Resource Allocation for QoS-Aware OFDMA Cellular Networks with Cooperative Relaying

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Abstract: - This paper proposes a QoS-aware resource allocation scheme, called as cooperative resource allocation scheme (CoRA), for relay-based OFDMA systems with both QoS and Best-effort (BE) users. Using CoRA, cooperative transmission between the Base-station (BS) and the Relay-station (RS) is employed to fulfill the requirements of QoS users. Moreover, resource scheduling is allowed not only at the BS but also at each RS to fully exploit time-varying channel state and multiuser diversity, so as to achieve efficient radio resource utilization. The design of subchannel and power allocation in CoRA can be formulated as a non-linear combinatorial optimization problem, which aims at maximizing the system utility while satisfying the data rate requirements of QoS users and the separate transmission power constraints at both BS and RSs. To address the prohibitively high computational complexity in solving the optimization problem, we decompose the global optimization problem in CoRA into subchannel allocation and power control sub-problems as a suboptimal solution. Numerical results demonstrate that our proposed CoRA scheme is able to significantly improve the network performance in terms of power saving, user utilities, system throughput, as well as the number of admitted users.

Key-Words: - Cooperative transmission, Subchannel allocation, Power control, Quality-of-service

1 Introduction

Orthogonal frequency-division multiple access (OFDMA) has been chosen to be the multicarrier technique for the current and future cellular systems such as the mobile Worldwide Interoperability of Microwave Access (WiMAX) networks and the Third-Generation Partnership Project Long-Term Evolution (3GPP LTE) [1][2] system, due to its inherent robustness against frequency-selective fading and its capacity for achieving high spectral efficiency. However, in traditional cellular architecture, the increase of the capacity along with the coverage would require the deployment of a large number of Base Stations (BSs) with high cost. Recently, Relay Stations (RSs) are introduced in each cell to alleviate this problem since the RS can offer high data rate in the remote areas of the cell while keeping a low cost of infrastructure[3][4]. Intensive research has been conducted and it has shown that deploying RSs in conventional cellular networks can achieve significant performance improvement including cell capacity enhancement, cell coverage extension, and transmission power saving [5][6]. Consequently, the combination of relay-based network architecture and OFDMA has been considered as a promising architecture for future wireless broadband networks.

In recent years, cooperative transmission technique has gained much attention as an advanced emerging transmitting strategy for future wireless networks. This technique efficiently utilizes the broadcast nature of wireless networks to exploit the inherent spatial and multiuser diversity through node cooperation when limited amounts of antennas are deployed at each node. Significant amount of research on cooperative communication techniques [7]-[12] has been done to allow stations to cooperate in transmissions to improve the overall performance of the network.

Inspired by the notable performance gain brought by the technique of relay and cooperation, we propose to introduce cooperative transmission mechanism between BS and RS into relay-based OFDMA cellular networks and address the key issue of resource allocation in the corresponding system. Compared with traditional OFDMA cellular networks, resource allocation in cooperative relay-based OFDMA system is much more challenging, as a number of issues are involved and are inherently associated with each other, such as relay selection, subchannel allocation and power control at BS and
the cooperative RSs.

In literatures, the authors of [13], [14] and [15] investigate only the relay selection problem for relay-enhanced cellular networks. In [16], M. Kaneko and P. Popovski develop a subchannel allocation algorithm that is performed at BS and an RS. Based on the channel state and buffered packets at the RS, the RS allocates subchannels to Mobile Stations (MSs) and sends control messages to the BS for the packet transmission of these MSs. Nevertheless, this paper proposes only a heuristic solution for a simple system model with only one RS. The authors in [17] study resource allocation using network coordination, in which the cooperation among different BSs of neighboring cellular is considered. The proposed resource allocation method applies network coordination to provide the QoS of users with given data rate requirements and to increase the capacity of BE users. However, the fairness between users is not addressed, and cooperation among different BSs could introduce more system overhead compared to the cooperation between BS and RS. In [18], B. G. Kim and J. W. Lee investigate the opportunistic power scheduling problem for the downlink in the OFDMA cellular network with fixed relay stations. They formulate a stochastic optimization problem to maximize the sum-rate for MSs considering the time varying channel state of each link. In addition, the spectral efficiency performance of opportunistic scheduling and spectrum re-use techniques for relay-based cellular networks in the downlink mode is analyzed in [19]. Subcarrier and power allocation schemes are investigated considering fairness issues for a cooperative OFDMA uplink system in [20]. The authors in [21] focus on the subcarrier-pair based resource allocation problem for cooperative OFDM systems composed of one source node, one destination node and multiple relay nodes, with the objective of maximizing the system transmission rate subject to individual power constraints on each node or a total network power constraint. In [22], the authors study the end-to-end capacity of a cooperative relaying scheme using OFDM modulation, under power constraints for both the base station and the relay station.

In this paper, we propose a QoS-aware resource allocation scheme, referred to as cooperative resource allocation scheme (CoRA) by jointly optimizing subchannel and power allocation, for cooperative relay-based OFDMA cellular networks with both QoS and BE users, under the constraints of the transmission power of BS and RSs, and data rate requirements of QoS users. The proposed scheme solves the relay-selection, subchannel-allocation and power-control problem with the consideration of efficiency and fairness of resource allocation. The following salient features of our work exist, thus making it different from the existing schemes.

- Our proposed CoRA scheme takes into account the cooperative transmission between BS and RS, as well as the resource scheduling at both BS and RS, so as to fully exploit time-varying channel state of each link and multiuser diversity of OFDM system. As a result, the radio resource utilization can be maximized.

- To reduce the computational complexity in solving the non-linear combinatorial optimization problem, we decompose the joint optimization problem into subchannel allocation and power control sub-problems. We formulate the subchannel allocation problem as a non-linear integer programming, and then show that it can be transformed into a convex problem. Furthermore, we devise a low complexity subchannel assignment algorithm.

