Dynamics Modeling and Trajectory Tracking Control for Humanoid Jumping Robot

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Abstract: - Jumping or running belong to a non-regular motion in humanoid robot field. Stance phase has only the holonomic constraints, nevertheless flight phase has the holonomic and non-holonomic constraints. The relationship between angular moment and cyclic coordinate is analyzed by dynamics equations. The impact model is expatiated by transferring dynamics equation from flight phase to stance phase, and the impact force and velocity is obtained by Jacobian matrix. The method of eliminating shock is discussed. Finally, trajectory tracking using hybrid model of the adaptive fuzzy control and computed torques is studied on model with parametric uncertainty. The hybrid control system proves to be asymptotical stabilization by Lyapunov stability theory. The numerical simulation and experimental results show computed torque controller is valid for jumping movement.

Key-Words: - jumping robot, dynamics modeling, non-regular motion, trajectory tracking

1 Introduction

Legged robots have better mobility, versatility and autonomous capability on discontinuous or uneven environment contrasting to wheeled or tracked vehicles among mobile robotics family. Recently, legged robots, especially humanoid robots [1], have been extensively studied and applied, such as HRP Project in Japan, Bip Plan in France, Shdow Project in England, MIT Leg Laboratory in USA and Humanoid Plan in China etc. Humanoid robots can be divided into regular and non-regular motion on locomotive mode. Walking may be so-called regular motion, because it is viewed as a periodic mechanism which intermittently interacts with environment through its feet. However, the motions such as jumping or running [2] belong to non-regular motion, which haven't supported legs during ballistic flight phase and have many contact points during landing phase. The control of jumping or running machines [3] is a difficult problem, because they have highly nonlinear dynamics, and in the case of monopeds or bipeds, they are static imbalance [4]. They can only keep balance through their motion taking into account the inertial forces [5]. Although jumping robot has a static imbalance, jumping motion can over an obstacle which size is similar or times to itself. So jumping robot improves its movement space in non-structural environment. More and more researchers have approached and engaged in this topic.

Raibert and his group are probably the best known dynamically balanced mechanism [6]. They have pursued and studied hopping or running robot, and their telescopic legs were driven by hydraulic or pneumatic actuator. Their basic control algorithm can be included three parts: upright height is modulated by driven legs, forwards speed is controlled by position of the legs during the flight phase, and body posture is regulated during the stance phase. Li and Ur-Rehman et al. used energy balance method and a stable limit cycle, furthermore, they achieved the orientation control of one-legged hopping robot during the flight phase by designing a state observation [7, 8]. Hyon et al. considered hopping machine as an equivalent mass-spring model, and showed a period doubling and chaotic behavior in 1-Dimension and 2-Dimension vertical jumping motion through applying return maps on a stable limit cycle [9]. Kusano et al. based on an equivalent mass-spring model, controlled vertical hopping using neural networks, and moreover, they optimized the angular velocity and vertical height [10].

The crucial steps in designing and controlling jumping robots are the dynamic model and trajectory planning and tracking. Although some researchers have started to pursue control algorithms based on dynamics, they mostly aim at equivalent mass-spring model [9], telescopic legs mode [8] and spring load inverted pendulum [5]. Moreover, a few researchers have studied multi-angular hopping [11], while they almost make the hypothesis that the whole gravity center lies in the coxa of a torso body [12].

This paper addresses dynamics characteristics of under-actuated and non-holonomic [13], and applied multibody and floated-basis space to establish the dynamics model. In order to track and control jumping motion, we study a hybrid system of computed torque and adaptive fuzzy control [14, 15]. An asymptotical stabilization using hybrid model is obtained by Lyapunov stability theory. Finally, numerical simulation and experiments are implemented. This paper is organized as follows. Section 2 briefly establishes an unified dynamics equations including flight, stance and landing impact phases, and discusses the methods of eliminating shock. In section 3, adaptive fuzzy trajectory tracking control is described, and it is proved to be asymptotical stable using hybrid model by Lyapunov stability theory. Section 4 dedicates to numerical simulation and experiments using dynamics equations, computed torque control and adaptive fuzzy trajectory tracking control. Section 5 gives some conclusions.

2 Dynamics Model

Humanoid robot motion space can be decoupled into sagittal and lateral plane. To study the dynamics characteristic, the jumping process can be divided into three phases: stance phase, flight phase and landing contact phase. Each phase has a dynamics equation because of their different constraint conditions. So the dynamics of jumping motion belongs to the dynamics of a various constraint system.

During the flight phase, the reference base is free-floating, and there is a non-holonomic dynamics because of the conservation of angular momentum with respect to the center of gravity (COG) of robot system. The mathematical model floated-basis space is more difficult than it on fixed-basis space. Only with microgravity or without gravity, there are many kinetic and dynamics modeling methods for space robots, such as Lagrange, Newton-Euler, Hamiton, Barycenter, Virtual and Equivalent Manipulator. However, the gravity has a significant influence to dynamics of jumping robot, so we can't apply those modeling methods directly without any modification.

The configuration of multibody is shown in Fig. 1. XOY is the inertia reference frame, and $X_{cm}O_{cm}Y_{cm}$ is a floated-basis frame which is fixed to the mass center of jumping robot. The vector of body coordinates $\mathbf{q}_b$ consists of the relative angles $(q_1, q_2, q_3, \cdots, q_{n-1})^T$, which describes the relative of the robot. The direction of anticlockwise is positive. The absolute orientation of robot is given by q_n. The robot's absolute position of COG $\mathbf{r}_{cm}$ is specified by the Cartesian coordinates $(x_{cm}, y_{cm})^T$. The vector of generalized coordinates $\mathbf{q}_f$ is denoted as $(q_b^T, q_n, r_{cm}^T)^T$. The mass of i-th rigid body is m_i, its length is l_i, the moment of inertia around its center of mass is I_i, and the position of its center of mass is given by $\overline{O_iC_i} = \alpha_i l_i$.

