

Markov Models and Their Use for Calculations of Important Traffic Parameters of Contact Center

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Abstract: - The appropriate model and consecutive design is the basis for building of successful Contact center. Excessive complexity of model can be sometimes contra-productive. The main goal of this work is to analyze of Markov model $M/M/m/\infty$ properties in Contact center environment. This model offers wide range of important traffic parameters calculations of the system, while keeping transparent and simple structure – the key factor in design of Contact center. The second part of the paper deals with the modelling of contact center equipped with Interactive Voice Response (IVR) system. IVR system is modeled by Markov model $M/M/\infty/\infty$. It results in the less number of needed agents in contact center.

Key-Words: - $M/M/m/\infty$, $M/M/\infty/\infty$, Erlang C formula, Automatic Call Distribution, Interactive Voice Response, Quality of Service

1 Introduction

Contact center is complex communication system which arises as upgrade of private branch Exchange and offers integrated system for communication with customers whether by telephone call, e-mail or even by text chat with contact center agent.

Contact center [17] consist of: Private Branch eXchange (PBX), Automatic Call Distribution (ACD), Computer Telephony Integration (CTI), Interactive Voice Response (IVR), Call Management System (CMS), Voice Recording (VR), Campaign manager (CM), agents and supervisors workplaces, email and web server (see Fig. 1).

It is very difficult to describe such complex system by mathematical models [11]. Therefore in the next we will see under term Contact center only the system for processing of telephone calls [9].

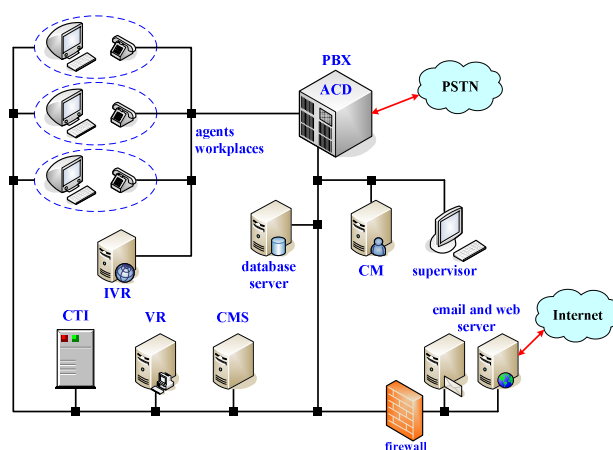


Fig. 1. Contact center architecture.

2 Contact Center as Queuing System

The system in which some number of service stations serves large volume of requests is termed as queuing system [15]. The Contact center is such queuing system. In this case the requests are represented by customers calling to Contact center

and service stations are represented by Contact center agents. The task of these agents is to handle customer requests. The general example of queuing system is depicted in the Fig. 2.

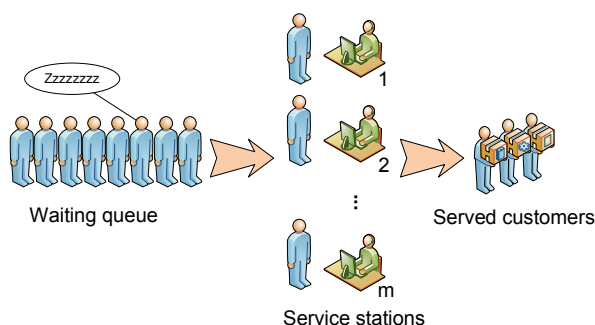


Fig. 2. Illustration of Queuing system.

Let us assume that system in the Fig. 2 is Contact center with m service stations or m agents. Requests are generated by customers randomly and independently each other calling into Contact center. Agents are serving callers one by one and in the case if all agents are busy, another customers have to wait in the waiting queue until some agent will be free. Call handling time is random and is not dependent on handling time of other calls. In this case we can this Contact center name as stochastic queuing system.

Statistical observation as shown that incoming telephone calls into Contact center simulate Poisson distribution of random variable, handling time is represented by exponential distribution [1]. Behaviour of such system can be described by Markov processes.

2.1 Automatic Call Distribution

Automatic Call Distribution (ACD) belongs to the basic software features, that is needed to the contact center realization.

Agents assigned to the individual service groups (Fig. 3) process the same type of incoming calls. ACD classifies and routes incoming calls according to programmed rules – e.g. according to the called number, arrival time of call, length of the waiting queue, etc. Then the calls are routed to the agent service groups (sales department, technical department, customer service, etc.). Through the ACD the calls can be routed to the next free, or to the longest free agent of the particular service group. The result is the uniform distribution of working load on all agents of the service group or processing the major of phone calls by one of the best agents.