- Power control problem is decoupled into two sub-problems in accordance with the types of users. Through defining the equivalent signal-to-noise ratio (SNR) for each wireless link, we show that the power control for QoS users can be formulated as a geometric programming problem. On the other hand, the power control for BE users is formulated as a convex optimization problem, and then we employ a utility-based water-filling method to solve it.

The rest of this paper is organized as follows. Section 2 describes the system model and basic assumptions. In Section 3, we present the proposed QoS-aware joint resource allocation scheme in detail. Section 4 elaborates the proposed subchannel allocation and power control algorithms. We evaluate the performances of the proposed resource allocation scheme in Section 5 and finally conclude this paper in Section 6.

2 System Model

As illustrated in Fig.1, we consider a single cell downlink OFDMA system with a BS, $M$ RSs and $K$ users. Among the $K$ users, $K_0$ QoS users have strict demands on resource requirements, and $K_B$ BE users have no any bandwidth requirement, where $K_0 + K_B = K$. The downlink channel is divided into $N$ orthogonal subchannels. Assume that the wireless channel is time-varying and frequency-selective. However, it is assumed to be flat within a subchannel and unchangeable during a scheduling period [16]-[18].
To improve the communication reliability and guarantee the requirements of QoS users, cooperative transmission between BS and RS is employed for QoS users while direct transmission is used for BE users. Each QoS user selects an RS as its cooperative relay according to current channel state of the link between BS and RS and the link between the RS and the user. It is assumed that all RSs operate in decode-and-forward (DF) mode, and an RS cannot transmit and receive data simultaneously. With cooperative transmission, each scheduling period is divided into two phases with the same duration. In the first phase, only the BS can transmit data to RSs and users. In the second phase, the BS and RSs can simultaneously transmit data to BE users and QoS users on different subchannels respectively, so as to make full use of radio resource.

Fig.1 Cooperative relay-based cellular network

Instead of using subchannel matching method [21] to forward data to the destination, we assume that all RSs can perform bit reallocation independently, i.e., each RS decodes the received data and reallocates the data among subchannels corresponding to the current channel state, and then forwards the data to the helped QoS users. It is worth mentioning that by adaptively allocating subchannel and power at both BS and RS, and thus exploring the time-varying channel state and multiuser diversity of OFDMA system, we can significantly improve resource utilization.

We develop a utility-based resource allocation framework to balance the efficiency and fairness of resource allocation. The degree of the satisfaction on the network service for QoS and BE users can be described by different utility functions with respect to the allocated data rate. The utility function of QoS user is a unit-step function with the value of 1 so long as the data rate requirement is met [23], \( U_q(R_q) = 1 \) if \( R_q \geq R_q^\max \), otherwise \( U_q(R_q) = 0 \). On the other hand, the utility function of BE user is a non-decreasing concave function [24], \( U_e(R_e) = 1 - e^{-0.1 R_e} \), \( R_e > 0 \).

Each direct communication link is denoted by \((k, m)\) where \( k \in \{1, 2, ..., K\} \), \( m \in \{0, 1, 2, ..., M\} \), and \( m = 0 \) represents the BS. Let \( H_{n,k,m} \) denote the channel gain of link \((k, m)\) on \(n^{th}\) subchannel, which is assumed to remain unchanged during a scheduling period and be independent for different links. Let \( \sigma^2 \) denote the noise power. Accordingly, the signal to noise ratio (SNR) per unit of transmission power for link \((k, m)\) on \(n^{th}\) subchannel is given by

\[
\eta_{n,k,m} = \frac{|H_{n,k,m}|^2}{\sigma^2}. \tag{1}
\]

Define \( c_{n,k,m}^{(r)} \) to be the subchannel allocation indicator, where \( c_{n,k,m}^{(r)} = 1 \) indicates that \(n^{th}\) subchannel is allocated to link \((k, m)\) for packet transmission in \(\tau^{th}\) phase, and \( c_{n,k,m}^{(r)} = 0 \) otherwise.

In order to avoid co-channel interference, each subchannel should be assigned to at most one link in each phase. Thus,

\[
\sum_{k=1}^{K} \sum_{m=0}^{M} c_{n,k,m}^{(r)} = 1, \forall n, \tau. \tag{2}
\]

Let \( p_{n,k,m}^{(r)} \) be the allocated transmission power for link \((k, m)\) on \(n^{th}\) subchannel in \(\tau^{th}\) phase. According to Shannon theory, the achievable rate of link \((k, m)\) for user \(k\) in \(\tau^{th}\) phase is given by

\[
R_{n,k,m}^{(r)} = \sum_{n=1}^{N} c_{n,k,m}^{(r)} \log_2 \left( 1 + p_{n,k,m}^{(r)} \eta_{n,k,m} \right). \tag{3}
\]

And in a high SNR wireless scenario, \( R_{n,k,m}^{(r)} \) can be approximately expressed as

\[
R_{n,k,m}^{(r)} \approx \sum_{n=1}^{N} c_{n,k,m}^{(r)} \log_2 \left( p_{n,k,m}^{(r)} \eta_{n,k,m} \right) \approx \log_2 \left( 1 + \sum_{n=1}^{N} \left( p_{n,k,m}^{(r)} \eta_{n,k,m} \right)^{c_{n,k,m}^{(r)}} \right). \tag{4}
\]

Accordingly, we define the equivalent SNR for link \((k, m)\) in \(\tau^{th}\) phase as

\[
SNR_{n,k,m}^{(r)} = \prod_{n=1}^{N} \left( p_{n,k,m}^{(r)} \eta_{n,k,m} \right)^{c_{n,k,m}^{(r)}}. \tag{5}
\]

It can be seen that the equivalent SNR for a link in a phase depends on the assigned subchannels, as well as the corresponding power on the allocated subchannels.

