A Survey of Automata on Three-Dimensional Input Tapes

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Abstract: The question of whether processing three-dimensional digital patterns is much more difficult than two-dimensional ones is of great interest from the theoretical and practical standpoints. Recently, due to the advances in many application areas such as computer vision, robotics, and so forth, it has become increasingly apparent that the study of three-dimensional pattern processing has been of crucial importance. Thus, the research of three-dimensional automata as computational models of three-dimensional pattern processing has also been meaningful. The main purpose of this paper is to survey the definitions and properties of various three-dimensional automata.

Key–Words: Computation, Constructibility, Finite Automaton, Inkdot, Marker, Recognizability, Three-Dimension, Turing Machine

1 Introduction

Blum and Hewitt first proposed two-dimensional automata as computational models of two-dimensional pattern processing, and investigated their pattern recognition abilities [3]. Since then, many researchers in this field have been investigating a lot of properties about automata on a two-dimensional tape. Recently, due to the advances in computer vision, robotics, and so on, the study of three-dimensional information processing has been of great importance. For instance, three-dimensional image is now needed in visual communication, such as virtual reality systems. Even in the Internet environment, new protocols have been proposed for virtual reality communication on the WWW [53]. In the medical field, we can easily get the precise three-dimensional volumetric image of a human body by excellent equipments such as X-ray CT scanner and MRI scanner. Thus, the study of three-dimensional automata has been meaningful as the computational model of three-dimensional information processing. In this paper, we show a survey of three-dimensional automata. Section 2 observes a historical overview, and provides a background and a motive for the study of three-dimensional automata. Section 3 concerns deterministic, nondeterministic, alternating with only universal states, and alternating three-dimensional Turing machines (including finite automata). In Section 4, we deal with three-dimensionally space constructibility and space hierarchy. In Section 5, we show some results about recognizability of three-dimensional connected pictures. In Section 6, we introduce other topics of three-dimensional automata. Finally, in Section 7, we conclude this paper by summarizing the results.

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2 Historical Background

Computer science has two major components : first, the fundamental mathematics and theories underlying computing, and second, engineering techniques for the design of computer systems — hardware and software.

Theoretical computer science falls under the first area of the two major components. It had its beginnings in various field: physics, mathematics, linguistics, electric and electronic engineering, physiology, and so on. Out of these studies came important ideas and models that are central to theoretical computer science [1,14].

In theoretical computer science, Turing machine has played a number of important roles in understanding and exploiting basic concepts and mechanisms in computing and information processing. It is a simple mathematical model of computers which was introduced by Turing in 1936 to answer fundamental problems of computer science — 'What kind of logical work can we effectively perform ?' [59]. If the restrictions in its structure and move are placed on Turing machine, the restricted Turing machine is less powerful than the original one. However, it has become increasingly apparent that the characterization and classification of powers of the restricted Turing machines should be of great importance. Such a study was active in 1950's and 1960's. On the other hand, many researchers have been making their efforts to investigate another fundamental problems of computer science — 'How complicated is it to perform a given logical work?'. The concept of computational complexity is a formalization of such difficulty of logical works.

In the study of computational complexity, the complexity measures are of great importance. In general, it is well known that the computational complexity has originated in a study of considering how the computational powers of various types of automata are characterized by the complexity measures such as space complexity, time complexity, or some other related measures. Especially, the concept of complexity is very useful to characterize various types of automata from a point of view of memory requirements [34]. This study was motivated by Stearns, Hartmanis, and Lewis in 1965 [54]. They introduced an $L(m)$ space-bounded one-dimensional Turing machine to formalize the notion of space complexity, and investigated its computing ability. Some results were refined by Hopcroft and Ullman [12-14]. Moreover, Chandra, Kozen, and Stockmeyer introduced an alternating Turing machine as a theoretical model of parallel computation in 1981 [5]. An alternating Turing machine, whose state set is partitioned into two disjoint sets, the set of universal states and the set of existential states, is a generalization of a nondeterministic Turing machine. A nondeterministic Turing machine is an alternating Turing machine which has only existential states. In related paper [11,15,29-31,39,40, 45,55], several investigations of these machines have been continued.

