Using Computer-aided Techniques in the Dynamic Modeling of the Human Upper Limb

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Abstract: - The wide mobility of human body leads to the necessity of modeling the osteoarticular system as a mechanism with a large number of degrees of freedom. Dynamic modeling of osteoarticular system is necessary because the exertion of various actions and natural physiological movements are essentially dynamic. Very often we use a simplified model because the phenomena produced are so complex that accurate mathematical reproduction is practically impossible. A dynamic model must provide a good estimate of total weight and mass distribution as well as transmissibility and amortization proprieties for bones, muscles, joints, blood and skin.

The paper presents a dynamic functional model considering the human upper limb as a mechanic system with 5 degrees of freedom in the case the segments are moved by their own weight forces. The bones and the muscles were modeled in Solid Works, the model of the upper limb obtained being very close as form to the real one. Based on this model, the calculus of mass proprieties was made. The differential equations of motion obtained were solved using Lagrange formalism.

Key-Words: - Modeling, Human upper limb, Dynamic model, Osteoarticular system

1 Introduction

The research regarding the mechanical aspect of the human body's osteoarticular system can be successfully made through classical and modern calculus and experimental engineering methods. Thus, the osteoarticular system can be seen by the engineer as being a deformable special structure, with a considerable complexity regarding its geometry, elastic properties and its functions.

Although the biomechanical modeling of the osteoarticular system is subject to the same general laws and principles which are used in engineering, we still have to consider the fact that there are some differences which limit the possibilities of this research method. As a consequence a model must be conceived and investigated as to determine within certain very accurate limits, the behavior of the original system. The inbred variability of geometry and of the mechanical properties of the osteoarticular system, from person to person, is one of the aspects that generate considerable supplementary difficulties regarding the accomplishment of biomechanical researches and which always must be taken into consideration.

The dynamic modeling of the osteoarticular system is necessary because the performance of some actions and common physiological motions is in its essence dynamic.

The high mobility of the human body leads to the necessity of the modeling of the osteoarticular system as a mechanism with a high degree of flexibility. The differential equation system which is obtained is complex and it implies numerical integration. In most cases, we use a simplified model because the occurring phenomena are so complex that a precise mathematical replica is virtually impossible.

The 3D model determination of some fundamental sizes in the study of dynamics is necessary for: improving the prosthesis and the metallic implants of the upper limb and their making in accordance with the particularities of each case, the assessment in which bone fractures are made (in most of the cases dynamic: falls, slips, impacts), with implications in forensic medicine; knowing the circumstances in which the fractures occur through shock etc.

In order to carry out a functional dynamic model of the human upper limb as closely as possible to the real one, it is necessary to study the proprieties of the biological materials that make up the human body, so that the measures involved in the differential equations are as correct as possible [5], [10], [26].

The studies carried out by different researchers are based especially on models made in FEM programs.

2 Bone Structure and Proprieties

In order to perform the locomotion function and to maintain its shape, the human body must possess a bone structure with certain mechanical properties which would adapt to external factors and environmental conditions.

The bones of the human skeleton are of different shapes and sizes. They fit into four categories: long, short, flat, and irregular.

From a microscopic point of view, bones consist 30-40% of organic material, of which almost 95% are compressed collagen fibers, and 60-70% of inorganic material, of which approximately 85% is made up of calcium phosphate, 6-10% of calcium carbonate, and the rest of alkaline salts. This composition leads to the preservation of bone rigidity and resistance to several stresses [5].

In figure 1, the structure of a long bone is represented [1], [17], [24], [25].

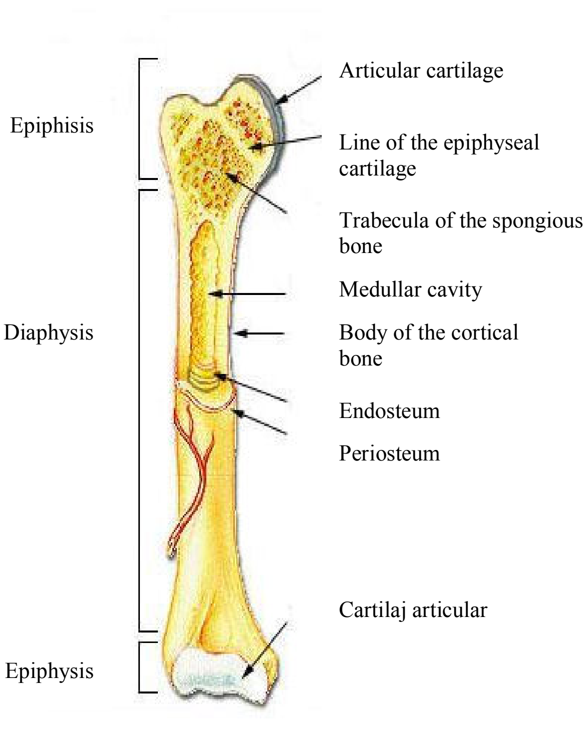

Fig.1. Structure of a long bone

From a macroscopic point of view, bones are of two types: cortical (compact) and spongious (trabecular). The **cortical bone** is very dense, intensely calcified, created to resist compression. It can also resist torsion and bending, but to a much lesser extent. The cortical bone is usually found in the diaphyses of long bones. They feature a central cavity medullar canal or medullar cavity. At the end of the long bones and surrounding tendons and attached ligaments, the bone structure is more porous and bears the name of **spongious bone** [15]. The passage from the cortical bone to the spongious one is gradual.

