Allocation Algorithm for Optimizing MC-CDMA Over Correlated Channel

Hassan EL GHAZI, Hamza DAHMOUNI and Nawal BOUNOUADER

Department of Signal and Communications,
Institut National des Postes et Télécommunications (INPT),
2. av Allal El Fassé, Madinat El Irfane, Rabat
MOROCCO.

elghazi@inpt.ac.ma , hamza.dahmouni@inpt.ac.ma , bounouader@inpt.ac.ma

Abstract: In this paper, we address the problem of MAI (Multiple Access Interference) degrading the performance of a downlink MC-CDMA system in a frequency selective fading channel. Our objective is to develop a code allocation strategy subject to use any code family and any equalizer. This allocation problem is formulated as a minimization problem of a cost function related to MAI. Firstly, we build the cost function from an approximation of the analytical expressions of the MAI power for MRC, EGC and MMSE equalization. Then we propose an iterative algorithm based on the min-max criterion to select the spreading code of the active users. Simulation results show the efficiency of our allocation strategy versus key parameters, in particular the correlation degree between the faded subcarriers.

Keywords: MC-CDMA, channel frequency correlation function, MAI minimization, MAI power approximation, spreading code allocation

1. Introduction

The MC-CDMA (Multi-Carrier Code Division Multiple Access) transmission [1]–[3] is a promising solution for the physical layer of future broadband wireless communication systems which will have to support multimedia services. By combining OFDM (Orthogonal Frequency Division Multiplexing) and CDMA, we obtain a high speed transmission capability in multipath environments and large multiple access capacity. Unlike CDMA, MC-CDMA performs the spreading operation in the frequency domain, mapping each chip of the user spreading code on one subcarrier, and thus introduces frequency diversity. However when the MC-CDMA signal propagates through a frequency selective fading channel, the code orthogonality is destroyed and the resulting MAI limits the system performance. Several approaches have been proposed to mitigate MAI and to improve signal detection. The conventional single-user detection techniques, applying per subcarrier equalization as in OFDM systems and then correlation with the code of the desired user, offer poor performance.

Indeed, by channel inversion, ZF (Zero Forcing) can eliminate MAI but in return-noise amplified on deeply faded subcarriers. The other techniques including MRC (Maximum Ratio Combining), EGC (Equal Gain Combining) and MMSE (Minimum Mean Square Error) can not restore the orthogonality of codes and lead to residual MAI. Therefore more advanced methods such as MUD (Multi-User Detection) [4]–[7] have been developed.

By exploiting the information from all users, they achieve better performance, but the main drawback is to strongly increase the receiver computational complexity, so the implementation in a mobile device is still a technical issue. Another approach for improving MC-CDMA performance without increasing complexity at the receiver side consists in selecting the set of spreading codes allocated to the active users, which has a great influence on the MAI level in a system transmitting through a realistic channel with correlated fading on subcarriers [8], [9]. The problem is then to define a code selection strategy with the following properties:

- possible use with any code family and with any equalization technique,

- low complexity in order to select the best codes with an acceptable computational cost,
- quick management of load variations which occur very frequently in a cellular system,
- the code allocation should not start each time from scratch.

The proposed solutions in the scientific literature for code selection do not possess the three properties simultaneously. Among the algorithms valid for any code family and equalization technique, the one proposed by Mottier and Castelain in [10] and then refined in [11] uses a indicator of the interference level, but performs an exhaustive search among all possible combinations of codes, so it has a very high computational cost.

To reduce the number of examined combinations examine, Mourad et *al.* [9] propose an iterative algorithm, but the power of the interference between users is calculated at each iteration and this calculation requires the knowledge of the channel FCF (Frequency Correlation Function) after equalization. The authors not provide any analytical expression, so an additional stage is needed to estimate the FCF, which introduces more complexity. Other techniques can easily and quickly select the best sets of spreading codes, but they are dedicated to specific MC-CDMA systems: Shi and Latva-Aho [12] use a property only observed for Walsh codes in the frequency domain and the work of Tsai et *al.* on MAI cancellation [13] concerns a system using Walsh codes and MRC equalization.

