

# The Curling Vector Field Transform of Gray-Scale Images: A Magneto-Static Inspired Approach

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**Abstract:** - For image structure representation and feature extraction, the curling vector field transform is proposed based on the magneto-static analogy. The digital image is taken as the source of the vector field, and the vector field transform of the image is presented imitating the form of the physical magnetic field, which has a distinctive feature of rotating whorl pattern. The simulation results indicate that the curling vector field can represent the image's structure feature, which can be applied in image segmentation. The experimental results show that image segmentation can be effectively implemented based on the image structure feature extracted by the curling vector field transform.

**Key-Words:** - vector field transform, curling vector field, image structure, image segmentation

## 1 Introduction

In digital image processing, the image transform is a fundamental technique. Generally, the transform maps the original image to the representation in the transformed domain, such as discrete Fourier transform, discrete cosine transform, wavelet transform, etc [1]. These transforms are the conversions from the image space to the coefficient space (which are all scalar). Compared with a scalar space, a vector space includes not only the intensity of vectors but also the direction information, which may give a deeper and more detailed representation of the image structure. Research work has been done in image processing methods by imitating physical vector fields, such as the electro-static field or the magnetic field [2-8]. Because a field is determined by the field source and thus reflects the properties of the field source, vector field transforms may reveal detailed feature of the source (i.e. the image). This has become a novel way to design image processing algorithms and has potential impact on the development of image transform research. Currently, there is relatively less work on the imitation of magnetic field in image processing, and there is also no general and uniform definition of vector transform imitating the magnetic field [7,8].

In this paper, a novel vector field transform for images is proposed imitating the static magnetic field, which is named the curling vector field transform. The properties of the proposed transform are investigated experimentally, and the transform is

applied in image segmentation. The experimental results indicate the proposed method can produce detailed representation of the image structure, based on which image segmentation can be effectively implemented.

## 2 The Curling Vector Field Transform

### 2.1 The physical magnetic field

The magnetic induction generated by an electric current element is defined by the Biot-Savart Law [9]:

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I d\vec{l} \times \vec{r}}{r^3} \quad (1)$$

where  $dB$  is the magnitude of the magnetic induction at a point in the space.  $I$  is the magnitude of the electric current element;  $d\vec{l}$  is a unit vector which represents the direction of the electric current element;  $\vec{r}$  is the radius vector from the electric current element to the point in the space.

In the physical magnetic field, the magnetic induction lines are close loops. The geometric shape of the magnetic induction lines are determined by the mathematical form of the vector field defined in (1). In this paper, the mathematical form of the magnetic field is imitated in image vector transform, based on which the curling vector field is proposed.

## 2.2 The definition of the curling vector

If the image is taken as the field source, the vector field can be generated by the imitation of the mathematical definition of the magnetic field. Suppose there is an electric current element on each image point, which is at a right angle to the image plane. Then the virtual field source will generate a vector field on the image plane. Furthermore, in order to extract the image structure information, the intensity of the vector field on each point is determined by the gray-scale values of related image points. In this paper, to reflect the gray-scale difference between the points in two different image regions, the vector generated by point  $(i,j)$  on the point  $(x,y)$  is related to the gray-scale difference between the two points:

$$\vec{V} = \frac{|g(i,j) - g(x,y)|}{r_{(i,j) \rightarrow (x,y)}^2} \cdot \frac{\vec{n}_0 \times \vec{r}_{(i,j) \rightarrow (x,y)}}{r_{(i,j) \rightarrow (x,y)}} \quad (2)$$

where  $\vec{V}$  is the vector generated by point  $(i,j)$  on the point  $(x,y)$ ;  $g(i,j)$  and  $g(x,y)$  are the gray-scale values of the two points;  $\vec{n}_0$  is the unit vector at a right angle to the image plane;  $\vec{r}_{(i,j) \rightarrow (x,y)}$  is the radius vector from  $(i,j)$  to  $(x,y)$ .

