken by the network in its way to the global minimum. The color assignment to the clusters is performed based on combination of information from the color palette used in the raw image and the last distribution of the pixels among clusters. Applying the system to a gold standard color image, the results show that HNN natural color segmentation accuracy can be significantly improved if we control its step size when modifying its weights between its neurons each iteration. The color matching process shows a lot of consistency when tested with natural color images as shown in the results presented here.
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Key-Words: - Hopfield Neural Network, Sensitivity analysis, Segmentation, Natural Color Image matching, RGB Color Space

1 Introduction

The division of natural images like rock, stone, clouds, ice, or vegetation into classes based on their visual similarity is a common task in many machine vision and image analysis solutions. Classification of natural images is demanding, because in the nature the objects are seldom homogenous. For example, when the images of rock surface are inspected, there are often strong differences in directionality, granularity, or color of the rock; even the images represented the same rock type. These kinds of variations make it difficult to classify these images accurately [1]. There are many papers dealing with segmentation of images using color, see [2] survey. Several authors are applying different techniques for color in order to improve the final result of the segmentation and some of them are mentioned in [3].

In [4], we have used HNN for segmentation of pathological liver color images obtained using needle biopsy. The segmentation results have been appreciated by pathologists as it helped to provide quantitative diagnosis of liver cancer. The algorithm is as follows:
The HNN classifier structure consists of a grid of N×M neurons with each row representing a pixel and each column representing a cluster. The network classifies the image composed of N pixels with P features among M clusters, in a way that the assignment of the pixels minimizes the following criterion function:

$$E = \frac{1}{2}\sum_{k=1}^{N}\sum_{l=1}^{M} R_{kl}^{2} V_{kl}^{2} \qquad (1)$$

Where R_{kl} is the Mahalanobis distance measure between the k^{th} pixel and the centroid of class l. R_{kl} is also equivalent to the error committed when a pixel k is assigned to a class l. Note that we have removed the term of white noise from the equation (2) in [4] in order to remove random effects in this study.
The minimization is achieved using HNN by solving the motion equations satisfying:

$$\frac{\partial U_{kl}}{\partial t} = -\mu(t)\frac{\partial E}{\partial V_{kl}} \qquad (2)$$

where U_{kl} is the input of the k^{th} neuron, and μ(t) is a scalar positive function of time, used as heuristically motivated stopping criterion of HNN, and is defined as in [5] by:

$$\mu(t) = t(T_s - t) \qquad (3)$$

where t is the iteration step and T_s is the pre-specified convergence time of the network which has been found to be 120 iterations [5]. The network classifies the feature space, without teacher, based on the compactness of each cluster calculated using Mahalanobis distance measured between the k^{th} pixel and the centroid of class l as given by:

$$R_{kl} = \left\| X_k - \overline{X}_l \right\|_{\Sigma_l^{-1}} = \left(X_k - \overline{X}_l \right) \Sigma_l^{-1} \left(X_k - \overline{X}_l \right) \tag{4}$$

$$1 \le k \le N \text{ and } 1 \le l \le M$$

Where X_k is the P-dimensional feature vector of the k^{th} pixel (here P = 3 with respect to the RGB color space components), is the P-dimensional centroid vector of class l, and $\sum_l$ is the covariance matrix of class l. The segmentation algorithm is described as follows in our previous work [4]:

Step1: Initialize the input of the neurons to random values.

Step2: Apply the following input-output relation, establishing the assignment of each pixel to only one class:

$$\begin{aligned} &if\ U_{km} = Max\left[U_{kl}(t), \forall l \right] \text{then } V_{km}(t+1) = 1, \\ &else\ V_{km}(t+1) = 0, \\ &1 \le k \le N, \text{and } 1 \le l \le M \end{aligned} \tag{5}$$

Step3: Compute the centroid $\overline{X}_l$ and the covariance matrix Σ_l of each class l, respectively, as follows:

$$\overline{X}_l = \sum_{k=1}^{N} X_k V_{kl} \Big/ n_l \qquad 1 \le l \le M \tag{6}$$

$$\sum\nolimits_l = V_{kl} \left(X_k - \overline{X}_l \right)^T \Big/ n_l - 1 \tag{7}$$

where n_l is the number of pixels in class l, and the covariance matrix is then normalized by dividing each of its elements by $\left[\sum_l \right]^{1/p}$.