The topic of this paper is to propose a code allocation strategy meeting all requirements previously cited. To design an algorithm independently of the code family and of the equalization technique, the problem is formulated in a general way. It then consists in trying to find the optimal code sets that minimize a cost function related to the MAI affecting the active users of the MC-CDMA system, as in [9]–[11]. Since this optimization problem has no analytical solution and the exhaustive search suffers from a prohibitive computational cost, it is necessary to develop a faster and simpler solving technique.

To meet the low complexity requirement, we first build a cost function which can be evaluated with few calculations from an analysis of MAI in a downlink MC-CDMA system. The analytical expression of the MAI power is provided from the exact calculation of the channel FCF after MRC, EGC and MMSE equalization in the case of a typical exponential PDP (Power Delay Profile). The cost function is then derived from an approximation of the exact expression. Low complexity and handling of load fluctuations are strong arguments in favor of an iterative solving process. The proposed algorithm is based on the min-max criterion to guarantee that no communication will be affected by strong interference. We show that the algorithm can be used whatever the code family and the equalizer, and thanks to its reduced complexity and to its iterative nature, it can also efficiently manage load variations.

The remaining of the paper is organized as follows. Section II presents the MC-CDMA system model and the channel model. Section III focuses on the MAI power and the formulation of the code allocation problem as a minimization problem. The cost function is defined from an approximation of the MAI power in section IV. In section V, an iterative algorithm is proposed to select the optimal spreading code allocated to users and is compared to different existing algorithms. Results obtained for Walsh codes are presented in section VI. The last section draws some conclusions.

2. MC-CDMA model

We consider the downlink MC-CDMA system represented by Fig. 1. The system is synchronous, so we consider orthogonal code families including common binary codes: Walsh, Golay, orthogonal Gold and also complex CI (Carrier Interferometry) codes [14]. The channel is assumed to be slowly time varying and frequency selective over the total bandwidth. In multicarrier systems, it is essential that each subcarrier experiences a flat fading, so Δf must be chose shorter than the channel coherence bandwidth. This requirement has two consequences. For high data rates, serial-to-parallel conversion must be performed before spreading, as proposed in [15]. Moreover fadings affecting the different subcarriers are inherently correlated.

2.1 Transmitter model

Input data symbols are assumed to form an i.i.d. zero mean random process with unitary power. After serial-to-parallel conversion of P symbols, the p^{th} symbol of user x_p^u is spread with the user code

$c^{z_u} = \{c_k^{z_u}, k = 0,...,N_c - 1\}$ where $|c_k^{z_u}|^2 = 1$. The components of the spread sequence $x_p^u c_k^{z_u}$ are then transmitted in parallel on N_c sub-carriers selected among a total amount of $N_s = pN_c$ orthogonal subcarriers. To achieve frequency diversity, the gap between the N_c subcarriers conveying the same symbol x_p^u is maximized, it is then equal to $P\Delta f$ as in [16]. Consequently the chips $x_p^u c_k^{z_u}$ are sent on subcarriers $p+kP$ for $k = 0,...,N_c-1$. After the inverse FFT, the MC-CDMA symbol is extended with a CP (Cyclic Prefix) longer than the channel maximum excess delay. Considering all previous assumptions, the system is prevented from inter-symbol and inter-subcarrier interference. Afterwards, to simplify notations and without loss of generality, the index p is omitted. The subcarriers used to transmit the symbol denoted x^u are just identified by index k varying from 0 to N_c -1

2.2 Correlated channel model

We assume a WSSUS (wide sense stationnary uncorrelated scattering) Rayleigh fading channel modeled as a tapped delay line filter. The impulse response is constant over the MC-CDMA symbol duration T_s and is written as:

