

Multimedia Interactive Environment for Study the Plane Analytical Geometry

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Abstract: - In this work will be presented the elaboration of an educational informatics system for studying plane analytical geometry elements. The achieved informatics system will be able to be used for teaching geometry, both in pre-university and in university education. Incorporating modern methods and techniques, the presented software will lead the user to obtain experience in understanding and handling the knowledge in geometry field and will grant easy and efficient acces to the newest information and knowledge.

Key-Words: - Interactive software, plane analytical geometry, Java, distance education.

1 Introduction

The interaction student-computer allows the diversification of the didactical strategy, facilitating the student's access to more ample information, more logically organised, variously structured, presented in different visualisation modes. In fact, not the computer itself as physical object, incorporating even multimedia configuration, produces immediate pedagogical effects, but the quality of the programs created and handled properly, of the informatics products, integrated by methodic efficiency criteria in the training activities [1], [2].

The efficiency of studying the geometry elements, applying the computer assisted training [3], [4], [5], depends on the methodological fundamenting degree of using this method, an important role having:

- the defining of optimal applying moments of the computer assisted training sequences;
- the adapting degree of the methodology elaborated at the disciplinary curriculum in mathematics;
- the application of the general computer assisted training principles;
- the flexibility degree of the educational programs and its application methodology against the educational level of the scholar population;
- the alternation of the computer assisted training sequences with traditional training sequences [6], [7].

1.1 Analysis of the existing training systems

Currently, in Romania there are national educational programs achieved within different education institutions, as well as a quite various series of foreign educational products. In each country the training process has a national character, either by the content of the study programs, or by the used didactical procedures, or even also by the study language [8], [9], [10].

Regardless the specific of the national teaching-learning-evaluation process, the modern educational programs should have a range of compulsory parameters:

- The completeness of the theoretical material (scientific veridicality, formulation's correctness, scholar program's suitability, historical knowldges, vocabulary's presence);
- Existance of the control block, control's performance method (test, interactive resolution, resolution by entering the response) and problems' selection;
- Teacher's possibility to form his personal course from the present theoretical material and managing the tests by using the existing database;
- Interactive resolution of problems, possibility of using help for resolution, error diagnosis;
- Simplesness to work with the program, with the separate components, program's aesthetic;
- Ergonomic qualities: on-screen information display (quantity and quality of the displayed information, number of windows opened simultaneously, pattern's quality and formulas'

representation, emphasizing the main elements by colours).

Each of the enumerated programs [11], [12], [13] in the next table satisfy at a larger or smaller extent the requirements previously enumerated, these being implemented in different programming languages and are destined for different geometry branches.

Crt. Nr.	Educational softwares	Authors	Geometry branches
1.	Geometria 8	Vasile Roman	- Space Geometry
2.	Java Applets on Mathematics	Walter Fendt	- Plane Geometry - Vectorial Geometry
3.	Cinderella	Jürgen Richter-Gebert, Ulrich Kortenkamp	- Euclidean Geometry - Projective Geometry
4.	The Geometry Applet	David E. Joyce	- Euclidean Geometry
5.	Geometria	GeoCentral	- Euclidean Geometry
6.	Pygeo	Arthur Siegel	- Euclidean Geometry - Projective Geometry
7.	Cabri Geometre II Plus	University „Joseph Fourier” of Grenoble	- Euclidean Geometry - Analytic Geometry
8.	Geogebra	Markus Hohenwarter	- Euclidean Geometry - Analytic Geometry
9.	EucliDraw	University of Crete	- Euclidean Geometry

Table 1 Educational software products

Analysing the software products, one can found as follows:

- The existing educational programs don't cover the entire curricular area of studying mathematics, especially in studying geometry by means of computer assisted training;
- Within programming elaboration for studying the geometry elements is necessary to take into account the optimal moments to apply the computer assisted training in studying mathematics: interactive resolution of problems and summative assessment.

2 Presentation of the proposed informatics systems

In achievement of the interactive informatics system for studying geometry is aimed to to reach the following pedagogical purposes:

- interactive presentation of the theoretical concepts and the main results;
- interactive presentation of applications with different difficulty levels for each required sub-domain;
- achievement of exact drawings by replacing the pencil and ruler with the mouse;
- solving of certain types of problems and presenting the used demonstration;
- performing of tests with different difficulty levels for assessment.

The presentation program of new knowledge will show the material based on an investigation dialog, the interaction being controlled by the user. It was aimed the achievement of a learning environment that should allow the undergoing of a natural dialog between the one who learns and the computer, the later having as basis the artificial intelligence.

The program for testing the knowledge will ensure the computer's intervention in one or more of the knowledge verification steps. In elaboration of the assessment instrument will be aimed to reach the following qualities:

- validity: measures what is ment to measure;
- fidelity: results almost constant during its successive application;
- aplicability: easy to manage and interpretation;
- objectivity: the concordance degree with the assessment results performed by other means is maximum.

For elaboration of the informatics educational system shall be used the Java programming language [14].

2.1 System's analysis

The informatics system will be described in a clear and concise manner by presenting the use-cases, using the UML unified modeling language [15].