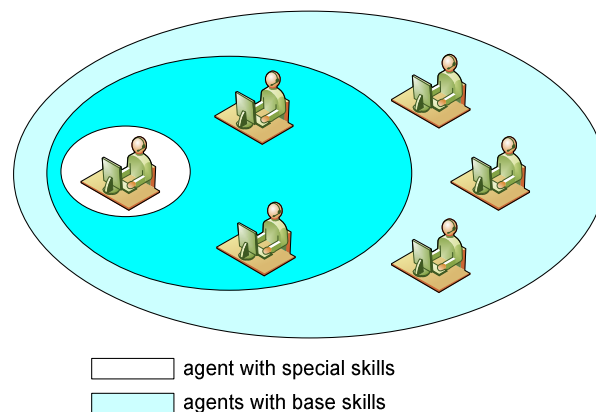


Fig. 3. Service group with special agent.

The task of the ACD is to connect the calling customer with appropriate agent - it is a complex operation with these important issues:

- customer request for the service,
- agent profil,
- agents state (free or busy),
- state of the waiting queues,
- system load in real time,
- alternative resource (in the case of load).

Contact centers are built in order to effective resources utilization (whether technical or human). Thereat, we have also to think about the situation that there is no free agent with appropriate skills for the customer. Therefore, there must be waiting queues in contact center.

2.2 Waiting Queues in Contact Center

On the basis of requested service the call is routed to the waiting queue assigned for the identified service. The behaviour of the call in the waiting queue is according to the FIFO (First In First Out) rule – thus, the call at the beginning of the waiting queue will be routed to the agent as the first. The special event occurs when the caller is specified as VIP (Very Important Person). Then the call is not placed to the end of the waiting queue, but to the beginning.

The calls rest in waiting queue till:

- release of some agent of called service group,
- expiration of time interval assigned for the customers waiting,
- session cancelation from the caller side.

For callers assigned into the waiting queue it is possible according to the predefined scenario to play up the different announcements (the position of the

caller in the waiting queue or forecast of the waiting time), music or ringing tone. The other possibility for the caller is to offer also information about new products without change of the position in the waiting queue. If the call is not processed till defined time, this call can be routed to other service group (i.e. call overflow), or it can be played another announcement.

In service group, the individual agent can have special skills (e.g. when the agent knows some languages) and for this agent, in case of free, is assigned call that needs his special skills with the maximum priority.

2.3 Poisson and Exponential Distribution

Number of incoming calls per time unit represents the discrete random variable. For description of random variable, we use its probability function defined in discrete form for random variable X , which takes values x as follows:

$$f(x) = P(X = x) \quad (1)$$

It is obvious, that $\sum_{x \in H} f(x) = 1$, where H is the set of all values which the random variable X can obtain.

For description of incoming requests into the Queuing System the *Poisson distribution* [3] is the most commonly used, for which is valid:

$$p_k = f(k) = P(X = k) = \frac{\lambda}{k!} e^{-\lambda}, \quad (2)$$

where $E(X) = \lambda$, $D(X) = \lambda$, λ represents the parameter of distribution and specifies mean number of requests per time unit.

The time which elapses between incoming of individual requests is also random variable. Based on the independence of the individual requests, it is possible for Poisson input flow to declare, that this continuous random variable has exponential distribution with mean value $1/\lambda$ [14]. Density of this random variable is defined as follows (t represents the inter-arrival time) [3]:

$$f(t) = \lambda e^{-\lambda t} \quad (3)$$

The random variable based on Poisson distribution is the most frequently used mathematical model for description of incoming calls into the contact center.

3 Markov Processes

Markov processes offer flexible and very efficient tool for description and analysis of dynamical properties of stochastic queuing systems [2], [3].

Let $T = \{t_0, t_1, \dots, t_n\}$ is set of various times, while $0 = t_0 < t_1 < \dots < t_n$ and let $S = \{s_1, s_2, \dots, s_m\}$ is set of all states in which the system can be in various times. Then such system can be described by Markov processes, if the following condition holds (4):

$$P(X_{t_{n+1}} \leq s_{n+1} | X_{t_n} = s_n) \quad (4)$$

i.e. probability that system is in time t_{n+1} in state s_{n+1} , depends only on assumption that in time t_n the system was in state s_n and does not depends on in which states and which times the system was prior to time t_n [2].