Step 4: Update the inputs of each neuron by solving the set of differential equations in (2) using Euler's approximation:

$$U_{kl}(t+1) = U_{kl}(t) + \frac{dU_{kl}}{dt} \qquad 1 \le k \le N, \text{ and } 1 \le l \le M \tag{8}$$

Step5: If $t < T_s$ repeat from Step2, else terminated.

2 The Sources

Only natural color images have been considered in this work with a gold standard color image to check the accuracy of the method. Each image can be thought of as a set of points in a three dimensional Euclidean space. Each pixel is represented as a point in this Euclidean space, where the three coordinates are the RGB components of the pixel color in the RGB color space. HNN classifies the pixels among a given number of clusters based on the mean and covariance matrix of each cluster without training data set. Figure1 shows the gold standard color image formed with five homogenous rectangular regions.

3 Segmentation Result

We have applied the above algorithm to segment the color image in Figure1 with a fixed number of clusters to five and the result is shown in Figure2. As it is seen in this result, HNN segments the image into three clear and homogenous clusters, and two other different regions non homogenous with the same color and other pixels dispersed with a special color among the two clusters. Figure3 shows the curve of HNN energy function in its way to the convergence state after 120 iterations. For this reason we decided to analyse the above algorithm's degrees of freedom in order to find the range of these parameters where we can ensure homogenous and accurate segmentation of the different objects of a natural scene in an RGB color space.

3.1 Sensitivity Analysis of HNN to its Degrees of Freedom

The above segmentation method of HNN has the desirable feature of rapid convergence to local optimum close to the global one without being trapped in early local optima. The convergence speed and location are controlled by two parameters [or degrees of freedom (DOF)] which are the initialization of neurons' inputs (Step1), and the update of neurons weights (Step4). Here, we chose a fixed random initialization of the neurons' input and we focus only on the analysis of the gradient-based update of HNN weights, given in equation (9). This updating of the neurons' input is a process for a control algorithm to find the optimal solution of the segmentation problem in a reliable manner. In our trial to improve the segmentation results of HNN to the gold standard image in Figure1, we have introduced a new control global term $\beta(E)$ to equation (3) in HNN algorithm as follows:

$$\frac{\partial U_{kl}}{\partial t} = -\mu(t)\beta(E)\frac{\partial E}{\partial V_{kl}} \tag{9}$$

$$\beta(E) = \frac{1}{1 + (Log(E)/t)^m} \tag{10}$$

Where E is the total Error at iteration t, and m controls the slope of the function $\beta(E)$. Figure5 shows the curves of HNN energy function during the segmentation process of the image in Figure1 with the same initialization matrix of neurons' inputs and with different values of the slope control parameter.

The interval [2.1, 2.3] includes convenient values of the slope control m that may insure better quality of the segmentation process. Figure6 shows the segmentation results corresponding to some of the curves in Figure5, it is clear from these results that the value 2.3 of the slope control parameter m is appropriate to give a better segmentation result with HNN.

Fig.1 Ground truth color image with five homogenous regions.

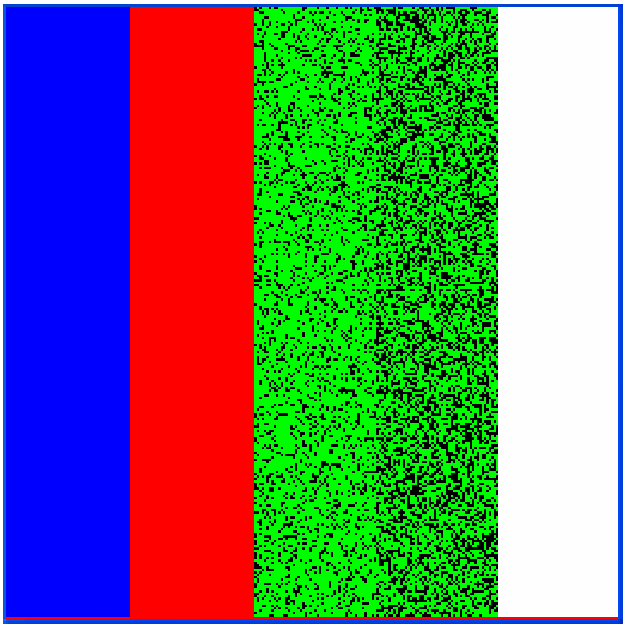


Fig.2 Result of HNN Segmentation to the ground truth color image in Figure1 with five clusters.