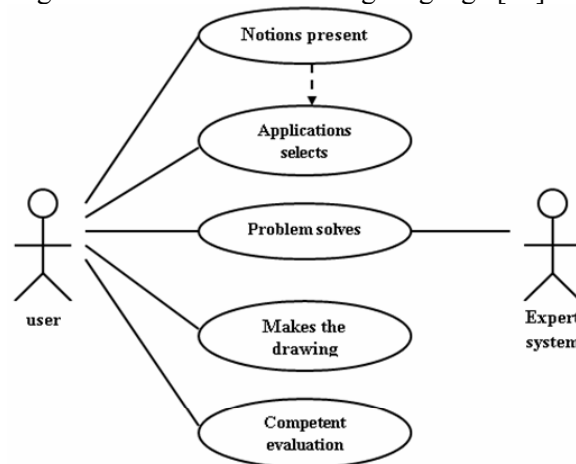


Fig.1 The use-cases diagram

Each case describes interactions between the user and the system. Representation of the use-cases diagram is shown in figure 1.

For each use-case presented in the previous diagram we'll build a diagram of activities. Each diagram will specify the processes or algorithms which are behind the analysed use-case. Figure 2 will present these diagrams.

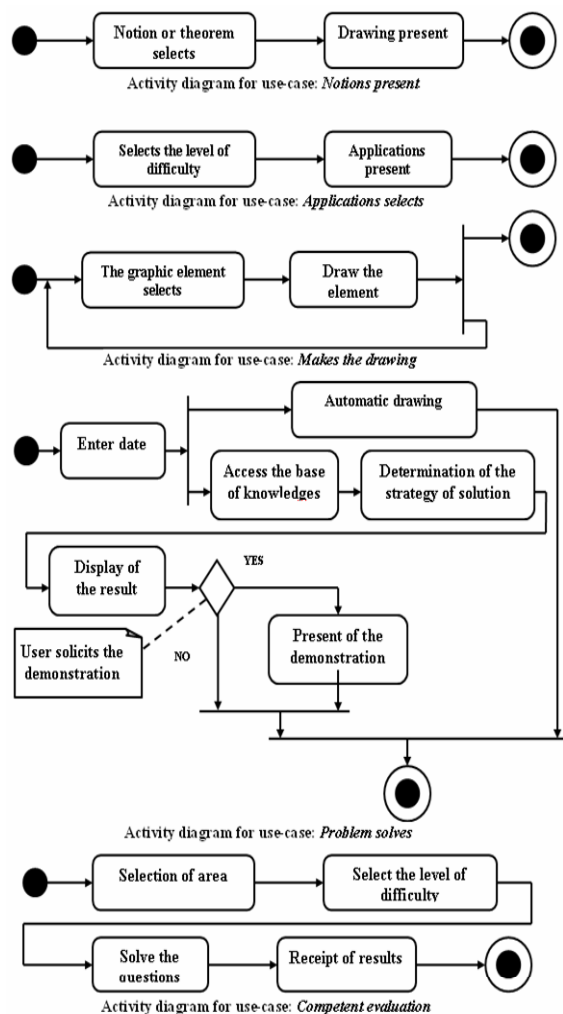


Fig.2 Activity diagrams

2.2 System's designing

The conceptual modeling allows the identification of the most important concepts for the informatics system [16]. Because classes means concepts, will be used the following classes to identify the plane geometry elements: point, line, semi-line, segment, semi-plane, conic, ellipse, circle, hyperbola, parabola, affine transformation, symmetry, rotation, translation, inversion.

The existing inheritance relationships between the classes previously presented can be represented

by means of relationship diagrams between classes (Fig. 3).

Between the classes' instances from the previously presented architecture there are mainly composition relationships. Further, we will present these relationships by means of the relationship diagrams between classes' instances (Fig. 4).

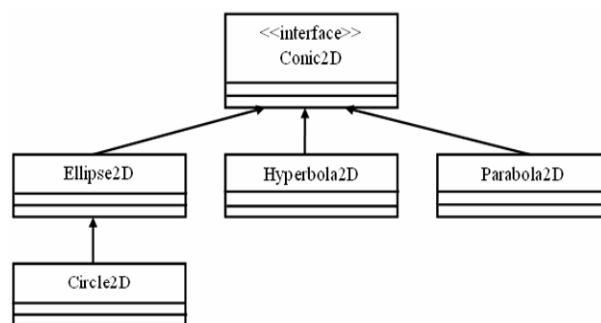


Fig. 3 Inheritance relationships

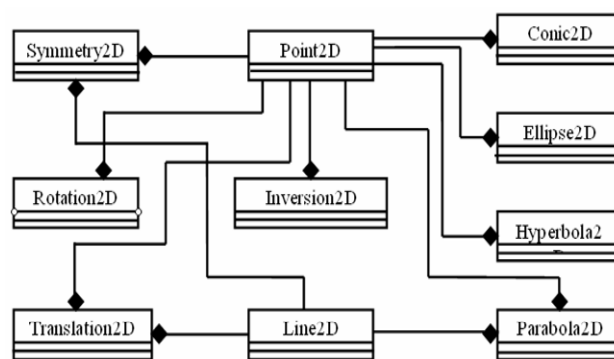


Fig. 4 Composition relationships

The presented classes' instances can be grouped by related properties and methods. The grouping is achieved by means of packages. In figure 5 are explicitly presented the structures of the packages which group objects from plane geometry. Between classes and classes' instances from a package there are inheritance and composition relationships, which were presented by means of the previous diagrams.