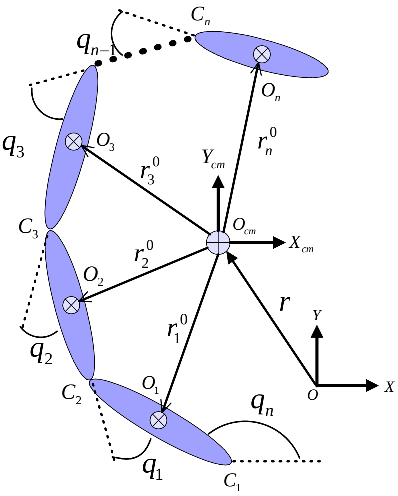


Fig.1. Multi-body model of jumping robot

2.1 Dynamics equations during flight phase

During flight phase, there are three constraints in COG, viz. two holonomic constraints resulting from the fact that the COG tracks a parabolic trajectory, and one non-holonomic resulting from the conservation of angular momentum with respect to COG. The dynamical model can be determined from Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial \mathbf{L}_f}{\partial \dot{\mathbf{q}}_f}\right) - \frac{\partial \mathbf{L}_f}{\partial \mathbf{q}_f} = \mathbf{\Gamma}_f \tag{1}$$

where Lagrangian is defined as $\mathbf{L}_f = \mathbf{K}_f - \mathbf{P}_f$, including the total kinetic energy K_f and the total potential energy P_f.

Defining variable ξ_i as the mass percent of i-th rigid body to the whole body, viz. $\xi_i = \frac{m_i}{\sum_{j=1}^{n} m_j} = \frac{m_i}{m}$.

The mass center equation can be denoted into $\sum_{i=1}^{n} \xi_i \mathbf{r}_i^0 = 0$ if $\mathbf{r}_i^i$ is the vector between O_{cm} and the i-th mass center. When analyzing the vector of i-th mass center in frame $X_{cm}O_{cm}Y_{cm}$ along the vector of whole mass center in frame XOY , it can yield $\sum m_i \mathbf{r}_i^0 \cos\langle \mathbf{r}_i^0, \mathbf{r} \rangle = 0$. So we decouple rotational motion from translational motion.

The kinetic energy of the system can be given by the following integral

$$\begin{aligned}\mathbf{K}_f &= \frac{1}{2}\sum_{i=1}^{n}\left(I_i \dot{q}_i^2 + m_i \left\|\dot{\mathbf{r}}_i\right\|^2\right) \\ &= \frac{1}{2}\sum_{i=1}^{n}\left(I_i \dot{q}_i^2 + m_i \left\|\dot{\mathbf{r}}_i^o\right\|^2\right) + \frac{1}{2}m\left\|\dot{\mathbf{r}}_{cm}\right\|^2 \\ &= \frac{1}{2}\dot{\mathbf{q}}_b^T \mathbf{M}(\mathbf{q}_b)\dot{\mathbf{q}}_b + \frac{1}{2}m\left\|\dot{\mathbf{r}}_{cm}\right\|^2 = \frac{1}{2}\dot{\mathbf{q}}_f^T \mathbf{D}_f(\mathbf{q}_b)\dot{\mathbf{q}}_f\end{aligned} \tag{2}$$

where

$$\mathbf{D}_f(\mathbf{q}_b) = \begin{bmatrix} \mathbf{M}(\mathbf{q}_b) & \mathbf{0}_{(n-1)\times 2} \\ \mathbf{0}_{2\times(n-1)} & m\mathbf{I}_{2\times 2} \end{bmatrix}$$

$$\theta_i = \begin{cases} \sum_{k=1}^{i} q_k & (i = 1, \cdots, n-1) \\ q_i & (i = n) \end{cases}$$

$$\mathbf{r}_j^0 = \begin{cases} \sum_{k=1}^{n}\left[\mathbf{R}_{z,\theta_k}\left[\begin{matrix}\left[(1-\alpha_k)\xi_k + \left(1 - \sum_{i=1}^{k}\xi_k\right)\right] l_k \\ 0\end{matrix}\right]\right] - \mathbf{R}_{z,\theta_j}\begin{bmatrix}(1-\alpha_j) l_j \\ 0\end{bmatrix} & (j = 1) \\ \mathbf{r}_{j-1}^0 - \mathbf{R}_{z,\theta_{j-1}}\begin{bmatrix}(1-\alpha_{j-1}) l_{j-1} \\ 0\end{bmatrix} - \mathbf{R}_{z,\theta_j}\begin{bmatrix}\alpha_{j-1} l_{j-1} \\ 0\end{bmatrix} & (j \neq 1) \end{cases}$$

and $\mathbf{R}_z$ is a 2D rotation matrix

$$\mathbf{R}_{z,\psi} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}$$

The potential energy of the system is

$$\mathbf{P}_f = mgy_{cm} \tag{3}$$

By the virtual principle, the general torques matrix is

$$\mathbf{\Gamma}_f = \begin{bmatrix}\tau_1 & \cdots & \tau_{n-1} & 0 & 0 & 0\end{bmatrix}^T \tag{4}$$

Using Eq. (2), (3) and (4), Lagrange's equation during flight phase becomes

$$\mathbf{D}_f(\mathbf{q}_b)\ddot{\mathbf{q}}_f + \mathbf{H}_f(\mathbf{q}_b, \dot{\mathbf{q}}_f)\dot{\mathbf{q}}_f + \mathbf{G}_f(\mathbf{q}_f) = \mathbf{\Gamma}_f \tag{5}$$

where $\mathbf{H}_f(\mathbf{q}_b, \dot{\mathbf{q}}_f)$ contains Coriolis and centrifugal terms

$$\mathbf{H}_f(\mathbf{q}_b, \dot{\mathbf{q}}_f) = \frac{\partial\left[\mathbf{D}_f(\mathbf{q}_b)\right]}{dt} - \frac{\partial\left[\frac{1}{2}\dot{\mathbf{q}}_f^T \mathbf{D}_f(\mathbf{q}_b)\dot{\mathbf{q}}_f\right]}{\partial \mathbf{q}_f}$$

and $\mathbf{G}_f(\mathbf{q}_f)$ is the gravity vector

$$\mathbf{G}_f(\mathbf{q}_f) = \begin{bmatrix}0 & \cdots & 0 & mg\end{bmatrix}^T$$

The Eq. (5) shows that Lagrangian $\mathbf{L}_f$ is independent of absolute orientation q_n , viz. $\frac{\partial \mathbf{L}}{\partial q_n} = \frac{d}{dt}\frac{\partial \mathbf{L}}{\partial \dot{q}_n} = 0$, so the angular momentum for flight phase can be expressed as

$$\begin{cases} \mathbf{M}_{cm} = \frac{\partial \mathbf{L}}{\partial \dot{q}_n} = \frac{\partial \mathbf{K}}{\partial \dot{q}_n} = const \\ \dot{\mathbf{M}}_{cm} = 0 \end{cases} \tag{6}$$

2.2. Dynamics equation for stance phase

Supposing without sliding and rotation under the robot foot, the constraint is holonomic during stance phase. The generalized coordinates can be taken as $\mathbf{q}_s = \left(\mathbf{q}_b^T, q_n\right)^T$. In the past, most people established dynamics during stance phase using a fixed space. Presently this paper contributes to describe dynamics on free-floating frame space.

