Determination of Insurance Policy Using a hybrid model of AHP, Fuzzy Logic, and Delphi Technique: A Case Study

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Abstract: -Based on a previous study, this paper presents evaluation models for selecting insurance policies. Five models have been built for five insurances, respectively, including life, annuity, health, accident, and investment-oriented insurances. The proposed models consist of analytical hierarchy process (AHP), fuzzy logic and the Delphi technique. The Delphi technique is employed to select inputs, define fuzzy expressions, and generate evaluation rules for the models. Four variables are selected as the inputs including age, annual income, educational level and risk preference. These inputs are transferred to fuzzy variables using trapezoidal membership functions and then fed to AHP. To build the models, we interviewed twenty domain experts with at least three years of working experience in insurance companies. To validate the performance, we designed a computer program and used 300 insurance purchase records to examine the evaluation results. Validation results and conclusive remarks are also provided at the end of this paper.

Key-words: - Insurance policy; Decision making; Fuzzy logic; AHP; Delphi technique; Evaluation model

1. Introduction

For insurance consultants, it is a little difficult to determine the type of insurance policy for their customers (insurance buyers) according to the different needs and backgrounds of the buyers. In general, there are five main types of insurances including life, annuity, health, accident, and investment-oriented insurances. The backgrounds of potential purchasers of the five insurances are quite different. In addition, the motivation of buying a particular insurance significantly varies from insurance to insurance. It is an important issue for insurance consultants to determine a suitable insurance for their clients. This study provides the evaluation models for insurance consultants to select appropriate polices among the five insurances. Below, we briefly introduce the characteristics of the five insurances.

A life insurance is a contract between an insurant and an insurer with a specific insured amount. In life insurance, the situation (live or dead) of an insurant determines the benefit payment. An insurant will be provided with the insured amount whenever the pre-set condition for the benefit payment is met. The annuity insurance regularly offers benefits to insurants in a certain period. Policy owners of annuity insurance will periodically get the benefits in their life time. The health insurance covers the medical expenditure and redeems the reduction of the income due to illness. The accident insurance shall pay the pre-specified amount if the insurant gets injured due to an accident. The investment-oriented insurance is a new product combing the functions of insurance and investment. The insurant should take the investment risk, and the insured amount varies based on the investment effects.

Originally proposed by Zadeh in 1965, fuzzy theory nowadays has been applied in many areas widely [1]. Unlike a traditional crisp set using binary values (0 and 1) to indicate the belonging relationship between an element (value) and a set (‘1’ means belonging and ‘0’ means not belonging), fuzzy logic uses a gradually changing value (matching degree μ) to describe the belonging relationship between an element and a set. The matching degree can conceptually be considered as the degree of how a particular value (element) belongs to a fuzzy set. Fuzzy variables are therefore employed to represent the linguistic expressions of human beings due to the property of uncertainty. A membership function is utilized to describe the matching degrees for a fuzzy expression. Two types of membership functions are often used in fuzzy logic: triangular and trapezoidal functions. The trapezoidal membership function is generally defined by four parameters: a, b, c and d, as shown in the following equations:

$$Trapezoid(x) = \begin{cases} 0 & x < a \\ (x-a)/(b-a) & a \le x < b \\ 1 & b \le x < c \\ (d-x)/(d-c) & c \le x < d \\ 0 & x \ge d \end{cases} \quad (1)$$

Fig. 1 shows a trapezoidal membership function for a fuzzy set of "*The car speed is moderate*". In the figure, a = 40, b = 60, c = 90, and d = 110. The matching degree for the speed of 50 km/hour is 0.5. The triangular membership function is conveniently considered as a special type of the trapezoidal membership function where b is equal to c.

Fuzzy logic has widely been employed in many areas. For example, Lin discussed how a fuzzy expert system was utilized to determine basketball zone defense patterns [2,3]. A general study on fuzzy-logic-based insurance policies has been proposed in [4]. Sanchez presented a fuzzy regression model to determine insurance claim and to evaluate the financial performances of insurance companies in mutant and uncertain environment [5]. An evaluation model has been applied to pure premiums of automobile bodily injury liability using fuzzy logic [6]. A fine-tune fuzzy logic model has been utilized to determine the insurance changing rates based on group health insurance data. [7].