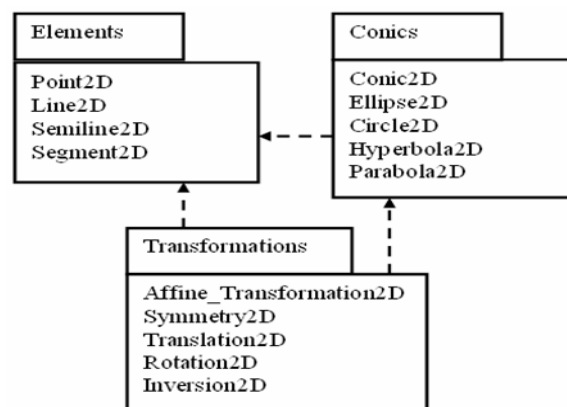


Fig. 5 Diagram of packages

Collaboration diagram focus upon the relationships between the objects [17]. They are very useful for visualizing the way several objects collaborate to get a job done and for comparing a dynamic model with a static model.

The sequence diagrams for this software, illustrates in figures 6, 7 and 8, are made with ArgoUML -0.24.

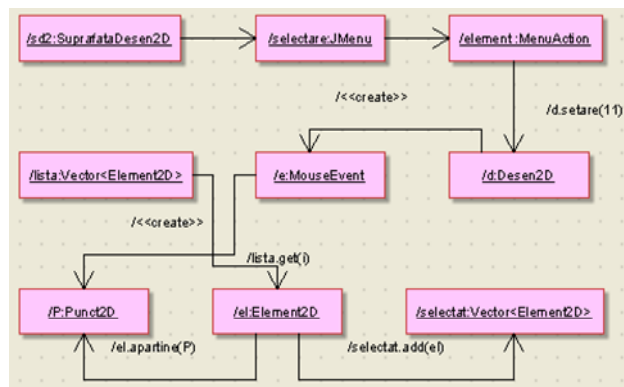


Fig.6 Collaboration diagram for selection of an element

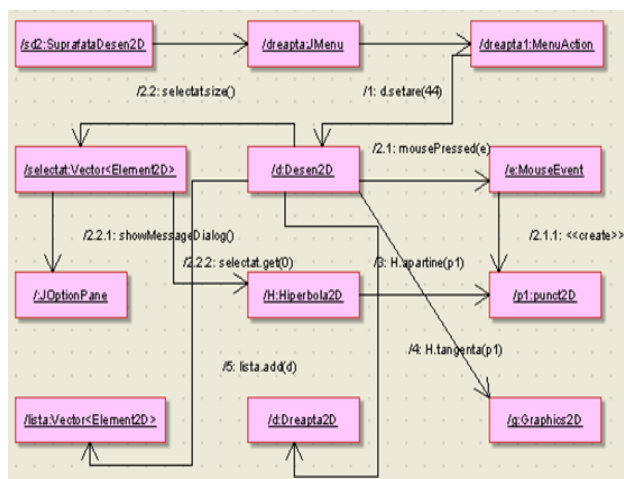


Fig.7 Collaboration diagram for drawing the tangent and the normal to a hyperbola

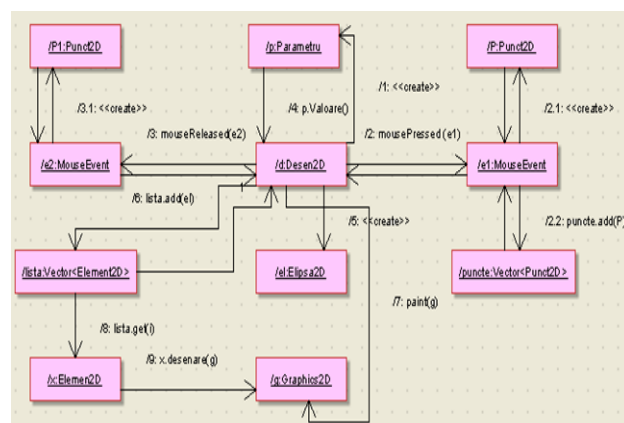


Fig.8 Collaboration diagram for drawing an ellipse

2.3 Component diagram

A component represents a modular, replaceable piece in the system [18]. Of primary importance are two well-defined interfaces: The required interface specifies formally which functionality the component expects from its environment.