According to the Cartesian coordinate space, the position of mass center during stance phase can be denoted

$$\begin{aligned}\mathbf{r}_{cm} &= \begin{bmatrix} x_{cm} \\ y_{cm} \end{bmatrix} \\ &= \mathbf{R}_{z,\theta_1}\begin{bmatrix}(1-2\alpha_1) l_1 \\ 0\end{bmatrix} - \sum_{k=1}^{n}\left[\mathbf{R}_{z,\theta_k}\left[\begin{matrix}\left[(1-\alpha_k)\xi_k + \left(1 - \sum_{i=1}^{k}\xi_k\right)\right] l_k \\ 0\end{matrix}\right]\right] \\ &= f_{cm}(\mathbf{q}_s)\end{aligned} \tag{7}$$

Hence, the relationship of generalized coordinates between flight phase and stance phase is

$$\dot{\mathbf{q}}_f = \begin{bmatrix} \mathbf{I}_{n\times n} \\ \frac{\partial f_{cm}}{\partial \mathbf{q}_s} \end{bmatrix}\dot{\mathbf{q}}_s \tag{8}$$

Substituting Eq. (8) into (2), the kinetic energy during stance phase yields

$$\mathbf{K}_s = \frac{1}{2}\dot{\mathbf{q}}_s^T \begin{bmatrix} \mathbf{I}_{n\times n} \\ \frac{\partial f_{cm}}{\partial \mathbf{q}_s} \end{bmatrix}^T \mathbf{D}_f(\mathbf{q}_b) \begin{bmatrix} \mathbf{I}_{n\times n} \\ \frac{\partial f_{cm}}{\partial \mathbf{q}_s} \end{bmatrix}\dot{\mathbf{q}}_s = \frac{1}{2}\dot{\mathbf{q}}_s^T \mathbf{D}_s(\mathbf{q}_b)\dot{\mathbf{q}}_s \tag{9}$$

where

$$\mathbf{D}_s(\mathbf{q}_b) = \mathbf{M}(\mathbf{q}_b) + m\left(\frac{\partial f_{cm}}{\partial \mathbf{q}_s}\right)^T \frac{\partial f_{cm}}{\partial \mathbf{q}_s}$$

Substituting Eq. (8) into Lagrange's equation yields

$$\mathbf{D}_s(\mathbf{q}_b)\ddot{\mathbf{q}}_s + \mathbf{H}_s(\mathbf{q}_b, \dot{\mathbf{q}}_s)\dot{\mathbf{q}}_s + \mathbf{G}_s(\mathbf{q}_s) = \mathbf{\Gamma}_s \qquad (10)$$

where $\mathbf{H}_s(\mathbf{q}_b, \dot{\mathbf{q}}_s)$ contains Coriolis and centrifugal terms

$$\mathbf{H}_s(\mathbf{q}_b, \dot{\mathbf{q}}_s) = \frac{\partial\left[\mathbf{D}_s(\mathbf{q}_b)\right]}{dt} - \frac{\partial\left[\frac{1}{2}\dot{\mathbf{q}}_s^T \mathbf{D}_s(\mathbf{q}_b)\dot{\mathbf{q}}_s\right]}{\partial \mathbf{q}_s}$$

and $\mathbf{G}_s(\mathbf{q}_s)$ is the gravity vector

$$\mathbf{G}_s(\mathbf{q}_s) = \frac{\partial(mgy_{cm})}{\partial \mathbf{q}_s}$$

The general torques matrix is

$$\mathbf{\Gamma}_f = \begin{bmatrix}\tau_1 & \cdots & \tau_n\end{bmatrix}^T$$

2.3. Dynamics equation for landing impact phase

Two types of impact models are often applied in impact problems [16]. One is point contact which happens to perfectly plastic impact [17] and the corresponsive time is instantaneous. The other is elastic impact [18] taking account of the coefficient of restitution e. During the impact process, it can be classified into external and internal impact. The external impact between the robot and environment is more effective and worthy than the internal impact among robot's joints in landing impact phase. This paper mainly studies the elastic external impact dynamics from flight phase to landing impact phase.

Supposing the robot's configuration $\mathbf{q}_f$ is unchanged, and only the velocity and acceleration change instantaneously during impact. During flight phase, the position of robot's foot tip is

$$\begin{bmatrix} x_{foot} \\ y_{foot} \end{bmatrix} = \begin{bmatrix} x_{cm} \\ y_{cm} \end{bmatrix} + \sum_{k=1}^{n}\left[\mathbf{R}_{z,\theta_k}\left[\begin{matrix}\left[(1-\alpha_k)\xi_k + \left(1-\sum_{i=1}^{k}\xi_k\right)\right] l_k \\ 0\end{matrix}\right]\right]$$
$$- \mathbf{R}_{z,\theta_1}\begin{bmatrix}(1-2\alpha_1) l_1 \\ 0\end{bmatrix}$$
$$= f_{foot}(\mathbf{q}_f) \qquad (11)$$

Using Jacobian matrix, yields

$$\begin{bmatrix} \dot{x}_{foot} \\ \dot{y}_{foot} \end{bmatrix} = \mathbf{J}\dot{\mathbf{q}}_f \qquad (12)$$

where $\mathbf{J}$ is the Jacobian matrix of the foot end

$$\mathbf{J} = \frac{\partial\left[\begin{bmatrix} x_{cm} \\ y_{cm} \end{bmatrix} + f_{foot}(\mathbf{q}_f)\right]}{\partial \mathbf{q}_f}$$

The virtual work done by the general torque $\mathbf{\Gamma}_\delta$ in the virtual displacement $\delta \mathbf{q}_f$ is $\delta \mathbf{w} = \delta \mathbf{q}_f \cdot \mathbf{\Gamma}_\delta$. The relationship between the general torque and the impact external force $\mathbf{F}_{ext}$ at the colliding foot tip is $\mathbf{\Gamma}_\delta = \mathbf{J}^T \mathbf{F}_{ext}$. Using the dynamics model in flight phase before landing impact phase, the impact dynamics model can be expressed as

$$\mathbf{D}_f(\mathbf{q}_b)\ddot{\mathbf{q}}_f + \mathbf{H}_f(\mathbf{q}_b, \dot{\mathbf{q}}_f)\dot{\mathbf{q}}_f + \mathbf{G}_f(\mathbf{q}_f) = \mathbf{\Gamma}_f + \mathbf{\Gamma}_\delta \qquad (13)$$