After that, the growth of the processing of pictorial information by computer was rapid in those days. Therefore, the problem of computational complexity was also arisen in the two-dimensional information processing. Blum and Hewitt first proposed two-dimensional automata — two-dimensional finite automata and marker automata, and investigated their abilities of pattern recognition in 1967 [3]. Since then, many researchers in this field have been investigating a lot of properties about automata on a two-dimensional tape. For example, Morita, Umeo, and Sugata proposed an $L(m, n)$ space-bounded two-dimensional Turing machine and its variants to formalize memory limited computations in the two-dimensional information processing [33]. Inoue, Takanami, and Taniguchi introduced two-dimensional alternating Turing machines as a generalization of two-dimensional nondeterministic Turing machines and as a mechanism to model parallel computation. Restricted version of two-dimensional alternating Turing machines were investigated [24,25]. Special types of two-dimensional Turing machines (two-dimensional pushdown automata, stack automata, multicounter automata, multihead automata, and marker automata) were investigated [16,21,51,52,56]. Moreover, cellular automata on a two-dimensional tape were investigated not only in the viewpoint of formal language theory, but also in the viewpoint of pattern recognition. Cellular automata on a two-dimensional tape can be classified into three types. The first type, called a two-dimensional cellular automata, is investigated [2,6,7]. Especially, many properties of two-dimensional on-line tessellation acceptors, which are restricted type of two-dimensional cellular automata, are investigated [17,18,20]. The second type of cellular automata on a two-dimensional tape is investigated [19,42,44,58]. Two typical models of this type are parallel / sequential array automata and one-dimensional bounded cellular acceptors. The third type, called a pyramid cellular acceptor, is investigated [22]. More detailed survey of two-dimensional automata theory is done by Inoue and Takanami [23].

By the way, the question of whether processing three-dimensional digital patters is much difficult than two-dimensional ones is of great interest from the theoretical and practical standpoints. In recent years, due to the advances in many application areas such as computer graphics, computer-aided design / manufacturing, computer vision, image processing, robotics, and so on, the study of three-dimensional pattern processing has been of crucial importance [9,10,41,43]. Thus, the reseach of three-dimensional automata as the computational model of three-dimensional pattern processing has been meaningful. However, it is conjectured that the three-dimensional pattern processing has its own difficulties not arising in two-dimensional case. One of these difficulties occurs in recognizing topological properties of three-dimensional patterns because the three-dimensional neighborhood is more complicated than two-dimensional case. Generally speaking, a property or relationship is topological only if it is preserved when an arbitrary 'rubber-sheet' distortion is applied to the pictures. For example, adjacency and connectedness are topological; area, elongatedness, convexity, straightness, etc. are not.

During the past about thirty years, automata on a three-dimensional tape have been proposed and several properties of such automata have been obtained. Inoue and Nakamura proposed an n-dimensional on-line tessellation acceptor which can determine whether an n-dimensional tape is accepted or not by the on-line and parallel processing [17]. Blum and Sakoda investigated the capability of finite automata in two-dimensional and three-dimensional space [4]. Yamamoto, Morita, and Sugata introduced a three-dimensional k-marker automaton, an $L(m)$ space-bounded three-dimensional Turing machine and an $L(m)$ space-bounded five-way three-dimensional Turing machine [60]. They studied the problem of recognizing connectedness of three-dimensional patterns by these machines. Taniguchi, Inoue, and Takanami investigated the relationship between the accepting powers of three-dimensional finite automata and five-way three-dimensional Turing machines [57]. They also proposed a k-neighborhood template $\mathcal{A}$-type two-dimensional bounded cellular acceptor which consists of a pair of a converter and a configuration-reader, as the computational model of three-dimensional pattern process. The converter converts the given three-dimensional tape to the two-dimensional configuration, and the configuration-reader determines the acceptance or nonacceptance of given three-dimensional tape, depending on whether or not the derived two-dimensional configuration is accepted [58]. Nakamura and Aizawa proposed the interlocking component which is a chainlike connectivity a new topological property of three-dimensional digital pictures, and investigated the recognizability of interlocking components [37]. Sakamoto proposed multi-dimensional automata, and showed their several properties [46,47,49,50]. Moreover, Ito et al. investigated about synchronized alternation and parallelism for three-dimensional automata [27, 28].