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l) \qquad (1)$$

Where L is the number of taps, h_l the complex gain and τ_l the delay of the l^{th} tap. Each gain h_l is a mutually uncorrelated complex Gaussian random process with zero mean and power σ_l^2. We consider an exponential PDP (Power Delay Profile) characterized by an RMS (Root Mean Square) delay spread σ_τ ,because it is the most commonly accepted PDP model for indoor channels in accordance with theory and experimental data [17],[18]. The delays are multiples of the sampling time $\tau_l = lT$, the maximum excess delay is then equal to $(L - 1)T$ which is assumed large enough compared to σ_τ , and σ_l^2 is related to the average value of the PDP within the interval $[LT, (L+1)T[$.

The FCF of the channel model is obtained by Fourier transform of the PDP. In the MC-CDMA system, the sampling time is $T=1/(Ns\Delta f)$ and the frequency spacing between two subcarriers k and $k+n$ transmitting the same symbol is equal to $nP\Delta f = n(Nc/Ns)\Delta f$. If we denote H_k the complex coefficient corresponding to the channel frequency response on subcarrier k, we get the FCF $R_H(n)$ between H_k and H_{k+n} defined as:

$$R_H(n) = E\left[H_k H_{k+n}^*\right] = \frac{1 - e^{-\frac{1}{N_s \Delta f \sigma_\tau}}}{1 - e^{-\left(\frac{1}{N_s \Delta f \sigma_\tau} + j2\pi \frac{n}{N_c}\right)}} \qquad (2)$$

2.3 Receiver model

Compared to the transmitter, the MC-CDMA receiver applies inverse operations. After CP removal and OFDM demodulation by FFT, the signal obtained on the k^{th} subcarrier is given by:

$$y_k = H_k \sum_{v=0}^{N_u - 1} x^v c_k^{z_v} + \eta_k \qquad (3)$$

Where η_k is the AWGN (additive white Gaussian noise) with power $N_u \sigma^2$ at the FFT output. Next single-user detection is performed to decode the signal of user u. After subcarrier by subcarrier equalization and dispreading with the user code, the decision variable y^u can be expressed as the summation of three terms which respectively correspond to the useful signal, the MAI and the additive Gaussian noise:

$$y^n = x^n \frac{1}{N_c} \sum_{k=0}^{N_c - 1} H_k g_k + x_{MAI}^{z_u} + \frac{1}{N_c} \sum_{k=0}^{N_c - 1} c_k^{z_u *} g_k \eta_k \qquad (4)$$

Where g_k is the equalization coefficient on the k^{th} subcarrier. The MAI term results from the mutual interference $x_{MAI}^{z_u z_v}$ between the detected user u and each interfering user v due to code orthogonality alteration by the channel and the equalizer:

$$x_{MAI}^{z_u} = \sum_{\substack{v=0 \\ v \neq u}}^{N_u - 1} x_{MAI}^{z_u z_v} = \sum_{\substack{v=0 \\ v \neq u}}^{N_u - 1} x^v \frac{1}{N_c} \sum_{k=0}^{N_c - 1} c_k^{z_u *} c_k^{z_v} H_k g_k \qquad (5)$$

Several equalization methods corresponding to different diversity combining strategies have been proposed in literature [15]. By channel inversion, ZF restores orthogonality and totally eliminates MAI when orthogonal codes are used. But the performance is degraded by noise amplification on subcarriers affected by deep fading. In the following we are interested in MAI minimization to improve the MC-CDMA system performance, so only MRC, EGC and MMSE equalizers are investigated.

