Construction of Virtual Backbone on Growth-Bounded Graph with Variable Transmission Range

Yanjing Sun, Xiangping Gu, Jiansheng Qian School of Information and Electrical Engineer China University of Mining and Technology Xuzhou, Jiangsu 221008 CHINA Yanjingsun_cn@hotmail.com

Abstract: - Virtual backbone has been used extensively in various aspects for wireless ad hoc or sensor networks recently. We propose an approximation solution to construct a virtual backbone based on a more generalized and realistic model of polynomial bounded growth. A localized distributed algorithm of MCDS_GBG for computing a Minimum Connected Dominating Sets as backbone in the growth-bounded graph is presented. This approach consists of three stages: firstly construct an MIS by network decomposition scheme; secondly compute a minimum dominating set in 2-separated collection with r transmission range and finally use Marking process and ruling k to reduce the virtual backbone with 3r transmission range. The computed Connected Dominating Set guarantees a constant stretch factor on the length of a shortest path and induces a subgraph of constant degree while the nodes only require direct neighborhood information. The efficiency of our approach is confirmed through both theoretical analysis and comparison study.

Key-Words: - Virtual backbone; Growth-bounded graph; Connected dominating sets; Maximal independent sets; Wireless ad hoc sensor network; Network decomposition

1 Introduction

Recently, the use of a virtual backbone in various applications in wireless ad hoc network has become popular [1-6]. These applications include topology control, point and area coverage in sensor networks, and routing protocol design. Researchers have proposed the use of a virtual backbone in the network as an alternative to a fixed routing infrastructure. Nodes in the virtual backbone act as a connected skeleton for the entire network. A recent overview can be found in [7]. When modeling the network as a graph, the most widely used concept for defining a backbone is the Connected Dominating Set (CDS). A dominating set (DS) [8]is a subset of nodes in the network where every node is either in the subset or a neighbor of a node in the subset. A CDS is a connected subset of the network nodes such that any node in the network is either part of the CDS or has a neighbor in the CDS.

A Connected Dominating Set is a natural candidate for virtual backbone infrastructure in ad hoc networks. For a virtual backbone to be effective, the underlying CDS must be small in size, have a low stretch (i.e., preserve the shortest paths in the original network), and must be computable using fast distributed local control algorithms. Since a maximal independent set (MIS) is a dominating set

and it is easy to construct, one usually constructs a maximal independent set at the first step.

Of the solutions to construct a virtual backbone in wireless networking, the usually used model of graph is unit disk graphs (UDG) [13]. While a UDG is too optimistic, the general graph (GG) [11] is often too pessimistic, as the connectivity of most networks is not arbitrary but obeys certain geometric constraints [14]. Since the growthbounded graph (GBG) model reflects reality quite well and is appropriate in many situations, related researches[10, 15-17]on approximation algorithms maximal of constructing independent set. dominating and minimum sets. connected dominating sets (MCDS) [26] on GBG model have been done recently without considering effective clustering or backbone formation. Proposed clustering and backbone formation methods [6, 18-21]mostly use the UDG model. Here, we propose a distributed algorithm for finding an approximation of a Minimum connected dominating sets to construct a virtual backbone in the growth-bounded graph. This approach constructs a MCDS by three phases: firstly construct an MIS by network decomposition; secondly find a minimum dominating set and finally use Marking process and ruling K to optimize the virtual backbone.

The paper is structured in the following way. Section 2 introduces some notation and preliminaries. A distributed algorithm to computer MCDS in growth-bounded graph is present in the section 3. Section 4 theoretically analyzes the proposed algorithm and makes comparison with others. The paper is conclude in section 5

2 Definition and model

The growth-bounded graph captures the intuitive notion that if many nodes are located close from each other, many of them must be within mutual transmission range. In graph theoretical terms, a graph is growth-bounded if the number of independent nodes in a node's *r*-neighborhood is bounded. We have the following definition.

Definition 1. (Growth-Bounded Graph) An undirected graph G=(V, E) is called a growthbounded if there exists a polynomial bounding function f(r) such that for every $v \in V$ and $r \ge 0$, the size of any maximal independent set in the *r*neighborhood $\Gamma_r(v)$ is at most f(r). Further, we say that *G* has polynomially bounded growth if f(r) is a polynomial p(r).

