Performance Evaluation of Full Rate Space-Time Block Code for Multiple Input Single Output (MISO) Wireless Communication System

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Abstract: - To enhance the capacity of a multiple input single output (MISO) system in wireless communications, the most widely used channel coding technique is the space time block coding (STBC). The outstanding orthogonal full rate STBC proposed by Alamouti is the most successful one and also very simple to implement, where only two transmit antennas and one receive antenna are used. As the number of antennas is increased beyond two, the orthogonality of the STBC is lost. Recent literature proposes a scheme known as the quasi-orthogonal STBC for full rate communication. It has been found that for ½ and ¾ rate, the orthogonality of the code can be maintained. In this paper, we propose a system with a full rate orthogonal STBC for four symbols and eight transmit antenna MISO case. After comparing the performance of the proposed system with the existing models, we have found that the proposed scheme is the best in context of the probability of bit error (BER) consideration.

Key-Words: - Rayleigh fading, Walsh matrix, quasi orthogonality, AOD, SNIR, Kronecker product.

1 Introduction
In wireless communication system the biggest challenge is to maximize the throughput under the influence of the fading environment. As the signal travels through open space in unguided mode, it is obvious that the signal can be contaminated with noise and multipath fading. To attain the space diversity, multiple-input-multiple-output (MIMO) system is successfully used in wireless communications. To lighten different types of fading in wireless communication systems, space diversity and space multiplexing techniques are broadly used while incorporation of MIMO is a simpler solution [1], [2]. Receive diversity involves multiple antennas at the receiving end and when transmitting end includes multiple antennas, it is referred to as the transmit diversity.

Multiple-antenna arrays are known to perform better than their single-antenna counterparts, because they can more effectively counter the effects of multipath fading and interference. However, the enhanced performance depends on the amount of channel information at the transmitter and on whether the transmitter is able to take advantage of this information. For example, it is well known that in a spatially uncorrelated Rayleigh-fading environment, for a multiple-antenna transmitter with perfect channel knowledge and a single-antenna receiver, i.e., for a multiple-input-single-output (MISO) case, the gain in throughput due to the optimization of transmission is roughly \( \log_2(n) \), where \( n \) is the number of transmitter antennas. The knowledge of the channel allows the transmitter to transmit with a signal covariance that maximizes the signal-to-noise ratio (SNR) at the receiver, thus increasing the mutual information.

The multipath fading of a MISO communication system can be reduced by space-time block codes (STBC). In a STBC, the signal transmission occurs in blocks which can be represented as a transmission matrix constructed with three parameters: number of transmitted symbols, antennas and the time slots in a data block. The construction procedure of a STBC can be defined in two categories such as complex orthogonal design and generalized complex orthogonal design. To achieve the full diversity with simple linear decoding, orthogonal space-time block codes (OSTBCs) are used. The first OSTBC presented by Alamouti [3] served as the pioneer for all research works based on square [4], [5] and nonsquare [6], [7] OSTBC to achieve maximum code rate. Furthermore, a square OSTBC can offer a minimized decoding delay and can be applied with different modulation techniques.

The historical and revolutionary work of S. Alamouti [3] as mentioned earlier, is the first STBC presenting the full rate and full diversity using a
simple model with two transmit antennas and one receive antenna. In [8], a Rayleigh fading multiple antenna communication is explained using a new approach, where data is represented using a STBC and then break the data into \( n \) streams which are then transmitted over \( n \) transmit antennas simultaneously. In [9], a quasi-STBC using a simple feedback scheme for MIMO channels with \( 2^n \) (\( n = 1, 2, \ldots \)) transmit antennas and 1 receive antenna, a MISO, is proposed, where data is represented using a STBC and then break the data into \( n \) streams which are then transmitted over \( n \) transmit antennas simultaneously. In [9], a quasilSTBC using a simple feedback scheme for MIMO channels with \( n \) transmit antennas and only one receive antenna, a MISO, is proposed, where the Alamouti scheme is extended with partial feedback. The authors of [9] showed an example with 4 transmit antennas and 1 receive antenna. Two STBCs differ only in the sign of the first column of the code matrix are compared for feedback to achieve the full diversity. In [10], authors have studied the performance of quasi-OSTBCs (QOSTBCs) on measured MIMO channels, i.e. receivers have knowledge about the channel condition, using four transmit and four receive antennas. For a complete reference, the text [2] on wireless communication may be useful. The features of the text include examples with detail solutions, problems, notes and references. It also includes theme examples which discuss significant issues of practical oriented problems. A group of QOSTBCs with full code rate for any number of transmit antennas, specifically, Four-Group QOSTBC (4Gp-QOSTBC) is analyzed in [11]. In 4Gp-QOSTBC, the received symbols can be separated into four independent groups during transmission through the maximal likelihood (ML) decoder. Reference [12] proposes two systematic ways of the construction of OSTBCs from amicable orthogonal designs (AODs) that avoid zero and irrational coefficients. The methods of the paper are based on the construction of higher order AODs or amicable families (AFs) from lower order AODs or AFs and authors have used them to construct new square OSTBCs. In [13], the authors have done a work on the performance of MIMO communication system where the effort is to minimize the average pairwise error probability. In [14] authors have performed an analytical research in order to find the bit error rate (BER) of an orthogonal frequency division multiplexing (OFDM) system and a space-time block coded OFDM system considering all three channel impairments: adaptive white Gaussian noise (AWGN), fading and jitter in fading environment including MIMO-OFDM system with switching/selection method for combining the signals at the multiple receiving antennas.