In order to reduce the implementation complexity of cooperative transmission between BS and RS, we restrict that each QoS user is allowed to select only one RS as its cooperative relay. Selecting an RS to cooperatively transmit for each QoS user plays an important role in resource allocation. With appropriate relay selection, not only the requirements of QoS users can be satisfied, but also the power of the BS can be saved as much as possible. Consequently, more power can be allocated to BE users to improve their utilities. Furthermore, the RS can bring gain if the quality of both links, from source to the RS and from the RS to the destination, is high.
to destination, is good enough. Otherwise, the performance of end-to-end communication will be deteriorated by channel impairments in either of relay links.

In this paper, we adopt a simple but efficient strategy to choose the relay with the best equivalent end-to-end channel gain for each QoS user. Let \( h_{n,j} \) and \( h_{n,k} \) denote the channel gain for the link between BS and \( j^{th} \) RS, and the link between \( j^{th} \) RS and QoS user \( k \), respectively. Let \( H_{n,j} \) be the channel gain for the link between BS and the \( j^{th} \) RS on \( n^{th} \) subchannel, and \( h_{n,k} = \| H_{n,j} \| \), respectively. Let \( g_{k}^{j} \) be the equivalent end-to-end channel gain for QoS user \( k \) with respect to \( j^{th} \) RS. Accordingly, the index of the selected cooperative RS for QoS user \( k \) can be given by

\[
g_{k} = \max \{ g_{k}^{j} \}, j \in \{1,2,\ldots,M\},
\]

where \( g_{k}^{j} = g (h_{n,j},h_{n,k}) = \frac{|h_{n,j}| |h_{n,k}|}{|h_{n,j}| + |h_{n,k}|} \).

Accordingly, the receiving rate at the RS \( g_{k} \) is

\[
R_{g_{k}} = \sum_{n=1}^{N} \log_{2} \left( 1 + \frac{g_{k}^{n} P_{rs} H_{n,k}}{I_{n} + g_{k}^{n} P_{rs} H_{n,k}} \right),
\]

where \( g_{k}^{n} = \| H_{n,k} \| / \sigma^{2} \), is the SNR per unit of transmission power for the link between BS and the RS \( g_{k} \) on \( n^{th} \) subchannel.

Define \( R_{g}^{*} \) to be the achievable cooperative rate for QoS user \( k \) according to [25], we have

\[
R_{g}^{*} = \log_{2} \left( 1 + SNR^{(1)} + SNR^{(2)} \right).
\]

As a result, the effective data rate for QoS user \( k \) and BE user \( l \) during a scheduling period is given by

\[
R_{k}^{QoS} = \frac{1}{2} \min \{ R_{g_{k}}, R_{g_{l}}^{*} \}, k \in \{1,2,\ldots,K_{Q}\},
\]

\[
R_{l}^{BE} = \frac{1}{2} \left( R_{g_{l}}^{*} + R_{g_{l}}^{*} \right), l \in \{K_{Q}+1,\ldots,K\}.
\]

The factor 1/2 in Eq.(9) and Eq.(10) is due to that there are two communication phases in each scheduling period.

Let \( P_{o} \) denote the maximum total transmission power of the BS for the first phase, and \( P_{2,m} \) \(( m \in \{0,1,2,\ldots,M\} \) be the maximum total transmission power of \( m^{th} \) transmitter for the second phase. Then, power allocation in the first and the second phase should satisfy the following conditions, respectively.

\[
\sum_{i=1}^{K} \sum_{n=1}^{N} p_{i,n,0}^{(1)} \leq P_{o}
\]

\[
\sum_{i=1}^{K} \sum_{n=1}^{N} p_{i,n,0}^{(2)} \leq P_{2,m}
\]

### 3 Problem Formulation

In this section, we propose a QoS-aware resource allocation scheme, referred to as cooperative resource allocation scheme (CoRA) by jointly optimizing subchannel and power allocation, for cooperative relay-based OFDMA cellular networks with both QoS and BE users.

As mentioned in previous section, we employ cooperative transmission between BS and RS, and resource scheduling at both BS and RSs to improve the overall performance. Accordingly, resource allocation should be coordinated between two communication phases and be also performed jointly at BS and the cooperative RSs. We formulate the design of joint subchannel and power allocation as a non-linear combinatorial programming, denoted by P1, with the objective of maximizing the system utility, subject to the separate transmission power constraints of BS and RS, and the data rate requirements of QoS users.

\[
P1: \max_{c_{n,m}^{(1)},P_{n,m}} \sum_{i=1}^{K} U_{i} (R_{i})
\]

s.t.

\[
R_{k}^{QoS} \geq R_{k}^{*}, k \in \{1,2,\ldots,K_{Q}\},
\]

\[
\sum_{i=1}^{K} \sum_{n=1}^{N} c_{n,k,m}^{(r)} = 1, c_{n,k,m}^{(r)} \in \{0,1\}, \forall n,r
\]

The optimization variables of programming P1 are \( c_{n,k,m}^{(r)} \) and \( p_{n,m}^{(r)} \). Eq.(14) represents the data rate requirements of QoS users. Eq.(15) means that \( c_{n,k,m}^{(r)} \) can only take the value 0 or 1, and each subchannel should be allocated to at most one link in each phase. It should be noted that the utility function of QoS user is a unit-step function, so that the sum utility of QoS users is a constant with the value of \( K_{Q} \), if Eq.(14) is met. Consequently, the optimization objectives is equivalent to maximizing the sum utility of BE users, that is

\[
\max_{c_{n,m}^{(1)},P_{n,m}} \sum_{i=1}^{K} U_{i} (R_{i}).
\]

Furthermore, for the sake of efficient resource allocation, according to Eq.(9) and Eq.(14), the data rate requirements of QoS users can be rewritten as

\[
\frac{1}{2} R_{i} = \frac{1}{2} R_{g_{i}}^{*} \geq R_{g_{i}}^{*}, k \in \{1,2,\ldots,K_{Q}\}.
\]

Programming P1 is a non-linear combinatorial programming problem, and can be proven to be NP-hard [26]. To address the high computational
complexity in solving P1, we decompose it into two sub-problems. The first sub-problem is to carry out subchannel assignment with fixed power allocation, while the second sub-problem is to perform power control based on the result of subchannel allocation.

