A Novel Field-source Reverse Transform for Image Structure Representation and Analysis

X. D. ZHUANG[1,2] and N. E. MASTORAKIS[1,3]
1. WSEAS Headquarters, Agiou Ioannou Theologou 17-23, 15773, Zografou, Athens, GREECE
xzhuang@worldses.org
2. Automation and Engineering College, Qingdao University, Qingdao 266003, CHINA
3. Department of Computer Science, Military Institutions of University Education, Hellenic Naval Academy, Terma Hatzikyriakou, 18539, Piraeus, GREECE
mastor@wseas.org http://www.wseas.org/mastorakis

Abstract: - The image source-reverse transform is proposed for image structure representation and analysis, which is based on an electro-static analogy. In the proposed transform, the image is taken as the potential field and the virtual source of the image is reversed imitating the Gauss's law. Region border detection is effectively implemented based on the virtual source representation of the image structure. Moreover, the energy concentration property of the proposed transform is investigated for promising application in lossy image compression. Experimental results indicate that the proposed source-reverse transform can achieve efficient representation of image structure, and has promising application in image processing tasks.

Key-Words: - Source-reverse transform, electro-static field, region border detection, lossy image compression

1 Introduction

Image transform is an important means for image analysis, and provides theoretic support to many image processing and analyzing methods [1]. Typical image transforms include the classic Fourier transform and the widely applied Wavelet Transform [1]. Reversible image transforms are based on the integral transform in mathematics, which can decompose and restore the image based on the kernel function. Besides those based on the mathematical integral transform, there are other kinds of techniques such as the Hotelling transform and Hough transform [1]. The constant emergence of new image transform techniques improves the development and application of image processing.

Currently, the research of new image transform techniques has attracted much attention to accomplish various image processing tasks. A typical category is the emergence of virtual field methods inspired by physical field, which has achieved promising and effective results [2-5]. These methods have an electro-static or magneto-static metaphor, which takes the image as the source of the virtual field. This category of methods has successfully applied in biometrics, corner detection, image segmentation, etc [2-9]. Because the processing tasks are implemented in the transform-domain, the reversibility is usually not taken into consideration in the design of the transform.

In physics, the field is determined by the source distribution [2,3,10,11]. Therefore, the source is an compact representation of the field and reflects structural feature of the field. This is the foundation of the effectiveness of the virtual-field based methods. Current research focuses on the virtual field generated by the image, i.e. the image is just taken as the field source [2-5].

To obtain interior structure representation of images, in this paper a novel image transform named source-reverse transform is proposed by taking the image as the electro-static potential field to reverse the source. Based on the relationship between the field and source in physics, the virtual source obtained by the proposed transform reflects structural feature of the image and can be the foundation for further processing tasks. In the experiments, the source-reverse transform is implemented for test images and real-world images. The analysis of the experimental results proves that the virtual field source is an interior representation of the region border structure, and the energy concentration property of the proposed transform can be exploited in lossy image compression.

2 The Relationship between the Electro-static Field and the Field Source

In physics, the electric field intensity is virtually the inverted gradient vector of the potential [10,11]:

$$\vec{E} = -\nabla V \qquad (1)$$

where $\vec{E}$ means the electric field intensity at a space point; V is the potential; ∇ is the Hamiltonian operator:

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k} \qquad (2)$$

where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are three base vectors.

Therefore, the electro-static field can be represented by either of the two equivalent forms: the form of vector field (i.e. the electric field intensity) and the form of scalar field (i.e. the electric potential). The electro-static field distribution is determined by the field source, i.e. the distribution of the charges. On the other hand, the source can be reversed from the field, which is well known as the Gauss's law in differential form [10,11]:

$$div\,\vec{E} = \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad (3)$$

where *div* means the divergence; ρ is the charge density at the same space point of $\vec{E}$, i.e. the distribution of the source; ε_0 is the permittivity of the vacuum. Therefore, the source distribution can be obtained by the following:

$$\rho = -\varepsilon_0 \cdot div(grad(V)) \qquad (4)$$

where *div* and *grad* mean the divergence and gradient operation respectively.