In this paper, we have been particularly interested to study the performance of a full rate OSTBC for a MISO wireless communication system under a Rayleigh fading multipath channel with 8 transmit antennas and 1 single receive antenna. For this, three different modulation schemes, viz., binary frequency-shift keying (BFSK), quadrature-phase-shift keying (QPSK), and 16 quadrature amplitude-shift keying (16-QAM) are considered. For the performance evaluation, the probability of bit error (BER) against the signal-to-noise ratio (SNR) is determined.

The paper is organized as follows. Section 2 deals with the literature review and the construction of full rate quasi-orthogonal coding, OSTBC, from AOD. Section 3 describes the proposed full rate orthogonal scheme while Section 4 gives the analytical expressions for different schemes along with the proposed scheme. Section 5 provides the results where comparison is made between the proposed scheme and some other schemes. Finally, Section 6 concludes the entire analysis of the work with some recommendations.

2 Literature Review and Problem Formulation

2.1 Quasi Orthogonal STBC

In [15] and [16], a double space/time diversity scheme is analyzed where two \( 2 \times 1 \) Alamouti systems are used in parallel to achieve full rate transmission of four symbols. The transmitter is equivalent to a \( 2 \times 4 \) stacked STBC as shown in Fig. 1.

The channel matrix of \( 4 \times 2 \) stacked STBC against four symbols (\( S_1, S_2, S_3 \) and \( S_4 \)) is taken as

\[
S/P \rightarrow\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4
\end{bmatrix}
\]

\[
\begin{array}{c}
\text{Alamouti STBC} \\
\text{Alamouti STBC}
\end{array}
\]

\[
\begin{pmatrix}
S_1 & -S_2^* \\
S_2 & S_1^*
\end{pmatrix}
\]

\[
\begin{pmatrix}
S_3 & -S_4^* \\
S_4 & S_3^*
\end{pmatrix}
\]

Fig.1 Stacked STBC transmitter.
and the decision statistics at the receiving end is evaluated as
\[ y_1 = H_1 S_1 + I_3 + I_4 + n_1, \]
\[ y_2 = H_1 S_2 + I_3 + I_4 + n_2, \]
\[ y_3 = H_2 S_3 + I_1 + I_2 + n_3, \]
\[ y_4 = H_2 S_4 + I_1 + I_2 + n_4, \]
where
\[ H_1 = \left[ \begin{array}{cccc} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\
-h_{11}^* & h_{12}^* & h_{13}^* & -h_{14}^* \\
-h_{21}^* & h_{22}^* & h_{23}^* & -h_{24}^* 
\end{array} \right], \]
and
\[ H_2 = \left[ \begin{array}{cccc} |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 \\ |h_{13}|^2 + |h_{14}|^2 + |h_{23}|^2 + |h_{24}|^2 \\
|h_{11}|^2 & |h_{12}|^2 & |h_{13}|^2 & |h_{14}|^2 \\
|h_{21}|^2 & |h_{22}|^2 & |h_{23}|^2 & |h_{24}|^2 
\end{array} \right] + |I_1|^2, \]
where \( I_1 \) is the interference from the \( i \)-th transmit antenna and \( n_i \) is the noise of \( i \)-th symbols.