The cortical and spongious bone densities do not differ greatly because both bone types consist of bone lamellas. Knowing the bone density is important for appreciating the sizes characterizing the distribution of masses in the osteoarticular system of the upper limb, without which the dynamic study of latter cannot be carried out [13].

3 The calculation of the center of gravity and of the moments of inertia on the 3D model of the human upper limb

Determining fundamental measurements in the study of dynamics on 3D models is necessary for: perfecting prostheses and metallic implants of the upper limb and their realization according to the particularities of each case, establishing the circumstances in which bone fractures are produced (in their vast majority dynamic cases: falling, slipping, impact), with implications in forensic medicine; knowing the conditions under which fractures are produced in stresses by shock, etc. [2], [11], [18], [38], [39], [40], [41].

In order to determine the center of gravity of a component of the bony skeleton of the human upper limb and the moments of inertia, one can employ the proper command of the programs Solid Works [36], **Mass Property,** or other programs can be used, into which these files are imported, such as Pro Engineer, AutoCad, etc.

In figures 2 - 4, examples of calculation in Solid Works are presented for the various components of the human upper limb skeleton, as well as the human upper limb as a whole. For the calculation of the moments of inertia, the reference frame was placed in the component's center of gravity.

Even though bones do not consist of a homogenous structure, the value of the mass is the one calculated with the average density: ρ = 1,3 g/cm^3. Approximation leads to results that are compatible to those in literature [3], [9], [10], [12], [31], [37].

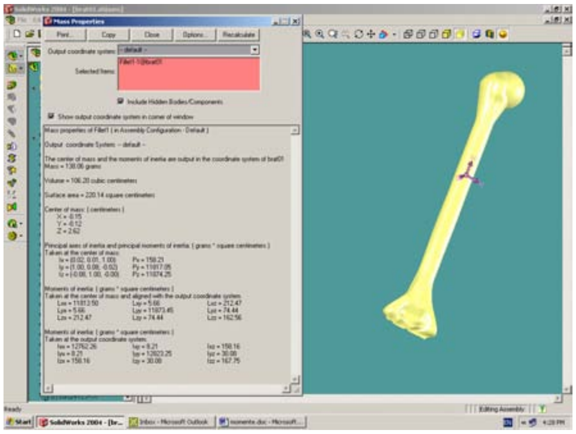

Fig.2. Calculation of mass proprieties for the humerus

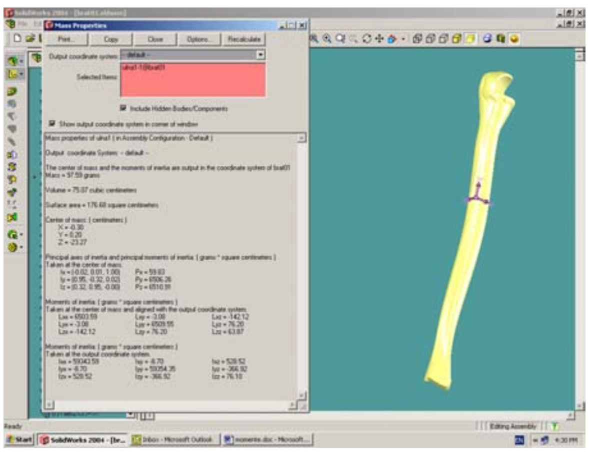

Fig.3. Calculation of mass proprieties for the ulna

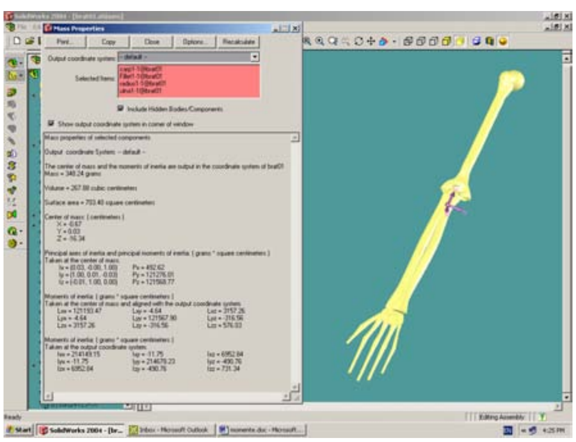

Fig.4. Calculation of mass proprieties for the for the skeleton of the human upper limb

For the arm with muscles, the muscular density ρ = 1,13 g/cm^3 was considered. The calculation of mass properties for this case is carried out while taking into account the fact that resulting system is no longer homogenous (figure 5).