The two components of  $V$  on  $x$  and  $y$  direction are:

$$V_x = \frac{|g(i,j) - g(x,y)|}{r_{(i,j) \rightarrow (x,y)}^2} \cdot \sin \theta_{(i,j) \rightarrow (x,y)} \quad (3)$$

$$V_y = \frac{|g(i,j) - g(x,y)|}{r_{(i,j) \rightarrow (x,y)}^2} \cdot \cos \theta_{(i,j) \rightarrow (x,y)} \quad (4)$$

where  $\theta$  is the direction angle of the radius vector  $\vec{r}_{(i,j) \rightarrow (x,y)}$ .

Therefore, if two image points are of the same gray-scale, the vector generated by one point on the other is a zero vector. Only if there is gray-scale difference between two image points, non-zero vector can be generated by one point on the other. According to the above definition, the vector generated by one point on another reflects the information of their gray-scale difference and their relative position (i.e. the distance and direction).

## 2.3 The definition of the curling vector field transform for images

Based on the definition of the curling vector, the curling vector field transform is proposed as follows. The transform of image  $g$  on the point  $(x,y)$  is defined as the sum of the vectors generated by all the other image points on  $(x,y)$ :

$$\vec{V}(x,y) = \sum_{i=1}^W \sum_{j=1}^H \frac{|g(i,j) - g(x,y)|}{r_{(i,j) \rightarrow (x,y)}^2} \cdot \frac{\vec{n}_0 \times \vec{r}_{(i,j) \rightarrow (x,y)}}{r_{(i,j) \rightarrow (x,y)}} \quad (5)$$

where  $\vec{V}(x,y)$  is the field vector on  $(x,y)$ ;  $W$  and  $H$  are the width and height of the image respectively.

The two components of  $\vec{V}(x,y)$  on  $x$  and  $y$  direction are:

$$V_x(x,y) = \sum_{i=1}^W \sum_{j=1}^H \frac{|g(i,j) - g(x,y)|}{r_{(i,j) \rightarrow (x,y)}^2} \cdot \sin \theta_{(i,j) \rightarrow (x,y)} \quad (6)$$

$$V_y(x,y) = \sum_{i=1}^W \sum_{j=1}^H \frac{|g(i,j) - g(x,y)|}{r_{(i,j) \rightarrow (x,y)}^2} \cdot \cos \theta_{(i,j) \rightarrow (x,y)} \quad (7)$$

The magnetic induction lines are close curves in physical magnetic field. This feature also exists in the curling vector field defined above. The curling vector transform is implemented for a group of test images to investigate the properties of the vector field. The test images are of the size  $32 \times 32$ . The experimental results show that the vectors inside each homogeneous region constitute a rotating whorl, which inspires the naming of the curling vector field. Three representative images of the test images and the corresponding experimental results are shown in Fig. 1 to Fig. 8.

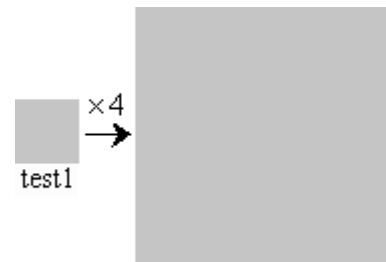


Fig. 1. The first image *test1* (the original image, and 4 times of original size on the right)

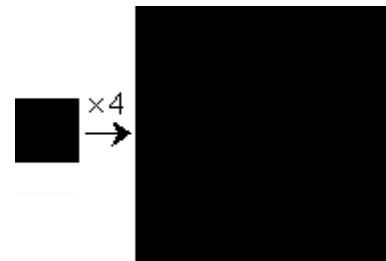


Fig. 2. The vector length in the curling vector field of *test1* (the original image; 4 times of original size on the right)

*Test1* is a homogeneous area as a whole. Therefore, the vectors in the transformed field are all zero vectors.