4 Markov Model $M/M/m/\infty$

From the large number of Markov models of queuing systems, it is possible to choose the most appropriate model for each system according to various criterions (complexity, computation costingness, etc.) [19]. Following part of our work tends to utilization of Markov model $M/M/m/\infty$ in the Contact center environment.

4.1 Basic Assumptions of Model

Let λ represents mean value of arrival rate of requests into system and let μ represents mean rate of request handling, while mean handling time of one requests is $T_{serv} = 1/\mu$. Graphical representation of $M/M/m/\infty$ system is depicted in the Fig. 4.

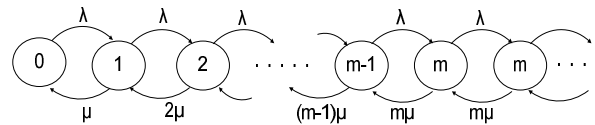


Fig. 4. Markov model $M/M/m/\infty$.

In the Fig. 4 we can see the behaviour of the system. The requests randomly arrive with Poisson distribution with mean value λ , these requests are handled, while handling time is random and can be described by exponential distribution with scale μ .

Therefore system is fulfilled with rate λ (transition rate from state k to state $k+1$, where k is number of requests in the system). On the other hand, system is emptying with rate $k\mu$ (from state $k+1$ to state k).

This behaviour is valid until state $k \leq m$, in case of more requests k in the system as number of

agents m , agents can handle requests only with rate $m\mu$ and every another incoming request have to wait in waiting queue for free agent, this waiting queue has unlimited capacity (length).

4.2 Calculation Capacity of Model

Through the basic equations of Markov model $M/M/m/\infty$ and their modifications it is possible to calculate large number of important traffic parameters of Contact center [2] - [5].

System stability ρ is as follows (5):

$$\rho = \frac{\lambda}{m\mu} \quad (5)$$

where m represents number of agents. The condition $\rho < 1$ must be valid in order to system stability.

Parameter P_0 represents the probability that calling customer will have to wait in the waiting queue for free agent (6):

$$P_0 = \frac{(m\rho)^m}{m!(1-\rho)} P_0 \quad (6)$$

where P_0 represents probability that system is empty (7):

$$P_0 = \left\{ \sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!} + \frac{(m\rho)^m}{m!} \left(\frac{1}{1-\rho} \right) \right\}^{-1} \quad (7)$$

Mean number of requests in the whole system is N (8):

$$N = \rho m + \frac{\rho}{(1-\rho)} P_0 \quad (8)$$

The request spend in the system mean time T (9):

$$T = \frac{1}{\mu} + \frac{P_0}{m\mu - \lambda} \quad (9)$$

Mean number of requests waiting in the queue Q is as follows (10):

$$Q = \frac{\rho}{(1-\rho)} P_0 \quad (10)$$

while request spend in the waiting queue mean time W (11):

$$W = \frac{\rho}{(1-\rho)} \frac{P_0}{\lambda} \quad (11)$$

From definition of parameters λ and μ we have (12):

$$A = \frac{\lambda}{\mu} \quad (12)$$

where A represents traffic load of the system in *Erlangs*.

4.3 Erlang C Formula and M/M/m/∞

Immediate rejection of call in case of occupation of all agents (as expected in Erlang B formula) is from point of provided services in contact center bad solution. This shortness is eliminated in second Erlang formula - Erlang C formula. In the case of the call can't be immediately served, the call is placed into the waiting queue with infinite length. If one of the agent is free, the call from waiting queue is automatically assigned to this agent. In the case of empty waiting queue, the agent is free and he waits for the next call.

Erlang C formula is defined as the function of two variables: number of agents N and traffic load A . Based on these values it is possible to determine the probability of P_C , that the incoming call will be not served immediately, but it have to wait in waiting queue.

By substitution of (12) into (5) and further substitution of (5) into (6) and (7) and by modification of equations we have (13):

$$P_C(m, A) = \frac{\frac{mA^m}{m!(m-A)}}{\sum_{k=0}^m \frac{A^k}{k!} + \frac{mA^m}{m!(m-A)}} \quad (13)$$

Equation (13) is known as *Erlang C formula* [1], where P_C represents probability that request will have to wait for handling in the waiting queue. It is obvious that equations (6) and (13) are the same.

From the analytical results we can see that Erlang C formula and Markov model are identically [10].

