Optimal Design of Virtual MIMO for WSN Performance Improvement

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Abstract

In this paper, we studied the performance of bit error rate (BER) for a virtual multiple-input multiple-output (MIMO) based communications architecture. As a case, wireless sensor networks with clusters were considered. Then an optimal transmitting power (TP) scheme for cooperative sensor nodes was presented. Specifically, by minimizing BER of the \(N_t \times 1\) virtual MIMO system, we derived the closed-form of the optimal TP for each cooperating node in one cluster. Through simulations, we compared this strategy with an equal TP assignment method. Its performance enhancement was verified by extensive simulations under different scenes. At the aim to energy efficient, a thorough explanation of optimally choosing number of cooperating nodes was also delivered by the aid of mathematical analysis as well as simulation verifications.

Keywords: energy efficiency, virtual MIMO, wireless sensor networks (WSNs)

1. Introduction

Wireless sensor networks (WSNs) have being research hotspots recently as they use cheap and low-power sensors to perform surveillance tasks. For example, environmental monitoring, military surveillance, animal tracking, and home applications. However, sensor nodes are usually battery operated and their operational life time should be maximized, hence energy consumption is a crucial issue in real WSN applications [1].

Since multiple-input-multiple-output (MIMO) can dramatically increase the channel capacity while also reduce transmission energy consumption in fading channels, schemes named virtual MIMO have been proposed for WSN to improve the system performance [2][6]. Similar to MIMO, in such a strategy, when a node has information to send, it cooperates with adjacent nodes tied by single-antenna to transmit its information to a certain destination, which forms a virtual antenna array. So the adjacent nodes who participate in cooperating act as the relay channels for the source node[2][3][4][6].

In [2], S. Cui analyzed the total energy consumption per bit of multi-antenna nodes. He represents that single-input-single-output (SISO) systems use more energy than MIMO as the communication scheme. And X. Li [6] proposed a virtual MIMO scheme using two transmitting sensors and space-time block code (STBC) to provide transmission diversity in WSN with neither antenna-array nor transmission synchronization.

Studies above have shown that cooperation among sensor nodes can lead to significant capacity increases. However, in these literatures, they both assumed that each node has the same channel fading conditions and equal transmitting power (TP). In fact, it may be reasonable to assume that the channel gains from each cooperating node to the destination are different, as sensor nodes are always placed randomly in a complex environment.

So setting the optimal transmitting power for each cooperating node in one cluster is necessary to maximize the system performance. In this paper, the optimality is determined in terms of minimizing the bit-error-rate (BER) of the system. As a case, an \(N_t \times 1\) virtual MIMO network topology is considered. And the closed-form solutions of optimal TP are provided.

To make the virtual MIMO network energy efficient, methods for choosing number of cooperating nodes is also delivered.

The rest of the paper is organized as follows. In Section II, related work about virtual MIMO is given. In Section III, we present the system model and provide exact BER expressions under a Nakagami fading narrowband channel with parameter \(m\), and we compute...
the optimal TP for each cooperating node. In Section IV, performance improvement of the proposed scheme is compared to traditional strategies through simulations, and the effect of the number of the cooperating nodes on the EPUB(Energy-per-useful-bit) is analyzed by the aid of mathematical analysis and simulation verifications. Finally, in Section V, we conclude the whole paper.

2. Related Work

The origin of cooperative communication can be traced back to the work of Cover and El Gamal [3], which presents numerical results for the relaying scenario of Gaussian channels setup. And in [4], authors set up the first information theoretic approach to cooperative for multi-hop transmission.

As real MIMO techniques require complex transceiver circuitry and large amount of signal processing which lead to increased power consumption at the circuit level, it is not a viable technology for energy-limited wireless sensor networks. However, it is showed in recent papers that implementing MIMO techniques in wireless sensor networks via cooperative communication techniques is possible instead of physically having multiple antennas at the sensor nodes. And such scheme is named by virtual MIMO[23].