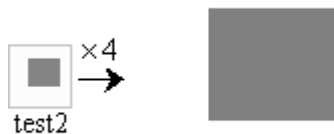


Fig. 3. The second image *test2* (the original image on the left, and 4 times of original size on the right)

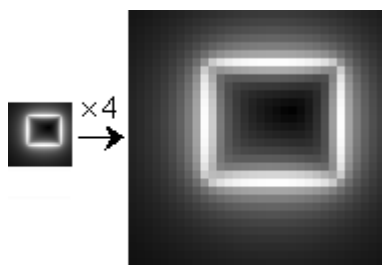


Fig. 4. The vector length in the curling vector field of *test2* (the original image; 4 times of original size on the right)

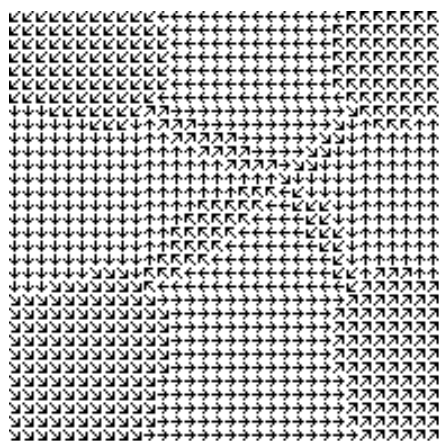


Fig. 5. The direction of each vector in the curling vector field of *test2*

*Test2* contains a rectangle region. In the vector field of *Test2*, the vectors in the rectangle region rotate clockwise, but the vectors in the background region rotate anti-clockwise.

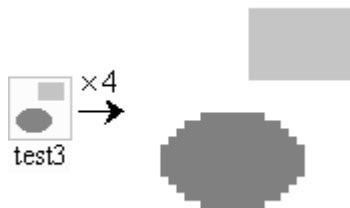


Fig. 6. The third image *test3* (the original image on the left, and 4 times of original size on the right)

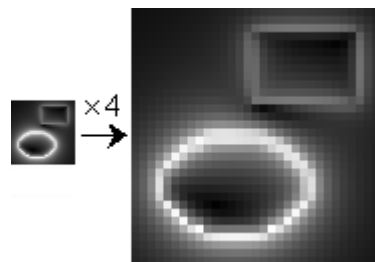


Fig. 7. The vector length in the curling vector field of *test3* (the original image; 4 times of original size on the right)

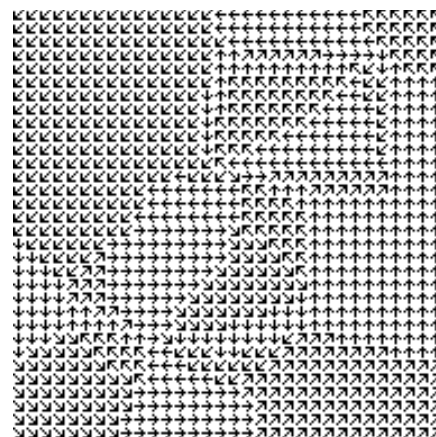


Fig. 8. The direction of each vector in the curling vector field of *test3*

There is a rectangle region and an ellipse region in *Test3*. In the vector field of *Test3*, the vectors in the rectangle region and the ellipse region rotate clockwise, but the vectors in the background region rotate anti-clockwise. The difference of rotating direction between the object regions and the background region is the base of segmentation in the curling vector field.

### 3 Image Segmentation Based on the Curling Vector Field

#### 3.1 The rotating direction and base points of rotating expansion

The experiments and analysis about the curling vector field show that the vectors inside a homogeneous region have the overall appearance of a rotating whorl. Moreover, if one region contains another, the rotating directions of them are opposite. This feature is determined by the definition of the vector field in Equation (5). Based on the rotating feature of the vectors inside homogeneous regions, any homogeneous region can be extracted by a kind of rotating expansion in the region. The points with definite rotating directions in a region are the starting points of the expansion. The region will be extracted by continuously expanding with a form of

rotating movement according to the rotating direction of the base points. In such rotating movement, the paths of movement will progressively cover the whole region. Thus the homogeneous region can be extracted.