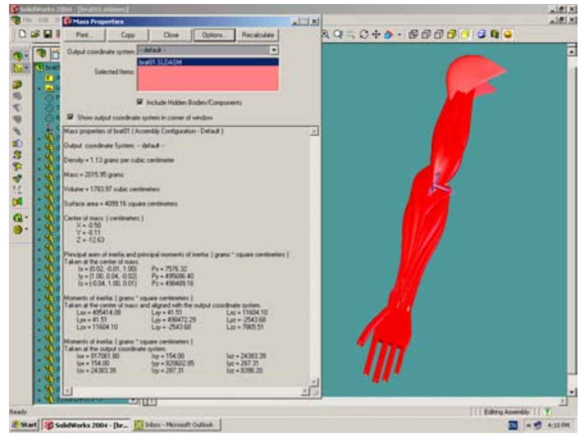

Fig.5. Calculation of mass proprieties for the for the skeleton of the human upper limb (with muscles)

4 The Dynamic Model of the Human Upper Limb

It is consider the upper limb's structure as a mechanical system with 5 degrees of freedom, subject only to its own friction forces, so that the differential equations of motion will be determined by using Lagrange's equations [4], [6], [7], [8] [16], [22], [33]:

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_i}\right) - \frac{\partial Ec}{\partial q_i} = Q_i \qquad i=\overline{1,5} \tag{1}$$

where:

Ec – total kinetic energy,
q_i - generalized variables corresponding to couples i,
Q_i – generalized forces.

Taking into consideration the simplified geometrical model and the spatial structure which physically models (molds) the upper human limb, the total kinetic energy is obtained:

$$E_C = \frac{1}{2}\cdot J_{Zo}^{(ms)}\cdot \dot{q}_1^2 + \frac{1}{2}\cdot J_{Z1}^{(ms)}\cdot \dot{q}_2^2 + \frac{1}{2}\cdot J_{Z2}^{(ms)}\cdot \dot{q}_3^2 + \frac{1}{2}\cdot J_{Z3}^{(r+p+d)}\cdot \dot{q}_4^2 + \frac{1}{2}\cdot J_{Z4}^{(p+d)}\cdot \dot{q}_5^2 \tag{2}$$

where z0, z1, z2 represent the rotation axis corresponding to the three motions at level of the shoulder (flexion-extension, abduction-adduction, rotation), z3 the axis corresponding to the flexion-extension motion at the elbow level, z4 the rotation axis corresponding to the flexion-extension motion at the level of the wrist, and J_{z0}, J_{z1}, J_{z2}, J_{z3}, J_{z4}, the inertia moments in connection to these axis.

The following notes have been made: *ms* – upper limb, *r* – radius, *p* – palm, *d* – fingers, also standing for the lengths of the afferent parts.

The inertia moments in connection with the axis z0, z1, z2, z3 and z4 are calculated by applying Steiner's formulas [3]:

$$J_{Z4}^{(p+d)} = J_{IIZ4,c(p+d)} + \left(\frac{p+d}{2}\right)^2 m_{(p+d)} = J_{IIZ4,c(p+d)}^{(p+d)} + \frac{(p+d)^2}{4} m_{(p+d)} \tag{3}$$

$$J_{Z3}^{(r+p+d)} = J_{IIZ3,c(p+d)}^{p+d} + \left(r + \frac{p+d}{2}\right)^2 m_{(p+d)} + J_{Z3}^{(r)} \tag{4}$$

$$J_{Z2}^{(ms)} = J_{IIZ2,c(p+d)}^{(p+d)} + \left(\frac{p+d}{2}\sin q_5\right)^2 m_{(p+d)} + J_{IIZ2,c(r)}^{(r)} + \left(\frac{r}{2}\sin q_4\right)^2 m_{(r)} + J_{IIZ2,c(h)}^{(h)} + \frac{h^2}{4} m_{(h)} \tag{5}$$

$$J_{Z1}^{(ms)} = J_{IIZ1,c(p+d)}^{(p+d)} + \left(\frac{p+d}{2}\cos q_5\right)^2 m_{(p+d)} + J_{IIZ1,c(r)}^{(r)} + \left(\frac{r}{2}\cos q_4\right)^2 m_{(r)} + J_{IIZ1,c(h)}^{(h)} + \frac{h^2}{4} m_{(h)} \tag{6}$$

$$J_{Z0}^{(ms)} = J_{IIZ0,c(p+d)}^{(p+d)} + \frac{(p+d)^2}{4} m_{(p+d)} + J_{IIZ0,c(r)}^{(r)} + \frac{r^2}{4} m_{(r)} + J_{IIZ0,c(h)}^{(h)} + \frac{h^2}{4} m_{(h)} \tag{7}$$

Although the bones do not show any homogeneous structure, the value of the weigh is that calculated based on the average density: ρ = 1,3 g/cm^3. Approximation leads to results compatible with those from literature [9], [19], [12]. For the arm with muscles the muscular density ρ = 1,13 g/cm^3 has been taken into consideration.