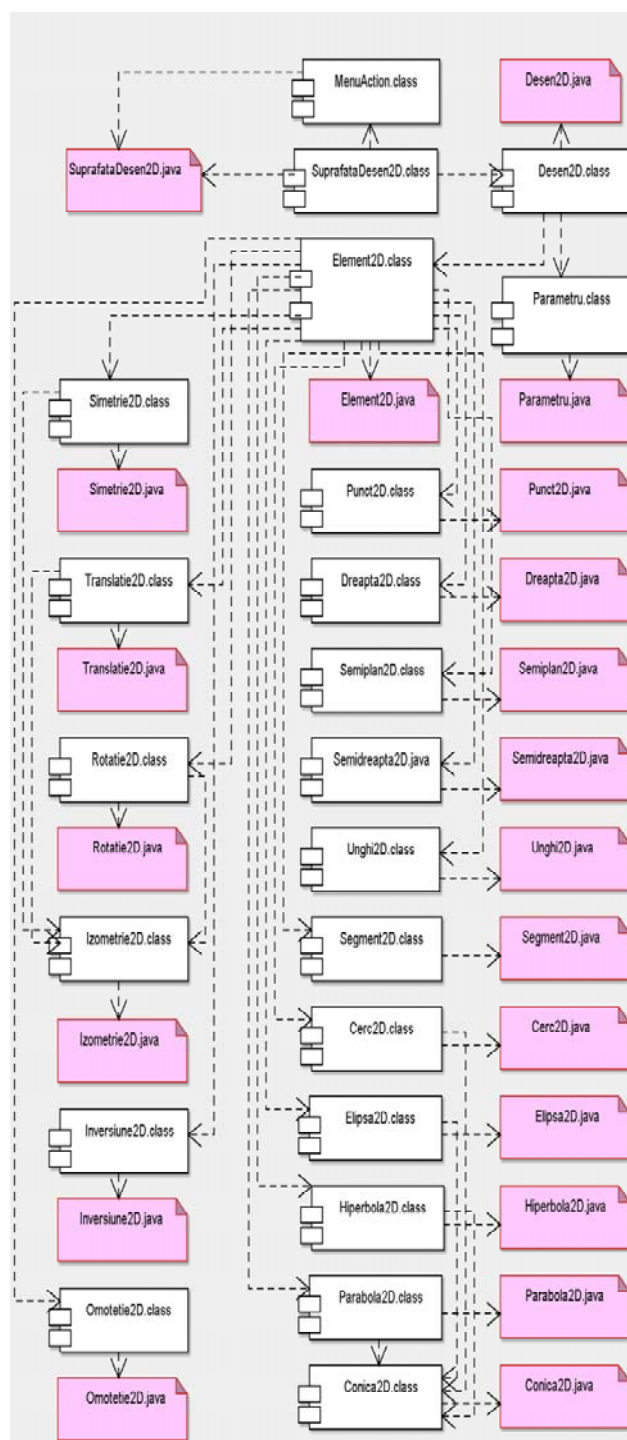


Fig. 9 Component diagram

The provided interface specifies the functionality the component is able to provide (to other components).

Ideally, the interfaces capture the required and provided functionality in such a detailed manner that allows to exchange two components conforming to the same interfaces freely. As a consequence, we could substitute one component by another component without changing anything else of the rest of the system (component is replaceable).

In principle, all parts of the system that are modeled by a component can also be modeled by a class. It is possible also for a class to express which interface it provides and which it requires.

In figure 9 is presented the component diagram with is made with ArgoUML-0.24.

2.4 The system's graphic components

The system's graphic interface is structured in five parts, corresponding to the five use-cases.

For presenting theoretical concepts/notions are achieved applets which allow the figures' movement by selecting and displacement of some geometrical elements. Moving figures on the screen help students to grasp the meaning of mathematical ideas intuitively.

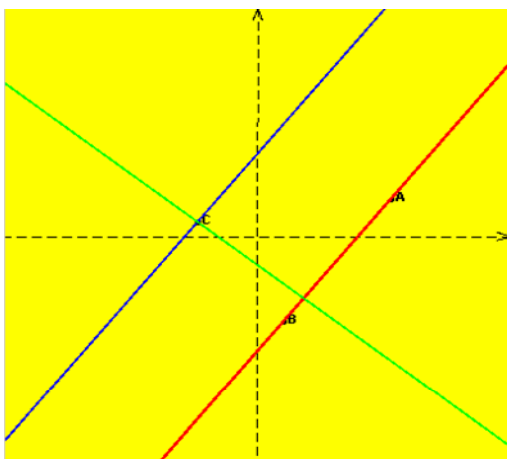


Fig.10 The parallel and the perpendicular to a line

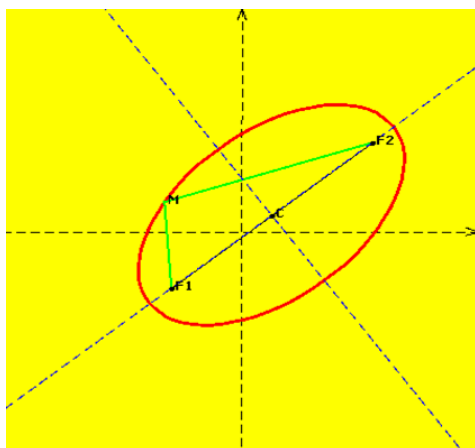


Fig. 11 The definition of an ellipse

From the application's main whindow can be selected elements of the plane analytical geometry [19] such as: line, ellipse, circle, hyperbola, parabola and geometrical transformations.

Through the selection of first option it is presented the line in plane and some of its properties. The figure 10 present the parallel and the perpendicular to a line take through a specified point.

If it is selected the second option, it is presented the ellipse and some of its mathematical properties. The standard equation of an ellipse is given by [20]:

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1, \text{ where } a, b \in \mathbb{R}_+^* \quad (1)$$

We can write the ellipse parametric equations as follows [20]:

$$\begin{cases} x = x_0 + a \cdot \cos \theta \\ y = y_0 + b \cdot \sin \theta \end{cases}, \text{ where } \theta \in [0, 2\pi] \quad (2)$$

The applet presented in figure 11 construct dynamically the ellipse as a locus of points in a plane such that the sum of the distances to two fixed points (foci) is a constant. The figure 12 present the tangent and the normal in a point to ellipsis, when the point goes through ellipsis.