Determining the rotating direction of each image point is the first step of region extraction in the curling vector field. In this paper, three kinds of rotating status on an image point are given as following. First, the vector directions are discretized into eight directions. Suppose that the vector direction on point A has the angle of zero, and B is the next point on A's vector direction. The rotating direction on A is defined according to the angle between the vectors on A and B. The clockwise rotating direction on point A is defined as the following two cases:

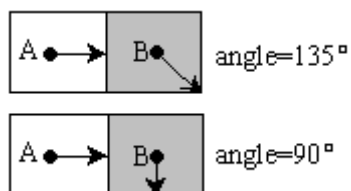


Fig. 9. The clockwise rotating direction on point A

The anti-clockwise rotating direction on point A is defined as the following two cases:

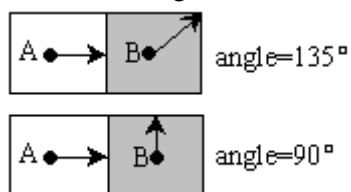


Fig. 10. The anti-clockwise rotating direction on point A

The uncertain rotating direction on point A is defined as the following four cases:

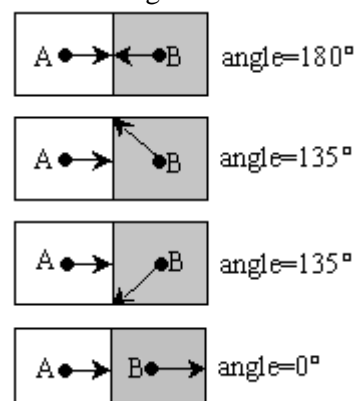


Fig. 11. The uncertain rotating direction on point A

If the vector on point A has the angle  $\alpha$ , and B is the next point on A's vector direction. The rotating direction can also be determined according

to the angle between the two vectors on A and B as the above cases.

Therefore, the image points can be divided into two classes: one with definite rotating direction and the other with uncertain rotating direction. Those with definite rotate directions are the starting points of rotating expansion, which are named the base points of rotating expansion.

The base point extraction is carried out for the test images. The experimental results are shown in Fig. 12 to Fig. 15. In Fig. 12 and Fig. 14, the white points represent the anti-clockwise rotating direction, the black points represent the clockwise rotating direction, and the gray points represent the uncertain ones. Because the vectors in the field of *Test1* are zero, there is no base extracted. Fig. 12 and Fig. 14 show the base points of *Test2* and *Test3* respectively. For a clearer view, the base points extracted are also shown in the discrete direction distribution of the vector fields as Fig. 13 and Fig. 15. The results of base point extraction indicate that these points form a kind of image structure representation, and can be the starting points for region extraction.

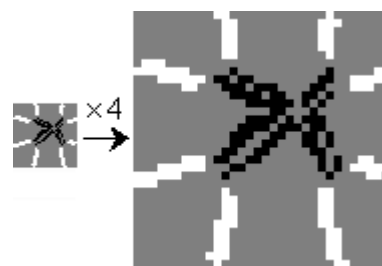


Fig. 12. The base points extracted for *Test2*

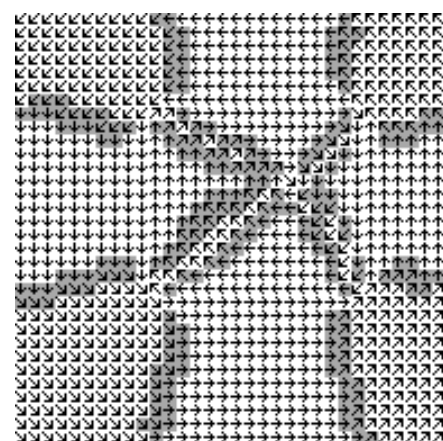
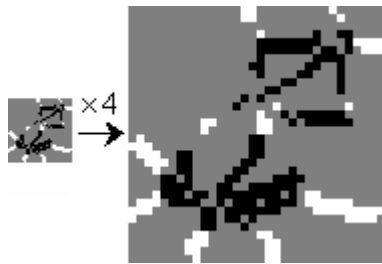
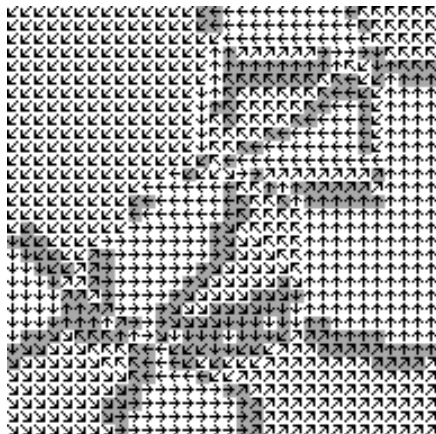