Analytical Hierarchy Process (AHP) was developed by Saaty to solve complicated decision-making problems with a lot of input variables under uncertain situations [8]. Basically, AHP uses a layered structure to hierarchically divide inputs into different layers (dimensions) to simplify a complicated problem. Each of layers consists of several nodes. Weights are used to connect the nodes in adjacent layers. AHP needs a specially designed questionnaire to perform pair-wise comparisons of importance among input variables (or dimensions). The results of the pair-wise comparisons are formulated in a matrix, called an ***importance matrix***. It is important to note that an importance matrix is symmetrical and reciprocal due to the property of pair-wise comparisons. Based on the maximum eigenvalue of the importance matrix, a consistency test is performed to examine the consistence of the results of the pair-wise comparisons. Weights are generated for AHP by taking the normalized eigenvector associated with the maximum eigenvalue of the importance matrix if a consistency test is passed.

AHP is widely utilized in many areas. For example, an AHP-based portfolio decision model has been presented to evaluate new products and assessing marketing [9]. AHP has also been employed to select life insurance contracts [10]. A hybrid framework using fuzzy logic and AHP has been proposed for route selection [11]. Huang presented a study on determining life and annuity insurances using AHP and fuzzy logic [12].

The Delphi technique is a method to get a consensus from a group of experts. It was originally developed by the RAND Corporation. The Delphi technique can be considered as a group decision-making methodology. It emphasizes forming the agreement from experts' opinions independently by showing anonymous ranking results. It avoids the mis-led conclusions dominated by some particular experts with their strong influences (authority) in the panel of the experts. Basically the Delphi technique uses questionnaires to collect the experts' opinions independently. This is an iterative process until the consensus is established. The Delphi technique has widely been applied in many fields [13-21]. For example, it was successfully applied in predicting the sale of a new product [13]. In addition, the Delphi technique has often been used to determine the budget allocations among different projects [15].

Based on the previous model using AHP and fuzzy logic [12], this paper presents extended models to determine the purchase policies for five main insurances using a hybrid framework of AHP, fuzzy logic, and the Delphi technique. Fig. 2 shows the conceptual diagram of the proposed models consisting of three units: the fuzzy logic unit, the AHP unit, and the Delphi technique unit. In the figure, four variables are utilized as inputs including age, annual income, educational level, and risk preference. Five evaluation models are established for determining the five insurances including life, annuity, health, accident, and investment-oriented insurances. In the fuzzy logic unit, trapezoidal functions are served to map the inputs to fuzzy variables. These fuzzy variables are then fed to the AHP unit. The evaluation models for the five insurance are final produced after employing the

AHP technique. The Delphi technique is employed in the phases of selecting the inputs, defining the fuzzy variables, designing the AHP questionnaire and generating the evaluation rules. Throughout the study, we interviewed twenty domain experts with at least three years working experience in insurance companies. These experts were also served to the domain experts for the Delphi technique.

In this paper, we present evaluation models for selecting insurance policies. Five models are built for five insurances, respectively, including life, annuity, health, accident, and investment-oriented insurances. The proposed models consist of analytical hierarchy process (AHP), fuzzy logic and the Delphi technique. The Delphi technique is employed to select inputs, define fuzzy expressions, and generate evaluation rules for the models. Four variables are selected as the inputs including age, annual income, educational level and risk preference. These inputs are transferred to fuzzy variables using trapezoidal membership functions and then fed to AHP. To build the models, we interviewed twenty domain experts with at least three years of working experience in insurance companies. To validate the performance, we designed a computer program and used 300 insurance purchase records to examine the evaluation results. Validation results and conclusive remarks are also provided at the end of this paper.

The evaluation models proposed by this study provide tools for insurance consultants to determine insurance purchasing policies for their customers which can reduce the subjective prejudice of the insurance consultants.