With the help of the Solid Works program [36], knowing the bone dimensions and the muscle and bone densities, these temporary figures are directly calculated by putting the reference systems directly into couples with the z axis orientated along the axis of those respective couples. Figure 6 shows the human upper limb model for which the inertia moment calculus is made, the axis in connection to which this calculus is made, as well as the results obtained [20], [21], [29], [30], [32], [34].

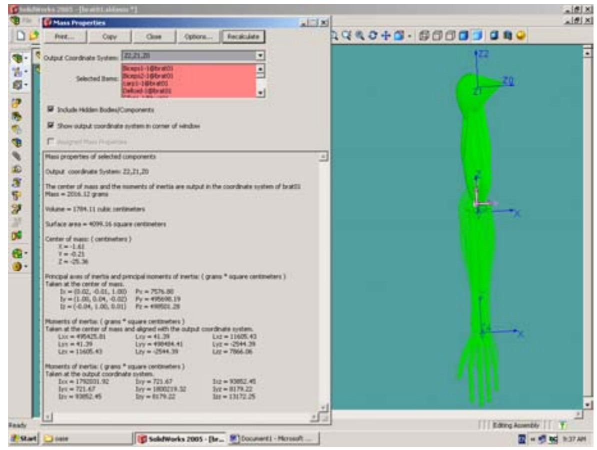

Fig.6. The calculus for the inertia moments for the whole arm (upper limb)

The kinetic energy derivates compared to generalized velocities and their derivates compared to time, as well as the derivates of the kinetic energy compared to the generalized coordinates become:

$$\frac{\partial E_C}{\partial \dot{q}_1} = J_{Zo}^{(ms)} \dot{q}_1 \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial E_C}{\partial \dot{q}_1}\right) = \dot{q}_1 \frac{dJ_{Zo}^{(ms)}}{dt} + J_{Zo}^{(ms)} \ddot{q}_1 = J_{Zo}^{(ms)} \ddot{q}_1 \tag{9}$$

$$\frac{\partial E_C}{\partial q_1} = 0 \tag{10}$$

$$\frac{\partial E_C}{\partial \dot{q}_2} = J_{Z1}^{(ms)} \dot{q}_2 \tag{11}$$

$$\frac{d}{dt}\left(\frac{\partial E_C}{\partial \dot{q}_2}\right) = \dot{q}_2 \left[-\frac{p+d}{2} \dot{q}_5 m_{(p+d)} \sin 2q_5 - \frac{r}{2} \dot{q}_4 m_{(r)} \sin 2q_4 \right] + J_{Z1}^{(ms)} \ddot{q}_2 \tag{12}$$

$$\frac{\partial E_C}{\partial q_2} = 0 \tag{13}$$

$$\frac{\partial E_C}{\partial \dot{q}_3} = J_{Z2}^{(ms)} \dot{q}_3 \tag{14}$$

$$\frac{d}{dt}\left(\frac{\partial E_C}{\partial \dot{q}_3}\right) = \dot{q}_3\left[\frac{p+d}{2}\dot{q}_5 m_{(p+d)}\sin 2q_5 + \frac{r}{2}\dot{q}_4 m_{(r)}\sin 2q_4\right] + J_{Z2}^{(ms)}\ddot{q}_3 \tag{15}$$

$$\frac{\partial E_C}{\partial q_3} = 0 \tag{16}$$

$$\frac{\partial E_C}{\partial \dot{q}_4} = J_{Z3}^{(r+p+d)}\dot{q}_4 \tag{17}$$

$$\frac{d}{dt}\left(\frac{\partial E_C}{\partial \dot{q}_4}\right) = J_{Z3}^{(r+p+d)}\ddot{q}_4 \tag{18}$$

$$\frac{\partial E_C}{\partial q_4} = \frac{1}{2}m_{(r)}\frac{r}{2}\sin 2q_4(\dot{q}_3^{\,2} - \dot{q}_2^{\,2}) \tag{19}$$

$$\frac{\partial E_C}{\partial \dot{q}_5} = J_{Z4}^{(p+d)}\dot{q}_5 \tag{20}$$

$$\frac{d}{dt}\left(\frac{\partial E_C}{\partial \dot{q}_5}\right) = J_{Z4}^{(p+d)}\ddot{q}_5 \tag{21}$$

$$\frac{\partial E_C}{\partial q_5} = \frac{1}{2}m_{(p+d)}\frac{p+d}{2}\sin 2q_5(\dot{q}_3^{\,2} - \dot{q}_2^{\,2}) \tag{22}$$