In [10], two QOSTBC matrices of four symbols are taken as
\[ S_1 = \left[ \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \\ S_2^* & -S_1^* & S_4^* & -S_3^* \\
S_3^* & S_4^* & -S_1^* & -S_2^* \\
S_4 & -S_3 & S_2 & S_1 \end{array} \right], \]
and
\[ S_2 = \left[ \begin{array}{cccc} -S_1 & S_2 & S_3 & S_4 \\ -S_2^* & -S_1^* & S_4^* & -S_3^* \\
-S_3^* & S_4^* & -S_1^* & -S_2^* \\
-S_4 & -S_3 & S_2 & S_1 \end{array} \right]. \]
The corresponding virtual channel matrices are
\[ H_{v1} = \left[ \begin{array}{cccc} h_{11} & h_{12} & h_{13} & h_{14} \\ -h_{11}^* & h_{12}^* & h_{13} & -h_{14} \\
-h_{11} & h_{12}^* & h_{13}^* & h_{14}^* \\
-h_{11} & h_{12} & h_{13} & h_{14} \end{array} \right], \]
and
\[ H_{v2} = \left[ \begin{array}{cccc} -h_{11} & h_{12} & h_{13} & h_{14} \\ -h_{11}^* & -h_{12}^* & -h_{13}^* & h_{14}^* \\
-h_{11} & -h_{12}^* & -h_{13} & h_{14} \\
-h_{11} & -h_{12} & -h_{13} & h_{14} \end{array} \right]. \]
The quasi orthogonal matrix is
\[ H_{v1}^H H_{v1} = H_{v1}^H H_{v2}^H = h^2 \left[ \begin{array}{cccc} 1 & 0 & 0 & X \\ 0 & 1 & -X & 0 \\
0 & -X & 1 & 0 \\
X & 0 & 0 & 1 \end{array} \right], \]
where the channel gain is
\[ h^2 = \left( |h_{11}|^2 + |h_{12}|^2 + |h_{13}|^2 + |h_{14}|^2 \right), \]
and the channel dependent interference parameter is
\[ X = 2 \text{Re}\left( h_{12}^* h_{14}^* - h_{12} h_{14}^* / h^2 \right). \]
The authors of [10] propose two different symbol matrices and switching any one of them according to the condition of the channel.

Similar analysis is also done in [17]-[19]. The performance of the QOSTBC was improved by phase-shifting the constellations of the symbols [20].

### 2.2 Construction of OSTBC from Amicable Orthogonal Design (AOD)

Authors of [12] have proposed two different and efficient ways of constructing OSTBCs from AODs. These methods are proposed here based on the construction of higher order AODs or AFs from the lower order AODs or AFs and use them to construct new square OSTBCs with no zero or irrational entries.

Let us consider a family of \( k \times k \) matrices \( A_1, A_2, \ldots, A_n \) and another group of matrices \( B_1, B_2, \ldots, B_m \). The relations among the matrices are:
\[ A_i \circ A_j = 0, \quad 1 \leq i \neq j \leq n; \]
\[ B_i \circ B_j = 0, \quad 1 \leq i \neq j \leq m; \]
where the operator ‘o’ is the Hadamard product. We also have
\[ A_i^T A_j + A_j^T A_j = 0, \quad 1 \leq i \neq j \leq n, \]
\[ B_i^T B_j + B_j^T B_i = 0, \quad 1 \leq i \neq j \leq m, \]
\[ A_i^T B_j - B_j^T A_i = 0, \quad 1 \leq i \neq j \leq n; 1 \leq i \neq j \leq m; \]
\[ A_i^T A_i = r_i I_k, \]
\[ B_i^T B_i = v_i I_k, \]
where \( r_i \) and \( v_i \) are constants.