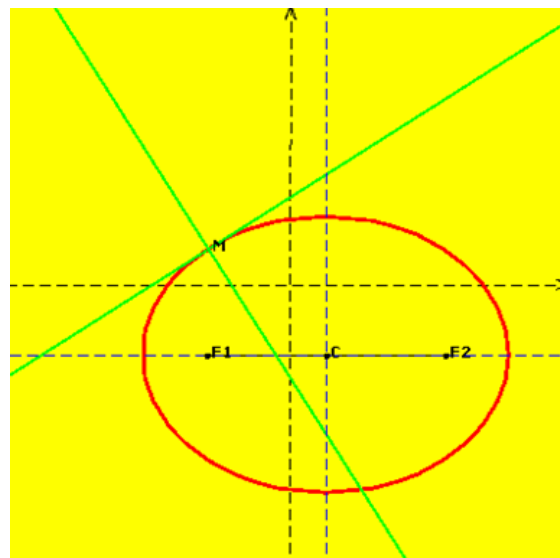


Fig. 12 The tangent and the normal to ellipsis

Through the selection of the next option it is presented the circle in plane and some of its properties.

The standard equation of a circle is given by [21]:

$$(x - x_0)^2 + (y - y_0)^2 = r^2, \text{ where } r \in \mathbb{R}_+^* \quad (3)$$

We can write the circle parametric equations as follows [22]:

$$\begin{cases} x = x_0 + r \cdot \cos \theta \\ y = y_0 + r \cdot \sin \theta \end{cases}, \text{ where } \theta \in [0, 2\pi] \quad (4)$$

The figure 13 present the tangent and the normal in a point to circle, when the point goes through circle. In figure 14 are presented two secant circles and in figure 15 are presented two tangent circles.

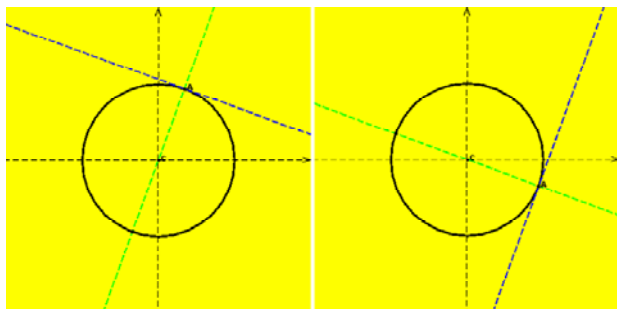


Fig. 13 The tangent and the normal to circle

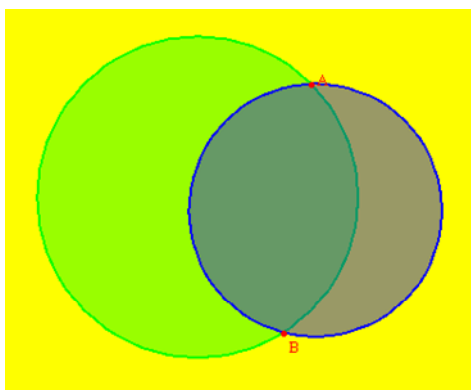


Fig. 14 Two secant circles

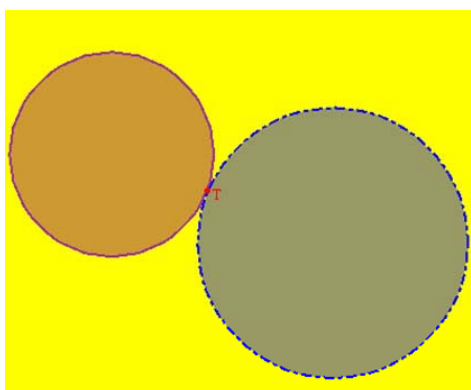


Fig. 15 Two tangent circles

If it is selected the option „hyperbola”, it is presented some of its mathematical properties (fig.16). The figure 17 present the tangent and the normal in a point to hyperbola, when the point goes through hyperbola. The standard equation of a hyperbola is given by [23]:

$$\frac{(x - x_0)^2}{a^2} - \frac{(y - y_0)^2}{b^2} = 1, \text{ where } a, b \in \mathbb{R}_+^* \quad (5)$$

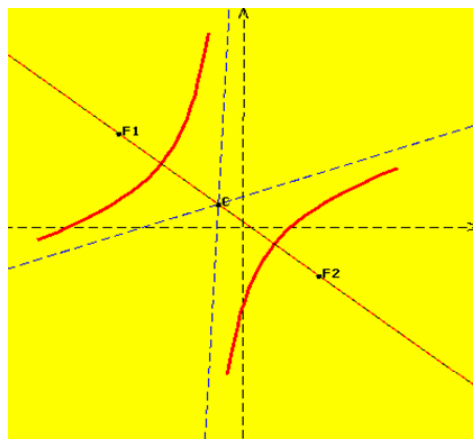


Fig. 16 The definition of a hyperbola

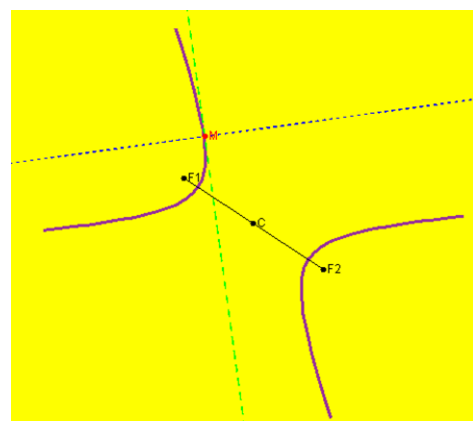


Fig. 17 The tangent and the normal to a hyperbola

Through the selection of the next option it is presented parabola in plane and some of its mathematical properties. The standard equation of a parabola is given by [22]:

$$(y - y_0)^2 = 2p \cdot (x - x_0), \text{ where } p \in \mathbb{R}_+^* \quad (6)$$