By replacing the equations (8)-(22) in (1), taking into account the expressions of generalized forces for $i=\overline{1,5}$, we obtain the following equations:

$$J_{Zo}^{(ms)}\ddot{q}_1 = -m_h g\frac{h}{2}\cos q_1 - m_r g(h\cos q_1 + \frac{r}{2}\cos q_4) - m_{(p+d)}g(h\cos q_1 + r\cos q_4 + \frac{p+d}{2}\cos q_5) \tag{23}$$

$$\dot{q}_2\left[-\frac{p+d}{2}\dot{q}_5 m_{(p+d)}\sin 2q_5 - \frac{r}{2}\dot{q}_4 m_{(r)}\sin 2q_4\right] + J_{Z1}^{(ms)}\ddot{q}_2 = -m_h g\frac{h}{2}\cos q_1\cos q_2 - m_r g(h\cos q_1\cos q_2 + \frac{r}{2}\cos q_4\cos q_2) m_{(p+d)}g(h\cos q_1\cos q_2 + r\cos q_4\cos q_2 + \frac{p+d}{2}\cos q_5\cos q_2) \tag{24}$$

$$\dot{q}_3\left[\frac{p+d}{2}\dot{q}_5 m_{(p+d)}\sin 2q_5 + \frac{r}{2}\dot{q}_4 m_{(r)}\sin 2q_4\right] + J_{Z2}^{(ms)}\ddot{q}_3 = 0 \tag{25}$$

$$J_{Z3}^{(r+p+d)}\ddot{q}_4 - \frac{1}{2}m_{(r)}\frac{r}{2}\sin 2q_4(\dot{q}_3^{\,2} - \dot{q}_2^{\,2}) = -m_r g\frac{r}{2}\cos q_4 - m_{(p+d)}g(r\cos q_4 + \frac{p+d}{2}\cos q_5) \tag{26}$$

$$J_{Z4}^{(p+d)}\ddot{q}_5 - \frac{1}{2}m_{(p+d)}\frac{p+d}{2}\sin 2q_5(\dot{q}_3^{\,2} - \dot{q}_2^{\,2}) = -m_{(p+d)}g(\frac{p+d}{2})\cos q_5 \tag{27}$$

By replacing the equations (23)-(27) the values for the inertia moments:

$$\begin{aligned} I_{z0}^{(m\,s)} &= I_{xx} = 0{,}179203192 \text{ kg*m}^2 \\ I_{z1}^{(m\,s)} &= I_{yy} = 0{,}180021932 \text{ kg*m}^2 \\ I_{z2}^{(m\,s)} &= I_{zz} = 0{,}001317225 \text{ kg*m}^2 \\ I_{z3}^{(r+p+d)} &= I_{xx} = 0{,}023420715 \text{ kg*m}^2 \\ I_{z4}^{(p+d)} &= I_{xx} = 0{,}000662244 \text{ kg*m}^2, \end{aligned} \tag{28}$$

weights: $m_{(p+d)}$=0,14366 kg (hand), $m_{(r)}$=0,83461 kg (forearm), $m_{(h)}$=1,03785 kg (upper part of the human upper limb), $m_{(ms)}$=2,01612 kg (upper limb) and the bone sizes h=0,29 m, r=0,235 m, $p+d$=0,16 m, the following differential equations system is obtained [24], [25]:

$$\ddot{q}_1 = -23{,}78\cos q_1 - 7{,}22\cos q_4 - 0{,}62\cos q_5 \tag{29}$$

$$\ddot{q}_2 = 0{,}06\dot{q}_2\dot{q}_5\sin 2q_5 + 0{,}54\dot{q}_2\dot{q}_4\sin 2q_4 - 21{,}60\cos q_1\cos q_2 - 71{,}83\cos q_2\cos q4 - 0{,}62\cos q_2\cos q_5 \tag{30}$$

$$\ddot{q}_3 = -8{,}46\dot{q}_3\dot{q}_5\sin 2q_5 - 75{,}38\dot{q}_3\dot{q}_4\sin 2q_4 \tag{31}$$

$$\ddot{q}_4 = 2{,}09\dot{q}_3^{\,2}\sin 2q_4 - 2{,}09\dot{q}_2^{\,2}\sin 2q_4 - 55{,}25\cos q_4 - 4{,}78\cos q_5 \tag{32}$$

$$\ddot{q}_5 = 8{,}33\dot{q}_3^{\,2}\sin 2q_5 - 8{,}33\dot{q}_2^{\,2}\sin 2q_5 - 186{,}66\cos q_5 \tag{33}$$

5 Solving the Differential Equation System for the Simplified Model of the Human Upper Limb

The integration of the differential equations system that represents the reduced model of the upper limb has been made in MatLab 7.01, using the following notations [14], [19], [23], [27], [28], [35]:

$$\dot{q}_1 = y(1),\ \dot{q}_2 = y(2),\ \dot{q}_3 = y(3),$$
$$\dot{q}_4 = y(4),\ \dot{q}_5 = y(5) \tag{34}$$

$$\ddot{q}_1 = y(6),\ \ddot{q}_2 = y(7),\ \ddot{q}_3 = y(8),$$
$$\ddot{q}_4 = y(9),\ \ddot{q}_5 = y(10) \tag{35}$$