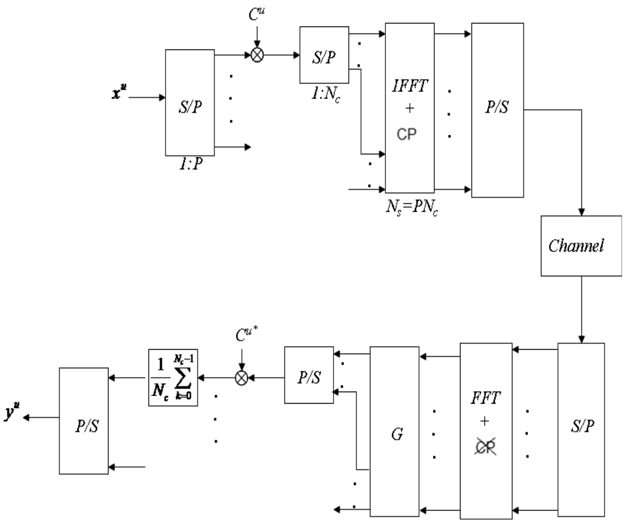

Fig. 1. Block diagram of the MC-CDMA system for user u

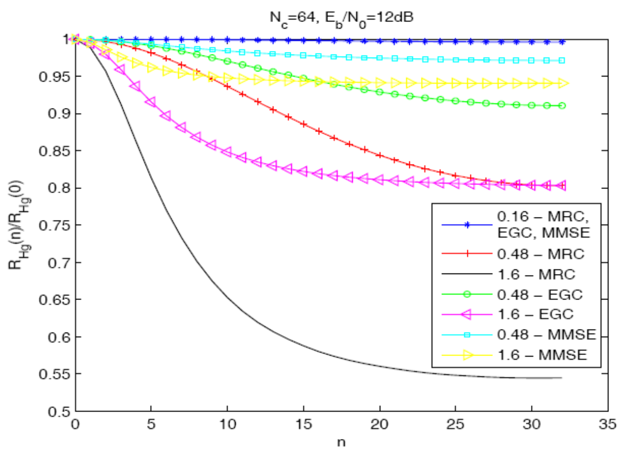

Fig. 2. FCF of the equalized channel for MRC, EGC and MMSE equalization and for several values of $Ns\Delta f\sigma_\tau$.

3. MAI power and problem formulation

In this section, we first derive the analytical expression of the MAI power from the channel FCF after MRC, EGC and MMSE equalization in the case of a typical exponential PDP. Then the code selection problem is expressed in a very general way with the aim of designing an algorithm independent of the code family and of the equalizer.

3.1 MAI power

From (5) and statistical properties of the input symbols, the power of the MAI affecting user u is written as:

$$P_{MAI}^{z_u} = \sum_{\substack{v=0 \\ v\neq u}}^{N_c-1} P_{MAI}^{z_u,z_v} \tag{6}$$

Where $P_{MAI}^{z_u z_v}$ is the power of the mutual interference between users u and v:

$$P_{MAI}^{z_u,z_v} = \frac{1}{N_c^{\,2}} \sum_{k=0}^{N_c-1} \sum_{k'=0}^{N_c-1} R_{H_g}(k-k') c_k^{z_u *} c_k^{z_v} c_{k'}^{z_u} c_{k'}^{z_v *} \tag{7}$$

With $R_{Hg}(n)$=E[$H_k g_k H_{k+n}{}^* g_{k+n}$] the channel FCF after equalization. Whatever the equalizer, the coefficient $H_k g_k$ affecting the k^{th} subcarrier can always be written as a function of $|H_k|$. Consequently $R_{Hg}(n)$ can be derived from the joint density of Rayleigh channel envelopes given by Jakes [17]. After mathematical formulations, we obtains the frequency channel expressions for respectively MRC, EGC and MMSE detection, $R_{Hg}(n)$ is related to the absolute square of the channel FCF before equalization. As $R_H(n)$ is also even and periodic with period N_c:

$$R_{Hg}(n)=R_{Hg}(-n)= R_{Hg}(N_c-n) \tag{8}$$

Figure 2 represents the normalized FCF $R_{Hg}(n)/R_H(0)$ for the different equalizers and for the following values of $Ns\Delta f\sigma_\tau$: 0.16, 0.48 and 1.6. For all equalizers, we can consider that $Ns\Delta f\sigma_\tau$ = 0.16 corresponds to highly correlated subcarriers, because the FCF is almost constant. For other values, the FCF decreases, when the subcarrier spacing n increases, from its maximum value at n=0 to get closer to a horizontal asymptote $y=R_{Hg}(N_c/2)$. The decay is all the more sharp since the product $Ns\Delta f\sigma_\tau$ is high. In MMSE case, it also depends on the signal to noise ratio:

$$\gamma = \frac{1}{\sigma^2} = \frac{2N_u}{N_c}\frac{E_b}{N_0}$$

The figure also shows that the range of values taken by the FCF depends on the equalization technique; it is larger with MRC and very tighter with MMSE. Using (8), the expression (7) can be rewritten for an even value of Nc as:

$$P_{MAI}^{z_u,z_v} = R_{Hg}(0) + \frac{1}{N_c} R_{Hg}(N_c/2)\Re[R_{\omega}^{z_u,z_v}(N_c/2)] + \frac{2}{N_c}\sum_{n=1}^{N_c/2-1} R_{Hg}(n)\Re[R_{\omega}^{z_u,z_v}(n)] \quad (11)$$

Where $R_{\omega}^{z_u,z_v}(n)$ is the PCF (periodic correlation function) of the chip by chip product $\omega_k^{z_u,z_v} = c_k^{z_u*} c_k^{z_v}$ of the spreading codes of users u and v defined by:

$$R_{\omega}^{z_u,z_v}(n) = \frac{1}{N_c}\sum_{k=0}^{N_c-1} \omega_k^{z_u,z_v}\omega_{k+n}^{z_u,z_v*} \quad (12)$$

Finally the MAI power obtained by substituting (11) in (6) depends on the following parameters: the code length N_c, the number of users N_u, the product $Ns\Delta f\sigma_\tau$, the ratio: E_b/N_0 for the MMSE technique, and the code product PCF governed by the spreading codes allocated to the active users. At every moment, the first parameters are fixed while there is a degree of freedom to select the N_u user codes among all available codes. Indeed, a CDMA based system almost never operates at full load because the capacity is first limited by MAI.

3 3.2 Problem formulation

In the most general way, the code allocation problem is a MAI minimization problem which can be expressed as follows. Consider a spreading code family with N codes of length N_c identified by a number ranging from 0 to $N-1$. In most cases (for Walsh, orthogonal Gold), $N=N_c$. For CI codes, N can take values up to $2N_c$. Let $S=\{0,1,\ldots,N-1\}$ be the set of all codes and $S_{Nu}=\{z_0,z_1,\ldots,z_{Nu-1}\}\subseteq S$ a subset of Nu codes that can be allocated to the Nu active users of the MC-CDMA system. We look for the optimal code subsets $S_{Nu}^{(opt)}$ that minimize a cost function f(S_{Nu}):

$$S_{Nu}^{(opt)} = \arg_{S_{Nu}} \min f(S_{Nu}) \quad (13)$$

where f(S_{Nu}) is related to the powers of the mutual interference between the different pairs of codes from S_{Nu}. Because the solution to the minimization problem is a priori not unique, we denote $\Omega_{Nu}^{(opt)}$ the set including all code subsets $S_{Nu}^{(opt)}$ leading to the minimum value. According to the expression of $P_{MAI}^{z_u,z_v}$ (11), the equation (11) cannot be analytically solved whatever the exact cost function. Furthermore, solving (13) by exhaustive search requires to evaluate the cost function for the C_{Nu}^{N} combinations of codes. When the code length increases, the search space becomes important that the computational cost is too prohibitive. Therefore a simpler and faster solving technique is needed.

4. Proposed function

To build the cost function, we propose an approximation of the mutual interference power derived from the analysis of the equalized channel FCF.