Let us consider some set of matrices \( M_i \) and \( N_i \) to construct the OSTBC. These two matrices \( M_i \) and \( N_i \) must satisfy the following conditions:
\[ M_i \circ M_j = 0, \quad 1 \leq i \neq j \leq 3; \]
\[ N_i \circ N_j = 0, \quad 1 \leq i \neq j \leq 3; \]
\[ M_i^T M_i = u_i I_4, \quad 1 \leq i \leq 3; \]
\[ N_i^T N_i = v_i I_4, \quad 1 \leq i \leq 3; \]
\[ M_i^T M_j + M_j^T M_i = 0, \quad 1 \leq i \neq j \leq 3; \]
\[ N_i^T N_j + N_j^T N_i = 0, \quad 1 \leq i \neq j \leq 3; \]
\[
M_i^T N_j - N_j^T M_i = 0, \quad 1 \leq i \neq j \leq 3; \\
M_i \cdot I_4 = 0, \quad 1 \leq i \leq 3; \\
N_i \cdot I_4 = 0, \quad 1 \leq i \leq 3; \\
M_i^T + M_i = 0, \quad 1 \leq i \leq 3; \\
N_i^T + N_i = 0, \quad 1 \leq i \leq 3.
\]

According to the above constraints given for the matrices \(M_i\) and \(N_i\), we can come to a conclusion that both \(M_i\) and \(N_i\) are a set of AOD of order 4. We can construct higher order AODs (e.g., AOD of order 8) from these lower order AODs. Finally, using these AODs an OSTBC of \(\frac{3}{4}\) and \(\frac{1}{2}\) code rate can be constructed.

### 2.2.1 Construction Method 1

Using the matrices \(A_2, B_2, M_4\) and \(N_4\) the following matrices can be formed:

\[
S_1 = \{B_1 \otimes M_1, B_1 \otimes M_2, B_1 \otimes M_3, \}
\]

\[
A_2 \otimes I_4, A_3 \otimes I_4, \ldots, A_n \otimes I_4\}, \quad (16)
\]

\[
S_2 = \{B_1 \otimes N_1, B_1 \otimes N_2, B_1 \otimes N_3, \}
\]

\[
B_2 \otimes I_4, B_3 \otimes I_4, \ldots, B_n \otimes I_4\}. \quad (17)
\]

Higher order AODs \(A\) with symbols \(a_1, a_2, a_3, a_4\) and \(B\) with symbols \(b_1, b_2, b_3, b_4\) can be constructed according to the formula given below:

\[
A = a_1 \cdot B_1 \otimes M_1, a_2 \cdot B_1 \otimes M_2, a_3 \cdot B_1 \otimes M_3, \]

\[
a_4 \cdot A_2 \otimes I_4, \ldots, a_{n+2} \cdot A_n \otimes I_4, \quad (18)
\]

\[
B = b_1 \cdot A_1 \otimes N_1, b_2 \cdot A_1 \otimes N_2, b_3 \cdot A_1 \otimes N_3, \]

\[
b_4 \cdot B_2 \otimes I_4, \ldots, b_{m+2} \cdot B_n \otimes I_4. \quad (19)
\]

The symbol \(\otimes\) means Kronecker product of matrices. Finally, using equation (18) and (19), the OSTBC is formed as follows:

\[
G_L = A + jB, \quad (20)
\]

where \(G_L\) is a \(\frac{1}{2}\) rate OSTBC.

### 2.2.2 Construction Method 2

Let us consider another set of matrices constructed as:

\[
R_1 = \{B_1 \otimes N_1, A_1 \otimes I_2, A_2 \otimes I_2, \ldots, A_n \otimes I_2\}, \quad (21)
\]

and

\[
R_2 = \{B_1 \otimes N_2, B_1 \otimes N_3, B_2 \otimes I_2, \ldots, B_{m+2} \otimes I_2\}. \quad (22)
\]

We can now construct higher order AODs \(A\) with symbols \(a_1, a_2, a_3\) and \(B\) with symbols \(b_1, b_2, b_3\) according to the formula given below:

\[
A = a_1 \cdot B_1 \otimes N_1, a_2 \cdot A_1 \otimes I_2, a_3 \cdot A_2 \otimes I_2, \ldots, \]

\[
a_{m+2} \cdot A_n \otimes I_2, \quad (23)
\]

\[
B = b_1 \cdot B_1 \otimes N_2, b_2 \cdot B_1 \otimes N_3, b_3 \cdot B_2 \otimes I_2, \ldots, \]

\[
b_{m+2} \cdot B_n \otimes I_2. \quad (24)
\]

For a linear OSTBC, \(G_L\) can be constructed according to equation (20) given in the previous section. The OSTBC constructed using this method is a 4-by-4 matrix with \(\frac{3}{4}\) code rate where three different symbols are transmitted using four time slots through four antennas. Example of both methods is given in Appendix A.