By utilization of this knowledge and by extension of Erlang formula with parameter *GoS* (*Grade of Service*), representing level of offered services [1], [6], it is possible to extend the computing capacity of $M/M/m/\infty$ model with this parameter (14):

$$GoS = 1 - P_0 e^{-\mu(m-A)T_w} \quad (14)$$

where *GoS* represents ratio of incoming calls that will be assigned to agent until selected time T_w .

On the basis of realized calculations [16] it is possible to use Erlang B formula for calculations of following parameters of contact center:

- number of agents needed to handle defined traffic load (A) at certain probability of call blocking (P_B),
- maximum possible traffic load of contact center at constant number of agents,
- probability of call rejection at defined load and number of agents.

And it is possible to use Erlang C formula for calculations of following parameters of contact center:

- number of agents needed to handle A at defined maximum values for probability of call waiting in waiting queue (P_C) and P_B ,
- Grade of Service (GoS) at defined number of agents, defined A and average call processing time,
- maximum possible A at defined number of agents (contact center stability),
- P_B at defined A and number of agents,
- P_C at defined A and number of agents.

5 Calculation of Important Traffic Parameters of Contact Center

The right design of Contact center requires the precise estimation of basic traffic parameters of Contact center, e.g. mean rate of telephone call arrivals into system and mean handling time of one call.

Unless otherwise stated, all calculations of traffic parameters of Contact center are tied with basic values:

- $\lambda = 60$ incoming calls per hour,
- call handling time is $T_{serv} = 5$ minutes.

From (12) results that average traffic load of such Contact center is 5 Erl.

5.1 Calculation of Required Number of Agents

The number of agents is the basic traffic parameter needed in design of Contact center. In the case of accurately estimated values λ and T_{serv} the subsequent calculation of required number of agents according to (6) is relative simple.

From the Fig. 5 we can see the relation between probability of waiting in the queue and raising number of agents. Let maximal probability P_Q in modeled Contact center does not exceed 5 %. From

the Fig. 5 and Tab. 1 it is obvious that the first suitable number of agents is $m = 10$.

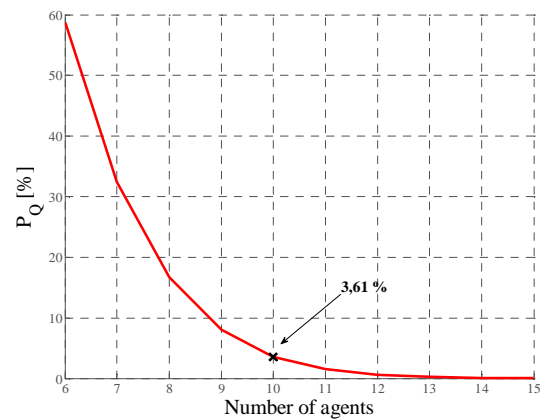


Fig. 5. Relation between P_Q [%] and number of agents.

Also from the Fig. 5 we can see that if we have 10 agents, the probability $P_Q = 3,61$ %. It means that from 100 calling customers into Contact center up to 4 customers have to wait in the waiting queue.

Other values of traffic parameters of Contact center are stated in the Tab. 1.

Table 1. Traffic parameters if $A = 5$ Erl.

m	P_0 [%]	N	T [min]	Q	W [min]	GoS [%]	ρ
8	16,73	5,28	5,28	0,28	0,28	87,61	0,63
9	8,05	5,10	5,10	0,10	0,10	94,60	0,56
10	3,61	5,04	5,04	0,04	0,04	97,81	0,50
11	1,51	5,01	5,01	0,01	0,01	99,17	0,45
12	0,59	5,00	5,00	0,00	0,00	99,71	0,42

Note 1: In the Tab. 1 we can see that traffic parameters N , T and Q , W reach up the same values. It is necessary to remind that $\lambda = 60$. In Markov model $M/M/m/\infty$, and similarly in others queuing systems following equation is valid (15):

$$\bar{N} = \lambda \cdot \bar{T} \quad (15)$$

In the original calculation all times are stated in hours. For better view, the times T and W in the Tab. 1 are stated in minutes, thus multiplied by 60, what is by coincidence also value of λ .

Equation (15) is called *Little law*, which says that average number of requests in the system equals mean time which the request spend in the system multiplied by mean rate of request arrivals [2], [5], [7].

5.2 Traffic Parameters and Traffic Load Variation

After calculation of required number of agents ($m = 10$) it is necessary to analyze the properties of designed Contact center at different traffic loads.

Firstly, it is necessary to know where the traffic limits of designed Contact center are. The basic representation is given in the Fig. 6.