3 Three-Dimensional Turing Machines

This section concerns alternating, nondeterministic, and deterministic three-dimensional Turing machines, including three-dimensional finite automata and marker automata.

3.1 Preliminaries

Definition 3.1. Let Σ be a finite set of symbols. A *three-dimensional input tape* over Σ is a three-dimensional rectangular array of elements of Σ. The set of all the three-dimensional input tapes over Σ is denoted by $\Sigma^{(3)}$.

Given an input tape $x \in \Sigma^{(3)}$, for each integer $j (1 \leq j \leq 3)$, we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(3)}$ with $l_1(x) = n_1$, $l_2(x) = n_2$ and $l_3(x) = n_3$ is denoted by $\Sigma^{(n_1,n_2,n_3)}$. When $1 \leq i_j \leq l_j(x)$ for each $j (1 \leq j \leq 3)$, let $x(i_1, i_2, i_3)$ denote the symbol in x with coordinates (i_1, i_2, i_3). Furthermore, we define $x[(i_1, i_2, i_3),(i'_1, i'_2, i'_3)]$, when $1 \leq i_j \leq i'_j \leq l_j(x)$ for integer $j (1 \leq j \leq 3)$, as the three-dimensional input tape y satisfying the following conditions:

(i) for each j $(1 \leq j \leq 3)$, $l_j(y) = i'_j - i_j + 1$;
(ii) for each r_1, r_2, r_3 $(1 \leq r_1 \leq l_1(y)$, $1 \leq r_2 \leq l_2(y)$, $1 \leq r_3 \leq l_3(y))$, y (r_1, r_2, r_3) = x $(r_1 + i_1 - 1$, $r_2 + i_2 - 1$, $r_3 + i_3 - 1)$.

Definition 3.2. A *three-dimensional alternating Turing machine* (3-ATM) is a seven-tuple $M = (Q, q_0, U, F, \Sigma, \Gamma, \delta)$, where

(i) Q is a finite set of *states*;
(ii) $q_0 \in Q$ is the *initial state*;
(iii) $U \subseteq Q$ is the set of *universal states*;
(iv) $F \subseteq Q$ is the set of *accepting states*;
(v) Σ is a finite *input alphabet* ($\# \notin \Sigma$ is the *boundary symbol*);
(vi) Γ is a finite *storage tape alphabet* ($B \in \Gamma$ is the *blank symbol*; and
(vii) $\delta \subseteq (Q \times (\Sigma \cup \{\#\}) \times \Gamma) \times (Q \times (\Gamma - \{B\}) \times$ *(east,west,south,north,up,down,no move)* $\times$ *(right,left,no move)* is the *next move relation*.

A state q in $Q - U$ is said to be *existential*. As shown in Fig.1, the machine M has a read-only rectangular input tape with boundary symbols '$\#$' and one semi-infinite storage tape, initially blank. Of course, M has a finite control, an input head, and a storage-tape head. A position is assigned to each cell of the storage tape, as shown in Fig.1.

A *step* of M consists of reading one symbol from each tape, writing a symbol on the storage tape, moving the input and storage heads in specified directions

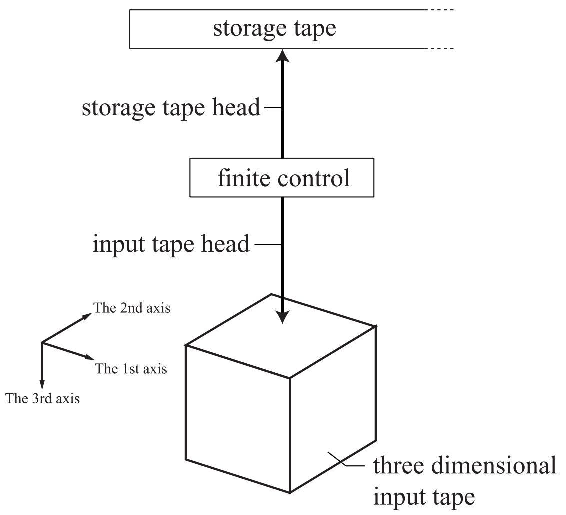

Fig. 1: Three-Dimensional Alternating Turing Machine.