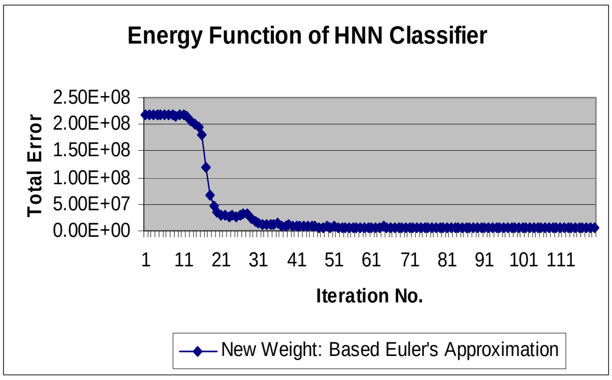


Fig.3 Energy function of HNN during the segmentation of the RGB color image in Fig.1.

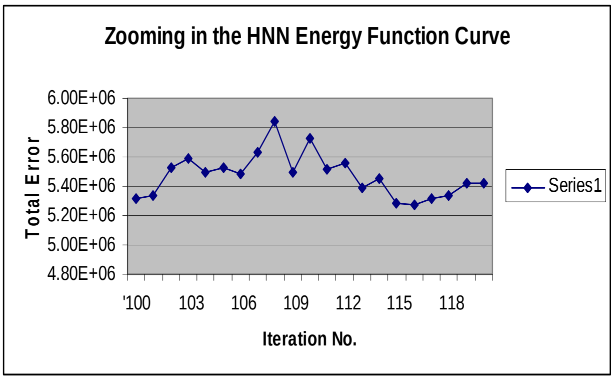


Fig.4 Zooming in the HNN Energy Function Curve for the last twenty iteration during the segmentation of the RGB color image in Fig.1.