The above equation can be regarded as the reverse process from field to source. Because the source determines the distribution of the field, the distribution of the source can be a compact representation of the field and contains the field's interior structure information. Therefore, in this paper a novel image transform is proposed by imitating the field source reverse principle for image structure representation and analysis.

3 The Source-reverse Transform for Digital Images

One of the ultimate goals of intelligent computer vision is the automatic recognition of the objects in the scene. Generally speaking, different objects occupy different regions in the image. Therefore, besides the image itself, an efficient representation of image structure is important for further analysis and recognition. In this paper, a novel image transform is presented based on the relationship between the field and the source, which takes the image as the field and reverse the source distribution. The properties of the source reverse transform are investigated experimentally, which can be applied in further image analysis and processing.

The Gauss's law in the electro-static field is for a continuous field in the space. However, the digital image is discrete. Therefore, to reverse from the image to the virtual source, discrete operator should be used to obtain the gradient and divergence of the digital image. Imitating the field source reverse in electro-static field, the source-reverse transform for an image $f(x, y)$ is as following:

$$F(x, y) = -div_d(grad_d(f(x, y))) \qquad (5)$$

where $F(x, y)$ is the virtual field source obtained by the transform; div_d and $grad_d$ are the discrete operators to get the estimation of the divergence and the gradient respectively. It is notable that the domain of $F(x, y)$ is still the two dimensional plane where the image is defined. Therefore, the spatial properties of $F(x, y)$ may have direct relationship with the image structure.

According to Equation (5), the source-reverse transform for an image includes two steps as following:

Step1: Estimate the virtual field intensity $\vec{E}(x, y)$ for each image point:

$$\vec{E}(x, y) = grad_d(f(x, y)) \qquad (6)$$

The operator $grad_d$ can get the two components of the discrete gradient on the x and y coordinates respectively. To obtain the gradient vector, the two partial derivatives of $f(x,y)$ should be estimated. In this paper, the Sobel operator is used to estimate the two partial derivatives, i.e. the components of gradient, which is shown in Fig. 1.

-1	0	1
-2	0	2
-1	0	1

The template to estmate the component on *x*-coordinate

1	2	1
0	0	0
-1	-2	-1

The template to estmate the component on *y*-coordinate

Fig. 1The two templates of Sobel operator to estimate the gradient

According to the above two image templates, the components of $\vec{E}(x,y)$ are estimated as following:

$E_x(x,y)=$
$[f(x+1,y-1)-f(x-1,y-1)]+2[f(x+1,y)-f(x-1,y)]+$
$[f(x+1,y+1)-f(x-1,y+1)]$ (7)

$E_y(x,y)=$
$[f(x-1,y-1)-f(x-1,y+1)]+2[f(x,y-1)-f(x,y+1)]+$
$[f(x+1,y-1)-f(x+1,y+1)]$ (8)

where $E_x(x,y)$ and $E_y(x,y)$ are the two components of the estimated virtual field intensity $\vec{E}(x,y)$.

Step2: Estimate the divergence of the virtual field intensity for each point as the virtual field source distribution $F(x, y)$:

$$F(x,y) = -div_d(\vec{E}(x,y)) \tag{9}$$

For the continuous vector field on the two dimensional plane, the divergence is defined as following:

$$div\,\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \tag{10}$$

where E_x and E_y are the two components of the vector field $\vec{E}(x,y)$.

Based on Equation (9), the estimation of the divergence of a discrete vector field should also use the discrete operator div_d, where the two partial derivatives in Equation (10) are still estimated by the Sobel operator as in **Step1**.

By the above two steps, the virtual source reverse can be implemented for a digital image taken as a potential field, and the virtual source is obtained as the result of the proposed image transform.

4 The Virtual Field Source as the Representation of Image Structure

The representation and analysis of image structure is important for many image-processing tasks [1,13]. Because the virtual source is still defined on the 2-D plane where the image is defined, the spatial properties of the virtual source may be closely related to the image structure. To investigate the properties of the proposed source-reverse transform, experiments are carried out for a group of test images and also a group of real world images. The principle to select proper images in the experiment is that the transform results for simple test images may distinctly show the basic characteristics of the source-reverse transform, while the transform results for real world images of much more complexity will reveal possible and promising applications of the method.