Integrating over the impact period Δt and carrying out the limiting process $\Delta t \to 0$ to the Eq. (13) yields

$$\lim_{\Delta t \to 0}\int_{t_0}^{t_0+\Delta t}\left[\mathbf{D}_f \ddot{\mathbf{q}}_f + \mathbf{H}_f \dot{\mathbf{q}}_f + \mathbf{G}_f\right]dt = \lim_{\Delta t \to 0}\int_{t_0}^{t_0+\Delta t}\left(\mathbf{\Gamma}_f + \mathbf{\Gamma}_\delta\right)dt \qquad (14)$$

However, the positions and velocities remain finite during impact, and when $\Delta t \to 0$, $\mathbf{\Gamma}_f$, $\mathbf{H}_f(\mathbf{q}_b, \dot{\mathbf{q}}_f)$ and $\mathbf{G}_f(\mathbf{q}_f)$ are equal to zero. Defined $\tilde{\mathbf{F}} = \lim_{\Delta t \to 0}\int_{t_0}^{t_0+\Delta t}\mathbf{F}_{ext}dt$, then the Eq. (14) becomes

$$\mathbf{D}_f\left[\dot{\mathbf{q}}_f^+ - \dot{\mathbf{q}}_f^-\right] = \mathbf{J}^T\tilde{\mathbf{F}} \qquad (15)$$

where the superscript "-" corresponds to the state before impact, and the superscript "+" corresponds to the state after impact.

Let's denote the velocity before impact as v_1 and v_2. Based on the elastic impact formula taking account of the coefficient of restitution e, yields

$$(\Delta v_1 - \Delta v_2)^T \vec{n} = -(1+e)(v_1 - v_2)^T \vec{n} \qquad (16)$$

where $\vec{n}$ is a normal vector.

The velocity increment of two contacting rigid bodies can be obtained by Eq. (15) and the following kinematic relationship

$$\Delta v_1 = \mathbf{J_1}\Delta\dot{\mathbf{q}}_f = \mathbf{J}_1\mathbf{D}_f^{-1}\mathbf{J}_1^T\tilde{\mathbf{F}} \qquad (17)$$

$$\Delta v_2 = \mathbf{J}_2\Delta\dot{\mathbf{q}}_f = \mathbf{J}_2\mathbf{D}_f^{-1}\mathbf{J}_2^T\tilde{\mathbf{F}} \qquad (18)$$

where $\mathbf{J}_1$ and $\mathbf{J}_2$ are Jacobian matrix of robot and environment respectively.

Substituting Eq. (17) and (18) into (16) yields

$$\left[\left(\mathbf{J}_1\mathbf{D}_f^{-1}\mathbf{J}_1^T \text{£}\ \mathbf{J}_2\mathbf{Ð}_f^{-1}\mathbf{J}_2^T\right)\tilde{\mathbf{F}}\right]^T \vec{n} = -(1+e)(v_1 - v_2)^T \vec{n} \qquad (19)$$

Simplifying the contact analysis, the friction on the contacting surface is negligible, impulse always acts along the normal vector $\vec{n}$. The contact force along vector $\vec{n}$ can be expressed as

$$\tilde{\mathbf{F}} = -\left[\vec{n}^T\left(\mathbf{J}_1\mathbf{D}_f^{-1}\mathbf{J}_1^T \text{£}\ \mathbf{J}_2\mathbf{Ð}_f^{-1}\mathbf{J}_2^T\right)^T \vec{n}\right]^{-1}(1+e)(v_1 - v_2)^T \vec{n} \qquad (20)$$

We study the impact between jumping robot and the ground in this paper, so $v_1 = \dot{\mathbf{q}}_f^-$, $v_2 = 0$ and $\mathbf{J}_2 = 0$. Using the Eq. (20) and (15) yields

$$\tilde{\mathbf{F}} = -\left[\vec{n}^T \mathbf{J}_1^T \mathbf{D}_f^{-1} \mathbf{J}_1 \vec{n}\right]^{-1} (1+e)\left(\mathbf{q}_f^-\right)^T \vec{n} \qquad (21)$$

$$\dot{\mathbf{q}}_f^+ = \mathbf{D}_f^{-1} \mathbf{J}_1^T \left\{-\left[\vec{n}^T \mathbf{J}_1^T \mathbf{D}_f^{-1} \mathbf{J}_1 \vec{n}\right]^{-1} (1+e)\left(\mathbf{q}_f^-\right)^T \vec{n}\right\} + \mathbf{q}_f^- \qquad (22)$$

2.4. Eliminating shock

The eliminating shock is always a problem in contact yield between robot and environment. For instance, when the rigid body model is landing on the ground, not only the foot toe but also all the joints experience impacts. The external impact can affect the posture stability, and the internal impact can destroy the robot machine structure. The shock during impact between robot and the ground influences the motion tracking and posture stability et al.

We can find the variables e, $\dot{\mathbf{q}}_f^-$ and $\vec{n}$ influence the external force from landing impact dynamics Eq. (21). Methods of eliminating shock and contact force may be summed up as follows

● Modulating the posture during flight phase. The posture includes the joint angles, and the value of vector $\mathbf{q}_f$ impacts the matrix $\mathbf{D}_f$, so the posture of jumping robot is a very important factor in impact. An appropriate posture can realize the soft landing.

● Reducing the velocity before impact. The velocity $\dot{\mathbf{q}}_f^-$ before impact affects the velocity $\dot{\mathbf{q}}_f^+$ after impact and the contact force $\tilde{\mathbf{F}}$. Reducing the value $\dot{\mathbf{q}}_f^-$, the contact force diminishes.

● Reducing the value of e. The coefficient of restitution concerns the impacting materials. The impact force increases by increasing the value e.

● Increasing the supposing area, and reducing the contact area of foot. The balance of humanoid robot can be improved through increasing the supposing area, and the robot can endure the tips. On the other hand, the impact force is related to the contact area between foot and environment. The impact force reduces if we reduce the contact area.

3 Trajectory Tracking Control

For trajectory tracking control [19], computed torque method is effective when the dynamics of system is accurately expressed [20]. However, there are some uncertainty factors in system model in fact. A variety of mechanism possess in nature parametric uncertainty or/and inaccuracy to some extent, such as a voluntary or unknown external force disturbance and variable loads etc. The exact mathematical model is difficult to be obtained from practical system. Aiming to an uncertainty problem, many researchers have studied some hybrid trajectory tracking control method uniting computed torque, such as variable structure sliding mode control (its contrary operation of inertia matrix is crucially complicated), learning control by neural networks as basis function, based on H_∞ robust control and model reference adaptive control et al.