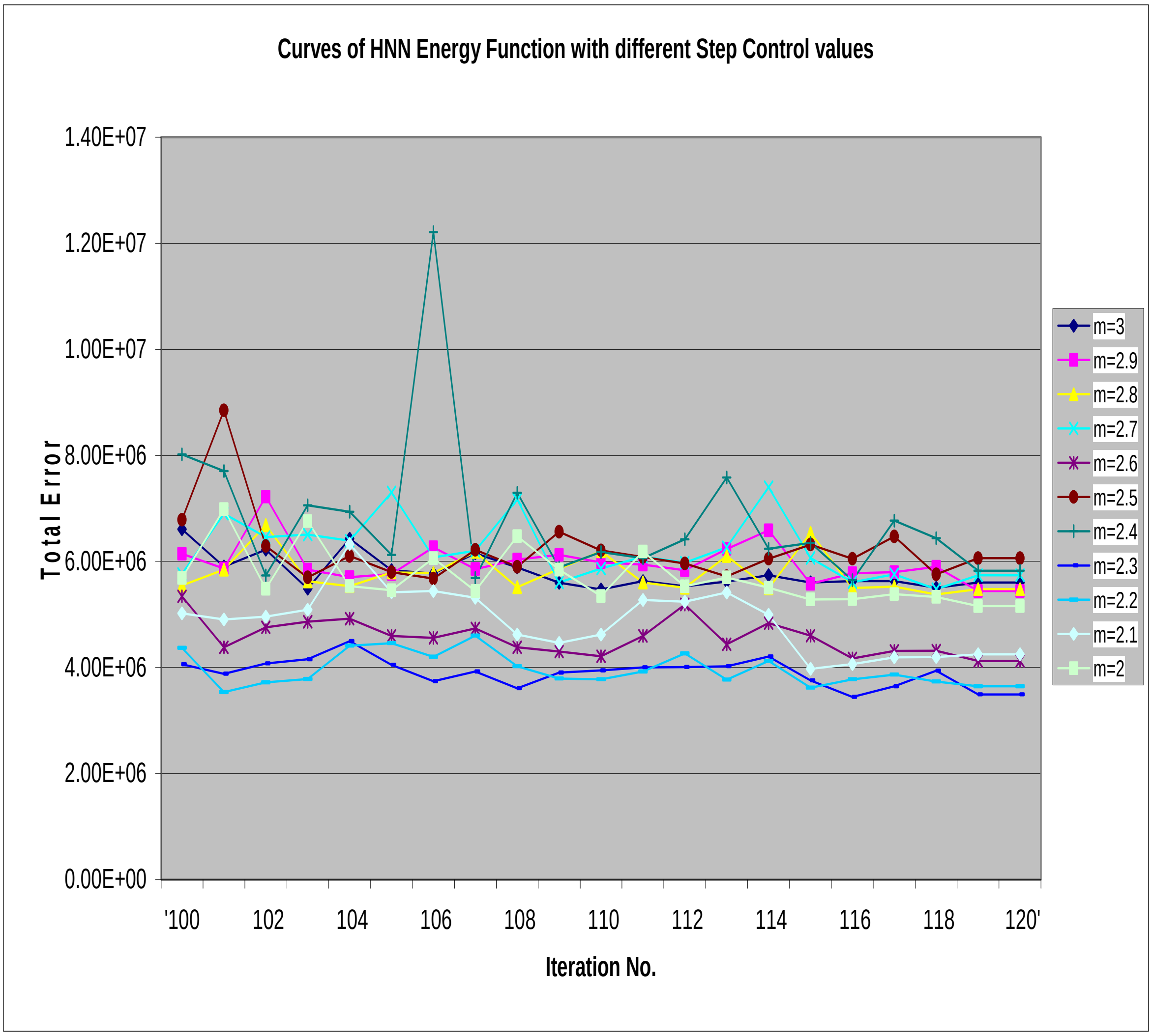


Fig.5 Curves of HNN energy function during the segmentation process of the image in Figure1 with the same initialization matrix of neurons' inputs and with different values of the slope control parameter.

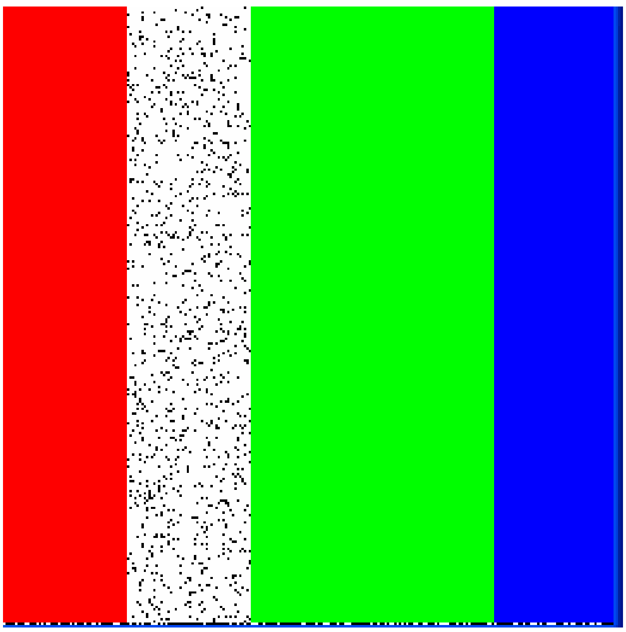


Fig.6(a)

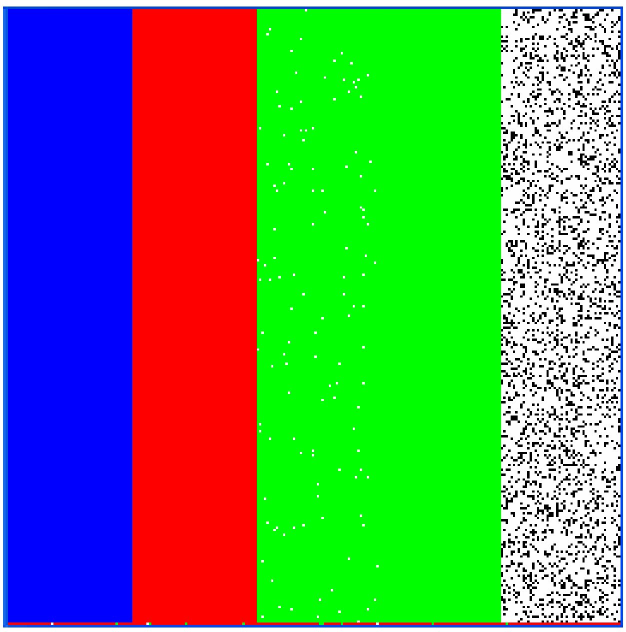


Fig.6(b)