In the experiments, the value of the source on each point is recorded. The results indicate that there are both positive and negative values in the source. To reveal the property of the virtual source, the source values $F(x, y)$, their absolute values $|F(x, y)|$ and the sign of each value $sgn(F(x, y))$ are visualized in the form of gray-scale images. An example is shown in Fig. 2, which is one of the simple test images. Fig. 3 shows the distribution of the absolute values of the source, where larger gray-scale corresponds to larger absolute value. Fig. 4 shows the sign of the value on each point, where the white points represent positive values, the black points represent negative values and the gray points represent the zero value. The values of $F(x, y)$ is shown in Fig. 5, where the larger the gray-scale the larger the value.

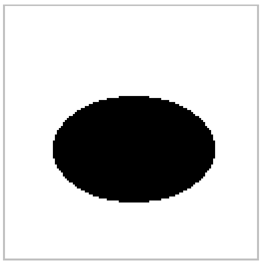


Fig. 2 One of the simple test images

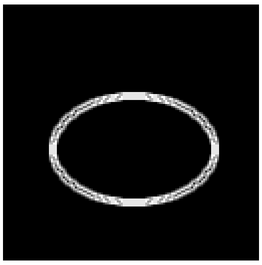


Fig. 3 The distribution of the absolute values of the source

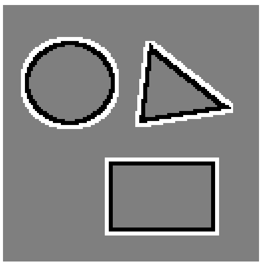


Fig. 4 The sign of the value on each point in the source

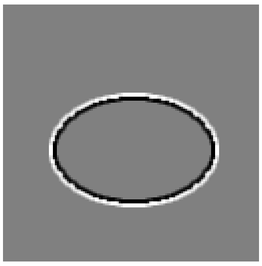


Fig. 5 The value of each point in the source

Fig. 4 shows that there are both regions of positive values and regions of negative values in the virtual field source. Fig. 6 and Fig. 7 show the borders of the positive regions and negative regions respectively, where the white points represent the border points. The experimental results show that for test images with simple objects, the borders of positive and negative regions can be the counters of the objects.

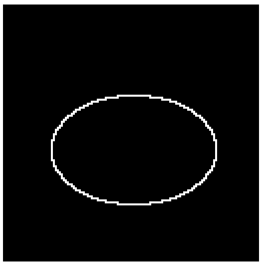


Fig. 6 The borders of the positive source regions

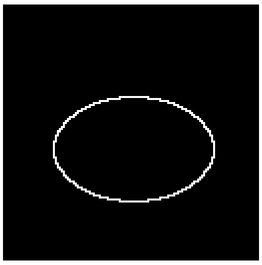


Fig. 7 The borders of the negative source regions

In the experimental results for simple test images, Fig. 4 shows that the source values in a homogeneous region are zero. Fig. 3, Fig. 4 and Fig. 5 show that the non-zero values in the virtual field source concentrate near the region borders, where there is more complex structure than the other parts of the image [12]. In another word, the energy in the virtual source concentrates on the borders of the homogeneous image regions, which is quite different from the Fourier transform in which the energy in the frequency domain concentrates in the area of low frequency. Moreover, Fig. 4 indicates that the source values on different sides of a region border are of different signs, which can be exploited in image structure representation and analysis. The experimental results for another test image are shown in Fig. 8 to Fig. 13, which also proves the above analysis.