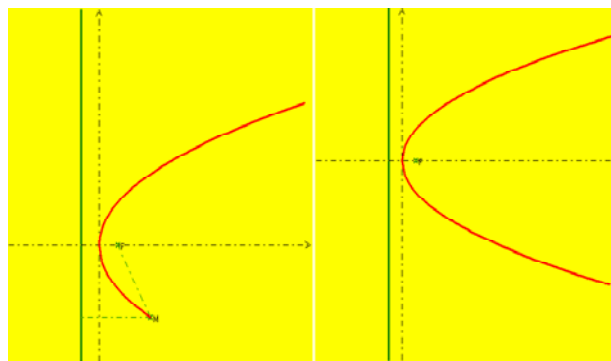


Fig. 18 The definition of a parabola

The applet presented in figure 18 construct dynamically the parabola as a locus of points in a

plane which are equidistant from a given point (focus) and a given line (directrix). The figure 19 present the tangent and the normal in a point to parabola, when the point goes through parabola.

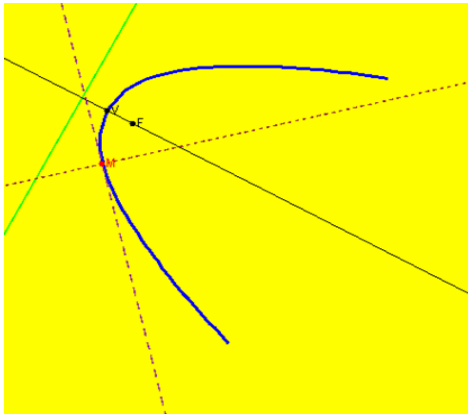


Fig. 19 The tangent and the normal to a parabola

Through the selection of last option they are presented the geometrical transformations in plane: symmetry, rotation, translation, inversion.

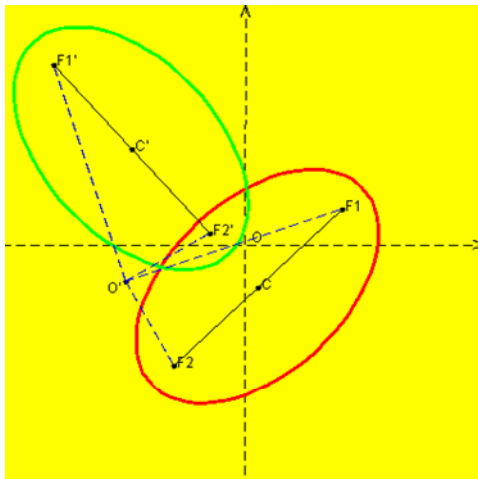


Fig. 20 The rotation of an ellipse

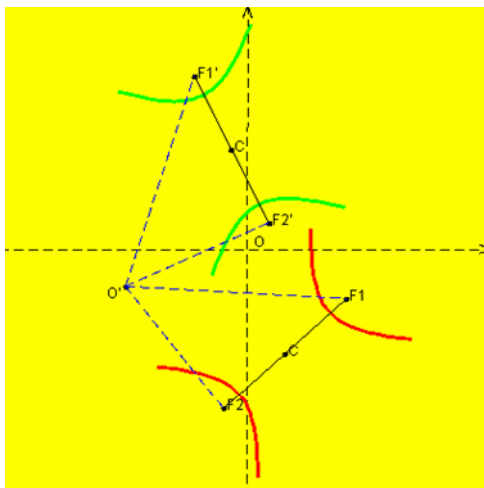


Fig. 21 The rotation of a hyperbola

The figure 20 present the rotation of an ellipse with 90 degrees. The figure 21 present the rotation of a hyperbola with 75 degrees. The figure 22 present the rotation of a circle with -60 degrees. The figure 23 present the rotation of a parabola with 30 degrees.

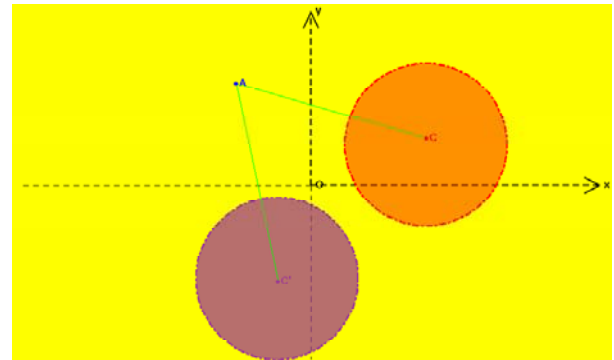


Fig. 22 The rotation of a circle

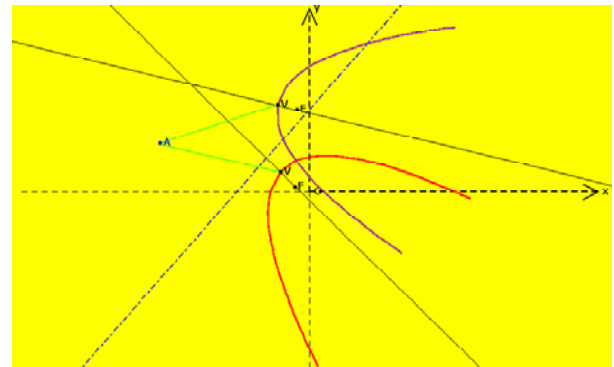


Fig. 23 The rotation of a parabola

The figure 24 present the translation of a circle, the figure 25 present the translation of an ellipse. In figure 26 is presented the translation of a hyperbola and figure 27 present the translation of a parabola.