2 Preliminaries

Basically, an AHP model is a hierarchically layered structure. Each of layers contains several nodes. Weights are used to connect nodes between adjacent layers. The first layer of an AHP model represents the inputs (factors) and the rest of layers are conceptually considered as internal dimensions. Fig. 3 shows a simple two-layer AHP with four inputs. The first layer contains four nodes associated with the four inputs, respectively. The second layer contains three nodes. Each of them represents an internal dimension between the inputs and the output. The mapping function for a node is calculated by the following equation [12]:

$$y_j = \sum_{i=1}^{n} w_{ij} x_i \qquad (1)$$

where

y_j = the output for node j in a particular layer.

x_i = the output of node i in the previous layer of this particular layer,

w_{ij} = the weight connecting node j in the particular layer and the node i in the previous layer.

n = the number of nodes in the previous layer.

The procedure of the AHP model is summarized as follows [8, 9]:

Step 1: Determine the structure of the AHP model. Normally, this step is done by interviewing domain experts.

Step 2: Design questionnaire. The questionnaire is specially designed to perform all of possible pair-wise comparisons among input factors (or dimensions). A nine-point scale is usually utilized to indicate the importance ratio of one factor to another [8,9]. Table 1 shows a nine-point scale used for AHP. Table 2 demonstrates a simple AHP questionnaire with three factors: factors A, B, and C. In the table, the first row shows two factors for comparison (the leftmost cell and the rightmost cell) and the values of comparison result of the two factors. It is important to note that the values shown in the first row are symmetrical. People use values in the left side if the factor in the leftmost cell is more important than the factor in the rightmost cell. The rest of rows indicate the pair-wise comparisons of importance among any pairs of two factors. For example, assume factor A is three times importance than factor B. We then mark "∨" in the cell associated with 3 (close to factor A which is located in the left side) in the second row. The ratio of factor A to factor B is 3:1. Similarly, the importance ratio of factor A to factor C is 4: 1, and the importance ratio of factor B to factor C is 1:7. These are all shown in Table 2.

Step 3: Use the questionnaire to collect the experts' opinions on the importance ratios among the factors and then generate an importance matrix. For example, the importance matrix associated with the comparison results in Table 2 is given by

$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 1 & 3 & 4 \\ 1/3 & 1 & 1/7 \\ 1/4 & 7 & 1 \end{bmatrix} \end{array} \qquad (2)$$

It is important to note that the importance matrix is symmetrically reciprocal with 1's in its diagonal items.

Step 4: Perform a consistency test

Constituency Ratio (CR) is used to determine if a questionnaire passes the consistency test. Before getting a CR, we need to compute Consistency Index (CI) using the following formula

$$CI = \frac{\lambda_{\max} - n}{n-1} \quad (3)$$

where n is the number of factors, $\lambda_{\max}$ is the maximum eigenvalue of the importance matrix.

CR is then calculated by

$$CR = \frac{CI}{RI} \quad (4)$$

where RI (Random Index) is defined in Table 3. If CR is less than or equal to 0.1, the questionnaire passes the consistence test and the weights in an AHP model are the elements of the normalized eigenvector associated with λ_{max}. If CR is greater than 0.1, the questionnaire fails.

The Delphi technique is basically an iterative procedure to reach the consensus of a panel of experts. It has some variants in different applications. The general procedure of the Delphi technique is described as follows [14,15].

Step 1: Establish a panel of experts

Step 2: Generate the criteria (or items) form the panel. The brainstorming methodology is often applied in this step to obtain the criteria (or items). **Step 3**: Rank the criteria with experts' opinions. Usually the criteria (or items) are ranked by a three-point scale with 1 (very important), 2 (somewhat important) and 3 (not important).

Step 4: Compute the mean and standard deviation of the scores ranked by the panel of experts and delete the criteria (or items) whose means are greater than or equal to 2.0. Show the ranking results anonymously to the panel and analyze possible reasons for those criteria (or items) with high standard deviations.

Step 5: Repeat Step 3 to Step 4 if necessary. The procedure may be repeated many times until in a stable situation which means a consensus has been reached among the experts.

3. Research design and model building

In this study, twenty people with at least three years working experience in insurance companies in Taiwan were selected as the domain experts. Five evaluation models were built for the five main insurances: life, annuity, health, accident, and investment-oriented insurances, respectively. Four inputs were used for the evaluation models including age, annual income, educational level and risk preference. The four inputs were expressed in fuzzy variables using four trapezoidal functions, as shown in Fig. 4 [12].