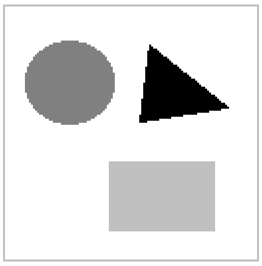


Fig. 8 Another simple test images

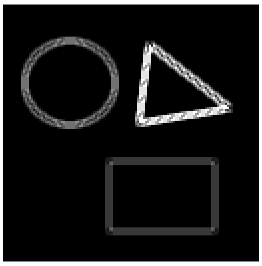


Fig. 9 The distribution of the absolute values of the source

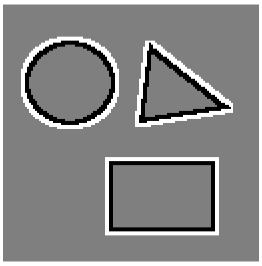


Fig. 10 The sign of the value on each point in the source

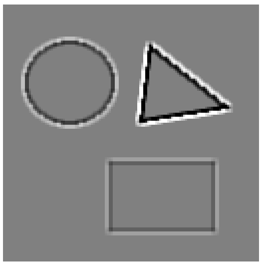


Fig. 11 The value of each point in the source

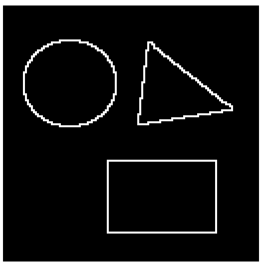


Fig. 12 The borders of the positive source regions

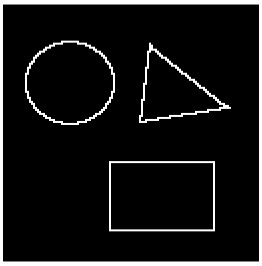


Fig. 13 The borders of the negative source regions

In order to investigate the possible application of the source-reverse transform, experiments are also carried out for real world images. The experimental results are shown in Fig. 14 to Fig. 31 for the broadcaster image, the brain image and the house image. The experimental results for real world images also indicate the properties of energy concentration and sign reverse across the region border in the virtual field source, which inspires a method of region border detection in Section 5.

Fig. 14 The image of the broadcaster

Fig. 15 The distribution of the absolute values of the source

Fig. 16 The sign of the value on each point in the source

Fig. 17 The value of each point in the source

Fig. 18 The borders of the positive source regions

Fig. 19 The borders of the negative source regions

Fig. 20 A medical image of the brain

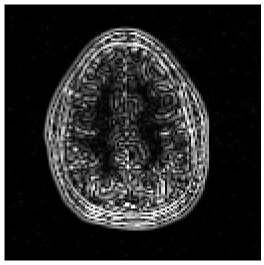


Fig. 21 The distribution of the absolute values of the source

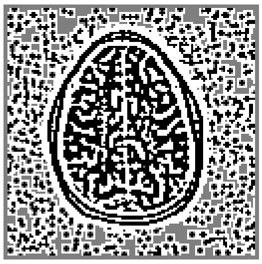


Fig. 22 The sign of the value on each point in the source

Fig. 23 The value of each point in the source

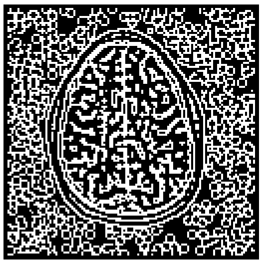


Fig. 24 The borders of the positive source regions

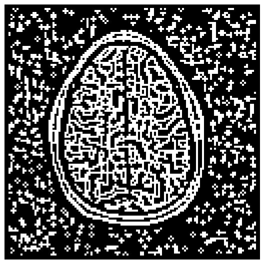


Fig. 25 The borders of the negative source regions

Fig. 26 The image of a house

Fig. 27 The distribution of the absolute values of the source

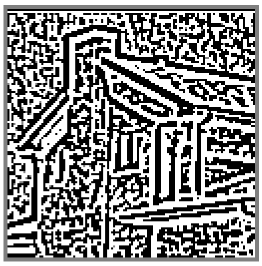


Fig. 28 The sign of the value on each point in the source

Fig. 29 The value of each point in the source

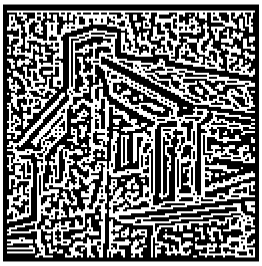


Fig. 30 The borders of the positive source regions

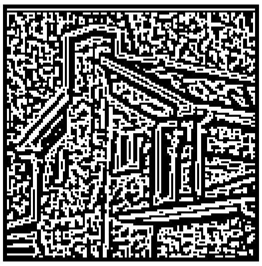


Fig. 31 The borders of the negative source regions

5 Region Border Detection Based on the Source-reverse Transform

In the above experimental results, the borders of the positive and negative source regions show the detail for all the image regions, while minor details may not be preferred in real world applications. Because the energy of the virtual source mainly concentrates near the region borders, the minor details of region borders can be eliminated with a threshold of absolute source value so that the main border of interest will be preserved for further analysis. Therefore, a region border detection method is proposed based on the virtual field source as following:

Step1: Implement the source-reverse transform for the image

Step2: Detect the points where the sign of source values reverse, i.e. find the points with different sign from neighboring points

Step3: For the points detected in **Step2**, eliminate the points with less absolute value than the threshold

The results of border detection for real world images are shown in Fig. 32 to Fig. 37.