Note that f(r) does not depend on the number of vertices in the graph, but on the radius of the neighborhoods only.

We model a wireless network as a growthbounded graph. Each node has a unique identifier and knows which nodes are within its transmission range[29]. When we say that a distributed algorithm computes a CDS, we mean that each node knows after executing its code whether it is part of the CDS or not. In the following, we need to introduce some notion and preliminaries for our description and analysis.

Let G = (V, E) be an undirected connected graph. For $V' \subseteq V$, we denote by G[V'] the subgraph of G induced by V': the vertex set of G[V'] is V' and the edge set consists of the edges of G with both endpoints in V'. The distance d(v, w) between two vertices $v, w \in V$ is the number of hops that must be traversed to go from v to w on a shortest path. Furthermore, we denote by $\Pi(v)$ the closed neighborhood of a vertex $v \in V$, i.e. $\Gamma(v) := \{u \in V \mid v \in V \mid v \in V \mid v \in V\}$ $d(v, w) \leq 1$ $\cup \{v\}$. Analogously, for $S \subset V$, let $\Gamma(S):=\bigcup_{s\in S} \Gamma(s)$ define the neighborhood of S. Likewise, the *i*-neighborhood of a set of vertices S is defined recursively as $\Gamma_1(S) := S$, and $\Gamma_1(S) := \Gamma(\Gamma_1)$ $_1(S)$) for $i \ge 1$. A maximal independent set M for a given graph G is a subset $M \subset V$ such that for every v, $w \in M$ we have $v \notin \Gamma(w)$ and furthermore no superset $M' \supset M$ with the latter property exists. For a subset $S \subseteq V$, MIS(S) is a maximum independent set on the induced subgraph G[S]. A dominating set (DS) D for a G is a subset $D \subseteq V$ such that for every $v \in V$ there is a $w \in D$ such that $v \in \Gamma(w)$.

Let $S \subseteq V$ be a subset of the nodes of a graph G =

(*V*, *E*). *S* is called *r*-ruling if for each node $u \in V \setminus S$, the distance to the closest node in *S* is at most *r*. If the set *S* in the above definition is an independent set, we speak of an r-ruling independent set [22]. For instance, a MIS is a 1-ruling independent set.

The concept of a 2-separated collection of subsets is introduced in [9]. For a graph G = (V, E), let $S := \{S_1, \ldots, S_k\}$ be a collection of subsets of vertices $S_i \subset V$, $i = 1, \ldots, k$, with the following property: for any two vertices $s \in S_i$ and $\bar{s} \in S_j$ with $i \neq j$, it is $d(s, \bar{s}) > 2$. We refer to *S* as a 2-separated collection of subsets.

For local neighborhoods, and subsets of the vertices in general, we now define local or partial solutions. Let P(V) denote the set of all subsets of vertices in G, and let $D: P(V) \rightarrow P$ be a function that returns for a set V' a minimum cardinality dominating set in G. In the following, we denote by D_{opt} an optimal solution to the MDS problem on G, in other words, $D_{opt} = D(V)$. The function D(.) is always computed with respect to the entire underlying graph G = (V, E). It may thus include vertices from outside the argument subset in its returned solution, i.e. , for a subset $V' \subset V$ the inclusion $D(V) \subseteq V'$ needs not to hold. [6]shows that the sum of the cardinalities of minimum dominating sets $D(S_i)$ for the subsets $S_i \in S$ of a 2separated collection forms a lower bound on the cardinality |D(V)| of a minimum dominating set in *G*.

Consider a subset $W \subseteq V$ of vertices called clusterheads. A clustergraph *G* of radius *c* is then given by introducing edges in the *c*-hop neighborhoods around $w \in W$. In this context, the vertices in *W* are used to identify each cluster. In other words, a clustergraph G = (V, E) can be described by the set V := W of vertices, and the edges are characterized by $(u, v) \in E \Leftrightarrow d_G(u, v) \leq c$ for any two $u, v \in W$.

3 MCDS on GBG algorithm

In the section, we present the algorithm of MCDS_GBG that yield a local approach to constructing a ρ -approximate connected dominating set in a connected graph. Inspired by [2, 23], the basic idea is to first reduce the network density

through clustering using a short transmission range. Neighboring clusterheads (clusterheads that are 2 or 3 hops away) are connected using a long (and normal) transmission range.