4 Subchannel Allocation and Power Control Algorithms

In this section, we will elaborate the proposed subchannel allocation and power control algorithms of the CoRA scheme.

4.1 Subchannel Allocation Algorithms

In each phase of the cooperative transmission, under a specific power allocation, say uniform power allocation, the subchannel allocation problem can be formulated as a non-linear integer programming, and can be further transformed into a convex optimization problem by relaxing the original optimization variables.

4.1.1 Subchannel Allocation for the First Phase of Cooperative Transmission

Assume that all subchannels are allocated with the equal power $p_0$ in the first phase of cooperative transmission. According to Eq.(16) and Eq.(17), subchannel allocation problem can be formulated as a programming, denoted by P2, as below

$$P2: \max_{\alpha,\beta} \sum_{k=1}^{K} U_k \left( \sum_{n=1}^{N} \left( c_{n,k,0}^{(i)} \log_2 \left( 1 + p_0 q_{n,k}^{(i)} \right) \right) \right)$$

s.t.

$$\frac{1}{2} \sum_{n=1}^{N} c_{n,k,0}^{(i)} \log_2 \left( 1 + p_0 q_{n,k}^{(i)} \right) \geq R_n^{req}, k \in \{1, \ldots, K\}$$

$$c_{n,k,0}^{(i)} \in \{0,1\}, k = \{1, \ldots, K\}$$

P2 is a non-linear integer programming problem. An exhaustive search over all combinations of subchannels is computationally impossible when the value of $N$ and $K$ are non-trivial. To solve P2, we first relax the constraint of exclusive subchannel assignment and replace $c_{n,k,0}^{(i)}$ with a real variable $\rho_{n,k}^{(i)}$, where $0 \leq \rho_{n,k} \leq 1(\forall n,k)$. Then it can be proven that the optimization problem becomes a convex optimization problem [27]. However, the optimal relaxing solutions $\rho_{n,k}^{(i)}$ are fractional, and we need to round them to the nearest integer values to facilitate subchannel assignment.

Based on the concavity of the objective function and the optimal relaxing solution of P2, we develop a low complexity suboptimal dynamic subchannel allocation algorithm, i.e. Algorithm 1, presented in Fig.2.

**Algorithm 1: Dynamic Subchannel Allocation Algorithm**

1) Define $S = \{n|1 \leq n \leq N\}$ to be the set of indexes for subchannel. Initialize the data rate of each user.

2) While(1)

   - $\rho_{n,k}^{(i)} = \max_{\alpha,\beta} \{\rho_{n,k}^{(i)}\}$; if $\rho_{n,k}^{(i)} = 0$, break;
   - If $R_{n}/2 \leq R_{n}^{req}$, $R_{n} = R_{n} + \log_2 \left( 1 + p_n q_{n,k} \right)$; $\rho_{n,k} = 1$;
   - $\rho_{n,k}^{(i)} = 0(\forall n,k)$. $S = S \setminus t$.
   - Else $\rho_{n,k}^{(i)} = 0$;

end-while

3) If the data rate requirement of every QoS user is met, go to Step 4; otherwise execute the compensation algorithm (listed in Fig.3);

4) Define $G = \{k|K_n + 1 \leq k \leq K\}$ to be the set of BE users, and compute the optimal rate $R_{n}^{opt}$ of each BE user based on the optimal solutions of programming P2; for $i = 1$ to $|S|$:

   - $L = \{l: l = \arg \max_{c\in C} R_{n}^{opt} \log_2 \left( 1 + p_n q_{n,k} \right), \forall n \in S\}$
   - If $|L| > 1$, then $l = \arg \max_{c\in C} R_{n}^{opt}$.
   - If $R_{n} < R_{n}^{opt}$, $R_{n} = R_{n} + \log_2 \left( 1 + p_n q_{n,k} \right)$;
   - $\rho_{n,k}^{(i)} = 1, \rho_{n,k}^{(i)} = 0(\forall n,k)$. $S = S \setminus n$.
   - Else $G = G \setminus l$.

end-for

**Fig.2 Dynamic subchannel allocation algorithm**

**Algorithm 2: Compensation Algorithm for QoS users**

Arrange the unsatisfied QoS users in increasing order of $\Delta R_{n} = R_{n}^{opt} - R_{n}$; for each user, iterate until $\Delta R_{n} \leq 0$

   - If $S = \emptyset$, stop;
   - $c := \arg \min_{n,k} \log_2 \left( 1 + p_n q_{n,k} \right) - \Delta R_{n}$;
   - $R_{n}^{opt} = 1, \rho_{n,k}^{(i)} = 0(\forall n,k)$. $S = S \setminus c$;
   - $R_{n} = R_{n} + \log_2 \left( 1 + p_n q_{n,k} \right)$;
   - $\Delta R_{n} = \Delta R_{n} - \log_2 \left( 1 + p_n q_{n,k} \right)$.