### 3. The Proposed Full Rate Scheme

The previous section shows the technique of formation of full rate quasi-orthogonal or \(1/2\) and 3/4 rate orthogonal STBCs. In this section, we propose a full rate orthogonal scheme of \(8 \times 1\) system i.e. 8 transmit antennas and one receive antenna. The code represents full rate where the number of complex symbols is four and they are transmitted using four time slots. Here, we use the concept of orthogonal rows of Walsh matrix used in code division multiple access (CDMA). For example, a 2-by-2 Walsh matrix is:

\[
H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \quad (25)
\]

We can apply recurrence relation to build higher order Walsh matrices such as \(H_4, H_8, H_{16}\) and so on. We can define a general form of the Walsh matrix as:

\[
H_{2n} = \begin{bmatrix} H_n & H_n \\ H_n & H_n \end{bmatrix}. \quad (26)
\]

Let us consider four symbols \(S_1, S_2, S_3\) and \(S_4\) having real parts \(a_1, a_2, a_3\) and \(a_4\) and the imaginary parts \(b_1, b_2, b_3\) and \(b_4\). We define a matrix,

\[
A' = \begin{bmatrix} H_2 & H_2 & H_2 & H_2 \\ H_2 & H_2 & H_2 & H_2 \end{bmatrix}. \quad (27)
\]

where \(H_2\) and \(H_2'\) are given as

\[
H_2 = \begin{bmatrix} a_1 & a_2 \\ a_2 & -a_1 \end{bmatrix}, \quad (28)
\]

and

\[
H_2' = \begin{bmatrix} a_3 & a_4 \\ a_4 & -a_3 \end{bmatrix}. \quad (29)
\]

Similarly, we can form another matrix \(B'\) using \(b_1, b_2, b_3\) and \(b_4\) considered as the imaginary part of the symbols \(S_1, S_2, S_3\) and \(S_4\). The complete matrices \(A'\) and \(B'\) based on equation (27) can be expressed as
The signal to noise plus interference ratio (SNIR) of a communication system is expressed as

\[
\text{SNIR} = \frac{\sum_{i=1}^{N} \alpha_i^2}{\sum_{i=1}^{N} \frac{\alpha_i}{\text{SNIR}} + \frac{1}{\text{SIR}}},
\]

where \( \alpha_i \) is the attenuation coefficient of the \( i \)-th path of a fading channel (\( N \) multi-path channel) and SIR is the signal to interference ratio. In this paper, three modulation schemes BFSK, QPSK and 16-QAM are considered for the determination of the BER against SNR in dB.

For the ideal case the probability of symbol error for BFSK is expressed as [21]:

\[
P_{b_{-BFSK}}(\text{SNR}) = Q\left(\frac{\sqrt{2} \alpha_a}{\text{SNR}}\right),
\]

and

\[
W = A' + jB',
\]

where \( W \) is a 4-by-8 orthogonal matrix and satisfies the following orthogonality property:

\[
W^T W = 2\left( |X_1|^2 + |X_2|^2 + |X_3|^2 + |X_4|^2 \right) I_4.
\]

where

\[
|X_1|^2 = |a_1|^2 + |b_1|^2,
\]

\[
|X_2|^2 = |a_2|^2 + |b_2|^2,
\]

\[
|X_3|^2 = |a_3|^2 + |b_3|^2,
\]

\[
|X_4|^2 = |a_4|^2 + |b_4|^2,
\]

and \( I_4 \) is the 4-by-4 identity matrix.

For the halfrate case,

\[
P_{b\frac{1}{2}}(\text{SNR}) = Q\left(\sqrt{\text{SNR} \left(\frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}\right)}\right),
\]

For \( \frac{3}{4} \) rate case,

\[
P_{b\frac{3}{4}}(\text{SNR}) = Q\left(\sqrt{\frac{2 \alpha_1 \alpha_4 - 2 \alpha_2 \alpha_3}{\text{SNR} \left(\frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}\right)}}\right),
\]

Finally, for proposed full rate orthogonal case,

\[
P_{b\text{proposed,BFSK}}(\text{SNR}) = Q\left(\sqrt{\frac{2 \alpha_1 \alpha_4 - 2 \alpha_2 \alpha_3}{\text{SNR} \left(\frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}\right)}}\right),
\]