In [2], Cui studied such distributed MIMO techniques and analyzed the total energy consumption per bit of SISO and MIMO, and then showed the feasibility of virtual MIMO in WSNs. They observed that virtual MIMO scheme can offer considerable energy savings in cooperative wireless sensor networks even after allowing for additional circuit power, communication and training overheads.

S. Jagannathan et al [8] investigated the effect of time synchronization errors on the performance of the cooperative MIMO systems, and concluded that the cooperative MIMO scheme has a good tolerance of up to 10% clock jitter. Based on the energy model proposed in [2], Yuan [9] presented an optimal cross-layer design of virtual MIMO for WSNs to ensure quality of service (QoS). J. N. Laneman did the research work on the system capacity analysis of the virtual MIMO scheme in [10].

As to the optimization of virtual MIMO, [11], [17] and [18] present power allocation optimization methods in the case of a single node transmitting at any time. In [19][20], with perfect channel knowledge at the transmitter under fixed nodes with no fading, they showed the investigation on the minimum energy consumption with the constraint that cooperating nodes are along the optimal non-cooperative route. And [21] proposes a minimum power cooperative routing algorithm in which, at any time, either a direct transmission or a single relay-aided transmission can occur. In [22] the choice of the number of cooperating transmitters and the cooperation strategy are investigated to exploit the diversity gain for an increase in either the range or the rate of the links or both. In [23], authors propose optimal cooperator selection policies for arbitrary topologies with links affected by path loss and multipath fading.

However, how to optimally choosing TP for each cooperating node in virtual MIMO under different channel gains has not been addressed in the previous literatures. The analysis of this paper extends the work in the previous literatures as it applies to general multi-hop WSN under different channel gains.

3. Model And Optimal Design

3.1. Network model

We consider a cluster based wireless sensor network, where collected data is to be transmitted from a source to a destination by $N_r \times 1$ virtual MIMO clusters. It is depicted in Fig.1. Considering the real wireless environment, we model the wireless link between any two nodes in the network as a Nakagami-$m$ fading narrowband channel. And parameter $m$ is for its routine mathematical manipulations and generality. The channel fades for different links are assumed to be statistically i.i.d. This is a reasonable assumption as the relays in scenes similar to Fig.1 are usually spatially well separated. The additive noise power at all receiving terminals is modeled as zero-mean complex Gaussian random variables with variance 1. To make it simple for medium access, the relays are assumed to transmit over orthogonal channels, thus no inter interference between relays is considered in the signal model, which is also assumed in [16][24].

![Fig.1 Cluster based virtual MIMO scheme for WSN[9]](image)

To make the virtual MIMO based system energy efficient, it is necessary to decrease the error probability, which means to reduce the retransmission times when nodes operating with low TP level.

As we assumed that the transmitted signal on the $i$th cooperating node, or equivalently virtual MIMO antenna, undergoes Nakagami-$m$ fading with fading parameter $m_i$ and average fading power $\bar{H}_i = E(H_i), \ i=1,2,...,N_r$, then we can get the instantaneous signal-to-noise ratio (SNR) per symbol of the $i$th diversity channel.

$$SNR = \frac{P_t}{\sigma^2}$$

where $P_t$ is the transmitted power and $\sigma^2$ is the variance of the noise.
\[ y_i = (H_i, T_i, p_I) / N_0 \]  

where \( N_0 \) is the single-sided thermal noise spectral density, \( H_i \) is the fading power of the \( i \)th diversity path, \( p_i \) is the transmitting power of the \( i \)th cooperating node, and \( T_i \) is the symbol duration, assumed to be constant. Let \( y_i \) be the average received SNR of the \( i \)th path, it can be given by

\[ y_i = \left( H_i, T_i, p_i \right) / N_0 = C_i p_i. \]  