The questionnaire for the study is shown in Table 4 [12].

Throughout this study, the Delphi technique was employed to reach the agreements among the twenty domain experts. First we employed the Delphi technique to determine the inputs and their associated membership functions. Second, we utilized it again to get the five evaluation rules for the five insurances. There were two iterative cycles to get the agreement in determining the inputs and their fuzzy expressions and three cycles in generating the five evaluation rules.

After reaching the agreements, we designed the questionnaire and sent it to the domain experts. Table 4 shows the questionnaire used in this study. We then collected the questionnaires answered by the domain experts and performed the AHP procedure to build the evaluation models. Below, we describe the AHP procedure step by step.

Step 1: Established the importance matrix and calculate the maximum eigenvalue (λ_{max}) of the matrix.

Step 2: Use Eqs. (3) and (4) to perform the consistency test.

Step 3: Get the weights for each of the questionnaires which pass the consistency test.

Step 4: Take geometrical averages of the weights and generate the five evaluation models for the five insurances, respectively.

Table 5 shows the evaluation rules obtained by performing the Delphi technique. Table 6 demonstrates the results of the consistency tests.

Finally, the evaluation models for the five insurance were built. They are shown as follows:

The evaluation model for the life insurance is given by

$$y = 0.36x_1 + 0.28x_2 + 0.10x_3 + 0.27x_4 \quad (5)$$

where

x_1 = the fuzzy variable of age,
x_2 = the fuzzy variable of annual income,
x_3 = the fuzzy variable of educational level, and
x_4 = the fuzzy variable of risk preference.

In Eq. (5), higher value of y means stronger recommendation on purchasing life insurance.

Using the same procedure, the rest of evaluation models are given by the following formulas:

Annuity insurance:

$$y = 0.28x_1 + 0.31x_2 + 0.10x_3 + 0.31x_4 \quad (6)$$

Health insurance:

$$y = 0.35x_1 + 0.26x_2 + 0.12x_3 + 0.27x_4 \quad (7)$$

Accident insurance:

$$y = 0.21x_1 + 0.29x_2 + 0.11x_3 + 0.39x_4 \quad (8)$$

Investment-oriented insurance:

$$y = 0.15x_1 + 0.32x_2 + 0.20x_3 + 0.33x_4 \quad (9)$$

In Eqs. (6) to (9), fuzzy variables x_1 to x_4 are the same as those in Eq. (5)

4. Validation

We designed a computer program and used a dataset of 300 insurance purchase records to validate the performance of the evaluation models. The computer program was implemented using java programming language. The 300 records were obtained from an insurance company in Taiwan.

There are three java classes in the compute program including ***FuzzyAh*****p**, ***Insurant***, and ***Membership*** classes. Figure 5 shows the class diagram (a Unified Modeling Language (UML) diagram) to describe the relationship among the three classes. In the Figure, ***FuzzyAHP*** is the main class to control the overall process in the computer program. Class ***Insurant*** is the class to get the outputs of the five evaluation models for an insurant in the data set. Class ***Membership*** is the class to generate the objects of the membership functions used in this study. The detailed descriptions on the variables and methods of the three java classes are demonstrated in Table 7.

The 300 samples are used to validate the performance of the proposed evaluation models. We applied Eqs. (5) to (9) to compute the evaluation results (y in each Eq.) for each sample in the dataset. Four threshold criteria are used to determine the purchases of the five insurance based on the computational results of the evaluation models. For each sample, we get the priorities of the five insurances based on the descending order of the evaluation results calculated by Eqs. (5) to (9). The four threshold criteria are shown as follows:

- Criterion 1: $y > 0.6$.
- Criterion 2: The first two insurances in the descending order of the evaluation results.
- Criterion 3: Satisfy Criterion 1 **or** Criterion 2.
- Criterion 4: $y > 0.5$

where y is calculated by Eqs. (5) to (9).

Table 8 shows the results of the validation. Based the validation results, in this study, we recommend Criterion 3 as the threshold criterion for the five proposed evaluation models.