end-for

**Fig.3 Compensation algorithm**

The main idea of Algorithm 1 is to guarantee the data rate requirements of QoS users with highest priority, and to allocate the remaining subchannels to BE users. First, the BS searches for the user with maximal non-zero $\rho_{n,k}^{(i)}$ among all the QoS users. If the data rate requirement of the QoS user is not met, the corresponding subchannel is allocated to it accordingly as stated in Step 2). Note that in Step 2), not all QoS users could be met on the data rate requirements, and thus a compensation algorithm, i.e.
Algorithm 2 given in Fig.3, is performed for unsatisfied QoS users to fulfill their demands as stated in Step 3. The compensation rule is that the unsatisfied QoS user $k$ is always assigned the subchannel with the capacity closest to its required data rate, until its data rate requirement is satisfied. Finally, the remaining subchannels are allocated to BE users as stated in Step 4). Subchannel $n$ is assigned to the BE user with the maximum value of $u_k(R^*_k)\log_2(1+p_k\eta_{s,k,0})$. If there is more than one user with the same maximum value, the subchannel $n$ is allocated to the user with maximum $\rho^*_{n,k}$. Furthermore, if a BE user has been allocated with a data rate equal to or more than $R^*_k$, it is not involved in the comparison any more.

4.1.2 Subchannel Allocation for the Second Phase of Cooperative Transmission

In the second phase, BS and the selected RSs transmit simultaneously to BE users and QoS users on different subchannels, respectively. To make full use of radio resource as much as possible, BS and RSs should coordinate to assign subchannels to users according to the channel state of the links between BS and BE users, and the links between RSs and the corresponding helped QoS users. In the proposed CoRA scheme, we develop a centralized subchannel allocation algorithm similar to that of the first phase but with some novel transformation.

Assume $p_{n,m}, m \in \{0,1,...,M\}$ to be the transmission power on each subchannel of transmitter $m$ in the second phase. The following condition should be satisfied according to Eq.(17) for each QoS user

$$\frac{1}{2}R^*_k \geq R^{\alpha_{ii}}, k \in \{1,2,...,K_Q\}. \quad (21)$$

It is equivalent to satisfying the following condition in accordance with Eq.(8)

$$R^{(2)}_{s_3} = \sum_{n=1}^{N} c_{s_3,n} \log_2\left(1+p_{n,s_3,n}\eta_{s_3,n}\right) \geq \log_2 \beta, \quad (22)$$

where $\beta = e^{2K_N} - 1 - \prod_{n=1}^{N} \left(p_{n,\eta_{s_3,n}}\right)^{2\lambda_{s_3}}$ and can be obtained from the result of subchannel allocation in the first phase.

Consequently, the subchannel allocation in the second phase can be also formulated as a non-linear integer programming, denoted by P3, given as follows

P3: max $\sum_{k=1}^{K} U_k \left(\sum_{n=1}^{N} c_{s_3,n,0} \log_2\left(1+p_{n,\eta_{s_3,n,0}}\right)\right)$

s.t. $\sum_{n=1}^{N} c_{s_3,n,0} \log_2\left(1+p_{n,\eta_{s_3,n,0}}\right) \geq \log_2 \beta, k \in \{1,...,K_Q\} \quad (24)$

$c_{s_3,n,m} \in \{0,1\}, \sum_{k=1}^{K} \sum_{n=0}^{N} c_{s_3,n,m} = 1, \forall n \quad (25)$

With the method similar to that of solving P2, we can accomplish the subchannel assignment for the second phase.

4.2 Power Control Algorithms

Recall that the design goal of resource allocation of the proposed CoRA scheme is to maximize the sum utility of BE users while providing the desired data rate to QoS users. It should be noted that the utility function of BE user is a non-decreasing concave function of the data rate, and the data rate is an increasing function of the allocated power. As a consequence, for a given subchannel allocation, the optimization objective of power control is equivalent to minimizing the power of BS required to satisfy all of the QoS users so as to maximize the power dedicated to BE users. Based on this consideration, we are inspired to decouple the power control problem into two sub-problems for QoS users and BE users, respectively.

4.2.1 Power Control for QoS users

Geometric programming (GP) is a well-investigated class of nonlinear, non-convex optimization problems with attractive theoretical and computational properties [28][29]. Since equivalent convex reformulation is possible for a GP problem, there exists no local optimum point but only global optimum. Moreover, the availability of large-scale software solvers makes GP more appealing.

With regard to the QoS users, the power control problem can be formulated as an optimization programming, denoted by P4, with the objective of minimizing the sum power consumed by QoS users in the first phase, subject to the transmission power constraint of each RS and the data rate requirements of QoS users.

$P4: \min_{p_{s_3,m}} \sum_{k=1}^{K_Q} \sum_{n=0}^{N} P_{s_3,n,k}^{(1)}$ \quad (26)

s.t. $\sum_{k=1}^{K_Q} \sum_{n=0}^{N} P_{s_3,n,k}^{(2)} \leq P_{s_3,m}, \forall m \in \{1,2,...,M\} \quad (28)$

By introducing the equivalent SNR for each wireless link as defined in Eq.(5), we show that programming P4 can be transformed into a geometric programming, denoted by P5 as follows, which can be readily turned into a convex optimization problem.