Algorithm: MCDS_GBG

Input: Connected Graph G = (V, E) of bounded growth, $\varepsilon > 0$, $\tilde{c} := c(\varepsilon) + 2$

Output: $(1+\varepsilon)$ -approximation Minimum Connected Dominating Set D

S1: Computer maximal independent set *I*;

S2: Use MIS *I* to construct cluster graph \overline{G} ;

- S3: Color \overline{G} using $\Delta_{\overline{G}} + 1$ colors;
- S4: $D := \phi$;

S5: for k = 1 to $\Delta_{\overline{G}} + 1$ do

S6: for each $v \in I$ with color k do in parallel

S7: while $\Gamma(v) \cap V \neq \phi$ do

S8: For some $u \in \Gamma(v) \cap V$, compute minimum dominating set $D_{\bar{r}}$ of $\Gamma_{\bar{r}}(v) \cap V$ such that $|D_{\bar{r}+2}| \leq (1+\varepsilon)|D_{\bar{r}}|$;

S9: Inform $\Gamma_{\tilde{c}}(v)$ about \bar{r} and $D_{\bar{r}+2}$;

S10: $D := D \cup D_{\bar{r}+2}(u);$

S11: $V := V \setminus \Gamma_{\overline{r}+2}(u)$;

End do

End for

End for

S12: Use D as clusterhead set and applies Marking process and self-pruning rule k in D; End

3.1 MIS algorithm

The distributed MIS construction for growthbounded graphs firstly computes a 3-ruling independent set and then induces an (O(1), O(1))decomposition [10, 22, 24]]which can be used to finally extend S' to a maximal independent set. Network decomposition is a very basic structure which can be used as the basis of distributed algorithms for a huge number of problems. A (d(n),c(n))-network decomposition of a graph G = (V, E)is a partition of V in disjoint clusters, such that the subgraph induced by each cluster is connected, the diameter of each cluster is in d(n), and the chromatic number of the resulting cluster graph is in c(n), where the cluster graph is obtained by contracting each cluster into a single node [5].

We apply algorithms from [24] to create a 3ruling independent set *S'* and turn *S'* into a MIS. Set *S'* induces a natural clustering of the nodes of *G*. For each node $u \in S'$, we define the cluster C_u to be the set of all nodes $v \in V$ for which *u* is the nearest node of *S'*. The cluster graph $G_{S'}$ induced by *S'* is then defined as follows. The node set of $G_{S'}$ is the set of clusters $\{C_u \mid u \in S_f^{\gamma}\}$. The clusters C_u and C_v are connected by an edge in $G_{S'}$ if and only if there are nodes $u' \in C_u$ and $v' \in C_v$ which are neighbors in the network graph G. Because S is a 3-ruling set, the distance between the centers u and v of two neighboring clusters C_u and C_v can be at most 7. The degree of $G_{S'}$ is therefore bounded by f(7) = O(1) if G is f-growth-bounded. Our MIS algorithm firstly compute $G_{S'}$ and color $G_{S'}$ with f(7) + 1 colors, resulting in a (O(1), O(1))-decomposition of G. Applying algorithms from [22, 24], this can be achieved in $O(\log^* n)$ rounds using messages of size $O(\log n)$.

After computing the decomposition, a MIS *M* of *G* is constructed by sequentially computing the contributions from each color of the coloring of $G_{S'}$. For each node *v*, let x_v be the color of *v*'s cluster. Using the cluster colors and the node identifiers, we define a lexicographic order \prec on the set *V* such that for $u, v \in V, u \prec v$ if and only if $x_u < x_v$ or if $x_u = x_v \land ID(u) < ID(v)$. Each node now proceeds as follows:

Initially, we set M = S'. All nodes v of S' inform their neighbors about the joining of M by sending a JOIN(v) message. If a node u receives a JOIN(v) message from a neighbor v, it cannot join the MIS any more and therefore sends a COVERED(u) message to all neighbors. If a node v has not received a JOIN(u) message but has received a COVERED(u) from all $u \in \Gamma(v)$ for which $u \prec v$, it can safely join M. Note that all neigbors $w \in \Gamma(v)$ with $w \prec v$, would need to receive a COVERED(v) message from v before joining M. If a node v joins M, it informs its neighbors by sending a JOIN(v) message.