Fig. 32 The region border detected based on Fig. 18

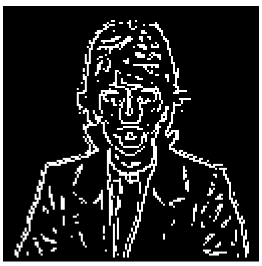


Fig. 33 The region border detected based on Fig. 19

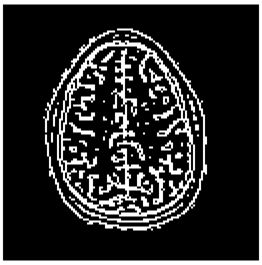


Fig. 34 The region border detected based on Fig. 24

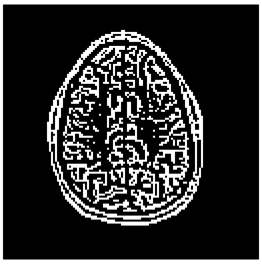


Fig. 35 The region border detected based on Fig. 25

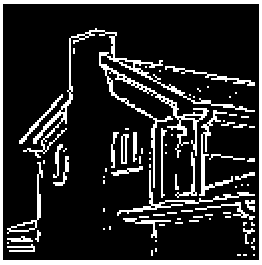


Fig. 36 The region border detected based on Fig. 30

Fig. 37 The region border detected based on Fig. 31

The experimental results indicate that the virtual source can be an efficient representation of image structure, based on which region border detection can be effectively implemented.

6 The Opposite Transform from the Virtual Source to the Image as a Virtual Potential Field

For any image transform, whether it is reversible is one of the basic characteristics. In this paper, although the analysis can be carried out just in the virtual source and its reversibility may not be considered for some applications, the opposite transform from virtual source to virtual potential field (i.e. the image) is discussed in this section.

For continuous electro-static field, the continuous source can be obtained by source reverse as Equation (4). On the other hand, the continuous potential field can also be generated by the source as following [11]:

$$V(x,y) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, dv}{r} \tag{11}$$

where ρ represents the charge density at a space point and r is the distance between that space point and (x, y). The integral in Equation (11) is carried out for the whole space where there is charge distribution. For continuous electro-static field in physics, the transform defined by Equation (4) and (11) is reversible.

However, for digital images, the opposite transform should be in a discrete form, i.e. the integral operation in Equation (11) should be replaced by summation as following:

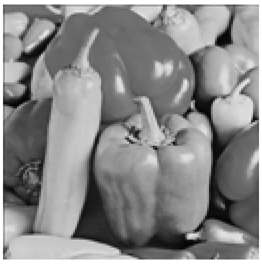
$$f'(x,y) = K \cdot \sum_{j=1}^{H}\sum_{i=1}^{W} \frac{F(i,j)}{r_{(i,j)\to(x,y)}} \tag{12}$$

where K is a positive constant; H and W are the height and width of the image respectively. $f'(x,y)$ is the virtual potential field (i.e. the restored image) obtained by the opposite transform; $F(i,j)$ is the virtual source.

Although the transform for continuous electro-static field is theoretically reversible, the discrete source-reverse transform includes operations of discretization which will introduce small errors in the transform process. Therefore, $f'(x,y)$ can be a nice approximation of the original image $f(x,y)$, and the source-reverse transform for digital images is not strictly reversible. The opposite transform is implemented for real world images. The experimental results for some of the real world images are shown in Fig. 38 to Fig. 41. The left one (a) of each pair of results is the visualization of the original data of $f'(x,y)$, and the right one (b) of each pair is the result of a contrast enhancement operation for $f'(x,y)$. The experimental results indicate that the quasi-reversible transform of source-reverse can provide nice approximation of the original image by the opposite transform, which may be exploited in lossy image compression.