$P5: \min_{\rho_{s_3,m}} \sum_{k=1}^{K_Q} \sum_{n=0}^{N} P_{s_3,n,k}^{(1)}$ \quad (29)

s.t. $\frac{1}{2}R^*_k = \frac{1}{2}R^\omega \geq R^{\alpha_{ii}}, k \in \{1,2,...,K_Q\}$
must satisfy the following conditions.

\[
A_k \left( \prod_{n=1}^{N} p_{k,n}^{(1)^j} \right) = B_k \left( \prod_{n=1}^{N} p_{k,n}^{(2)^j} \right) \tag{30}
\]

\[
\left( \prod_{n=1}^{N} p_{k,n}^{(1)^j} \right) \geq \lambda \alpha_k \quad k \in \{1, 2, \ldots, K\} \tag{31}
\]

\[
\sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n}^{(2)} \leq P_{2,0} \tag{32}
\]

where the coefficients \(A_k, B_k\) and \(C_k\) are given by

\[
A_k = \prod_{n=1}^{N} s_{n,k}^{(1)^j} \tag{33}
\]

\[
B_k = \prod_{n=1}^{N} \eta_{n,k}^{(1)^j} \tag{34}
\]

\[
C_k = \left( 2^{R_{k,0}^{(2)}} - 1 \right) \left( \prod_{n=1}^{N} s_{n,k}^{(1)^j} \right). \tag{35}
\]

With the above problem formulation, we can use some optimization tools, such as Matlab, Yalmip, and CVX, to obtain the optimal solutions \(p_{k,n,0}^{(1)^j}\) and \(p_{k,n,0}^{(2)^j}\) for QoS users.

### 4.2.2 Power Control for BE users

For BE users, the power allocation problem can be formulated as a convex programming problem, denoted by P6 as follows, with the objective of maximizing the sum utility of BE users, subject to the transmission power constraints of the first and the second phases.

\[
P6: \max_{p_{k,n,0}^{(1)^j}, p_{k,n,0}^{(2)^j}} \sum_{k=1}^{K} \sum_{n=1}^{N} U_k^{(1)}(r_k) \tag{36}
\]

s.t.

\[
\sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n}^{(1)} \leq P_1 - P_{QoS} \tag{37}
\]

\[
\sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n}^{(2)} \leq P_{2,0} \tag{38}
\]

where \(P_{QoS}\) is the overall power consumption of QoS users in the first phase, which can be derived from the solutions of programming P5.

By using Lagrange multiplier method, the closed-form solutions of programming P6 can be expressed as follows (refer to Appendix A for detailed derivation)

\[
p_{k,n,0}^{(1)^j} = c_{n,k,0}^{(1)^j} \left[ \frac{U_k^{(1)}(r_k)}{2 \lambda} - \frac{1}{\eta_{n,k,0}} \right]^+, \tag{39}
\]

\[
p_{k,n,0}^{(2)^j} = c_{n,k,0}^{(2)^j} \left[ \frac{U_k^{(2)}(r_k)}{2 \mu} - \frac{1}{\eta_{n,k,0}} \right]^+. \tag{40}
\]

This is indeed a utility-based water-filling for each phase. Here the Lagrange multipliers \(\lambda, \mu\), and the optimal data rate \(r_k^{(2)^j}\) must satisfy the following conditions.

\[
\sum_{k=1}^{K} \sum_{n=1}^{N} c_{n,k,0}^{(1)^j} \left[ \frac{U_k^{(1)}(r_k)}{2 \lambda} - \frac{1}{\eta_{n,k,0}} \right]^+ = P_1 - P_{QoS} \tag{41}
\]

\[
\sum_{k=1}^{K} \sum_{n=1}^{N} c_{n,k,0}^{(2)^j} \left[ \frac{U_k^{(2)}(r_k)}{2 \mu} - \frac{1}{\eta_{n,k,0}} \right]^+ = P_{2,0} \tag{42}
\]

When the subchannel assignment is fixed, the power optimization programming P6 can be resolved by solving a series of linear optimization problems by means of the sequential linear approximation algorithm (Frank–Wolfe method)[30], which can be summarized by Algorithm 3 in Fig.4. The algorithm consists of two parts, corresponding to the power control for the first and the second phase, respectively. Moreover, the power control in the second phase depends on the results of that in the first phase. In each phase, every iteration involves two steps. First, we solve an optimization problem with fixed marginal utilities, which is a regular water-filling problem, and then update their marginal utilities using a sub-gradient method. Intuitively, by solving the group of optimization problems with a linear objective of \(\max \sum_{k=1}^{K} u_k^m \), where \(u_k^m\) is the marginal utility of BE user \(k\), subject to the same constraints as those of the programming P6, we can trace out the entire boundary of the data rate region.

**Algorithm 3: Utility-based Water-filling Algorithm for BE users**

1. In the first phase, while \(\sum_{k=1}^{K} U_k^{(1)}(r_k^*) \geq \varepsilon\)
   1) Obtain the new power allocation from the linearization problem \(\max \sum_{k=1}^{K} u_k^m \) and the corresponding data rates
      \[
p_{k,n,0}^{(1)^j} = c_{n,k,0}^{(1)^j} \left[ \frac{U_k^{(1)}(r_k^*)}{2 \lambda} - \frac{1}{\eta_{n,k,0}} \right]^+, \tag{43}
\]
   2) Update \(u_k^m\) with a positive step size \(\mu \in (0, 1)\)
      \[
u_k^{(1)^j} \leftarrow (1 - \mu) v_k^{(1)^j} + \mu U_k^{(1)}(r_k^*), \tag{44}
\]
      end-while

2. Obtain the optimal power allocation \(p_{k,n,0}^{(2)^j}\) and the data rate \(r_k^{(2)^j}\). According to Eq.(10), we have
   \[
   \frac{1}{2} R_k^{(2)^j} \leftarrow r_k^{(2)^j}. \tag{45}
   \]

3. In the second phase, while \(\sum_{k=1}^{K} U_k^{(1)}(r_k^*) \geq \varepsilon\)
   1) Obtain the new power allocation from the linearization problem \(\max \sum_{k=1}^{K} u_k^m \) and the corresponding data rates
      \[
p_{k,n,0}^{(2)^j} = c_{n,k,0}^{(2)^j} \left[ \frac{U_k^{(2)}(r_k^*)}{2 \mu} - \frac{1}{\eta_{n,k,0}} \right]^+. \tag{46}
\]
      end-while

4. Repeat steps 2 and 3 until the system utility reaches a predefined threshold.
In order to verify the effectiveness of the CoRA scheme, we compare the performance of the CoRA scheme with that of other two schemes, i.e., 1) S-CoRA (Simple Cooperative Resource Allocation) scheme, in which RS use subchannel matching method to forward data and bit reallocation is not applied; 2) NCRA (Non-Cooperative Resource Allocation) scheme, in which there is no cooperative transmission, and joint subchannel and power allocation is performed at only BS.