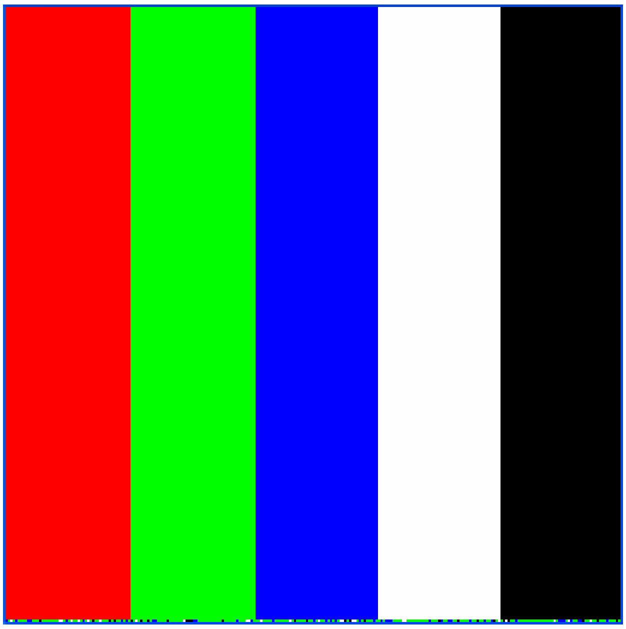


Fig.6(c)

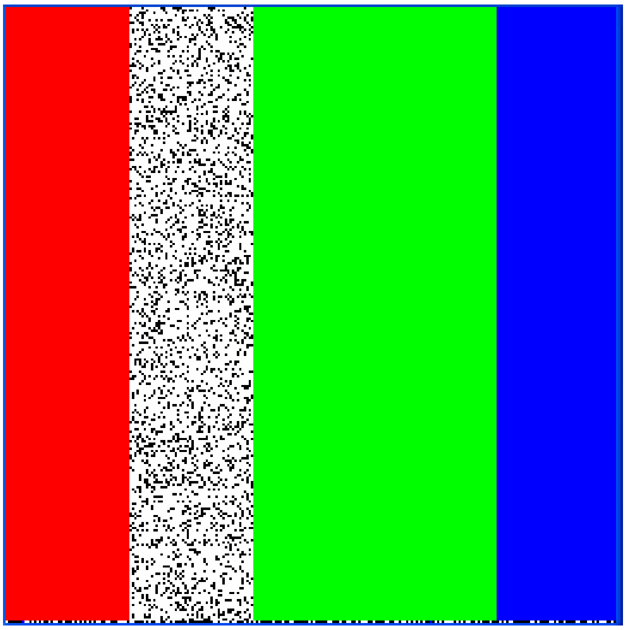


Fig.6(d)

Fig.6 (a), (b), (c), and (d) are the segmentation results of HNN with values of the parameter m in the control global term, m=2.1, m=2.2, m=2.3, m=2.6, respectively

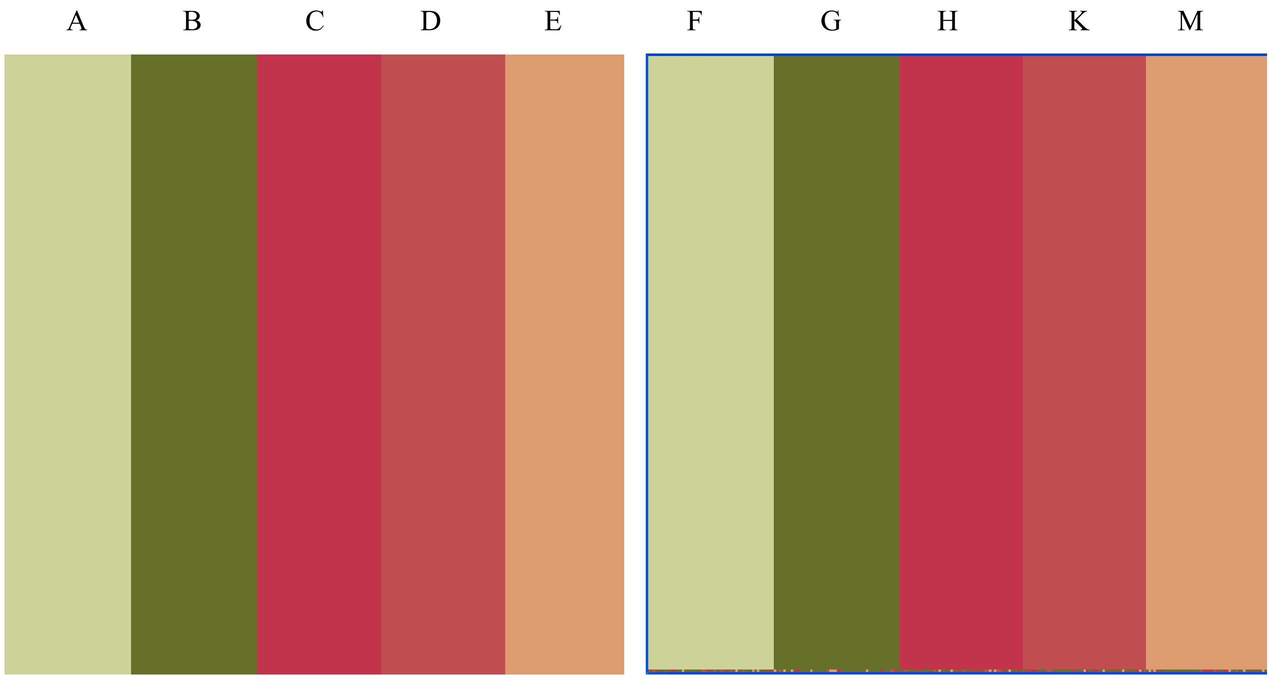
4 Discussion of HNN Sensitivity & Natural Color Matching