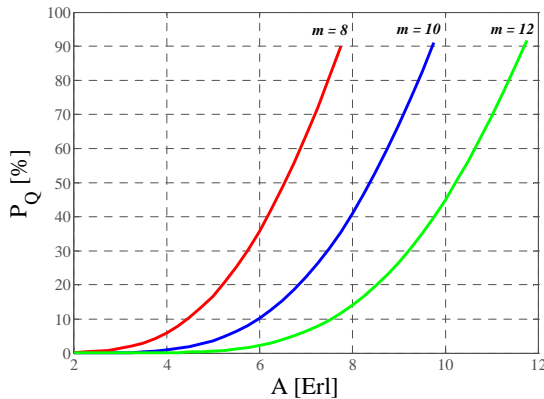


Fig. 6. Relation between P_Q [%] and A [Erl].

From the relation between P_Q [%] and A [Erl] we can see that with raising traffic load the probability that customer will have to wait for handling in the waiting queue is rising sharply.

For comparison, also dependencies between P_Q [%] and A [Erl] for the systems with 8 and 12 agents are depicted in the Fig. 6.

The better view of modeled Contact center behaviour give the graphs figure out in the Fig. 7 and Fig. 8.

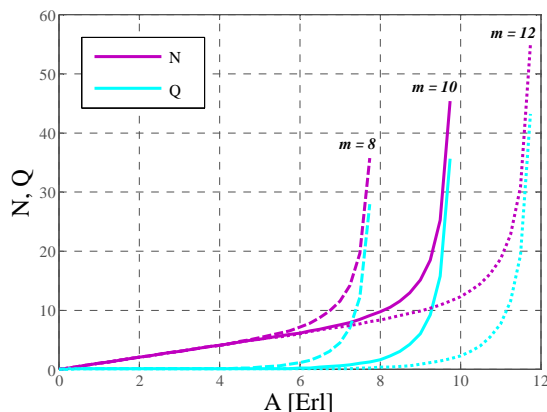


Fig. 7. Relation between number of requests in system/queue and A [Erl].

From the trace of dependencies between average number of requests in the system/queue and traffic load (see Fig. 7) we can see that initially it is increasing moderately and almost linearly until it reaches “breaking point” value of traffic load, then

increasing is very sharply. The same breaking point can be seen in Fig. 8 between dependencies of the mean time, that request will spend in the system, and traffic load.

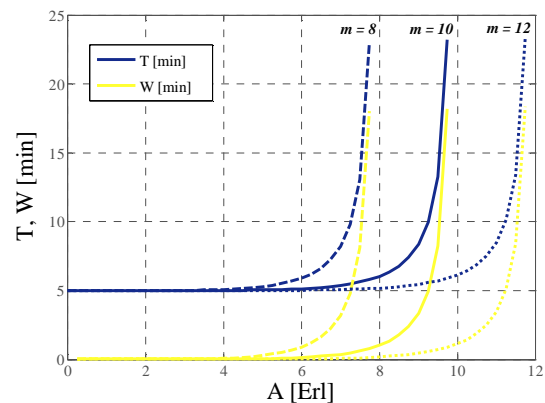


Fig. 8. Relation between the mean time that request spend in the system/queue [min] and A [Erl].

Breaking point position gives the maximum traffic load which can be handled by Contact center without change in the number of agents or in the call handling time. From the Fig. 7 and Fig. 8 it is obvious that breaking point for Contact center with number of agents $m = 10$ and mean call handling time $T_{serv} = 5$ minutes represents the value $A = 9$ Erl. Precise values are stated in the Tab. 2. Traffic load value 9 Erl means in other words 108 incoming calls per hour at mean call handling time 5 minutes.

Table 2. Traffic parameters if $m = 10$.

A [Erl]	λ	P_Q [%]	N	T [min]	Q	W [min]	m_{req}
4	48	0,88	4,01	5,01	0,01	0,01	9
5	60	3,61	5,04	5,04	0,04	0,04	10
6	72	10,13	6,15	5,13	0,15	0,13	11
7	84	22,17	7,52	5,37	0,52	0,37	13
8	96	40,92	9,64	6,02	1,64	1,02	14
9	108	66,87	15,02	8,34	6,02	3,34	15

The Tab. 2 also contains parameter m_{req} . This parameter says about required number of agents at given traffic load in order to preserve maximal probability P_Q under level of 5%.

Note 2: Little law (15) described in Note 1 is also valid here.