In QPSK scheme, the BER for 1/2 rate, 3/4 rate, full rate quasi-orthogonal and proposed full rate cases are:

\[
P_{b\frac{1}{2}}(\text{SNR}) = Q\left(\sqrt{\frac{2 \alpha_1 \alpha_3 + \alpha_4 \alpha_4}{\text{SNR} \left(\frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}\right)}}\right),
\]

\[
P_{b\frac{3}{4}}(\text{SNR}) = Q\left(\sqrt{\frac{2 \alpha_1 \alpha_3 + \alpha_4 \alpha_4}{\text{SNR} \left(\frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}\right)}}\right),
\]

\[
P_{b\text{quasi,orthQPSK}}(\text{SNR}) = Q\left(\sqrt{\frac{2 \alpha_1 \alpha_3 + \alpha_4 \alpha_4}{\text{SNR} \left(\frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}\right)}}\right),
\]

where

\[
d_1 = \left(\frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}{\text{SNR}}\right) + \left(\frac{2 \alpha_1 \alpha_4 - 2 \alpha_2 \alpha_3}{\text{SNR} \left(\frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}\right)}\right).
\]

and

\[
d_2 = \left(\frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}{\text{SNR}}\right) + \left(\frac{2 \alpha_1 \alpha_4 - 2 \alpha_2 \alpha_3}{\text{SNR} \left(\frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}\right)}\right).
\]
and

\[ P_{b, \text{proposed, QPSK}}(\text{SNR}) = Q \left[ \frac{4 \left( \frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4} \right)}{\text{SNR}} \right] ^{1/4} \]  

Similarly, expressions of the BER for the 16-QAM are:

\[ P_{b, \text{QAM}}(\text{SNR}) = Q \left[ \frac{d_3}{8 \left( \frac{\text{SNR}}{M - 1} \right)} \right] \times \frac{4 \left( \sqrt{M} - 1 \right)}{\sqrt{M} V} \]  

where

\[ d_3 = \left( \frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4} \right) , \]

and

\[ P_{b, \text{quasi-orth QAM}}(\text{SNR}) = Q \left[ \frac{d_5}{\sqrt{d_6}} \right] \times \frac{4 \left( \sqrt{M} - 1 \right)}{\sqrt{M} V} \]  

where

\[ d_5 = 2 \left( \alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 \right) \]

\[ d_6 = \left( \frac{3 \text{SNR}}{M - 1} \right) + \left( \frac{2 \alpha_1 \alpha_4 - 2 \alpha_2 \alpha_3}{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2} \right) \]

and

\[ P_{b, \text{proposed, QAM}}(\text{SNR}) = Q \left[ \frac{12 \left( \frac{3 \text{SNR}}{M - 1} \right) d_3}{\sqrt{M} v} \right] \times \frac{4 \left( \sqrt{M} - 1 \right)}{\sqrt{M} v} \]  

These analytical expressions are plotted in the next section including the ideal single line case.

**5 Results**

In this section, analytical result of a MISO system under multipath Rayleigh fading channel is analyzed for three different modulation schemes: BFSK, QPSK and 16-QAM. To evaluate the performance of the schemes, let us take the typical values of attenuation parameters of multipath channel: \( \alpha_1 = 0.24 \), \( \alpha_2 = 0.47 \), \( \alpha_3 = 0.12 \) and \( \alpha_4 = 0.32 \).
channels are also compared with the ideal single-input single-output (SISO) system of an AWGN channel. We know that the performance of a multipath fading channel is heavily degraded because of intersymbol interference (ISI) and $P_b$ will be much greater than that of the AWGN channel as is also visualized from Fig. 2 through Fig. 4. As has been mentioned earlier, the performance of the proposed scheme can be further enhanced with the increment of the number of transmit antennas to $2^n$ where $n$ is an integer greater than 3. The entire result of this section reveals that the fully orthogonal MISO technique is found as the best tool for combating the multipath fading of wireless channel.