The results obtained using the control global term $\beta(E)$ with HNN show that the control parameter can drive the network through a better path to reach a position closer to the global optima. The use of a ground truth color image can be considered as a piece of evidence for the capability of HNN in making a crisp classification for homogenous color regions. To display the segmentation result with its five clusters with colors similar or identical to those in the raw image, we compute the sum of each color component of the pixels assigned to each cluster, the average of each component is considered as final or matched color of that cluster.

Fig.7 (a) and (b) show a raw color image (the same as in Figure1) and its corresponding segmented image, respectively, after matching their regions' colors based on the average of the color space components of all pixels in the cluster. As it is clear from the results, the matching technique is perfect 100% for all regions in the raw image, in that region(A) has the same color as its corresponding cluster (F) in the segmented image, also the same for the rest of regions. The noise that appears in the last rows of the segmented image is an indication that HNN did improve the result of the segmentation process by converging to optima closer but not equal to the global optima.

Fig.8 shows anther color image (a) and its corresponding result (b) of our segmentation and color matching processes obtained with seven as clusters number given to HNN classifier. The discontinuity seen in the segmentation result is due to two main facts: the first fact is that the network did not reach a global minimum during the segmentation task, and the second is the intensity variation among the pixels of the same region in the raw images. The intensity variation did not exist in the ground truth color image with five homogenous regions shown in Figure1 and that is the reason why its corresponding segmentation and matching result, shown in Fig.7 (b), is much smoother than the result shown in Fig.8 (b) when compared to its corresponding raw image Fig.8 (a)

Fig.9 shows another color image and its corresponding segmentation results with different clusters number. This image contains big intensity variation among its regions, as it is clear in Fig.9 (a), the raising of the clusters number to ten did not help HNN to put the beak of the bird in a specific cluster, however, If we focus in the beak region in the raw image, we find that even this small region contains a lot of intensity variation among its representative pixels. For this reason, to improve the image segmentation results it is necessary to use a technique to maximize pixel consistency within true regions before we segment the whole image.



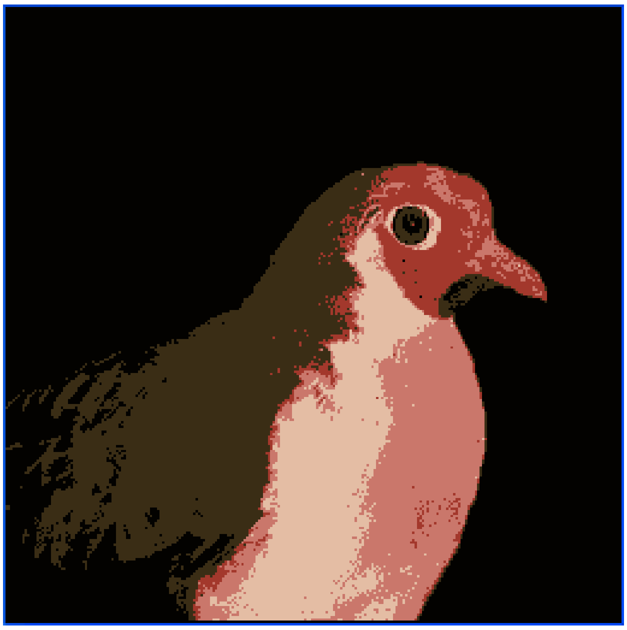
(a) (b)

Fig.7 (a) and (b) show the raw color image (the same as in Fig.1) and its corresponding segmentation result after matching the clusters' color with their regions in the raw image.