5. Conclusions

We proposed the evaluation models for purchasing five insurances including life, annuity, health, accident, and investment-oriented insurances. The proposed models were established using a hybrid framework consisting of AHP, fuzzy logic, and the Delphi technique. Four factors were used as the inputs of the models including age, annual income, educational level and risk preference. Twenty experts with at least three years of insurance consulting experience were selected as the domain experts for building the models. The fuzzy logic served to transfer the inputs to the fuzzy variables using appropriate membership functions. These fuzzy variables were then fed to the AHP model. The AHP model performed the consistence test and finally generated the evaluation models for the five insurances. The Delphi technique was employed to select the inputs, define the fuzzy variables, and determine the evaluation rules for the proposed models.

To validate the performance, we designed a computer program and used 300 insurance purchase records to examine the evaluation results. We showed the validation results and recommended the threshold criterion for the evaluation models of the five insurances.

As the directions for future studies, we suggest using more examples to build the evaluation models. In addition, fuzzy expert systems might be utilized to build another decision model for determining insurance polices.

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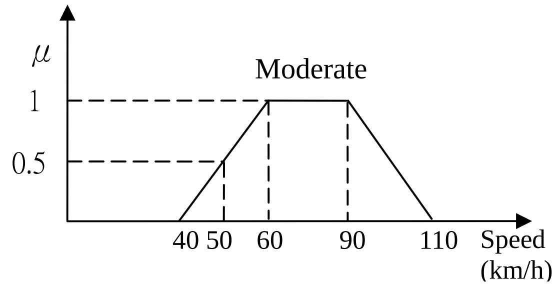


Fig. 1: Example of the membership function of “The car speed is moderate” .

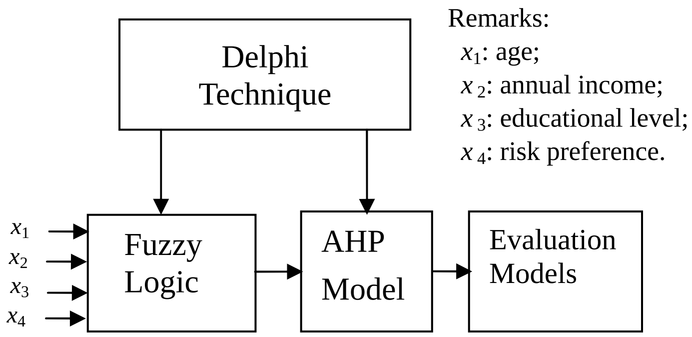


Fig. 2: The conceptual diagram of the propose models

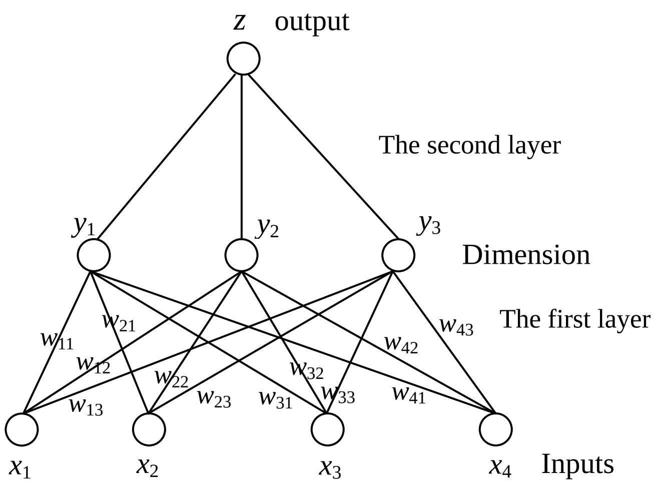
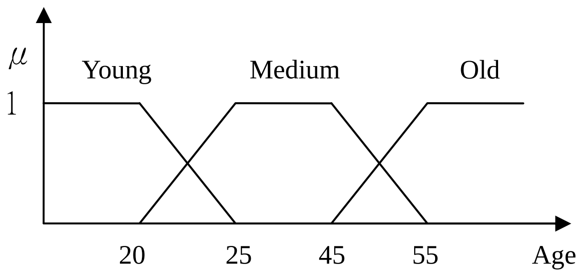
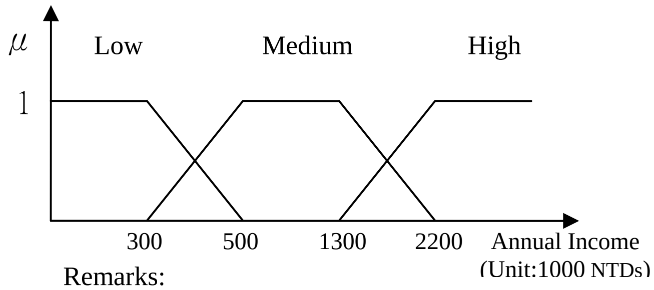