Table 1 Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of subchannel (N)</td>
<td>128</td>
</tr>
<tr>
<td>Number of RS (M)</td>
<td>6</td>
</tr>
<tr>
<td>Path loss exponent for BS-RS links</td>
<td>2</td>
</tr>
<tr>
<td>Path loss exponent for BS-MS and RS-MS links</td>
<td>4</td>
</tr>
<tr>
<td>Shadowing standard deviation</td>
<td>7 dB</td>
</tr>
<tr>
<td>Total transmission power of BS (E₀)</td>
<td>8 w</td>
</tr>
<tr>
<td>Noise power</td>
<td>-105 dB</td>
</tr>
</tbody>
</table>

In the first experiment, let there be $K_0 = 40$ BE users, while the number of QoS users $K_Q$ is variable, where $K_Q \in \{12, 16, 20, 24, 28, 32, 36\}$. In addition, the power of each BS is $E_i = 4$ w. The sum utility of BE users versus the number of QoS users is illustrated in Fig.5. It is obvious that the schemes with cooperative transmission substantially outperform the scheme without cooperative transmission. In particular, the sum utility of our proposed CoRA scheme is higher than that of the other two schemes. Moreover, the achieved performance gain increases with the number of QoS users. When the number of QoS users grows up to 36, the proposed CoRA scheme achieves approximately 5.41% utility gain than S-CoRA scheme, and up to 41.09% utility gain than NCRA scheme.

Fig.6 illustrates the sum throughput of BE users. It can be seen that the sum throughput of BE users decreases with the number of QoS users in all schemes. However, the proposed CoRA scheme achieves the highest sum throughput of BE users among the three schemes, which demonstrates the spectrum efficiency of the proposed CoRA scheme outperforms that of the other two schemes.

The sum power of BS consumed by QoS users in the three schemes is illustrated in Fig.7. It is clearly that the sum power of BS consumed by QoS users in the three schemes all increase with the number of QoS users. However, compared with the other two schemes, the proposed CoRA scheme has allocated the least power of BS to QoS users, hence it is the most efficient in power saving. In addition, with the gradual increase of the number of QoS users, the difference of sum power consumption between the schemes with cooperative relaying
and those without cooperative relaying becomes more significant accordingly.

Fig. 5 Sum utility of BE users vs. the number of QoS users

Fig. 6 Sum throughput of BE users vs. the number of QoS users

Fig. 7 Sum power of BS dedicated to QoS users vs. the number of QoS users

In the second experiment, we examine the rejection probability of BE users with and without cooperation between BS and RS. Table 2 shows eleven cases used in the simulation experiment. Each case has different numbers of QoS users and BE users. Fig. 8 illustrates the rejection percentage of BE users of the proposed CoRA scheme and NCRA schemes. It can be found that the cooperation technique decreases the rejection probability effectively. This significant improvement comes from the joint resource scheduling at both BS and RS. The proposed CoRA scheme decreases the power required to satisfy all QoS users’ data rate demands, so that more power is saved for BE users, and thus more BE users can be accommodated than the schemes without cooperation. It can be concluded that cooperation with resource scheduling at both BS and RS enhances the coverage of the networks.

In the third experiment, we study the effect of the maximum transmission power of each RS on the system utility. For $K_Q = 25$ and $K_B = 45$, Fig. 9 illustrates the system utility versus $E_1/E_0$, the ratio between the maximum transmission power of an RS and that of BS. As shown in Fig. 9, the proposed CoRA scheme always outperforms the other two schemes. With the increase of $E_1/E_0$, the achieved system utility of the schemes with cooperative transmission increases. While since the relay mechanism is not applied in the NCRA scheme, the system utility remains roughly stable except a small disturb due to the fading effect of the wireless channel. When the transmission power of relay, $E_1$, is small, the bit reallocation at RSs allows the system utility to increases remarkably due to the diversity of the wireless channel. However, as $E_1$ is increasing, the increased power compensates the lost space/user diversity gain of the S-CoRA algorithm so that the difference between the system utility of the CoRA scheme and that of the S-CoRA scheme is decreasing. Overall, the utility obtained by the CoRA scheme is always larger than that of the S-CoRA scheme.

| Table 2 Simulation parameters |
|-----------------------------|----------------|----------------|----------------|----------------|----------------|
| Case1 | Case2 | Case3 | Case4 | Case5 | Case6 |
| $K_Q$ | 20 | 25 | 30 | 30 | 30 |
| $K_B$ | 30 | 30 | 30 | 35 | 40 |
| Case7 | Case8 | Case9 | Case10 | Case11 |
| $K_Q$ | 30 | 30 | 30 | 35 |
| $K_B$ | 50 | 55 | 60 | 30 |

Fig. 8 Percentage of Rejection for BE users with different load
6 Conclusion

In this paper, we have developed a QoS-aware resource allocation scheme for the users with different QoS requirements in relay-based downlink OFDMA system. The proposed CoRA scheme solves the relay-selection, subchannel-allocation and power-control problem, and aims at maximizing the system sum utility while satisfying the data rate requirements of QoS users. Due to the cooperative transmission between BS and RS, as well as the resource scheduling at both BS and RS, the proposed CoRA scheme can fully exploit the time, spatial, frequency and multiuser diversity of system, and thus achieving significant performance improvement in terms of power saving, user utilities, system throughput, and the number of admitted users. Consequently, the proposed CoRA scheme is an efficient resource allocation method for cooperative relay-based downlink OFDM system with heterogeneous services. In the future, it could be an interesting research topic to combine admission control and scheduling with the CoRA scheme perfectly to further enhance the overall system performance.