(east, west, south, north, up, down, or no move for input head, and left, right, or no move for storage head), and entering a new state, in accordance with the next move relation δ.

Definition 3.3. A *configuration* of a 3-ATM M =(Q,q_0,U,F,Σ,Γ,δ) is a pair of an element of $\Sigma^{(3)}$ and an element of $C_M = (\boldsymbol{N} \cup \{0\})^3 \times S_M$, where $S_M = Q \times (\Gamma - \{B\})^* \times \boldsymbol{N}$, and $\boldsymbol{N}$ denotes the set of all positive integers. The first component x of a configuration $c = (x, ((i_1, i_2, i_3)$,(q,α,j))) represents the input to M. The second component (i_1, i_2, i_3) of c represents the input-head position. The third component (q, α, j) of c represents the state of the finite control, nonblank contents of the storage tape, and the storage-head position. An element of C_M is called a *semi-configuration* of M and an element of S_M is called a storage state of M. If q is the state associated with configuration c, then c is said to be a universal (existential, accepting) configuration if q is a *universal (existential, accepting)* state. The initial configuration of M on input x is $I_M(x) = (x, ((1, 1, 1), (q_0, \lambda, 1)))$, where λ is the null string.

Definition 3.4. Given $M = (Q, q_0, U, F, \Sigma, \Gamma, \delta)$, we write $c \vdash_M c'$ and c' is a *successor* of c if configuration c' follows from configuration c in one step of M, according to the transition rules δ. $\vdash_M^*$ denotes the reflexive transitive closure of $\vdash_M$. The relation $\vdash_M$ is not necessarily single-valued, because δ is not. A *computation path* of M on x is a sequence $c_0 \vdash_M c_1 \vdash_M \cdots \vdash_M c_n (n \geq 0)$, where $c_0 = I_M(x)$.

A computation tree of M is a finite, nonempty labeled tree with the following properties:

(i) each node v of the tree is labeled with a configuration $l(v)$;
(ii) if v is an internal node (a nonleaf) of the tree, $l(v)$ is universal and $c|l(v) \vdash_M c$ =c_1,$\cdots$,c_k, then v has exactly k children $v_1, \cdots, v_k$ such that $l(v_i) = c_i$ $(1 \leq i \leq k)$; and
(iii) if v is an internal node of the tree and $l(v)$ is existential, then v has exactly one child u such that

$$l(v) \vdash_M l(u).$$

A computational tree of M on input x is a computation tree of M whose root is labeled with $I_M(x)$. An *accepting computation tree* of M on x is a computation tree of M on x whose leaves are labeled with accepting configurations. We say that M accepts x if there is an accepting computation tree of M on input x. Define

$$T(M) = \{x \in \Sigma^{(3)} \mid M \ accepts \ x\}.$$

We next define a five-way three-dimensional alternating Turing machine, which can be considered as an alternating version of a five-way three-dimensional Turing machine [49].

Definition 3.5. A five-way three-dimensional alternating Turing machine ($FV3$-ATM) is a 3-ATM $M = (Q, q_0, U, F, \Sigma, \Gamma, \delta)$ such that
$\delta \subseteq (Q \times (\Sigma \cup \{\#\}) \times \Gamma) \times (Q \times (\Gamma - B) \times$ $\{$*east,west,south, north, down, no move*$\}$ $\times$ $\{$*right, left, no move*$\}$).

That is, an $FV3$-ATM is a 3-ATM whose input head can move east, west, south, north, or down, but not up.

The set of states of alternating automaton is partitioned into two nonintersecting sets, i.e., the set of existential states and the set of universal states. A *three-dimensional nondeterministic Turing machine* (3-NTM) (a *five-way three-dimensional nondeterministic Turing machine* ($FV3$-NTM)) is a 3-ATM ($FV3$-ATM) which has no universal state. Conversely, a 3-ATM ($FV3$-ATM) which has no existential states is called a *three-dimensional alternating Turing machine with only universal states* (3-UTM) (a *five-way three-dimensional alternating Turing machine with only universal states* ($FV3$-UTM)). Of course, a *three-dimensional deterministic Turing machine* (3-DTM) (a *five-way three-dimensional deterministic Turing machine* ($FV3$-DTM)) is a special case of 3-ATM ($FV3$-ATM), i.e., each of whose configurations has at most one successor.