6 Contact Center as Multi-phase Queuing System

The previous part of the paper was dealt with Contact center as system in which incoming calls are served directly by Contact center agents, respectively these calls have been waiting in the queue for free agent. In fact, there are often Contact centers in which the first contact with customer is made by IVR (*Interactive Voice Response*) unit [7].

The main tasks of IVR unit are identification of calling customer, to recognize the matter of his problem and after then the call is transferred to the most skilled agent that will solve given problem. This way can effectively shorten the call handling time by time needed for customer identification and find out the matter of his problem.

Modeling of Contact center with IVR unit is very complicated, the wide range of statistical calculations and detailed knowledge of Markov processes are needed [8].

The more simple way is utilization of multi-phase queuing system (QS) theory (Fig. 9).

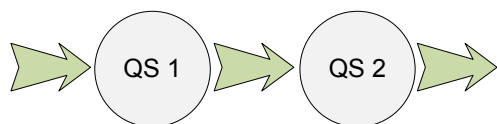


Fig. 9. Multi-phase queuing system.

Let QS 2 is Contact center modeled by Markov model $M/M/m/\infty$ as till now, with number of agents $m = 10$. Let QS 1 represents the IVR unit modeled by Markov model $M/M/\infty/\infty$.

6.1 Interactive Voice Response

Interactive Voice response (IVR) represents the abbreviation for the electronically synthesized voice output system, that will greet the customer and provide him the possibility of the session with service or agent. The system can be used for communication in more languages. Therefore the IVR application can at the beginning of the session request the caller to choose the communication language. Through the IVR function the selection of needed service and identification of the caller can be automatically recognized. After connection of the caller with the agent, the agent automatically see on the desktop of his PC the most important data about the customer. Thus, the agent of contact center is competent to serve the caller.

The success of the IVR system is based on the caller's experience. The IVR application must give

the caller always the possibility of choice. The IVR system enables:

- minimizing of interruption in realized session,
- providing of interactive customer oriented services, which are available not only within working hours,
- providing of information on demand by voice, fax and e-mail,
- providing of IVR services for automatic transactions,
- automatic voice recognition,
- serving the calls also when the agents are busy,
- providing the information about estimated waiting time,
- interactive waiting queues, that offer services - e.g. ordering within waiting (the caller save his order in waiting queue).

6.2 Markov Model $M/M/\infty/\infty$

Contrary to $M/M/m/\infty$ model, this model assumes unlimited number of service stations. It means that every request incoming to system is immediately handled. In this system no waiting queue exists and infinite number of requests can be simultaneously in the system [3].

It is obvious that such system is always stable, i.e. the system will never be overloaded by requests that it can not handle.

$$N_1 = \frac{\lambda_1}{\mu_1} \quad (16)$$

Mean time which the request will spend in the system is (17):

$$T_1 = \frac{1}{\mu_1} = T_{obs} \quad (17)$$

6.3 Multi-phase Queuing system $M/M/\infty/\infty + M/M/m/\infty$

Let traffic parameters of queuing system QS 1 represented by $M/M/\infty/\infty$ model has index 1 and traffic parameters of queuing system QS 2 $M/M/m/\infty$ has index 2.

The one of the assumptions of both systems is unlimited requests population. Therefore such multi-phase queuing system is called open. In the open system the following equation is valid [3]:

$$\lambda_1 = \lambda_2 = \lambda \quad (18)$$

Total number of requests in the system is (19):

$$N = N_1 + N_2. \quad (19)$$

Total time which request will spend in the system is (18):

$$T = T_1 + T_2. \quad (20)$$

6.4 Contact Center model with IVR

Let IVR in Contact center is represented by $M/M/\infty/\infty$ model and Contact center itself is represented by $M/M/m/\infty$ model described in the previous part.

Let the mean handling time of one call $T_{serv} = 5$ min. is divided as follows: the request spend in IVR unit average time $T_{serv1} = 1$ min. Average time of call handling by agent is shortened by IVR to $T_{serv2} = 4$ min. Average number of requests incoming to Contact center is $\lambda = 60$ calls per hour.

Model of this system is depicted in the Fig. 10.

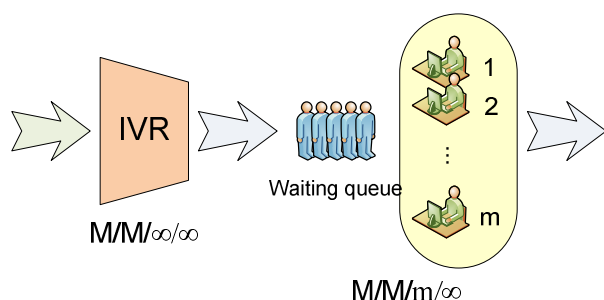


Fig. 10. Contact center model with IVR.