The code used for calculus is:

```
function dy=antoanela_1(t,y)

dy=zeros(10,1);
dy(1) = y(6);
dy(2) = y(7);
dy(3) = y(8);
dy(4) = y(9);
dy(5) = y(10);
dy(6) = -23.78*cos(y(1))-
7.22*cos(y(4))-0.62*cos(y(5));
dy(7) =
0.06*y(7)*y(10)*sin(2*(y(5)))+0.54*y(7
)*y(9)*sin(2*(y(4)))-
21.60*cos(y(1))*cos(y(2))-
71.83*cos(y(2))*cos(y(4))-
0.62*cos(y(2))*cos(y(5));
dy(8) = -
8.46*y(8)*y(10)*sin(2*(y(5)))-
75.38*y(8)*y(9)*sin(2*(y(4)));
dy(9) = 2.09*y(8)*y(8)*sin(2*(y(4)))-
2.09*y(7)*y(7)*sin(2*(y(4)))-
55.25*cos(y(4))-4.78*cos(y(5));
dy(10) = 8.33*y(8)*y(8)*sin(2*(y(5)))-
8.33*y(7)*y(7)*sin(2*(y(5)))-
186.66*cos(y(5));
```

The system integration was achieved through the sequence of lines:

```
>> tspan=[0 0.4];
y0=[-1.57 0 -3.14 0 -1.57 0 0 -5 0
10];
[t,ySol]=ode45('antoanela_1',tspan,
y0);
ySol(:,1:10)
plot(t, ySol(:,1:10))
legend('y_1','y_2',
'y_3','y_4','y_5','y_6','y_7',
'y_8','y_9','y_10')
```

On the basis of the numerical values and the articulator variables and the articulator velocities obtained as a result of integrating for t∈[0,0.4] and the initial conditions different from zero, the graphic representations of the motion laws and velocities corresponding to these results have been obtained (figure 7).

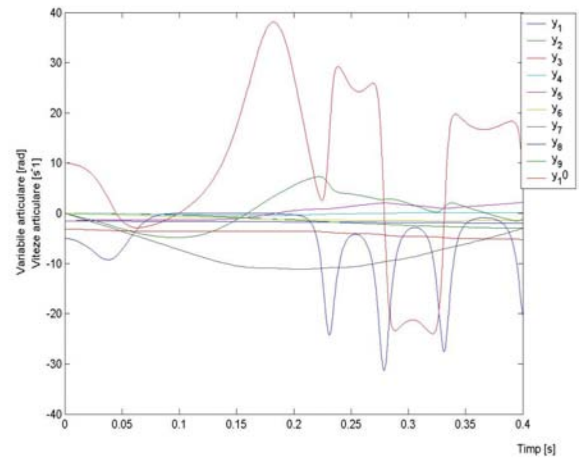

Fig.7. Graphic diagram of the motion laws and velocities, t∈[0,0.4]

As solving of the differential equation system is numerical, the statistical processing of the data symbolized in figure 6 is particularly important for the variation of the articulator variables q_i written down as y_i in the MatLab program. The main statistical estimates are represented in figure 8 from which results: the mean values during the specified time, the median values and the global variation interval indicated as such through the respective extreme values.

Statistics for y_1 / y_2 / y_3 / y_4 / y_5 — Check to plot statistics on figure

	X	Y (y_1)	Y (y_2)	Y (y_3)	Y (y_4)	Y (y_5)
min	0	-1.986	-3	-5.234	-0.4752	-1.57
max	0.4	-1.57	0	-3.14	0.1515	2.15
mean	0.1991	-1.729	-1.406	-3.989	-0.119	0.1802
median	0.2023	-1.706	-1.381	-3.604	-0.06063	0.574
std	0.1172	0.1284	1.024	0.6269	0.2075	1.355
range	0.4	0.4156	3	2.094	0.6267	3.72

Fig.8. Statistic estimator for variation or articular variables qi, t∈[0,0.4]

In order for the results of the dynamic modeling to be usable also for the realization of upper-limb prostheses, on the basis of the solutions obtained by integrating differential equations, one also performed the approximation of functions $y\_i = f(t)$ through the method of orthogonal polynomials. As soon as they are obtained, one can apply the reverse method in the differential equations of motion (29-33), with a view to calculating generalized forces which can be concretized by using activation engines for couplings that are going to develop active moments in accordance to what results from the above-mentioned equations.

Thus, also in MatLab the polynomial functions approximating the laws of motion were obtained, having in view that the degree of the approximation polynomial should correspond to as little deviations as possible. One can observe in figures 9 – 13 that a polynomial function ensuring an adequate approximation could not be obtained for all of the articular variables.