Fig. 13. The base points for *Test2* in the direction distribution of the vector field

Fig. 14. The base points extracted for *Test3*Fig. 15. The base points for *Test3* in the direction distribution of the vector field

On the other hand, the results also indicate that there may be more than one group of base points in a single homogeneous region, especially in a large region of the image. This may cause the decomposition of a large region into smaller sub-regions in the region extraction process, and makes region-merging as a necessary post-processing step.

### 3.2 Primitive region extraction from the curling vector field

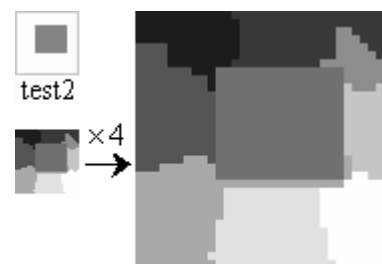
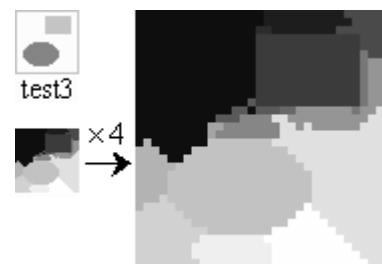
In the region extraction process, each group of base points will expand to a corresponding region, which is named the primitive region. The primitive regions can be regarded as the elements of an image, which constitute a kind of representation of image structure. The primitive regions in the image can be extracted with a rotating expansion process in the curling vector field, which starts from the base points and expands according to the rotating direction of the base points. In the rotating expansion process, each group of base points will expand to a corresponding primitive region. Given the curling vector field and the base points extracted, the rotating expansion process is described in steps as following:

**Step1:** Gather the base points into groups. In each group, the base points are connected to each other. In another word, two neighboring base points are of the same group. Assign each

group of base points a group number. Initially, the image points with uncertain rotating direction do not belong to any group.

**Step2:** For each image point with a group number, carry out the rotating expansion operation. If the current point  $p_c$  belongs to a certain group, investigate the next point  $p_{n1}$  on  $p_c$ 's vector direction, and also investigate the next point  $p_{n2}$  on  $p_c$ 's rotating direction. If  $p_{n1}$  or  $p_{n2}$  does not belong to any group, add it to  $p_c$ 's group, which is an occurrence of new classification. **Step2** is repeated until no new classification happens.

The results of primitive region extraction for the test images are shown in Fig. 16 and Fig. 17:

Fig. 16. The primitive regions extracted for *Test2*Fig. 17. The primitive regions extracted for *Test3*

In the experimental results of *test2* and *test3*, the object regions are extracted completely (the rectangle in *test2*, and the rectangle and circle in *test3*). Moreover, the background areas in *test2* and *test3* are split into several sub-regions. Therefore, a region may be split into several sub-regions in the rotating expansion process, especially the background region or the large object regions in real world images. In order to obtain effective segmentation result, a region-merging step is needed. The method of real world image segmentation based on the curling vector field is presented in Section 3.3.

### 3.3. Real world image segmentation based on the curling vector field

Experiments are also carried out for real world images to extract primitive regions. The images are with the size of  $128 \times 128$ . The experimental results

of the broadcaster image, the pepper image, the house image and the brain image are shown as follows. The experimental results indicate that the number of primitive regions is large because of the complexity of real world images. There are 274 primitive regions for the broadcaster image, 628 primitive regions for the pepper image, 398 primitive regions for the house image, and 423 primitive regions for the brain image.