6.5 Impact of IVR on Modeled Contact Center

Impact of IVR system on modeled Contact center is figure out in the Fig. 11.

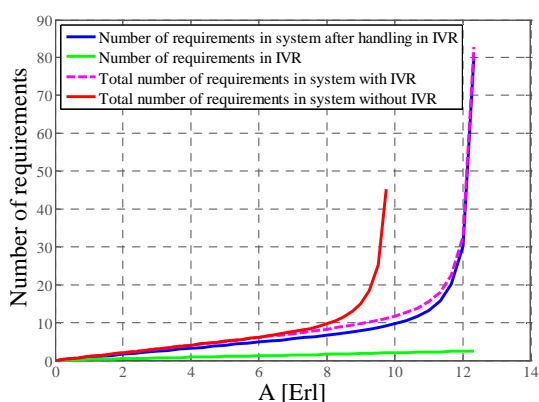


Fig. 11. Impact of IVR on modeled Contact Center.

From the course of dependencies between average number of requests in the system and traffic

load A [Erl] it is obvious that breaking point has moved till behind value $A = 11$ Erl, contrary to 9 Erl in the system without IVR unit. The Contact center with IVR is hence stable with this volume of traffic load, when the system without IVR can not be already used.

Through the IVR unit the Contact center is able to handle traffic load up to 11 Erl, what represents 132 incoming calls per hour at mean call handling time 5 minutes.

It is obvious that if the modeled Contact center was designed for handling traffic load up to 9 Erl, while required number of agents was 10, after adding the IVR unit into the system, the lower number of agents will be needed under the same conditions, what means great savings of operational costs. Precise values are stated in the Tab. 3.

Table 3. Comparison of systems with/without IVR.

	A [Erl]	N_1	N_2	N	m	P_0 [%]	Savings [%]
without IVR	5	–	–	5,04	10	3,61	–
with IVR	5	1,00	4,06	5,06	8	5,90	20
with IVR	5	1,00	4,02	5,02	9	2,38	10

From the Tab. 3 we can see that if $A = 5$ Erl approximately the same number of requests on average is in the system with or without IVR unit. The difference lies in the different number of required agents. The system with IVR unit is almost the same as the system without IVR when 8 agents are employed, but such system fails in condition $P_0 < 5\%$. The system with IVR with 9 agents actually overreaches the system without IVR with 10 agents in value P_0 . One agent in such system represents 10 % of total number of agents. The use of IVR unit brings relative high savings in needed number of agents. The savings have mainly financial character, because the costs for contact centers agent are every month periodically repeated. This fact is stated in the Tab. 3 as parameter *Savings*.

7 Conclusion

The basic assumptions for success achieving are above quality of service [18]. In case of services provided by contact center are agents very important and therefore it is necessary to determine their required number. When there are a few agents, it causes reduction the number of customers (the customers are not disposed to wait in waiting queue

too long). On the other hand, when there are lot of agents, so they are unused and they also increases the financial costs of contact center traffic. It is therefore necessary to give attention to optimal number of agents in contact center.

Markov model $M/M/m/\infty$ offers interesting tool for Contact Center analysis. The main advantage of this model is the wide spectrum of important traffic parameters of Contact Center calculations:

- P_Q - probability that calling customer will have to wait in the waiting queue for free agent,
- P_0 - probability of empty system,
- N - mean number of requests in the whole system,
- T - mean time that the request spend in the system,
- W - mean time that the request spend in the waiting queue,
- Q - mean number of requests waiting in the queue.

Other possibilities bring interconnection of Markov model with Erlang C model, especially the estimation of Grade of Service, which can be one of the main parameters in QoS evaluation of modeled Contact Center [12], [13].

Among the disadvantages of this model there are some restrictive basic assumptions of the model, mainly assumptions of unlimited requests population and infinite capacity of waiting queue. In spite of these limitations, we can minimize them, for example by chosen maximal value of traffic load A or through T_w parameter in GoS estimation.

Interesting results offer adding of IVR unit into modeled Contact Center. Using multi-phase queuing system theory and interconnection of $M/M/m/\infty$ model with $M/M/\infty/\infty$ model creates from Markov $M/M/m/\infty$ model ultimately complex tool for Contact Center analysis.