(a) Visualization of $f'(x,y)$

(b) Result of contrast enhancement



(c) The original peppers image

Fig. 38 The restored results and the original image of the peppers

(a) Visualization of $f'(x,y)$

(b) Result of contrast enhancement

(c) The original house image

Fig. 39 The restored results and the original image of the house

(a) Visualization of $f'(x,y)$

(b) Result of contrast enhancement

(c) The original boat image

Fig. 40 The restored results and the original image of the boat

(a) Visualization of $f'(x,y)$

(b) Result of contrast enhancement

(c) The original bridge image

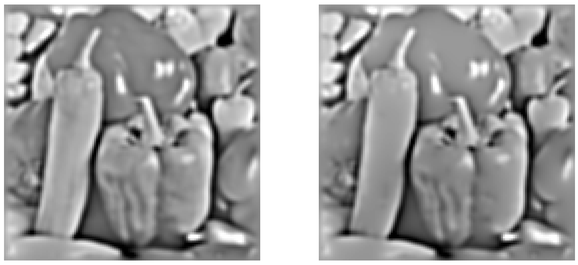
Fig. 41 The restored results and the original image of the bridge

7 Data Reduction of Virtual Field Source for Lossy Image Compression

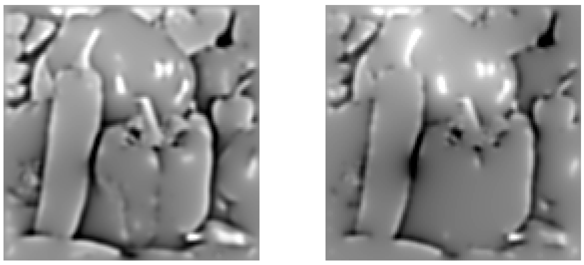
The experimental results have indicated that the energy in the virtual field source concentrates near the border of the homogeneous image regions, which may be exploited in lossy image compression. Because a large part of the values in the source are relatively small, experiments are carried out to investigate the effect of eliminating small source values on the restoration of the field (i.e. the image).

The experimental results are shown in Fig. 42 to Fig. 45. In the experiments, the threshold to eliminate small values in the virtual source is determined by a certain percentage of the maximum of the absolute values. For each real world image, the results show the effect of assigning 1%, 5%, 10% and 20% of the maximum absolute value to the threshold respectively. If the absolute value on a point is smaller than the threshold, that value is set to zero.

Then the virtual potential field (i.e. the image) is restored from the reduced source with small values eliminated. The experiments show the different effect of eliminating small values in the source with increasing the threshold value. The original images are of the size 128×128. Therefore, the uncompressed virtual source has totally 16384 values. The results indicate that the subjective visual perception of the restored image is still acceptable when a large part of the values in the virtual source are reduced. But when most of the small values are eliminated, the quality of the image becomes unacceptable for visual perception, which is shown in (c) and (d) of each group of results. The results indicate that the source-reverse transform has potential and promising application in lossy image compression.



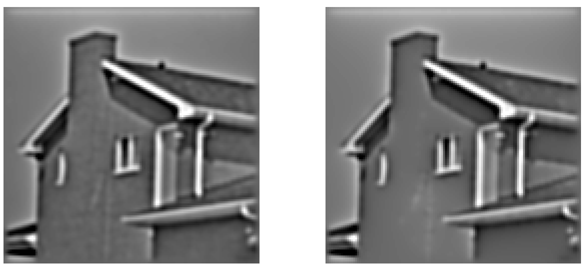
(a) (b)



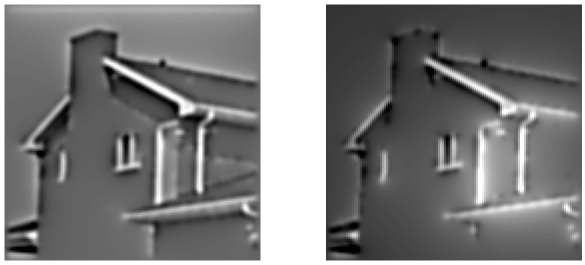
(c) (d)