Definition 3.6. Let $L(m)$: $\boldsymbol{N} \rightarrow \boldsymbol{R}$ be a function with one variable m, where $\boldsymbol{N}$ is the set of all positive integers and $\boldsymbol{R}$ is the set of all nonnegative real numbers. With a 3-ATM M we associate *a space complexity function* SPACE that takes configurations to natural numbers. That is, for each configuration $c = (x, ((i_1, i_2, i_3), (q, \alpha, j)))$, let SPACE($c$) $= |\alpha|$. We say that M is $L(m)$ *space-bounded* if for all $m \geq 1$ and for each x with $l_1(x) = l_2(x) = l_3(x) = m$, if x is accepted by M, then there is an accepting computation tree of M on input x such that for each node v of the tree, SPACE($l(v)$) is smaller than or equal to the smallest integer which is greater than or equal to $L(m)$. By '3-$ATM(L(m))$' ('$FV3$-$ATM(L(m))$', '3-$UTM(L(m))$', '$FV3$-$UTM(L(m))$', '3-$NTM(L(m))$', '$FV3$-$NTM(L(m))$', '3-$DTM(L(m))$', ('$FV3$-$DTM(L(m))$') we denote an $L(m)$ space-bounded 3-ATM ($FV3$-ATM, 3-UTM, $FV3$-UTM, 3-NTM, $FV3$-NTM, 3-DTM, $FV3$-DTM).

A 3-ATM(0) ($FV3$-ATM(0)) is called a *three-dimensional alternating finite automaton (five-way three-dimensional alternating finite automaton)*, and denoted by 3-AFA ($FV3$-AFA). Similarly, a 3-UTM(0) (3-NTM(0), 3-DTM(0)) is called a three-dimensional alternating finite automaton with only universal states (three-dimensional nondeterministic finite automaton, three-dimensional deterministic finite automaton), denoted by '3-UFA' ('3-NFA', '3-DFA'). A five-way 3-UFA (3-NFA, 3-DFA) is denoted by '$FV3$-UFA' ('$FV3$-NFA', '$FV3$-DFA').

Moreover, we introduce two automata for the improvement of picture recognizability of the finite automaton. One is a multi-marker automaton. It is a finite automaton which keeps marks as 'pebbles' in the finite control, and cannot rewrite any input symbols but can make marks on its input with the restriction that only a bounded number of these marks can exist at any given time [32, 49]. The other is a multi-inkdot automaton. This automaton is a conventional finite automaton capable of dropping an inkdot on a given input tape for a landmark, but unable to further pick it up. For a three-dimensional k-marker automaton (three-dimensional k-inkdot automaton) is denoted by 3-XMA_k (3-XIA_k) for each $X \in \{D, N, U, A\}$, where D means determinism, N means nondeterminism, U means alternation with only universal states, and A means alternation.

For each $X \in \{D, N, U, A\}$, we denote a 3-XTM ($FV3$-XTM, 3-$XTM(L(m))$, $FV3$-$XTM(L(m))$, 3-XFA, $FV3$-XFA) whose input tapes are restricted to cubic ones by 3-XTM^c. $FV3$-XTM^c, 3-$XTM^c(L(m))$, etc. have the same meaning.

For each $X \in \{D, N, U, A\}$, we denote by $\mathcal{L}$[3-XTM] the class of sets of all three-dimensional tapes accepted by 3-XTM's. That is, $\mathcal{L}$[3-XTM] $=T|T = T(M)$ for some 3-XTM M. $\mathcal{L}[FV3$-$XTM]$, $\mathcal{L}$[3-$XTM(L(m))$], etc. are defined similarly.

3.2 Main Accepting Powers

This subsection states the accepting powers of three-dimensional Turing machines [49].

Theorem 3.1. If $L(m) = o(logm)$, then $\mathcal{L}$[3-$DTM^c(L(m))$] $\subsetneq$ $\mathcal{L}$[3-$UTM^c(L(m))$] $\subsetneq$ $\mathcal{L}$[3-$ATM^c(L(m))$].