Adaptive fuzzy control is introduced to jumping motion, and an adaptive law is obtained by uniting fuzzy control with computed torque method. The effective trajectory is designed in posture tracking control.

3.1. Fuzzy logic and fuzzy basis function

Considering a multi-input and single-output fuzzy system [21-24], its input is $\mathbf{x} = \left[x_1, x_2, \cdots, x_n\right]^T$, its output is $\mathbf{y}$. The fuzzy rule base consists of a set of linguistic rules in the form of "IF a set of conditions are satisfied, THEN a set of consequences are inferred." We consider the case where the fuzzy rule base consists of n rules in the following form

R_i: if x_1 is $\mathbf{A}_1^i$ and x_2 is $\mathbf{A}_2^i$ and $\cdots$ and x_n is $\mathbf{A}_n^i$ then $\mathbf{y}$ is $\mathbf{B}^i$

where the fuzzy set function of input x_i is $\mathbf{A}_j^i$, and its membership function is $\mu_{A_j^i}\left(x_i\right)$. The fuzzy set function of output $\mathbf{y}$ is $\mathbf{B}^i$, and its membership function is $\mu_{B^i}\left(\mathbf{y}\right)$.

Fuzzy system is structured of product inference engine, singleton fuzzifier, center average defuzzifier, and Gaussian membership function. So yielding

$$\mathbf{y} = f\left(\mathbf{x}\right) = \frac{\sum_{i=1}^{M} y^i \left[\prod_{i=1}^{n} \mu_{A_j^i}\left(x_i\right)\right]}{\sum_{i=1}^{M} \left[\prod_{i=1}^{n} \mu_{A_j^i}\left(x_i\right)\right]} = \mathbf{\Theta}_i^T \zeta\left(\mathbf{x}\right) \qquad (23)$$

3.2. Universal approximation theorem

For any continuous function $g\left(\mathbf{x}\right)$ on the compact set $U \in R^n$ and arbitrary $\varepsilon > 0$, there exists a fuzzy system $f\left(\mathbf{x}\right)$ in the form of Eq. (24) such that

$$\sup_{x \in U} \left|g\left(\mathbf{x}\right) - f\left(\mathbf{x}\right)\right| < \varepsilon \qquad (24)$$

Universal approximation theorem shows that the fuzzy system can approximate to a given function in arbitrary accuracy. It supplies a proof to issue the nonlinear modeling by using fuzzy system.

3.3. Design of adaptive fuzzy controller

We can find that the jumping robot have a type like Eq. (25) in flight phase and stance phase through the dynamics analysis in section 2.

$$\mathbf{D}\ddot{\mathbf{q}}+\mathbf{H}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{G}(\mathbf{q})=\boldsymbol{\tau} \tag{25}$$

Supposing the tracking error equation $\mathbf{e}=\mathbf{q}-\mathbf{q}_d$, and where $\mathbf{q}_d$ is the pre-defined reference trajectory. When the dynamics model is accurate, then the rule of trajectory tracking control can be denoted by computed torque methodology without unknown perturbation.

$$\boldsymbol{\tau}_a=\mathbf{M}_a(\mathbf{q})(\ddot{\mathbf{q}}_d-\mathbf{K}_v\dot{\mathbf{e}}-\mathbf{K}_p\mathbf{e})+\mathbf{H}_a(\mathbf{q},\dot{\mathbf{q}}) \tag{26}$$

Where the subscript a denotes exact model matrix, K_v and K_p are constant matrix of controller. Generally, they are diagonal matrix.

Substituting Eq. (26) into (25), the error equation can be expressed as

$$\ddot{\mathbf{e}}-\mathbf{K}_v\dot{\mathbf{e}}-\mathbf{K}_p\mathbf{e}=0 \tag{27}$$

However, most of the dynamics models have some uncertainty and inaccuracy by the mathematic modeling methods. Suppose the uncertainty perturbations of jumping robot are $\Delta\mathbf{D}(\mathbf{q})$, $\Delta\mathbf{H}(\mathbf{q})$ and $\Delta\mathbf{G}(\mathbf{q})$ in dynamics equations. Substituting Eq. (26) into (25), yields

$$\ddot{\mathbf{e}}-\mathbf{K}_v\dot{\mathbf{e}}-\mathbf{K}_p\mathbf{e}=\mathbf{M}_a^{-1}[\Delta\mathbf{M}\ddot{\mathbf{q}}+\Delta\mathbf{H}]=\rho(\mathbf{x}) \tag{28}$$

where $\rho(\mathbf{x})$ is centralized uncertain part of robot system.

In this paper, we study the hybrid control system of both computed torque and fuzzy compensatory controller, and use the control method to track the designed trajectory. The computed torque is defined as Eq. (26). By universal approximation theorem, we design adaptive fuzzy compensatory torque controller $\boldsymbol{\tau}_c=-\mathbf{M}_c(\mathbf{q})\hat{\rho}(\mathbf{x})$, and use adaptive fuzzy to approximate to the centralized uncertainty $\rho(\mathbf{x})$, where $\hat{\rho}(\mathbf{x})$ is the fuzzy approximated estimating value of $\rho(\mathbf{x})$. So the hybrid controller can be written as

$$\boldsymbol{\tau}=\boldsymbol{\tau}_a+\boldsymbol{\tau}_c \tag{29}$$

3.4. Adaptive fuzzy control algorithm

Defining the fuzzy approximated estimating value of centralized uncertainty as

$$\hat{\rho}(\mathbf{x}|\mathbf{W})=\mathbf{W}\boldsymbol{\zeta} \tag{30}$$

where $\mathbf{W}$ is a power matrix, and $\boldsymbol{\zeta}$ is a vector of fuzzy basis function.

The optimization value $\mathbf{W}^*$ of $\mathbf{W}$ is a constant matrix, and satisfies with

$$\mathbf{W}^*=\arg\min_{W\in\Omega_f}\left[\sup_{x\in U}\left|\hat{\rho}(\mathbf{x}|\mathbf{W})-\rho(\mathbf{x})\right|\right] \tag{31}$$

The robot's centralized uncertainty can be denoted by fuzzy system as

$$\rho(\mathbf{x}|\mathbf{W})=\mathbf{W}^*\boldsymbol{\zeta}+\boldsymbol{\varepsilon} \tag{32}$$

where $\boldsymbol{\varepsilon}$ is a remnant error.