### 6 Conclusion

The popularity of wireless technology is escalating with time in spite of its high rate of data loss. For wireless communication environment, multiple antenna mode has provided a huge improvement in alleviating multipath fading of channels. Only full rate, completely orthogonal with the simplest decoding techniques transmission code, namely, space-time block code scheme is given by Alamouti [3] for two transmission antennas and one receiver antenna is now in existence. In post Alamouti period, a number of other schemes have been proposed. Existing models are half (1/2) rate or three fourth (3/4) rate fully orthogonal schemes for multiple antenna mode systems. There are also some full rate quasi-orthogonal schemes presented by various researchers, experiencing huge inter-symbol interference (ISI). In this paper, we have attempted to construct a scheme which would be able to triumph over all the inconveniences in wireless communication systems. Our proposed model is fully orthogonal for full rate at the expense of increment of the number of antennas. As a future extension of the study, we still have the opportunity to enhance the suggested scheme and observe its performance for minimum shift keying (MSK) and Gaussian minimum shift keying (GMSK) modulation schemes. These two modulation schemes are considered to be the most popular schemes for wireless mobile communications. Another important extension of this work could be implementing water-filling model of power constraints into it. The widest application of the proposed fully orthogonal MISO technique will be in the wireless voice over Internet protocol (VoIP) system under dynamic modulation and coding schemes [23], [24].
Appendix A

Examples Construction 1: The construction method discussed in section 2.2 is utilized here for constructing a half (½) rate OSTBC. At first let construct an AOD of order 8, type (2, 2, 2, 2; 2, 2, 2, 2) and lower order amicable families (AF),

\[
\begin{bmatrix}
1 & -1 \\
-1 & -1 \\
1 & 1 \\
1 & 1
\end{bmatrix}
\]

where each of type (2, 2; 2, 2) can be expressed as

\[
\begin{bmatrix}
1 & -1 \\
-1 & 1 \\
1 & 0 \\
0 & 1
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
1 & 1 \\
1 & -1 \\
1 & 1 \\
-1 & -1
\end{bmatrix}
\]

Considering a set of matrices \(\{M, N\}\) satisfying the conditions listed in equations (10) to (15) are given below:

\[
M_1 = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0
\end{bmatrix},
\]

\[
N_1 = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0
\end{bmatrix},
\]

\[
M_2 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix},
\]

\[
N_2 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{bmatrix},
\]

\[
M_3 = \begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
-1 & 0 & 0 & 0
\end{bmatrix},
\]

\[
N_3 = \begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0
\end{bmatrix},
\]

Using equations (16) to (19), matrices \(A\) and \(B\) can be constructed as

\[
A = \begin{bmatrix}
A_1 & A_2 \\
A_3 & A_4
\end{bmatrix},
\]

where

\[
A_1 = \begin{bmatrix}
a_1 & a_4 & a_3 & a_2 \\
-a_4 & a_1 & a_2 & -a_3 \\
-a_3 & -a_2 & a_1 & a_4 \\
-a_2 & a_3 & -a_4 & a_1
\end{bmatrix},
\]

\[
A_2 = \begin{bmatrix}
a_1 & -a_4 & -a_3 & -a_2 \\
a_4 & a_1 & -a_2 & a_3 \\
a_3 & a_2 & a_1 & -a_4 \\
a_2 & -a_3 & a_4 & a_1
\end{bmatrix},
\]

\[
A_3 = \begin{bmatrix}
a_1 & a_4 & a_3 & a_2 \\
-a_4 & a_1 & a_2 & -a_3 \\
-a_3 & -a_2 & a_1 & a_4 \\
-a_2 & a_3 & -a_4 & a_1
\end{bmatrix},
\]

\[
A_4 = \begin{bmatrix}
a_1 & a_4 & a_3 & a_2 \\
-a_4 & a_1 & a_2 & -a_3 \\
-a_3 & -a_2 & a_1 & a_4 \\
-a_2 & a_3 & -a_4 & a_1
\end{bmatrix},
\]

and

\[
B = \begin{bmatrix}
B_1 & B_2 \\
B_3 & B_4
\end{bmatrix},
\]

where

\[
B_1 = \begin{bmatrix}
-b_1 & b_4 & b_3 & b_2 \\
-b_4 & -b_1 & -b_2 & b_3 \\
-b_3 & b_2 & -b_1 & -b_4 \\
-b_2 & -b_3 & b_4 & -b_1
\end{bmatrix},
\]

\[
B_2 = \begin{bmatrix}
-b_1 & -b_4 & -b_3 & -b_2 \\
b_4 & -b_1 & b_2 & -b_3 \\
b_3 & -b_2 & -b_1 & b_4 \\
b_2 & b_3 & -b_4 & -b_1
\end{bmatrix},
\]

\[
B_3 = \begin{bmatrix}
b_1 & -b_4 & -b_3 & -b_2 \\
b_4 & b_1 & b_2 & -b_3 \\
b_3 & -b_2 & b_1 & b_4 \\
b_2 & b_3 & -b_4 & b_1
\end{bmatrix},
\]
According to equation (15) an OSTBC, $G_L$, can be represented as:

$$G_L = A + jB.$$  
(A.1)