Corollary 3.1. $\mathcal{L}$[3-DFA^c] $\subsetneq$ $\mathcal{L}$[3-NFA^c] $\subsetneq$ $\mathcal{L}$[3-AFA^c].

Theorem 3.2. If (i) $L(m)=o(m^2)$, or (ii) $L(m) \geq logm$ and $L(m)=o(m^3)$, then $\mathcal{L}[FV3$-$DTM^c(L(m))]$ $\subsetneq$ $\mathcal{L}[FV3$-$UTM^c(L(m))]$ $\subsetneq$ $\mathcal{L}[FV3$-$ATM^c(L(m))]$, and $\mathcal{L}[FV3$-$UTM^c(L(m))]$ and $\mathcal{L}[FV3$-$NTM^c(L(m))]$ are incomparable.

Corollary 3.2. (i) $\mathcal{L}[FV3$-$UFA^c]$ $\subsetneq$ $\mathcal{L}[FV3$-$AFA^c]$. (ii) $\mathcal{L}[FV3$-$UFA^c]$ is incomparable with $\mathcal{L}[FV3$-$NFA^c]$. (iii) $\mathcal{L}[FV3$-$DFA^c]$ $\subsetneq$ $\mathcal{L}[FV3$-$UFA^c]$.

Theorem 3.3. If (i) $L(m) = o(m^2)$, or (ii) $L(m) \geq logm$ and $L(m) = o(m^3)$, then $\mathcal{L}[FV3$-$UTM^c(L(m))]$ $\subsetneq$ $\mathcal{L}$[3-$UTM^c(L(m))$].

Corollary 3.3. $\mathcal{L}[FV3$-$UFA^c]$ $\subsetneq$ $\mathcal{L}$[3-UFA^c].

Theorem 3.4. $\mathcal{L}[FV3$-$UFA^c]$ $\subsetneq$ $\mathcal{L}[FV3$-$DTM^c(m^2)]$, and space m^2 is necessary and sufficient for $FV3$-DTM^c's and $FV3$-NTM^c's to simulate $FV3$-UFA^c's.

Theorem 3.5. $\mathcal{L}$[3-UFA^c] $\subsetneq$ $\mathcal{L}[FV3$-$DTM^c(m^3)]$, and space m^3 is necessary and sufficient for $FV3$-DTM^c's to simulate 3-UFA^c's.

Remark 3.1. We conjecture that $\mathcal{L}$[3-UFA^c] $\subsetneq$ $\mathcal{L}[FV3$-$NTM^c(m^2)]$, but we have not completed the proof of this conjecture yet.

Theorem 3.6. Space m^3 is necessary and sufficient for $FV3$-DTM^c's to simulate $FV3$-AFA^c's and 3-AFA^c's.

Open Problems 3.1. (i) Is $\mathcal{L}$[3-$NTM^c(L(m))$] incomparable with $\mathcal{L}$[3-$UTM^c(L(m))$] for any L such that $L(m) = o(logm)$? (ii) $\mathcal{L}[3\text{-}DTM^c(L(m))] \subsetneq \mathcal{L}[3\text{-}NTM^c(L(m))] \subsetneq \mathcal{L}[3\text{-}ATM^c(L(m))]$ for any $L(m) \geq logm$?

4 Three-Dimensionally Space Constructibility and Space Hierarchy

This section concerns three-dimensionally space constructible functions and space complexity hierarchy of three-dimensional Turing machines whose input tapes are restricted to cubic ones.

4.1 Main Accepting Powers

Definition 4.1. A function $L(m)$: $\boldsymbol{N} \rightarrow \boldsymbol{R}$ is called *three-dimensionally space constructible* if there is a 3-$DTM(L(m))^c$ M such that for each $m \geq 1$, there exists some input tape x with $l_1(x) = l_2(x) = l_3(x) = m$ on which M halts after its storage head has marked off exactly the greatest integer cells which is smaller than or equal to $L(m)$. (In this case, we say that M constructs the function L in the storage tape.)