To obtain meaningful segmentation result, the region-merging step is proposed according to least gray-scale difference criterion. First, an expected number of remaining regions after merging is given. Then the following steps are carried out to merge regions until the expected region number is reached:

**Step1:** For each region in the image, calculate its average gray-scale value.

**Step2:** Find the pair of neighboring regions with the least difference of the average gray-scale, and merge them into one region.

**Step3:** If current region number is larger than the expected region number, return to **Step1**; otherwise, end the merging process.

Based on the above sections, a method of image segmentation with the curling vector field transform is proposed as following:

**Step1:** Implement the curling vector field transform for the gray-scale image.

**Step2:** Judge the rotating direction of each image point, and extract the base points in the curling vector field.

**Step3:** Gather the neighboring base points into corresponding group, and each group is assigned a group number.

**Step4:** Extract the primitive regions by the rotating expansion, which starts from the base points.

**Step5:** Merge neighboring primitive regions according to the least average gray-scale difference criterion, and obtain the segmentation result with a pre-defined number of remaining regions.

The proposed segmentation method is applied to a group of real world images. The results of the broadcaster image, the pepper image, the house image and the brain image are shown in Fig. 18 to Fig. 25. Fig. 18, Fig. 20, Fig. 22 and Fig. 24 are the original images. Fig. 19 is the segmentation result of the broadcaster image with 20 regions remained after merging. Fig. 21 is the segmentation result of the pepper image with 40 regions remained after merging. Fig. 23 is the segmentation result of the house image with 50 regions remained after merging. Fig. 25 is the segmentation result of the brain image with 60 regions remained after merging. In Fig. 19, Fig. 21, Fig. 23 and Fig. 25, different

regions are differentiated from each other by different gray-scale values. The experimental results show that the gray-scale image segmentation can be effectively implemented based on the curling vector field.



Fig. 18. The broadcaster image



Fig. 19. Segmentation result of the broadcaster image (20 regions remained after merging)



Fig. 20. The pepper image

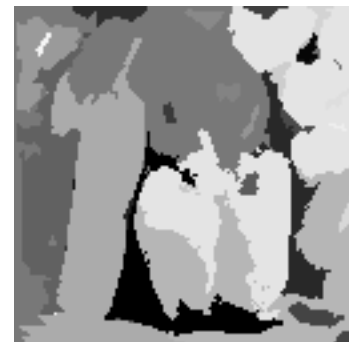


Fig. 21. Segmentation result of the pepper image (40 regions remained after merging)



Fig. 22. The house image



Fig. 23. Segmentation result of the house image (50 regions remained after merging)

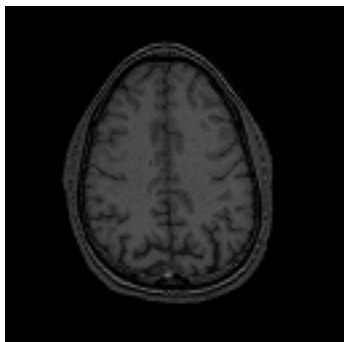


Fig. 24. The brain image



Fig. 25. Segmentation result of the brain image (60 regions remained after merging)

## 4 Conclusion

In this paper, a novel vector field transform for gray-scale images is proposed, which is named the curling vector field transform. It is defined by imitating the physical magnetic field and based on

the idea that the image is taken as the field source in the vector field transform. The gray-scale difference between image points is introduced into the definition of the vector field. Thus the curling vector field has the unique feature that inside each homogeneous region the vectors display a rotating whorl pattern, which is a kind of representation for image structure. In this paper, this feature of the curling vector field is exploited in image segmentation. Experimental results indicate that image segmentation can be effectively be implemented based on the curling vector field. Further research will investigate other properties of the curling vector transform, and its application in other image processing tasks will also be studied.

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