Fig. 3: A simple two-layer AHP



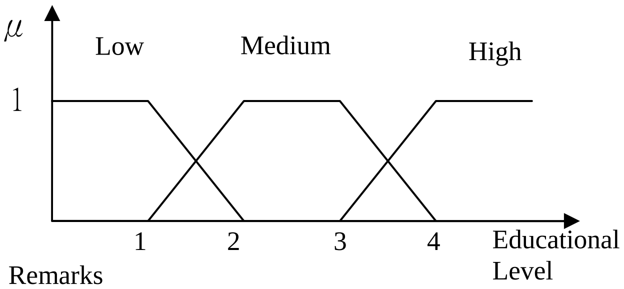
(a) Fuzzy variable of age



Remarks:

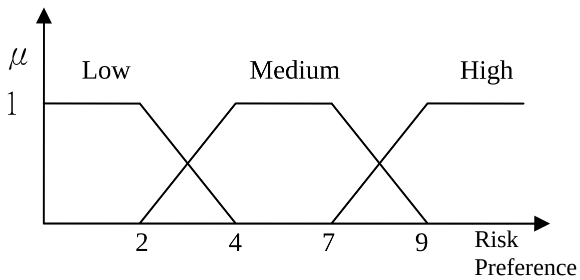
1. The exchange rate of NTD (New Taiwan Dollar) to USD (United States Dollar) is approximately 32: 1.
2. The per capita GNP (Gross National Product) in Taiwan is 567,923 NTD in 2007 [22].

(b) Fuzzy variable of annual income



Remarks
1: Junior High School or below
2: Senior high or Vocational School
3: College or Jiunor College (Community college)
4: Graduate School (Master or Doctoral degree)

(c) Fuzzy variable of educational level



Remarks:
Risk preference is scored by a ten-point scale and measured by the insurance consultants.

(d) Fuzzy variable of risk preference

Fig. 4: The four fuzzy variables for the proposed models [12]

Table 1: The definition and explanation of AHP 9-point scale (taken from [8, 9, 12])

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgment slightly favor one activity over another
5	Essential or strong importance	Experience and judgment strongly favor one activity over another
7	Demonstrated importance	An activity is strongly favored and its dominance is demonstrated in practice
9	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between the two adjacent judgments	When compromise is needed

Table 2: A simple example of questionnaire

Factor	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Factor
A							∨											B
A						∨												C
B															∨			C

Table 3: Random Index (taken from [8, 9, 12])

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
R.I.	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Remark: *n* is the number of factors

Table 4: Questionnaire for the study [12]

Factor	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Factor
Age																		Annual Income
Age																		Risk Preference
Age																		Educational Level
Annual Income																		Risk Preference
Annual Income																		Educational Level
Risk Preference																		Educational Level

Table 5: The evaluation rules

Insurance	Evaluation rule
Life	The prospect's age is medium The prospect's annual income is medium The prospect's education is medium The prospect's risk preference is low
Annuity	The prospect's age is high The prospect's annual income is high The prospect's education is medium The prospect's risk preference is low
Health	The prospect's age is low The prospect's annual income is low The prospect's education is medium The prospect's risk preference is medium
Accident	The prospect's age is medium The prospect's annual income is low The prospect's education is medium The prospect's risk preference is medium
Investment-oriented	The prospect's age is medium The prospect's annual income is medium The prospect's education is medium The prospect's risk preference is high