Definition 4.2. A function $L(m)$: $\boldsymbol{N} \rightarrow \boldsymbol{R}$ is called *three-dimensionally fully space constructible* if there is a 3-$DTM(L(m))^c$ M which, for each $m \geq 1$ and for each input tape x with $l_1(x) = l_2(x) = l_3(x) = m$, makes use of exactly the greatest integer cells which is smaller than or equal to $L(m)$ and halts.

4.2 Three-Dimensionally Space Construtible Functions and Complexity Results

In this subsection, we show three-dimensionally fully space constructibility and space complexity hierarchies of three-dimensional Turing machines whose input tapes are restricted to cubic ones [49].

Theorem 4.1. We consider the following three functions :

$log^{(0)}m = m$,
$log^{(k)}m = log(log^{(k-1)}m)$, for $k \geq 1$, and
$log^{*}m = min\{x | log^{(k)}m \leq 1\}$.

Then, the functions $log^{(k)}m$ (k: any natural number) and $log^{*}m$ are three-dimensionally fully space constructible.

Theorem 4.2. For any $X \in \{D, N, U, A\}$, for any function $L(m)$: $\boldsymbol{N} \rightarrow \boldsymbol{R}$, and for any constant $d > 0$,

$$\mathcal{L}[3\text{-}XTM^c(L(m))] = \mathcal{L}[3\text{-}XTM^c(L(m)+d)].$$

Theorem 4.3. For any $X \in \{D, N, U, A\}$, for any function $L(m)$: $\boldsymbol{N} \rightarrow \boldsymbol{R}$, and for any constant $d > 0$,

$$\mathcal{L}[3\text{-}XTM^c(L(m))] = \mathcal{L}[3\text{-}XTM^c(dL(m))].$$

Theorem 4.4. Let $L_1(m)$ and $L_2(m)$ be any functions such that (i) $L_2(m)$ is three-dimensionally space constructible, (ii) $lim_{i\rightarrow\infty} L_1(m_i)/L_2(m_i) = 0$, and (iii) $L_2(m_i)/logm_i > k$ (i=1,2,...) for some increasing sequence of natural numbers m_i and for some constant $k > 0$. Then there exists a set T in $\mathcal{L}[3\text{-}XTM^c(L_2(m))]$, but not in $\mathcal{L}[3\text{-}XTM^c(L_1(m))]$ for any $X \in \{D, N\}$.

Theorem 4.5. For any functions $L_1(m)$ and $L_2(m)$ such that (i) $L_2(m)$ is three-dimensionally space constructible, (ii) $L_1(m) = o(L_2(m))$, there exists a set in $\mathcal{L}[3\text{-}DTM^c(L_2(m))]$, but not in $\mathcal{L}[3\text{-}NTM^c(L_1(m))]$.

Open Problems 4.1. (i) Are the functions $log^{(k)}m$ ($k \geq 3$) and $log^{*}m$ fully space constructible by one-dimensional deterministic two-head Turing machines or by two-dimensional deterministic Turing machines with square inputs? (ii) Is there any other unbounded function below $logm$ which is three-dimensionally fully space constructible? (iii) Is there an infinite tight hierarchy for 3-$ATM^c(L(m))$'s with $L(m) \geq logm$? (iv) Is there an infinite space hierarchy for 3-$ATM^c(L(m))'$ with $L(m) \leq loglogm$?

5 Recognizability of Connected Pictures

The recognition of the connectedness of digital pictures is one of the most fundamental problems in picture processing. There have been various results related to this problem. Especially, to recognize three-dimensional connectedness seems to be much more difficult than the two-dimensional case, because of intrinsic characteristics of three-dimensional pictures. This section mainly show the recognizability of three-dimensional connected tapes by three-dimensional automata. Let T_C be the set of all three-dimensional connected pictures. It is interesting to investigate how much space is required for three-dimensional Turing machines to accept T_C. For this problem, we have

Theorem 5.1. (i) $T_C \in \mathcal{L}[3\text{-}AFA^c]$. (ii) $logm$ space is necessary and sufficient for $FV3\text{-}ATM$'s to recognize T_C.

Theorem 5.2. $T_C \in \mathcal{L}[3\text{-}NMA_1]$ [60].