Here, we consider a M-QAM transmission through the \( i \)th path which can be seen as an antenna. So its conditional BER (conditioned on the instantaneous SNR) can be expressed by

\[ P_e(y_i | y) \approx 4 \int \frac{1}{b} \left( 1 - \frac{1}{2} \right) \left( \frac{3y_i}{2} \right)^{\frac{1}{2}} Q \left( \frac{3y_i}{2} \right), \]  

in which \( b \) is the number of bits per symbol and \( b = \log_2 M \), \( Q(\cdot) \) is the Q-function defined as \( Q(x) = \left( 1 / \sqrt{2\pi} \right) e^{-x^2} dx \). We assume that clock and carrier frequency information is available at the receiver, the BER can be well approximated by Chernoff bound [12,13].

\[ P_e(y_i | y) = a_1 \exp(-a_2 y_i) \]  

for \( b = 4 \), here \( a_1 \) and \( a_2 \) are constants.

As we know, for NMF channels, the probability distribution function (PDF) of the channel gain is an expression of gamma function. The received SNR, is then gamma distributed according to the PDF, given by

\[ p_{\gamma}(\gamma) = \left( \frac{m}{\gamma} \right)^{m-1} \exp(-m \gamma) \frac{\gamma^{m-1}}{\Gamma(m)} \]  

\( m \) is the Nakagami fading parameter (\( m > 1/2 \)), and \( \Gamma(\cdot) \) is the gamma function[12].

So the average BER in the \( i \)th path can be given by

\[ P_{e_i} = \int \int P_e(y_i | y) p_{\gamma}(\gamma) dy \]  

\[ = \int_{0}^{\infty} a_i e^{-\lambda_i \left( \frac{m_i}{\gamma_i} \right)^{\frac{m_i-1}{\gamma_i}} \exp(-m_i \gamma_i / \gamma_i)} \]  

where \( x = y_i / \gamma_i \).

By simple algebra of Eq.(4), we have

\[ P_{e_i} = a_i (1 + \frac{a_2 \gamma_i}{m_i})^{-m_i} \]  

Assuming the channel fading for different links to be statistically independent, then the average BER of \( N_i \) diversity channels would be the product of BERs for each path where we have from (5), that is

\[ P_e = a_i \prod_{i=1}^{N} (1 + \frac{a_2 \gamma_i}{m_i})^{-m_i} \]  

3.2 Problem Formulation

To make the virtual MIMO energy efficient, the problem can be stated as maximizing the throughput under the total transmitting power constraint. As the throughput is determined by the BER performance, minimizing the average BER of the virtual MIMO system would be favorable and we use it to calculate the optimal transmitting power for each node (just like one antenna).

Letting \( p_i \) denote the transmitting power allocated to diversity path \( i \) by the cluster head, the problem of interest is thus for each cluster head to determine the optimal \( N_i \times 1 \) transmitting power allocation vector \( \Phi = \{ p_1, p_2, \cdots, p_{N_i} \} \), subject to total power constraints.

The transmitting powers allocation vector \( \Phi \) is trivially constrained to the nonnegative orthant, i.e. \( \Phi > 0 \), as transmitting powers are non-negative.

Considering the total transmit power for a cluster is limited to a fixed \( P \), the transmitting power allocation vector is also constrained as \( 1^T \Phi \leq P \). These constraints define the convex space of feasible allocation vectors \( \Phi \) characterizing transmitting power allocation solutions for each cluster head.

Combining the function in (6) with the set of constraints defined above yields the following transmitting power allocation optimization problem which aims to find the globally optimal allocation over the set of cluster heads.

\[ \Phi^* = \arg \min_{\Phi \in \mathcal{P}} P_e \]  

subject to \( 1^T \Phi \leq P \)  

\[ \Phi \geq 0 \]  

Theorem 1. The optimal transmitting power for the cooperative system given in \( N_i \times 1 \) virtual MIMO clusters is

\[ p_i,_{\text{optimal}} = m_i \max \left( \frac{1}{a_2 \sum_{i=1}^{N_i} m_i}, \frac{1}{a_2 \sum_{i=1}^{N_i} m_i \gamma_i / C_i} \right) \]  

The proof is given in Appendix.