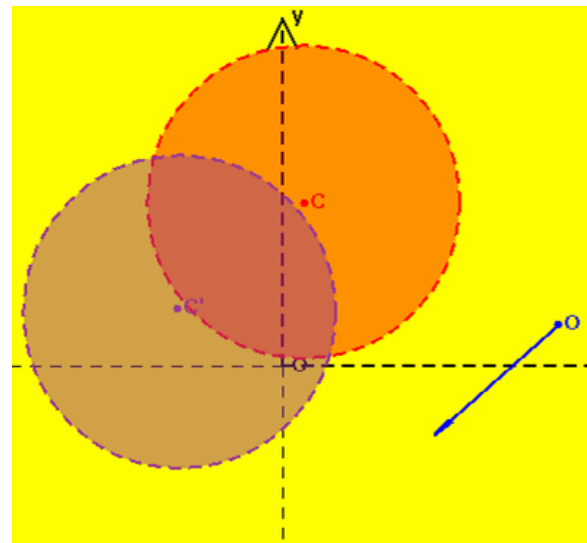


Fig. 24 The translation of a circle

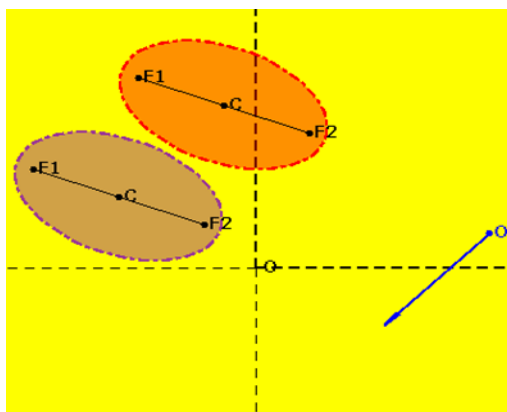


Fig. 25 The translation of an ellipse

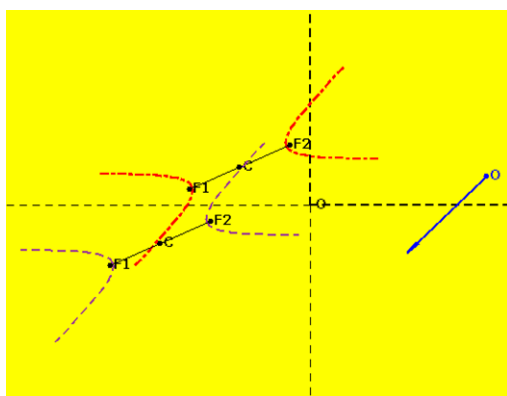


Fig. 26 The translation of a hyperbola

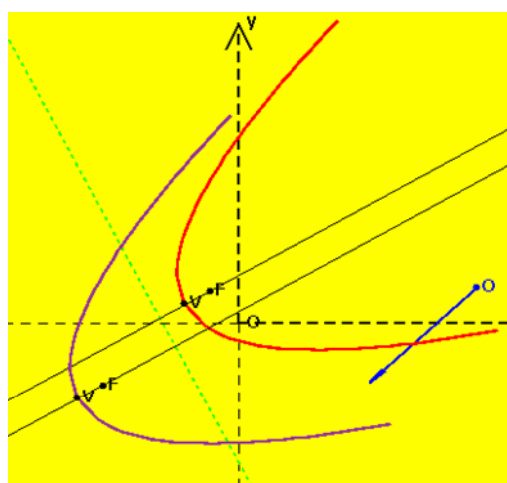


Fig. 27 The translation of a parabola

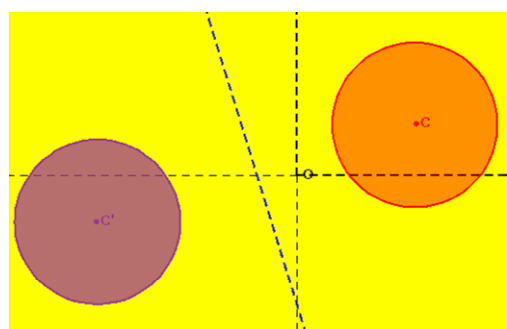


Fig. 28 The symmetry of a circle

The figure 28 present the symmetry of a circle, the figure 29 present the symmetry of an ellipse. In figure 30 is presented the symmetry of a hyperbola and figure 31 present the symmetry of a parabola. In figure 32 is present the inversion of a circle.