Substituting the total control law Eq. (32) into perturbation system Eq. (28), the closed system error can be expressed as

$$\ddot{\mathbf{e}}-\mathbf{K}_v\dot{\mathbf{e}}-\mathbf{K}_p\mathbf{e}=\mathbf{W}^*\boldsymbol{\zeta}+\boldsymbol{\varepsilon} \tag{33}$$

Selected the state variable $\mathbf{x}=\left[\mathbf{e}^T,\dot{\mathbf{e}}^T\right]^T$ and $\mathbf{e}^T=[e_1,e_2,\cdots,e_n]$, the Eq. (33) yields

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}(\tilde{\mathbf{W}}\boldsymbol{\zeta}+\boldsymbol{\varepsilon})=\begin{bmatrix}0 & I\\ -\mathbf{K}_p & -\mathbf{K}_v\end{bmatrix}\mathbf{x}+\begin{bmatrix}0\\ \mathbf{I}\end{bmatrix}(\tilde{\mathbf{W}}\boldsymbol{\zeta}+\boldsymbol{\varepsilon}) \tag{34}$$

Theorem 3.1. *Selected an adaptive control law, the closed control system is asymptotically stable when the hybrid jumping robot is controlled byhybrid model of computed torque and adaptive fuzzy controller. Where* $\tilde{\Gamma}_i$ *is a scalar value, and* $\tilde{\boldsymbol{\Gamma}}=diag(\tilde{\Gamma}_1,\tilde{\Gamma}_2,\cdots,\tilde{\Gamma}_n)$. *The matrix* $\mathbf{P}$ *is the solution of the following Riccati-like equation*

$$\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}+\mathbf{Q}+\mathbf{P}\mathbf{B}\mathbf{B}^T\mathbf{P}=0 \tag{35}$$

Proof. Let us choose a Lyapunov function

$$V=\mathbf{x}^T\mathbf{P}\mathbf{x}+Tr(\tilde{\mathbf{W}}^T\tilde{\boldsymbol{\Gamma}}\tilde{\mathbf{W}})$$

where the symbol "Tr" denotes the matrix rank.

The time derivative of V is

$$\begin{aligned}\dot{V}&=\mathbf{x}^T(\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A})\mathbf{x}+2\mathbf{x}^T\mathbf{P}\mathbf{B}\rho+2\mathbf{Tr}(\dot{\tilde{\mathbf{W}}}^T\tilde{\boldsymbol{\Gamma}}\tilde{\mathbf{W}})\\&=-\mathbf{x}^T(\mathbf{P}^T\mathbf{B}\mathbf{B}^T\mathbf{P}+\mathbf{Q})\mathbf{x}+2\mathbf{x}^T\mathbf{P}\mathbf{B}\boldsymbol{\varepsilon}+\\&\quad 2\mathbf{x}^T\mathbf{P}\mathbf{B}\tilde{\mathbf{W}}\boldsymbol{\zeta}-2\mathbf{Tr}(\boldsymbol{\zeta}\mathbf{x}^T\mathbf{P}\mathbf{B}\tilde{\mathbf{W}})\end{aligned}$$

By the fact $\mathbf{x}^T\mathbf{P}\mathbf{B}\tilde{\mathbf{W}}\boldsymbol{\zeta}=Tr(\boldsymbol{\zeta}\mathbf{x}^T\mathbf{P}\mathbf{B}\tilde{\mathbf{W}})$, the above equation becomes

$$\begin{aligned}\dot{V}&=-\mathbf{x}^T(\mathbf{P}^T\mathbf{B}\mathbf{B}^T\mathbf{P}+\mathbf{Q})\mathbf{x}+2\mathbf{x}^T\mathbf{P}\mathbf{B}\boldsymbol{\varepsilon}\\&=-\mathbf{x}^T\mathbf{Q}\mathbf{x}-(\mathbf{B}^T\mathbf{P}\mathbf{x}-\boldsymbol{\varepsilon})^T(\mathbf{B}^T\mathbf{P}\mathbf{x}-\boldsymbol{\varepsilon})+\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}\\&\le-\mathbf{x}^T\mathbf{Q}\mathbf{x}+\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}\end{aligned}$$

Integrating the above equation from $t=0$ to $t=T$ yields

$$\int_0^T\dot{V}dt=V(T)-V(0)\le\int_0^T\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}dt-\int_0^T\mathbf{x}^T\mathbf{Q}\mathbf{x}dt<\infty$$

Since $V(T)\ge0$, the above inequality implies the following inequality

$$\int_0^T\mathbf{x}^T\mathbf{Q}\mathbf{x}dt\le V(0)+\int_0^T\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}dt$$

And the above inequality is equivalent to the following

$$\int_0^T \|\tilde{\mathbf{x}}\|^2 dt \le \frac{V(0)+\int_0^T \boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon} dt}{\lambda_{\min}(\mathbf{Q})}$$

where $\lambda_{\min}(\mathbf{Q})$ is the minimal eigenvalue of matrix $\mathbf{Q}$. By the Barbalet lemma, $\lim_{t\to\infty}\tilde{\mathbf{x}}(t)=0$ when $\mathbf{x}\in L_2$. Hence, the closed control system is asymptotically stable when the hybrid jumping robot is controlled by computed torque and adaptive fuzzy controller.

□

4 Simulations of Trajectory Tracking Control

To verify the feasibility of the dynamics model and suggested control scheme, we designed a two-joint and three-rigidbody jumping robot (foot, crus and thigh) with rotate joints. The inertia parameters are given in table 1. The mechanism is independently driven by servo motor for each joint. The type of servo motors is Tower Pro MG995, which made in Taiwan of China. Its rated velocity is $0.13s/60^\circ(6.0V)$, viz. $8.05rad/s$, and its rated torque is $11Kg\cdot cm$.