4. Numerical Results

In this section, we evaluate the performance improvement of the proposed scheme.

Fig. 2 shows the average BER performance of the virtual MIMO based on the total power using optimal TP scheme and equally common one, respectively. The comparison is for \( N_i = 3 \) cooperating nodes where, for example, we set \( C_1 : C_2 : C_3 = 10:5:1 \). As it can be seen, the performance degradation of the equal TP scheme is more severe than that of optimal power allocation. That is to say, for a required BER performance, optimal TP scheme has less power consumption. For example, when using optimal transmitting power for each different cooperating node, at \( BER = 10^{-3} \), optimal scheme needs 36% less power. It also can be seen that when the total power is low, the optimal strategy tends...
to assign most of power to the best branch, which improves the communication reliability between clusters.

$$E_{\text{bit}} = (P_{\text{pa}} + P_{c})/R_b,$$

where $P_{\text{pa}}$ is power consumption of the power amplifier (PA) and $P_c$ is total values for DAC, the mixer, the low-noise amplifier (LNA), the intermediate frequency amplifier (IFA), the active filters et al., we use the model introduced in [5].

From [15], $P_{\text{pa}} = (1 + \alpha)P$, in which $\alpha$ is the efficiency of the power amplifier, then,

$$P_{\text{c}} = N_t(P_{\text{dac}} + P_{\text{mix}} + P_{\text{fil}} + P_{\text{syn}}) + (P_{\text{pre}} + P_{\text{mix}} + P_{\text{fil}} + P_{\text{syn}} + P_{\text{adc}})$$

To make clear comparison, we use the energy-per-useful-bit (EPUB) metric, which was presented in [11]. By Eq.(6), the EPUB can be stated as follows:

$$\text{EPUB} = E_{\text{bit}}/(1 - P_c)$$

By Eq.(6) and Eq.(9), we can get

$$\text{EPUB} = E_{\text{bit}}/(1 - a_i \prod_{k=1}^{N_t} (1 + \alpha_2 y_k)^{-m_k})$$

$$= \left(\frac{P}{\eta} + P_c\right)/(R_b (1 - a_i \prod_{k=1}^{N_t} (1 + \alpha_2 y_k)^{-m_k}))$$

$$= \left(\frac{P}{\eta} + P_c(N_t)/(R_b (1 - P_c(N_t,P)))$$

Taking the values in [5], we can describe the plot of EPUB versus total TP with different number of cooperating nodes in Fig.5. As we can see, when $P_c$ dominates the power consumption, the more number of cooperating nodes leads to less energy efficient. However, when the total TP is high enough, the number of cooperating nodes make little effect on the EPUB.

Table 1 Values for node’s physical layer [5]

<table>
<thead>
<tr>
<th>$f_c$</th>
<th>2.5 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{mix}}$</td>
<td>30.3mW</td>
</tr>
<tr>
<td>$P_{\text{fil}}$</td>
<td>2.5mW</td>
</tr>
<tr>
<td>$P_{\text{filr}}$</td>
<td>2.5 mW</td>
</tr>
<tr>
<td>$P_{\text{pre}}$</td>
<td>20mW</td>
</tr>
<tr>
<td>$P_{\text{syn}}$</td>
<td>50mW</td>
</tr>
<tr>
<td>$R_b$</td>
<td>1Mbps</td>
</tr>
<tr>
<td>$P_{\text{adc}}$</td>
<td>7mW</td>
</tr>
<tr>
<td>$P_{\text{dac}}$</td>
<td>15.4mW</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