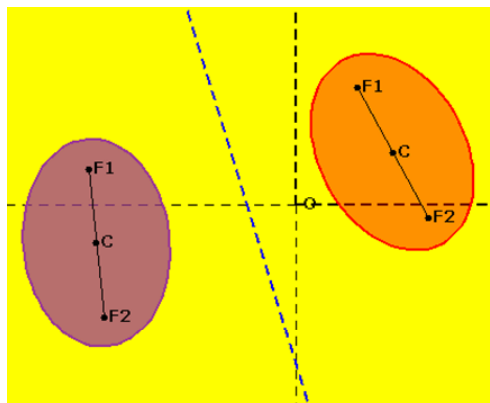


Fig. 29 The symmetry of an ellipse

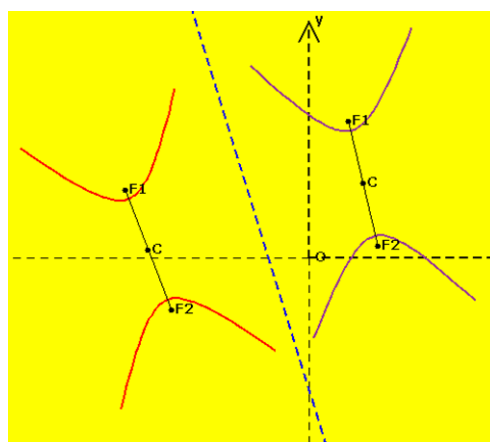


Fig. 30 The symmetry of a hyperbola

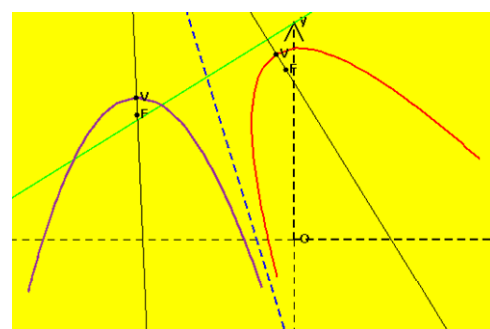


Fig. 31 The symmetry of a parabola

The program represents a modern drawing instrument, simulating the drawing surface, the ruler, the compass and offering other specialized instruments, that allow the drawing-up of complicate figures, instantaneously and with a great accuracy [24]. In figure 33 is presented an example for a construction that is easy with this software, but hard to do with ruler and compass on a sheet of paper.

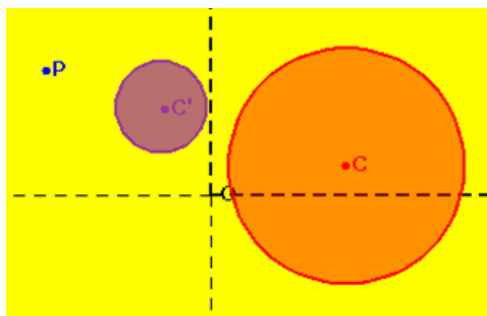


Fig. 32 The inversion of a circle

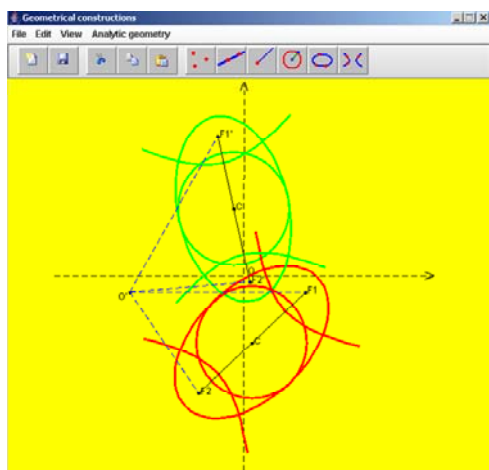


Fig. 33 An example for a construction that is easy with this software

The paper is replaced by the computer's screen, and the pencil by the mouse. For drawing-up a figure, first must be selected the instrument and then used the mouse.

The program's interface includes a menu bar, a bar with buttons corresponding to the most important operations and the drawing surface. Among the most important operations we mention :

- drawing-up of free points or points with certain properties, i.e. the middle of a segment ;
- drawing-up of certain lines or lines which fulfill certain conditions, i.e. the parallel to a line or the perpendicular to a line;
- drawing-up of circles when is specified the centre and radius, or when are selected three points which will identify the circle;
- possibility to move the geometrical construction;
- possibility to undertake parts of a geometrical construction and keeping them in the button's bar for a possible further reutilisation.

The instruments range of the graphic interface allows also operations as:

- determination of some measures: lengths of segments, measures of angles;

- consolidation of knowledge accumulated by using the solved problems grouped by complexity levels;
- interactive resolution of some categories of problems [25], [26], [27];
- current and summative assessment of the knowledge accumulated by using some tests grouped by different difficulty levels.

3 Conclusion

The advantages of the presented software are:

- By the multitude of the offered facilities, this mathematic software can be used successfully for computer assisted training at geometry, both in the pre-university and university environment;
- The software is useful both in the step of acquiring new knowledges, the step of consolidating the acquired knowledge, and in the assessment step;
- Is achieved a high technical level, being taken into account all the methodical requirements;
- Are intended few exposure levels, depending on the schoolchildren's and students' preparation level;
- The concepts/notions are presented dynamically, by means of *applets*;
- The theorems or presented results are accompanied by demonstration;
- The interesting and attractive mode of presenting the information or the gradual access to information;
- By the multitude of the offered options, the application can replace successfully the paper and the pencil for making the geometrical constructions;
- Allows the interactive resolution of some categories of problems;
- The presented scientific content is well structured, additional material being available.

Utilisation of the presented informatics system in studying geometry will contribute to build-up and develop the students' informational culture. The computer assisted training in the geometry elements' study process is also an efficient method to increase the learning motivation of this discipline and the quality of its acquiring.

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