3.2 MDS construction

A locally optimal dominating set is computed by the central vertices of the MIS *I* created by the first stage. Using the same method as [9], we construct suitable subsets $T_i \subset V$, which contain a 2-separated collection $S_i \subset T_i$, i=1,...,k. corresponding sets $S_i \subset$ T_i together with a bound of $(1 \pm \varepsilon)$ for the local

 T_i together with a bound of $(1 + \varepsilon)$ for the local dominating sets $D(S_i)$ and $D(T_i)$, then the union of the respective local dominating sets also dominates the entire set of vertices, resulting in a global $(1+\varepsilon)$ approximation for the MDS. The basic idea of the construction is simple: we compute a local dominating set for a neighborhood of a vertex, and expand this neighborhood until we have formed sets S and $T \supset S$ which satisfy a desired bound. Then, we eliminate the current neighborhood and continue the same steps for the remaining graph. The algorithm works as follows. We start with an arbitrary vertex $v \in V$ and consider for r = 0, 1, 2,, the *r*-th neighborhoods $\Gamma_r(v)$. Starting with $\Gamma(v) = v$, we compute dominating sets of minimum cardinality for these neighborhoods as long as (1) holds.

 $|D(\Gamma_{r+2}(v))| > (1+\varepsilon) |D(\Gamma_r(v))| \tag{1}$

Denote by \overline{r} the smallest r for which (1) is violated.

For the dominating set, $\Gamma_{\bar{r}+2}(v)$ is removed from G, and $D_{\bar{r}+2}$ is added to the partial solution, before again going on with the remaining graph. Considering the $(\bar{r}+2)$ -neighborhoods is due to the fact that $D_r \subset \Gamma_r$ needs not hold, but $D_r \subset \Gamma_{r+1}$ is clearly satisfied. The combinations of these local subsets dominate the whole graph G and obey the desired approximation ratio. During the pre-processing part of Algorithm, the graph is clustered using balls of radius $2\tilde{c}$ around vertices that form a maximal independent set I in G. The set I can be created by a local algorithm presented above. The resulting cluster graph \overline{G} then has bounded degree, which allows for an efficient ($\Delta_{\overline{G}}$ +1)-coloring by a local algorithm [15].

3.3 MP and Rule k

After constructing a $(1+\varepsilon)$ -approximation MDS as clusters within the $r_1=r$ transmission range, we apply the $r_2=3r$ transmission range to connect clusterheads by which avoiding a selection process of gateway nodes as regular clustering approach [27]. Each clusterhead uses a transmission range of r_2 for MP [24] and Rule k [25]. Thus, the backbone is constructed based on clusterheads using a transmission range of r_2 . A transmission range of r_2 ensures that all neighboring clusterheads (i.e., clusterheads within 3 hops) are directly connected under a transmission range of 3r.

4 Analysis and comparison

From (1), let Γ_i , i=1,...,k, denote the respective neighborhoods when (1) is violated, i.e. $\Gamma_i = \Gamma_{\overline{r}_i+2}(v_i)$, for the collection of neighborhoods $\{\Gamma_1,...,\Gamma_k\}$, as [9], the union $D:=\bigcup_{i=1}^k D(\Gamma_i)$ forms a dominating set for the graph.

Lemma 1. Let D_{opt} denote an optimal solution to the Minimum dominating set on *G*, the $D \subset V$ computed by the algorithm satisfies $|D| \leq \rho |D_{opt}|, \rho = 1 + \varepsilon$.

Proof. As proven in [9], the subsets $\Gamma_{\bar{i}_i}(v_i)$, i=1,...k, created by the algorithm form a 2-

separated collection Γ' in *G*. Additionally, the criterion for stopping expanding the neighborhood guarantees that each pair of local dominating sets satisfies

$$|D(T_i)| \le \rho |D(\Gamma_{\overline{r_i}}(v_i))| (i=1,...,k)$$

Clearly, $|D| = |\bigcup_{i=1}^k D(\Gamma_i)| \le \sum_{i=1}^k |D(\Gamma_i)| \le \rho \sum_{i=1}^k |D(\Gamma_i)| \le \rho |D(\Gamma')| = \rho |D_{\text{opt}}|$

Lemma2. Let G = (V, E) be a graph of (polynomially) bounded growth, $I \subset V$ be an independent set on G, and c be a constant. Then, the maximum degree of the clustergraph \overline{G} given by the clusters $\Gamma_c(v), v \in I$, is bounded by O(f(c)).