In the second experiment, we compared both equal and optimal power allocation strategies on different channel scenarios, for example, a Rayleigh fading channel ($m=1$) and a $m=8$ Nakagami one. The results are showed in Fig. 3, where we can observe that the amount of performance improvement is greater in better channel conditions. The third clear comparison could be made on the number of cooperating nodes, that is $N_t$. To this aim, we considered a Nakagami-$m$ scenario with different $N_t$. The channel conditions are set to the same for all experiments and are equally distributed. As we can see from Fig. 4, the BER performance improvement is higher for larger number of cooperating nodes. But it should be noticed that the more the number of cooperating sensor nodes is the bigger the cluster of them should be formed. This will require more number of nodes to maintain synchronization and more energy consumption on nodes’ physical circuits, where might be an issue.

By [15], the energy consumption per bit can be given by
5. Conclusion

In this paper, we have introduced a new optimal TP strategy for each cooperating node by evaluating BER performance of a cluster based virtual MIMO wireless sensor network. Closed-form expressions for (i) the average BER experienced by each diversity branch, (ii) the optimal TP for each cooperating node have been derived. The BER performance of a network with equal TP scheme has been compared with that of a network with optimal TP one. It has been shown that the BER performance of a network with an optimal TP for each cooperating node strategy which intends to minimize BER of the system can achieve better than that of a network with equal common TP ones. This brings less power consumption for forward, so that a highly limited resource sensor network could achieve a longer life. As all the overhead in the scheme is mainly in software energy used to digital processing and the energy exchanged in the initial stages of transmission scenario, the proposed scheme even doesn’t make any other additional overhead, but just few software calculations for the TP values when in initialization. Finally, the impact of the number of the cooperating nodes on the EPUB is analyzed by the aid of mathematical analysis as well as simulation verifications.

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Appendix

This appendix outlines the proof for Eq. (8).

Since the total TP is fixed, the constraint in this case is

$$P = \sum_{i=1}^{N_i} p_i \quad (p_i \geq 0)$$

where $P$ is the value of total TP.

Equivalently, one can minimize the logarithm of the objective function, obtaining the following program:

$$\begin{align*}
\text{minimize} & \quad \log(P_i) = \log(1) - \sum_{i=1}^{N_i} m_i \log(1 + \frac{a_iC_i}{m_i} p_i) \\
\text{subject to} & \quad P = \sum_{i=1}^{N_i} p_i \\
& \quad p_i \geq 0 \quad \forall i \in I
\end{align*}$$

The function can be written as

$$Z = \log(a_i) - \sum_{i=1}^{N_i} m_i \log(1 + \frac{a_iC_i}{m_i} p_i) + \xi \sum_{i=1}^{N_i} p_i$$

where $\xi$ is the Lagrange multiplier. The partial derivative, with respect to $p_i$, of the Lagrangian yields:

$$\frac{\partial Z}{\partial p_i} = \xi - \frac{a_iC_i}{1 + \frac{a_iC_i}{m_i} p_i}$$

Setting the partial derivative to 0 and solving for $\xi$, one gets

$$\xi = \frac{a_iC_i}{1 + \frac{a_iC_i}{m_i} p_i}$$

The next step is to solve for $p_i$ in terms of $\xi$ and substitute it back in the constraint equation. Let $p_i(\xi)$ be the inverse of equation (A.4), it can be given by

$$p_i(\xi) = m_i(1/\xi - 1/\alpha_iC_i)$$

and substitute this value back into the constraint equation

$$P = \sum_{i=1}^{N_i} p_i \quad (p_i \geq 0)$$

obtaining:
\[
\frac{1}{\xi} = \frac{p}{\sum_{a=1}^{N} m_a} + \frac{\sum_{n=1}^{N} (m_n/C_n)}{\sum_{n=1}^{N} m_n} \quad (A.6)
\]
then by Eq.(A.6) and (A.5), we can lead to Eq.(8).

REFERENCES
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