Theorem 5.3. (i) the necessary and sufficient space for $FV3$-DTM's to simulate 3-DTM_1's (3-NTM_1's) is $2^{lmloglm}$ (2^k, where $k = l^2m^2$). (ii) the necessary and sufficient space for $FV3$-NTM's to simulate 3-DTM_1's (3-NTM_1's) is $lmloglm$ (l^2m^2), where $l(m)$ is the number of rows (columns) on each plane of three-dimensional rectangular input tapes.

Theorem 5.4. $T_C \notin \mathcal{L}[3\text{-}NIA_k]$ [32].

Remark 5.1. $[3\text{-}NIA_k] \subsetneq \mathcal{L}[3\text{-}AIA_k]$ for any integer k.

Open Problems 5.1. (i) $T_C \notin \mathcal{L}[3\text{-}DTM(L(m))]$ or $T_C \notin \mathcal{L}[3\text{-}NTM(L(m))]$ for $L(m) = o(logm)$? (ii) $T_C \in \mathcal{L}[3\text{-}UIA_1]$? (iii) Is T_C accepted by a 3-DTM_1?

By the way, *Digital geometry* has played an important role in computer image analysis and recognition [43]. In particular, there is a well-developed theory of *topological* properties such as *connectedness* and *holes* for two-dimensional arrays [44]. On the other hand, three-dimensional information processing has also become of increasing interest with the rapid growth of computed tomography, robotics, and so on. Thus it has become desirable to study the geometrical properties such as *interlocking components* and *cavities* for three-dimensional arrays [37]. Interlocking components was proposed as a new topological property of three-dimensional digital pictures in [37]. Let S_1 and S_2 be two subsets of the same three-dimensional digital picture. S_1 and S_2 are said to be interlocked when they satisfy the following conditions:

(1) S_1 and S_2 are *tori*,
(2) S_1 goes through a *hole* of S_2,
(3) S_2 goes through a *hole* of S_1.

The interlocking of S_1 and S_2 is illustrated in Fig.2. This relation may be considered as a chainlike connectivity. Generally speaking, a property or relationship is *topological* if it is preserved when an arbitrary 'rubber-sheet ' distortion is applied to the pictures. For example, adjacency and connectedness are topological; area, elongatedness, convexity, straightness, etc. are not.

It is proved that no one-marker automaton can recognize interlocking components in a three-dimensional input tape [37]. Moreover, in [35], three-dimensional one-marker automata are investigated in terms of the space complexities that five-way three-dimensional Turing machines require and suffice to recognize interlocking components.

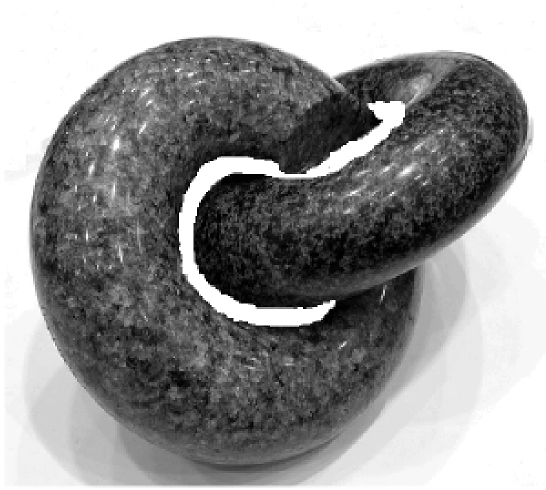

Fig. 2: Interlocking Components.

6 Other Topics

In this section, we list up other topics and related references about three-dimensional automata.

(i) Properties of special types of three-dimensional Turing machines (leaf-size bounded automata, parallel automata, multi-counter automata, etc. on three-dimensional tapes) [28,48,49].
(ii) Cellular types of three-dimensional automata [8,17,22,58].
(iii) Closure properties [14,23,49].
(iv) Recognizability of topological properties [36-38].
(v) NP-complete problems [14,26,49].

7 Conclusions

In this paper, we surveyed several aspects of three-dimensional automata. Especially, we dealt with three-dimensional Turing machines, including finite automata, three-dimensionally space constructability, recognizability of three-dimensional connected pictures, and so on. We believe that there are many problems about three-dimensional automata to solve in the future. We hope that this survey will activate the investigation of three-dimensional automata theory.

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