Proof: According to the definition of neighborhood, for any two vertices $u, v \in V$, we denote by d(u, v) the shortest hop-distance between u and v. Using this, we call $\Gamma_r(v)$ the (closed) r-neighborhood of v, that is, for any $r \ge 0$, and $v \in V$, $\Gamma_r(v) := \{ u \in V \mid d(u; v) \le r \}$. Since G = (V, E) be a graph of (polynomially) bounded growth, $I \subset V$ be an independent set on G, the size of any independent set in the r-neighborhood $\Gamma_r(v)$ is at most f(r), for cluster graph \overline{G} with the constant c, obviously, Δ_G is bounded by O(f(c)).

The coloring of the cluster graph \overline{G} , due to its bounded degree (Lemma 2), takes $O(\Delta_{\overline{G}} \log^* n)$ rounds [9]. The maximum degree of \overline{G} depends on the constant $c = c(\varepsilon)$, since G has polynomially bounded growth, it is $\Delta_{\overline{G}} = O(1/\varepsilon^{O(1)})$, where the exponent of $1/\varepsilon$ depends on the polynomial bound of the graph itself. In while loop, the algorithm benefits from the fact that the different color cluster can be done completely parallel by the respective leaders of the same colors in the MIS. Note that every action performed is limited to the *c*-neighborhood of the respective vertices.

Theorem 1. Let G = (V, E) be a polynomially growth-bounded graph. Then, there exist local, distributed $(1+\varepsilon)$ approximation algorithms, $\varepsilon > 0$, for the Minimum Dominating Set problems on G. The number of communication rounds needed for the respective construction of the subsets is $O(T_{\text{MIS}}+\log^* n/\varepsilon^{O(1)})$.

Proof: The maximal independent set computation can be achieved deterministically in $T_{\text{MIS}} = O(\log\Delta\log^* n)$ [] communication rounds [21].The coloring procedure of the cluster graph \overline{G} , due to its bounded degree (Lemma 3), takes $O(\Delta_{\overline{G}} \log^* n)$ rounds as proven in [9]. Since $\Delta_{\overline{G}} = O(1/\varepsilon^{O(1)})$, we can therefore bound the number of communication rounds for the coloring part of the algorithm by $O(cf(1) \Delta_{\overline{G}}) = O(1/\varepsilon^{O(1)})$. Thus, the

Ν, Δ

D1

number	of	communication	rounds	needed	for	the

respectively: the name of the algorithm, the time

		*		-		
Algorithm	Time	Approximation	Stretch	Sync	Graph Model	Information
	Complexity	Ratio				
Wan et al.	O(n)	<i>O</i> (1)	O(n)	no	UDG	D1
Alzoubi et al.	O(n)	<i>O</i> (1)	<i>O</i> (1)	no	UDG	D1
Dai et al.	<i>O</i> (1)	O(n)	O(n)	no	undirected	D2
Wu	<i>O</i> (1)	O(n)	O(n)	no	directed	D2
Wu et al.	<i>O</i> (1)	O(n)	O(n)	no	undirected	D2
Kuhn et al.	$O(k^2)$	$O(k\Delta^{2/k}\log\Delta)$	O(n)	yes	undirected	Dk
Jia et al.	$O(\log n \log \Delta)$	$O(\log \Delta)$	O(n)	yes	undirected	D1
Dubashi et al.	$O(\log^2 n)$	$O(\log n)$	$O(\log n)$	yes	undirected	D1
CDSColor	$O(\Delta \log^2 n)$	$\tilde{O}(1)$	O(1)	ves	UDG	Ν. Δ

O(1)

 $O(1/\varepsilon^{\acute{O}(1)})$

ves

 $\tilde{O}(1)$

Table 1 Performance comparison of connected dominating sets algorithm for virtual backbone

MCDS GBG $O((1+TMIS + log^*n)/\varepsilon^{O(1)})$ $1+\varepsilon$ respective construction of the subsets is $O(T_{\text{MIS}})$ $+\log^* n/\varepsilon^{O(1)}$).