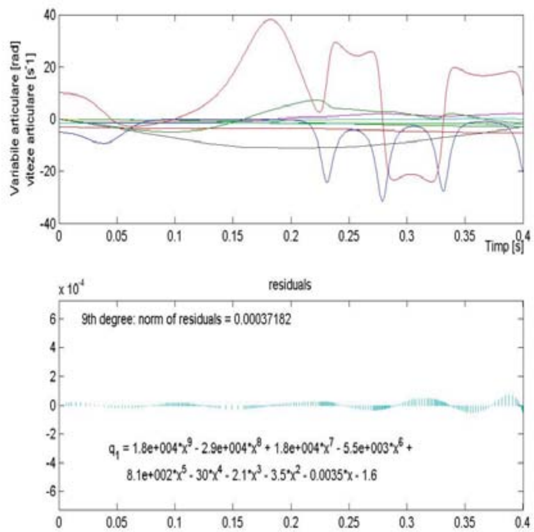

Fig.9. The polynomial function for q_1 articular variable

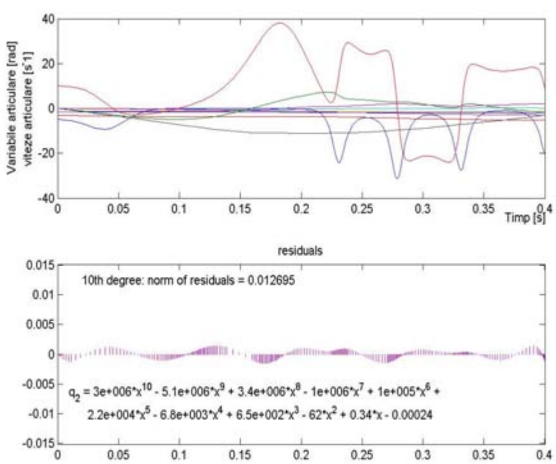

Fig.10. The polynomial function for q_2 articular variable

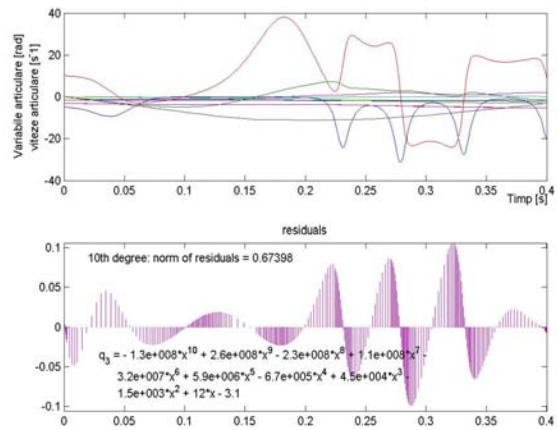

Fig.11. The polynomial function for q_3 articular variable

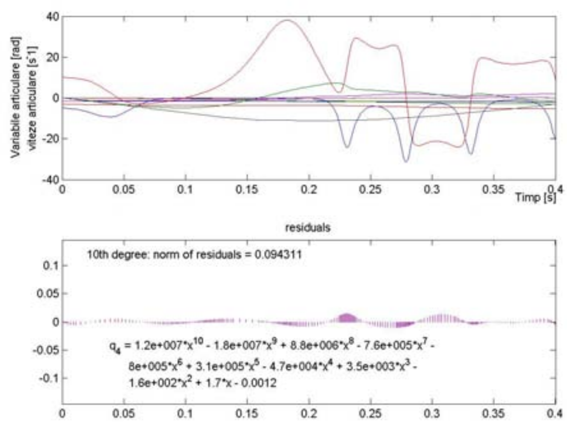

Fig.12. The polynomial function for q_4 articular variable

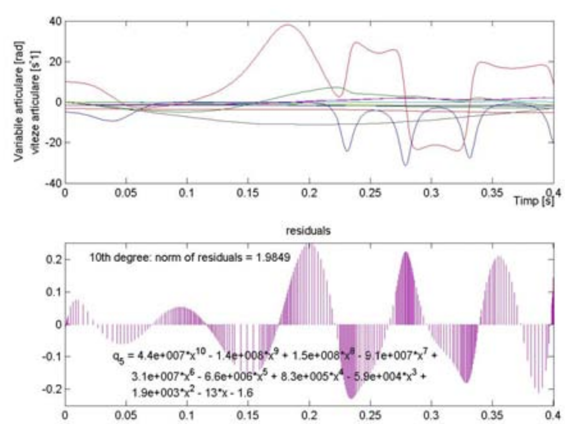

Fig.13. The polynomial function for q_5 articular variable

On the basis of the numerical values and the articulator variables and the articulator velocities obtained as a result of integrating for t∈[0,0.4] and the initial conditions equal to zero, the graphic representations of the motion laws and velocities corresponding to these results have been obtained (figure 14).