Table1. Inertial parameters of the hopping robot

i	l_i (m)	α_i	m_i (kg)	I_i ($kg\cdot m^2$)	τ_i ($kg\cdot cm$)
1	0.06	0.5	0.01	3.75×10^{-5}	11
2	0.11	0.4	0.08	6.67×10^{-4}	11
3	0.16	0.6	0.26	6.67×10^{-4}	none

The referenced tracking trajectory is $\mathbf{q}_d=[\sin(t/10),\cos(t/10)]$ with initial state selected as $q_1(0)=1$, $q_2(0)=0$, $\dot{q}_1(0)=0$, $\dot{q}_2(0)=0$. Select $\mathbf{K}_v=diag(1,1)$ and $\mathbf{K}_p=diag(2,2)$, the membership functions are

$$\mu_{F_i^1}(x_i)=\frac{1}{1+\exp[-5(x_i+0.6)]},$$

$$\mu_{F_i^7}(x_i)=\frac{1}{1+\exp[-5(x_i-0.6)]},$$

$$\mu_{F_i^2}(x_i)=\exp[-5(x_i+0.4)],$$

$$\mu_{F_i^6}(x_i)=\exp[-5(x_i-0.4)],$$

$$\mu_{F_i^3}(x_i)=\exp[-5(x_i+0.2)],$$

$$\mu_{F_i^5}(x_i)=\exp[-5(x_i-0.2)],$$

$$\mu_{F_i^4}(x_i)=\exp[-0.5x_i^2].$$

$i=1,2,3,4$

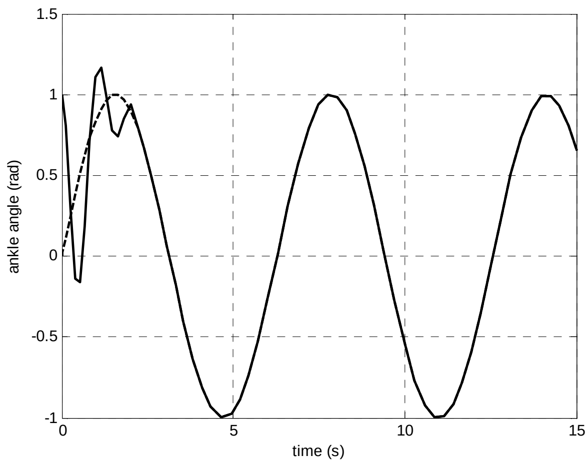
So the fuzzy basis function is

$$\zeta_i(\mathbf{x})=\frac{\mu_{F_1^i}(x_i)\mu_{F_2^i}(x_i)\mu_{F_3^i}(x_i)\mu_{F_4^i}(x_i)}{\sum_{j=1}^{7}\mu_{F_1^j}(x_i)\mu_{F_2^j}(x_i)\mu_{F_3^j}(x_i)\mu_{F_4^j}(x_i)}$$

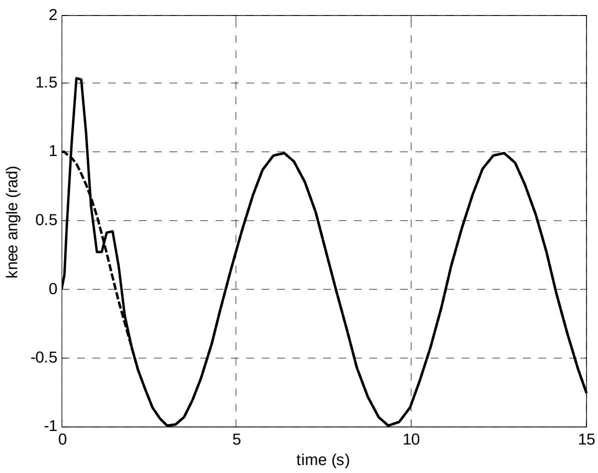
Supposing $\mathbf{Q}=diag[10,10,10,10]$ and $\boldsymbol{\Gamma}=diag[8,8]$, the solution of Riccati-like Eq. (35) can be expressed as

$$\mathbf{P}=\begin{bmatrix}12.2396 & 0.0000 & 1.7417 & 0.0000\\ 0.0000 & 12.2396 & 0.0000 & 1.7417\\ 1.7417 & 0.0000 & 2.8057 & 0.0000\\ 0.0000 & 1.7417 & 0.0000 & 2.8057\end{bmatrix}$$

During jumping motion, the angular joint trajectory applied computed torque controller to accurate dynamics model is shown in Fig 2. It shows when the dynamics model of jumping robot is accurate and without uncertainty, the actual trajectory can track the referenced trajectory very well under computed torque controller.



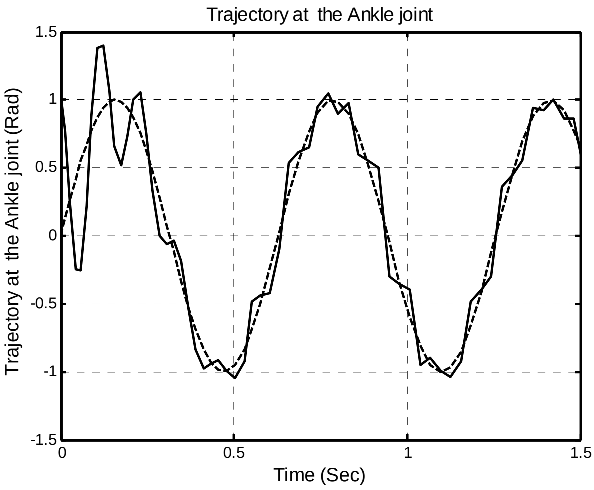
(a)



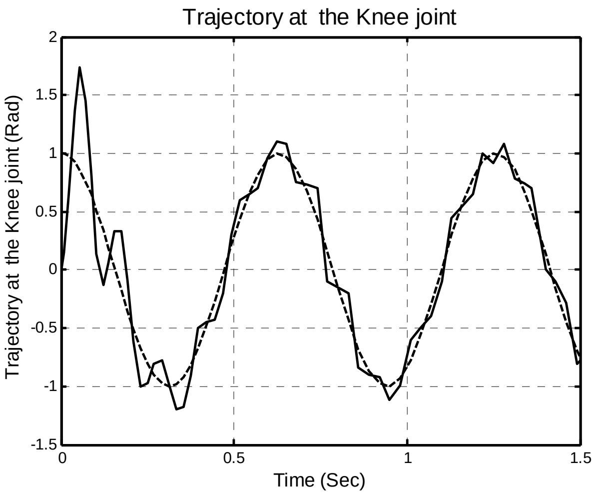
(b)

Fig.2. Angular joint trajectory applied computed torque controller to accurate dynamics model. (a) Angular joint trajectory at ankle joint; (b) Angular joint trajectory at knee joint. The solid line is the actual trajectory of ankle joint, and the dashed line is the referenced trajectory.

However, when the dynamics model of jumping robot is inaccurate and with uncertainty, Fig. 3 shows the actual trajectory doesn't track the referenced trajectory very well with computed torque controller.