4.1 MAI power approximation

Based on theoretical formulation according to the equation [20], we show that The channel FCF $R_{Hg}(n)$ after MRC, EGC and MMSE equalization decreases from its maximum value at n=0 to get closer to a horizontal asymptote y= $[R_{Hg}(N_c/2)]$ when n increases. To simplify expression (11), we suppose that from a gap d between the subcarriers, the equalized channel coefficients $H_k g_k$ have a minimum correlation: $R_{Hg}(n) \approx R_{Hg}(N_c/2) \quad \forall \quad n \rangle d$. Then the power of the mutual interference between two users u et v is approximated by:

$$P_{MAI}^{z_u,z_v} = R_{Hg}(0) + \frac{1}{N_c} R_{Hg}(N_c/2)\Re[R_{w}^{z_u,z_v}(N_c/2) + \frac{2}{N_c}\sum_{n=1}^{d} R_{Hg}(n)\Re[R_{w}^{z_u,z_v}(n)] \quad (14)$$

$P_{MAI}^{z_u,z_v(d)}$ is the d^{th} order approximation of the mutual interference power between two users. As shown in Appendix C, the code product PCF verifies the following property for the different code families (Walsh, orthogonal Gold, Golay and CI):

$$\sum_{n=1}^{N_c-1} R_{w}^{z_u,z_v}(n) = -1 \quad (15)$$

Using this property as well as the Hermitian nature and the periodicity with period N_c of the PCF, the d^{th} order approximation of the mutual interference power (12) becomes:

$$P_{MAI}^{z_u,z_v(d)} = R_{Hg}(0) - R_{Hg}(N_c/2) + \frac{2}{N_c}\sum_{n=1}^{d} \Re[R_w^{z_u,z_v}(n)](R_{Hg}(n) - R_{Hg}(N_c/2)) \quad (16)$$

Whatever the equalization technique and the approximation order, the deviation from the exact value of the MAI power is expected to decrease when $Ns\Delta f\sigma_\tau$ increases. Indeed the equalized channel FCF $R_{Hg}(n)$ decreases more rapidly towards the asymptote and the proposed approximation is more accurate. We can precise that the deviation is similar with MRC and EGC techniques. With MMSE based detection, the deviation is smaller because of the lower range of values taken by the FCF between n = 0 and n= $N_c/2$, therefore the proposed approximation is more accurate whatever the value of $Ns\Delta f\sigma_\tau$.

4.2 Selection criterion

The d^{th} order approximation of the mutual interference power is now used to define the cost function f(S_{Nu}) Since in many applications, the minimum maximum criterion is known to be more appropriate than the minimum average criterion [19], [20], f(S_{Nu}) is defined as the maximum value of the d^{th} power approximation over all couples (z_u,z_v) from S_{Nu} × S_{Nu} . Among all possible code combinations S_{Nu}, we look for the subsets which satisfy:

$$S_{Nu}^{d} = arg_{S_{Nu}}\ min\ max_{(z_u,z_v)\in S_{Nu}\times S_{Nu} z_u\langle z_v} P_{MAI}^{z_u,z_v(d)} \quad (17)$$

By minimizing the maximum power, no communication will be affected by strong interference and hence, the overall MAI will be kept at a low level. As previously, the solution to (15) is not necessarily unique and we denote $\Omega_{Nu}^{(d)}$ the set including all code subsets $S_{Nu}^{(d)}$ leading to the minimum value.

Figure 3 represents the BER of the MC-CDMA system with MRC detection and QPSK modulation when using the subsets of Walsh codes with length N_c= 16 provided by an exhaustive search from approximation orders d ranging from 1 to 7. More precisely it shows the maximum, average and minimum values of the BER obtained over the different code subset $S_{Nu}^{(d)}$ from $\Omega_{Nu}^{(d)}$ versus $Ns\Delta f\sigma\tau$. For d = 7, which corresponds to the exact value of the power, only one curve is represented. Indeed the three values are identical because all subsets $S_{Nu}^{(7)} = S_{Nu}^{(opt)}$ lead to the minimum BER which can be achieved.