The $G_L$ is an orthogonal STBC with $1/2$ code rate. To prove the orthogonal property of a STBC, it has to satisfy the following relation:

$$G_L G_L^H = \lambda I_N,$$  
(A.2)

where $G_L^H$ is the conjugate transpose of $G_L$, $X$ is a factor dependent on the symbols being transmitted and $I_N$ is an identity matrix of $N \times N$. For $N = 8$,

$$X = 2(a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 + b_4^2).$$

Example of Construction 2: Let us consider the construction method as has been discussed in section 2.2.2 for an AOD of order 4 and type (2, 2, 2, 2, 2, 2, 2, 2) indicated as $A^r_4$ and $B^r_4$ defined as:

$$A^r_4 = \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad A^r_3 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix},$$

$$B^r_4 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad B^r_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. $$

A set of matrices $N_j$, where $1 \leq j \leq 3$, can be represented as:

$$N_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad N_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

and a 2-by-2 identity matrix $I_2$:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. $$

We derive $A_{3/4}$ and $B_{3/4}$ matrices according to equations (21) to (24) as given below:

$$A_{3/4} = \begin{bmatrix} -a_2 + a_3 & a_1 & a_2 + a_3 & -a_1 \\ -a_1 & -a_2 + a_3 & a_1 & a_2 + a_3 \\ a_2 + a_3 & a_1 & a_2 - a_3 & a_1 \\ -a_1 & a_2 + a_3 & -a_1 & a_2 - a_3 \end{bmatrix},$$

and

$$B_{3/4} = \begin{bmatrix} b_2 + b_3 & b_1 & -b_2 + b_3 & -b_1 \\ b_1 & -b_2 + b_3 & -b_1 & b_2 + b_3 \\ b_2 - b_3 & b_1 & b_2 + b_3 & b_1 \\ b_1 & -b_2 + b_3 & b_1 & -b_2 + b_3 \end{bmatrix}. $$

According to equation (20) depicted in section 2.2.1, the OSTBC with $3/4$ code rate can be developed as:

$$G_{3/4} = A_{3/4} + jB_{3/4}. $$  
(A.3)

The matrix $G_{3/4}$ is an orthogonal STBC with $3/4$ code rate. The orthogonal property of a STBC can be proved by satisfying the equation (A.2). Here we need a 4-by-4 identity matrix $I_4$ as the transmission is occurring in 4 time slots. Equation (A.2) can be rewritten as matrix form:

$$G_{3/4} G_{3/4}^H = R,$$

where

$$R = 2(a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2).$$

Example of Eight antenna full rate 4 symbol case (proposed scheme): The proposed full rate orthogonal STBC (4 symbols, 4 time slot and 8 antenna case) expressed as:

$$W = [W_1, W_2],$$

where

$$W_1 = \begin{bmatrix} a_1 + ib_1 & a_2 + ib_2 & a_1 + ib & a_2 + ib_2 \\ a_2 - ib_2 & a_1 + ib & a_2 - ib_2 & -a_1 + ib \\ a_1 + ib_1 & a_2 + ib & -a_1 - ib_1 & -a_2 - ib_2 \\ a_2 - ib_2 & -a_1 + ib & -a_2 - ib_2 & a_1 - ib_1 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} a_3 + ib_3 & a_4 + ib_4 & a_3 + ib & a_4 + ib_4 \\ a_4 - ib_4 & a_3 + ib & a_4 - ib_4 & -a_3 - ib_3 \\ a_3 + ib_3 & a_4 + ib & -a_3 - ib_3 & -a_4 - ib_4 \\ a_4 - ib_4 & -a_3 + ib & -a_4 - ib_4 & a_3 - ib_3 \end{bmatrix}. $$

Taking $a_1 = 1, a_2 = 1, a_3 = -1, a_4 = -1, b_1 = 1, b_2 = 1, b_3 = -1$ and $b_4 = 1$ we get,

$$W W^H = 16.$$

References:


[11] Chau Yuen, Yong Liang Guan and Tjeng Thiang Tjhung, A Class Of Four-Group Quasi-Orthogonal STBC Achieving Full Rate And Full Diversity For Any Number Of Antennas, Accepted for PIMRC 2005.