 $O(\Delta \log^2 n)$

Theorem 2. The marking process and the restricted versions of Rule k have the communication complexity $O(\Delta_{\overline{G}})$ and the computation complexity $O(\Delta_{\overline{\alpha}}^{2})$, where $\Delta_{\overline{\alpha}}$ is the maximum vertex degree in the network.

Combining theorems and lemmas, we obtain the main theorem:

Theorem 3. Communication complexity of the algorithm is $O((1+TMIS + \log^* n)/\varepsilon^{O(1)})$.

Let G'(V', E') be the graph induced by the CDS of the (growth-bounded) graph G = (V, E) computed by presented algorithm. The backbone graph G' has maximum degree $O(1/\varepsilon^{O(1)})$ and guarantees a $O(1/\varepsilon^{O(1)})$ stretch with G' as a backbone.

Lemma 3. Under the growth-bounded graph model, a MDS of $G(r_1)$ is a MCDS of $G(r_2)$, if $G(r_1)$ is connected and $r_2 \ge 3r_1$.

Proof: Let V' be a MDS of $G(r_1)$. An alternative definition of a MCDS is that any node pair in the network is connected via nodes in the MCDS (i.e., the backbone nodes). For any two nodes *u* and *v*, we can construct a path $(u, w_1, w_2, ..., w_l, v)$ in $G(r_2)$, such that $w_i \in V'$ for $1 \le i \le l$. Since $G(r_1)$ is connected, a path ($u = x_1, x_2, ..., x_l = v$) exists in $G(r_1)$. For each $x_i \ (1 \le i \le l)$, there is a corresponding $w_i \in V'$ that is either x_i itself or a neighbor of x_i . The distance between x_i and w_i is $d(x_i, w_i) \leq r_1$. The distance between w_i and w_{i+1} is $d(w_i, w_{i+1}) \leq d(w_i, x_i) + d(x_i, y_i)$ x_{i+1}) + $d(x_{i+1}, w_{i+1}) \leq 3r_1 \leq r_2$. Therefore, (u, w_1, w_2, w_2) ..., w_l , v) is a valid path in $G(r_2)$.

Theorem 4. The clusterhead set V', derived from G(r) via clustering, is a MCDS of G(3r).

Let G'(3r) be the subgraph of G(3r) derived from V'. Since MP and Rule k preserve a CDS, we have:

Corollary 1. *V*["] derived from the MP and Rule *k* is a MCDS of G'(3r).

In order to show MCDS GBG algorithm efficiency, we compare it with other algorithms.In the table I, the columns from left to right correspond the following aspects of the algorithms to

complexity of the algorithm, the worst case approximation ratio of the size of the CDS, the worst case stretch of the CDS, whether the CDS algorithm requires the network to be synchronized, the graph classes to which the guarantees of the CDS algorithm apply, and the information required at each node during the execution of the algorithm. Hence, it is an important aspect about our result that the nodes do not require any position or distance information. That is, the only information available at a node is the connectivity information to its neighbors. Table 1 show that our construction algorithm works well.

GBG

5 Conclusion

In this paper, we present a distributed algorithm for computing minimum connected dominating sets to construct a virtual backbone in the growth-bounded graph. In different stages of the proposed algorithm, we adjust the transmission range of clusterhead nodes to avoid selection process of gateway nodes for constructing a connected dominating set. This approach constructs an MIS by network decomposition scheme, computes a minimum dominating set for clustering by r transmission range and then uses marking process and selfpruning ruling K to optimize the virtual backbone by 3r transmission range. The algorithms run locally and computer a $(1+\varepsilon)$ -approximation MCDS while adding the guarantee that the computed CDS has constant stretch and constant degree. Especially, the nodes only require direct neighborhood information The time complexity is $O((1+TMIS + \log^* n)/\varepsilon^{O(1)})$, where $T_{\rm MIS}$ is the time required to compute a maximal independent set in the graph and n denotes the number of nodes. The efficiency of our approach is confirmed through both theoretical analysis and comparison study.

Interesting future extensions include designing fast distributed and local algorithms which incorporate the effect of network dynamics such as node mobility[29], node removal and node addition.

Both these extensions present fundamental analytical challenges and has tremendous practical significance for large scale ad hoc wireless sensor networks.

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