(a) Result of restoration with the threshold defined as 1% of the maximum absolute value; 3684 values eliminated
(b) Result of restoration with the threshold defined as 5% of the maximum absolute value; 8969 values eliminated
(c) Result of restoration with the threshold defined as 10% of the maximum absolute value; 11473 values eliminated
(d) Result of restoration with the threshold defined as 20% of the maximum absolute value; 13858 values eliminated

Fig. 42 The effect of eliminating small source values on the restoration of the peppers image



(a) (b)

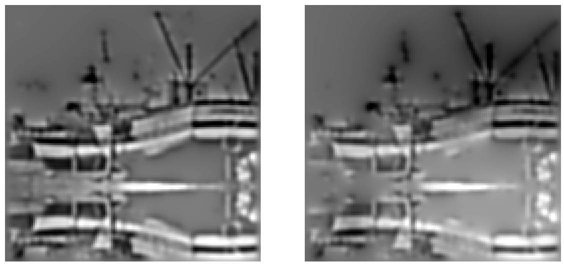


(c) (d)

(a) Result of restoration with the threshold defined as 1% of the maximum absolute value; 6870 values eliminated
(b) Result of restoration with the threshold defined as 5% of the maximum absolute value; 11347 values eliminated
(c) Result of restoration with the threshold defined as 10% of the maximum absolute value; 12688 values eliminated
(d) Result of restoration with the threshold defined as 20% of the maximum absolute value; 14356 values eliminated

Fig. 43 The effect of eliminating small source values on the restoration of the house image

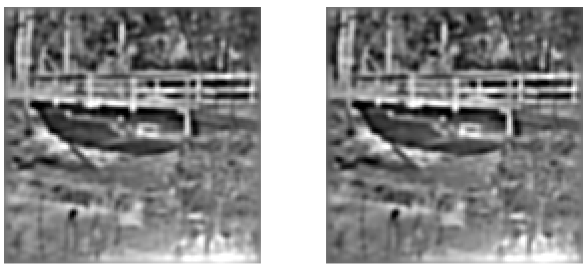
(a) (b)



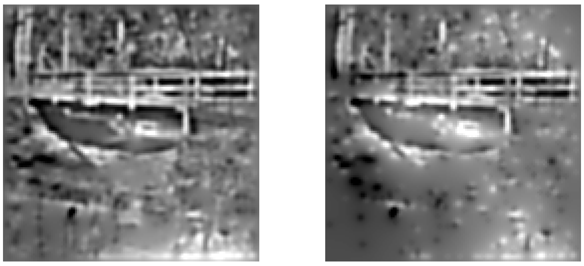
(c) (d)

(a) Result of restoration with the threshold defined as 1% of the maximum absolute value; 3176 values eliminated
(b) Result of restoration with the threshold defined as 5% of the maximum absolute value; 7526 values eliminated
(c) Result of restoration with the threshold defined as 10% of the maximum absolute value; 9690 values eliminated
(d) Result of restoration with the threshold defined as 20% of the maximum absolute value; 12361 values eliminated

Fig. 44 The effect of eliminating small source values on the restoration of the boat image



(a) (b)



(c) (d)

(a) Result of restoration with the threshold defined as 1% of the maximum absolute value; 1954 values eliminated
(b) Result of restoration with the threshold defined as 5% of the maximum absolute value; 5565 values eliminated
(c) Result of restoration with the threshold defined as 10% of the maximum absolute value; 9103 values eliminated
(d) Result of restoration with the threshold defined as 20% of the maximum absolute value; 13065 values eliminated

Fig. 45 The effect of eliminating small source values on the restoration of the bridge image

8 Conclusion

In this paper, a novel source-reverse transform is presented for digital images based on the relationship between the electro-static potential field and the field source in physics. The properties of the proposed transform are investigated and analyzed by experiments on groups of test images and real world images. A region border detecting method is proposed based on the virtual source representation of image structure. The quasi-reversible property of the proposed transform is also experimentally investigated and analyzed. The promising application of the transform in lossy image compression is also investigated based on the energy-concentration property in the virtual field source. Future work will consider how to overcome the small error between the original and restored images caused by discretization in the transform process, so that the quality of the restored image will be improved to a standard of strictly reversible transforms. Further research will also investigate the detailed characteristics of the source-reverse transform together with its potential application in other image processing tasks.

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