The system integration was achieved through the sequence of lines:

```
>> tspan=[0 0.4];
y0=[0 0 0 0 0 0 0 0 0 0];
[t,ySol]=ode45('antoanela_1',tspan,
y0);
ySol(:,1:10)
plot(t, ySol(:,1:5))
legend('y_1','y_2', 'y_3','y_4','y_5')
```

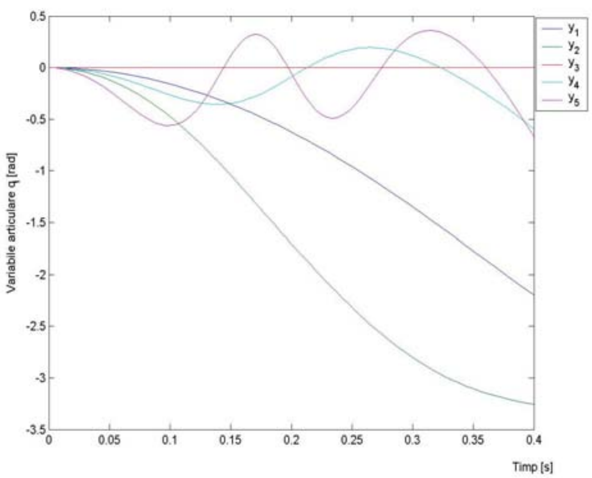

Fig.14. Graphic diagram of the motion laws for null initial conditions, t∈[0,0.4]

Changing the initial conditions and the period of time, the results are:

```
>> tspan=[0 1];
y0=[-0.785 -0.785 1.57 -0 -0.785 0 0 0
0 0];
[t,ySol]=ode45('antoanela_1',tspan,
y0);
ySol(:,1:10)
plot(t, ySol(:,1:5))
legend('y_1', 'y_2', 'y_3', 'y_4',
'y_5')
```

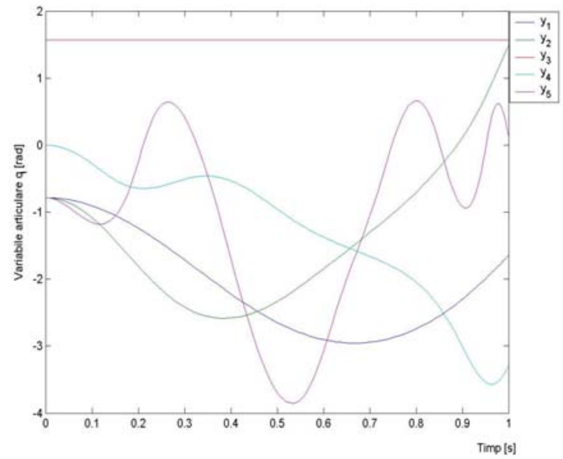

Fig. 15. Graphic diagram of the motion laws, t∈[0,1]

For this case, the polynomial functions were obtained, an example for q_1 articular variable being illustrated in figure 16.

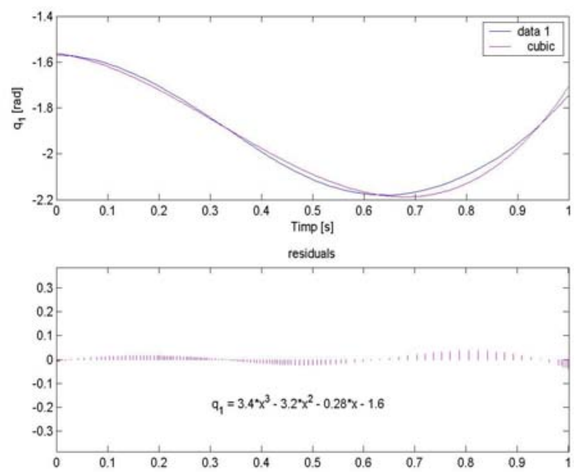

Fig.16. The polynomial function for q_1 articular variable for t∈[0,1]

The same method can be used for studying the complete model and for obtaining all laws of motions and velocities in joints through numerical integration of the differential equations resulting from the 2^{nd}-type Lagrange equations.

4 Conclusion

In order for the results of the dynamic modeling and to process a prosthetics for the upper limb based on outcome obtained by integrating the differential equations, the approximation of the functions $y\_i = f(t)$ can be made through the orthogonal polynomial method. After obtaining them, the opposed method can be applied in the differential equations of motion (29)-(33) to calculate the generalized forces that can be put in effect with the help of couple engines which will develop active moments in accordance with the results from the mentioned equations.

The variant studied in the paper can be extended to other cases in which the initial conditions are different and the time period is larger.

Both the obtained representations as well as the possible due to the changing of the initial conditions or the period of time, allow multiple subsequent uses.

The future work can be represented by realizing a dynamic model of human upper limb based on a mechanical system with more than 5 degrees of freedom.

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