Table 6: The CRs of the consistency tests for the five insurances

Expert	Insurance				
	Life	Annuity	Health	Accident	Investment-oriented
1	0.0324*	0.0686*	2.1268	0.1932	0.0979*
2	0.3869	0.1567	0.0694*	0.1259	0.2456
3	0.3791	0.3791	0.3862	0.3791	0.2531
4	1.5616	0.2616	0.4085	0.1498	0.0562*
5	0.0428*	0.0572*	0.0572*	0.0428*	0.0572*
6	1.7444	0.7599	0.0881*	0.3474	0.7499
7	0.3791	0.3791	0.0572*	0.3759	0.2011
8	0.0887*	0.3862	0.2531	0.3158	0.0953*
9	0.2697	0.6854	0.0603*	0.0797*	0.0797*
10	0.0797*	0.1836	0.0923*	0.2684	0.0923*
11	0.2853	1.0132	0.2597	0.3158	1.4289
12	0.1491	0.2753	0.2129	0.0161*	0.5776
13	0.3158	1.5517	0.3791	0.1836	1.4338
14	0.2550	0.1596	0.0939*	0.0738*	0.2597
15	0.1893	0.1560	0.0976*	0.0121*	0.2427
16	0.3791	0.4362	0.4362	0.3069	0.4979
17	0.4279	0.2767	0.8623	0.0572*	0.0000*
18	0.3211	0.2460	0.1836	0.0562*	0.0834*
19	0.4035	0.0890*	0.0999*	0.0337*	0.2567
20	0.2684	0.0339*	0.1567	0.0161*	0.2559

Remarks: * : pass the consistency tests.

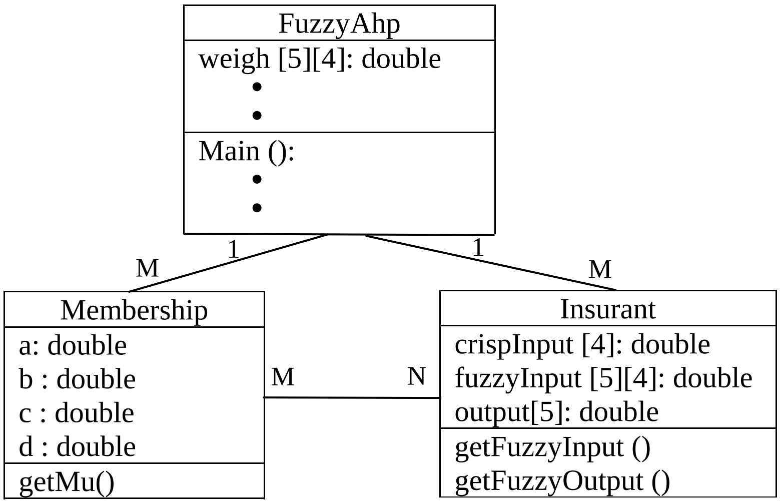


Fig. 5: The class diagram of the computer program

Table 7: The descriptions of the variables and methods for the three java classes in Figure 5.

Class	Variable / Method	Description
FuzzyAhp	weigh [5][4]	Storing the weights for the 5 evaluation rules; each with 4 weights.
	main ()	To control the main procedure. It is also the entering point of the computer program.
Membership	a, b, c, d	The 4 parameters to represent a trapezoidal membership function (see Eq. (1))
	getMu()	To perform fuzzy mapping: mapping a crisp number to a fuzzy number
Insurant	crispInput [4]	Storing the values of the four attributes (age, annual income, educational level and risk preference) for an insurant.
	fuzzyInput [5][4]	Storing the fuzzy numbers to be inputted to the 5 evaluation models; each with 4 fuzzy numbers
	output[5]	Storing the 5 outputs (evaluation results) for the 5 insurances.
	getFuzzyInput ()	To get the fuzzy numbers using the method of getMu() in Membership class
	getFuzzyOutput ()	To get the 5 outputs (evaluation results) for the 5 insurances.

Table 8: The results of the validation

Threshold criterion		Number of samples which are correctly determined	Evaluation accuracies
Number	Description		
1	$y > 0.6$	167	56%
2	The first two insurances in the descending order of the evaluation results.	188	63%
3	Satisfy Criterion 1 or Criterion 2	206	69%
4	$y > 0.5$	219	73%

Remarks: (1) Number of samples = 300; (2) y is obtained by Eqs. (5) to (9).