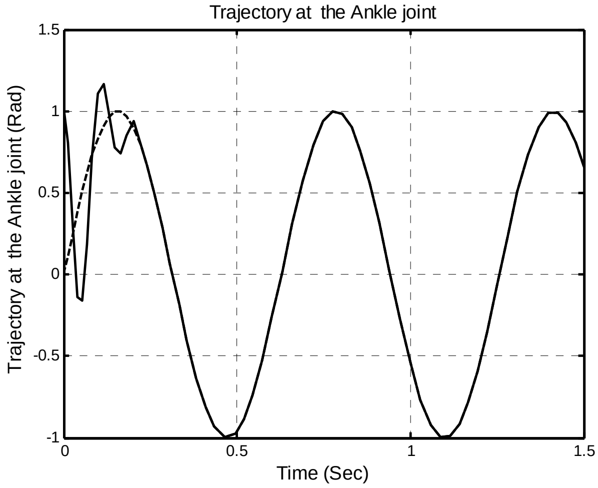
(a)



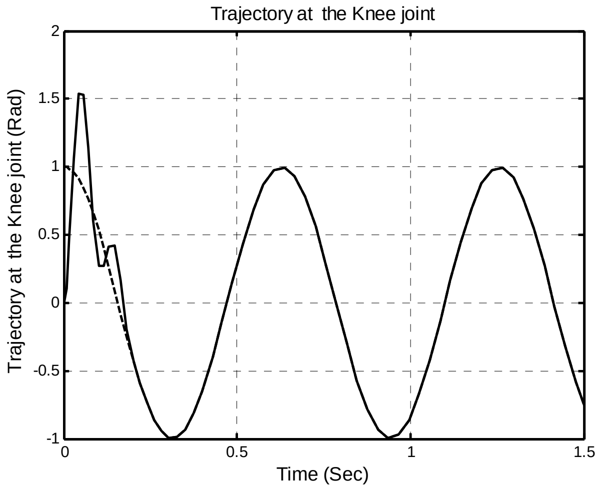
(b)

Fig.3. Angular joint trajectory applied computed torque controller to the dynamics model with uncertainty. (a) Angular joint trajectory at ankle joint; (b) Angular joint trajectory at knee joint. The solid line is the actual trajectory of ankle joint, and the dashed line is the referenced trajectory.

Although the track error under computed torque controller is decreasing gradually on time increase, the actual values always fluctuate along the referenced trajectory. In this case, the actual trajectory can also track the referenced trajectory very well and quickly with hybrid control system of both computed torque and fuzzy compensatory controller as shown in Fig. 4. It also shows the robustness of uncertainties system is improved by the hybrid controller.



(a)

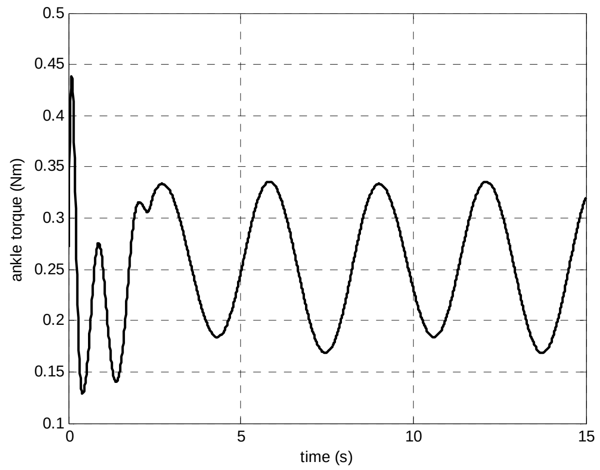


(b)

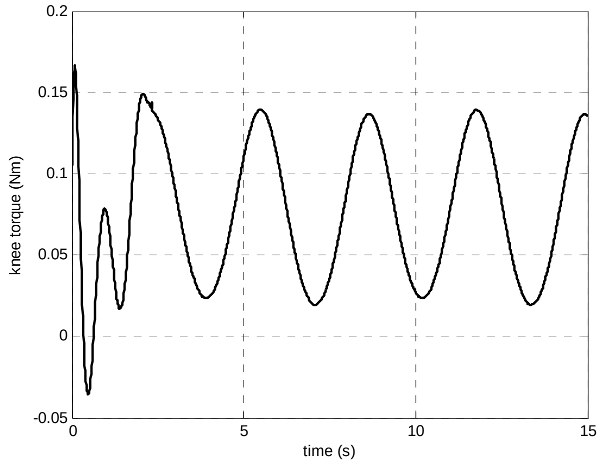
Fig.4. Angular joint trajectory applied hybrid model of computed torque and fuzzy compensatory controller to uncertainty dynamics model. (a) Angular joint trajectory at ankle joint; (b) Angular joint trajectory at knee joint. The solid line is the actual trajectory of ankle joint, and the dashed line is the referenced trajectory.

The simulation results reflect the computed torque controller is valid for accurate model, and it is invalid for the dynamic model with uncertainty. However, most mathematical models are inaccurate, and the systems are with uncertainty. The hybrid control system of both computed torque and fuzzy compensatory controller is valid and robust for the system with uncertainties.

Furthermore, Fig. 5 shows the torques of both ankle joint and knee joint satisfies all requirements of motor performance.



(a)



(b)

Fig.5. Torques trajectory under hybrid control system. (a) Torque trajectory at ankle joint. (b) Torque trajectory at knee joint.

5 Conclusions

Walking, jumping and running are the basic motion modes of human being. The walking belongs to a regular motion, while the jumping or running is a non-regular motion in humanoid robot field. Humanoid jumping motion includes stance phase, flight phase and landing impact phase. Jumping robot belongs to a variable constraints system because every phase has different constraint conditions. Stance phase has only the holonomic constraints, nevertheless flight phase has the holonomic and non-holonomic constraints. In this paper, the unified dynamics models of jumping robot including flight phase, stance phase and landing impact phase are achieved in detail. A method to compute impact force and velocity is obtained by Jacobian matrix. Furthermore, eliminating shock was analyzed. The methods of eliminating shock can help us to design and control the jumping robots. Finally, trajectory tracking control using the hybrid model of adaptive fuzzy control and computed torque was studied on models with parametric uncertainties. The hybrid control system proves to be asymptotical stabilization by Lyapunov stability theory. Numerical simulation and experimental results show computed torque controller is valid for accurate modeling, and it is invalid for the dynamic modeling with uncertainty. The hybrid system of both computed torque and fuzzy compensatory controller is valid for the system with uncertainties. The unified dynamics models adapts to the space robot in state of microgravity or weightlessness. The hybrid control is also available in posture and stability control for non-regular motion.

Furthermore, the contents of dynamics and control in jumping robot are very wide. In this paper, we study only dynamics and trajectory tracking control. In the future, we will develop the postural stability, softy landing with optimal energy, path or gait stability with external disturbances, and test the control arithmetic in experiment.

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