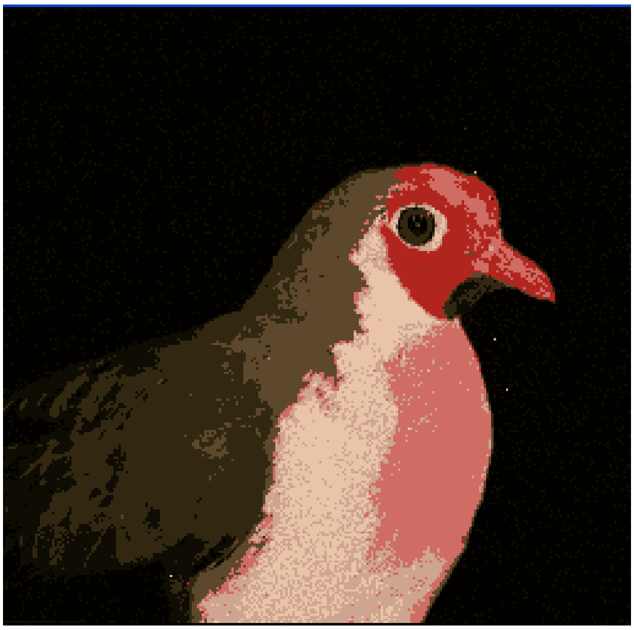
(a) (b)

Fig. 8 (a) and (b) show, respectively, a raw color image and its corresponding segmentation result with 7 clusters after automatically matching the clusters' colors with their regions in the raw image.

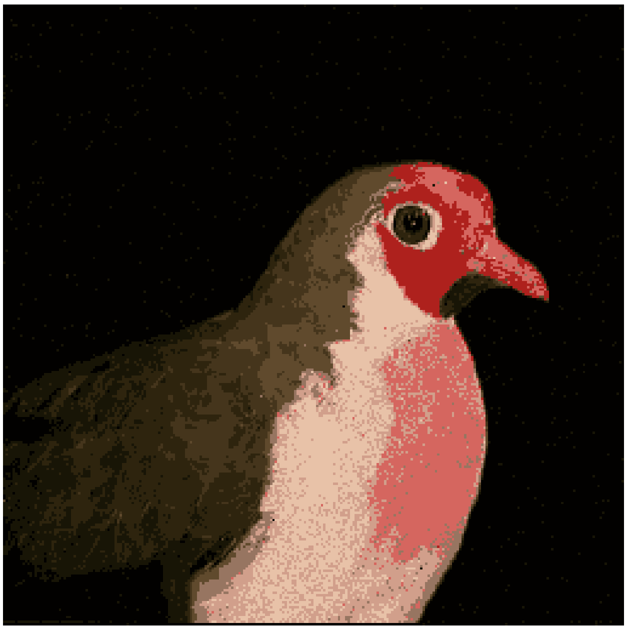
(a-raw image)



(b-5clusters)



(c-8 clusters)



(d-10 clusters)

Figure 9 shows a color image (a) and its corresponding segmentation results obtained using our method (b) with 5 clusters, (c) with 8 clusters, and (d) 10 clusters.

4 Conclusion

Herein we have proposed a new method to control the direction of HNN in its seeking of the global minimum when segmenting color images. As it is proven by the use of a ground truth color image, HNN did not reach the global optima, but produced a clear and better segmentation result when it is used with a step control parameter. After trying a large range of the control parameter m, we came to a conclusion that the latter is not the only parameter responsible or which decides the convergence point of HNN, but also the random initialization matrix used to initialize the inputs of HNN. From the case studies presented above it is clear that the matching approach added natural touches to the segmentation results by giving the clusters colors close to the raw image, this information can help in the pattern recognition field. In our future work, we will focus on the effects of neural network weights initialization and study their effect in seeking or reaching the global optimum.

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