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References:

- [1] DIAGNOSTIC STRATEGIES: Traffic Modeling and Resource Allocation in Call Centers, Available on Internet: <www.fer.hr/_download/repository/A4_1Traffic_Modeling.pdf>
- [2] G. Bolch, S. Greiner, H. de Meer, S. Trivedi, *Queuing Networks and Markov Chains*, John Wiley & Sons, Inc., Hoboken, New Jersey, 2006.
- [3] J. Polec and T. Karlubíková, *Stochastic models in telecommunications 1*, Published by Jozef Murgaš fund for telecommunications in FABER, 1999, 1. edition, ISBN 80-968125-0-5.
- [4] Washington University in St. Louis, The Department of Computer Science & Engineering: Queuing theory, Available on Internet: <<http://www.cse.wustl.edu/~praveenk/support/quick-qt.pdf>>
- [5] M. Veeraraghavan, “M/M/1 and M/M/m Queuing Systems”, 20. March 2004, <<http://www.ece.virginia.edu/mv/edu/715/lectures/QT.pdf>>
- [6] Ch. Hischinuma, M. Kanakubo and T. Goto, “An Agent Scheduling Optimization for Call Center”, In: *The 2nd IEEE Asia-Pacific Services Computing Conference*, 2007.
- [7] I. Baroňák and E. Chromý, “Contact center – part of modern communication infrastructure”, In: *Telekomunikace*, no. 11, November, 2004, pp. 22-26.
- [8] J. Wang and R. Srinivasan: Staffing a Call Center with Interactive Voice Response Units and Impatient Calls, In: *IEEE/SOLI 2008*, IEEE International Conference, 2008.
- [9] T. Mišuth and I. Baroňák, “Performance Forecast of Contact Centre with Differently Experienced Agents”, In: *Elektrorevue*. ISSN 1213-1539. Vol. 15, 16.11.2010 (2010), art. no. 97.
- [10] E. Chromy, T. Misuth and M. Kavacky, “Erlang C Formula and Its Use In the Call Centers”, In: *Journal AEEE - Information and Communication Technologies and Services*, Volume 9, Number 1, pp. 7-13, March 2011, ISSN 1804-3119.
- [11] M. Mrajca, J. Serafin and Z. Brabec, “Optimization of the NGOSS Change Management Process Simulation Model”, In: *The 11th International Conference KTTO 2011*, June 22-24, Szczyrk, Poland, 2011, pp. 14-18, ISBN 978-80-248-2399-7.

- [12] B. Kyrbashov, I. Baroňák, M. Kováčik and V. Janata, "Evaluation and Investigation of the Delay in VoIP Networks", In: *Radioengineering*, ISSN 1210-2512, Vol. 20, No. 2 (2011), pp. 540-547.
- [13] J. Mičuch and I. Baroňák, "Comparison of Two Probability Theories Providing QoS", In: *Elektrorevue*, ISSN 1213-1539, Vol. 15, 16.11.2010, art. no 102.
- [14] K. Vastola, "Interarrival times of a Poisson process", Troy (New York, USA): Rensselaer Polytechnical Institute, 15.March 1996. Available on Internet: <<http://networks.ecse.rpi.edu/~vastola/pslinks/perf/node32.html>>.
- [15] I. Baroňák and P. Kvačkaj, "A New CAC Method Using Queuing Theory", In: *Radioengineering*. ISSN 1210-2512. Vol. 17, No. 4 (2008), Part. II, pp. 62-74.
- [16] E. Chromy, J. Diezka and M. Kovacic, "Traffic Analysis in Contact Centers", In: *The 11th International Conference KTTO 2011*, June 22-24, Szczyrk, Poland, 2011, pp. 19-24, ISBN 978-80-248-2399-7.
- [17] M. Fernandes, C. Lima and J. Schimiguel, „Strategic Management Using VoIP Technology: a Case Study in a Call Center Company”, In: *WSEAS TRANSACTIONS on COMMUNICATIONS*, Issue 1, Volume 10, January 2011, pp. 34-43, ISSN: 1109-2742.
- [18] M. Voznak and J. Rozhon, „Methodology for SIP Infrastructure Performance Testing”, In: *WSEAS TRANSACTIONS on COMPUTERS*, Issue 11, Volume 9, September 2010, pp. 1012-1021, ISSN 1109-2750.
- [19] D. Tran and T. Pham, "A Combined Markov and Noise Clustering Modeling Method For Cell Phase Classification", In: *WSEAS TRANSACTIONS on BIOLOGY and BIOMEDICINE*, Issue 3, Volume 3, March 2006, pp. 161-166, ISSN 1109-9518.

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