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References:


Appendix A Closed-form solutions of power control for BE users in CoRA

As mentioned in Section 4, the power control problem for BE users can be formulated as a convex programming P6, which is defined by Eq.(36)-Eq.(38). Therefore, the Lagrangian is

\[ L(p_{(1)}, p_{(2)}) = \sum_{k=K_{k-1}}^{K} U_k(R_k) \]

\[ + \lambda \left( P_o - P_{Q,k} - \sum_{k=K_{k-1}}^{K} \sum_{n=1}^{N} p_{n,k,0} \right) \]

\[ + \mu \left( P_{Q,0} - \sum_{k=K_{k-1}}^{K} \sum_{n=1}^{N} p_{n,k,0}^{(2)} \right) \]

\[ + \sum_{k=K_{k-1}}^{K} \sum_{n=1}^{N} \nu_{n,k} p_{n,k,0} \]

\[ + \sum_{k=K_{k-1}}^{K} \sum_{n=1}^{N} \nu_{n,k} p_{n,k,0}^{(2)} \]

where \( \lambda \), \( \mu \), \( \nu_{n,k} \) and \( \nu_{n,k} \) are Lagrange multipliers, and for each BE user \( k \in \{K_0 + 1, \ldots, K\} \), we have

\[ R_k = \frac{1}{2} \sum_{n=1}^{N} \left( p_{n,k,0} + p_{n,k,0}^{(2)} \right) \log_2 \left( 1 + \frac{p_{n,k,0} + p_{n,k,0}^{(2)}}{\eta_{n,k,0}} \right) \]

\[ + \frac{1}{2} \sum_{n=1}^{N} \left( p_{n,k,0} + p_{n,k,0}^{(2)} \right) \log_2 \left( 1 + \frac{p_{n,k,0} + p_{n,k,0}^{(2)}}{\eta_{n,k,0}} \right) . \]

Setting the derivative of \( L(p_{(1)}, p_{(2)}) \) with regard to \( p_{n,k,0} \) and \( p_{n,k,0}^{(2)} \), respectively, leads to the following equations

\[ \frac{U_k^*(R_k)}{2} \frac{\eta_{n,k,0}}{1 + \eta_{n,k,0}^{(2)}} - \lambda + \nu_{n,k} = 0 , \]

\[ \frac{U_k^*(R_k)}{2} \frac{\eta_{n,k,0}}{1 + \eta_{n,k,0}^{(2)}} - \mu + \nu_{n,k} = 0 . \]
The Karush-Kuhn-Tucker (KKT) conditions [31] impose that \( \nu_{n,k} \geq 0 \) and \( \nu_{n,k} \geq 0 \). Consequently,

\[
\lambda \geq \frac{U_s'(R_s')}{2} \frac{\eta_{n,k,0}}{1 + p_{n,k,0}^{(1)} \eta_{n,k,0}}, \tag{48}
\]

\[
\mu \geq \frac{U_s'(R_s')}{2} \frac{\eta_{n,k,0}}{1 + p_{n,k,0}^{(2)} \eta_{n,k,0}}. \tag{49}
\]

On the other hand, the KKT conditions \( \nu_{n,k} p_{n,k,0}^{(1)} = 0 \) and \( \nu_{n,k} p_{n,k,0}^{(2)} = 0 \) provide the following equations,

\[
p_{n,k,0}^{(1)} \left( \lambda - \frac{U_s'(R_s')}{2} \frac{\eta_{n,k,0}}{1 + p_{n,k,0}^{(1)} \eta_{n,k,0}} \right) = 0, \tag{50}
\]

\[
p_{n,k,0}^{(2)} \left( \mu - \frac{U_s'(R_s')}{2} \frac{\eta_{n,k,0}}{1 + p_{n,k,0}^{(2)} \eta_{n,k,0}} \right) = 0. \tag{51}
\]

If \( \lambda > \frac{U_s'(R_s')}{2} \frac{\eta_{n,k,0}}{1 + p_{n,k,0}^{(1)} \eta_{n,k,0}} \), condition Eq.(50) is only met if \( p_{n,k,0}^{(1)} = 0 \). Else, \( p_{n,k,0}^{(1)} = \frac{U_s'(R_s')}{2\lambda} \frac{1}{\eta_{n,k,0}} \). In the same way, if \( \mu > \frac{U_s'(R_s')}{2} \frac{\eta_{n,k,0}}{1 + p_{n,k,0}^{(2)} \eta_{n,k,0}} \), condition Eq.(51) is only met if \( p_{n,k,0}^{(2)} = 0 \). Else, \( p_{n,k,0}^{(2)} = \frac{U_s'(R_s')}{2\mu} \frac{1}{\eta_{n,k,0}} \).

As a result, the closed-form solutions of the power control problem for BE users are given by

\[
p_{n,k,0}^{(1)} = \left[ \frac{U_s'(R_s')}{2\lambda} \frac{1}{\eta_{n,k,0}} \right]^+ \tag{52}
\]

\[
p_{n,k,0}^{(2)} = \left[ \frac{U_s'(R_s')}{2\mu} \frac{1}{\eta_{n,k,0}} \right]^+. \tag{53}
\]

where \( [x]^+ = \max\{0,x\} \). The constants \( \lambda \) and \( \mu \) are chosen to meet the transmission power constraints Eq.(37) and Eq.(38), respectively.

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