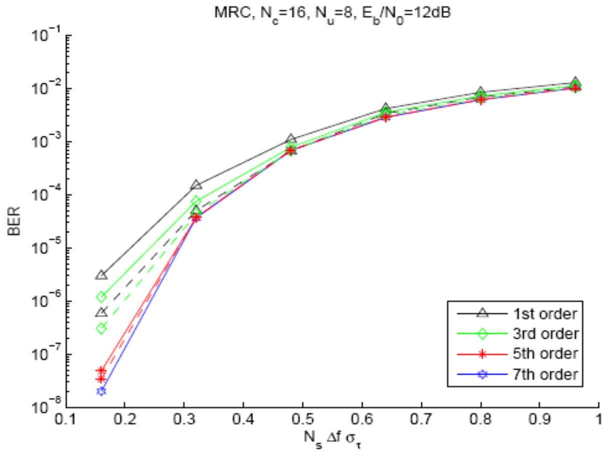

Fig. 3. BER versus $Ns\Delta f\sigma_\tau$ for several values of the approximation order d. Maximum BER is plotted with a solid line and average BER with a dashed line.

For d < 7, only the curves of average and maximum BER are plotted, since the minimum BER over the subsets $S_{Nu}^{(d)}$ is equal to the BER obtained with the optimal subset, which means that the following inclusion is always verified: $S_{Nu}^{(opt)} \subset \Omega_{Nu}^{(d)}$. Whatever the approximation order, when $Ns\Delta f\sigma\tau$ increases, the differences between the BER curves get reduced because the accuracy of the approximations improves. For this reason, we consider values of $Ns\Delta f\sigma_\tau$ below 1, beyond the differences are very small. The curves of average BER and maximum BER get closer to the minimum BER curve when the approximation order increases because the search for code subsets is refined. More precisely, from the 5th order, the BER is close to the minimum BER whatever the value of $Ns\Delta f\sigma_\tau$ 0.16. We can make the same observations with EGC detection, but in MMSE case, all BER curves almost merge because the different approximations are more accurate. Whatever the degree of correlation between subcarriers, the first order approximation is sufficient to get a BER close to the minimum BER.

5. Spreading code allocation algorithm

The cost function defined by approaching the mutual interference power can be evaluated with few calculations, but solving the minimization problem (17) by exhaustive search still has a prohibitive computational cost. For low complexity and to react quickly to load changes, we propose an iterative algorithm as in [9].

5.1 Proposed iterative algorithm

The principle is to start from an initial state, here a spreading code z_0 selected from the set S of the N available codes. Then the following process is repeated: at each iteration k, we compare, for each of the $N-k$ remaining codes, the values of the approximate power of the mutual interference with the k codes already chosen in previous iterations. We select the remaining code z_k which minimizes the maximum power over the first k codes. After Nu-1 iterations, we get a subset of N_u codes that represents a suboptimal solution. To improve the effectiveness of the method, the process is applied several times with different initial conditions. In total, N subsets of N_u codes $(S_{Nu}^{(d)})_{n=1\ldots N} = \{z_0 = n, \ldots z_{Nu-1}\}$ are built from the N available codes. The final solution is the best of the suboptimal solutions, i.e. the solution that meets the minimum maximum criterion. The code allocation algorithm, described in detail by table I, actually meets all requirements: independence of the code family and of the equalizer, low complexity and quick management of load variations.

More precisely, the prohibitive complexity of the exhaustive search is reduced to an acceptable quadratic complexity versus N and N_u. Assuming the: $N_u(N_u$-1)/2 values of the mutual power are known, the exhaustive search [10], [11] evaluates the cost function for the $\binom{N}{Nu}$ combinations of codes and at each evaluation, the maximum over all the power values is calculated. The complexity is then equal to $o((\binom{N}{Nu})N_u^2)$.

In the case of the iterative algorithm, the cost function is only evaluated on the N code subsets which are built. So the second stage of the algorithm presents a complexity equal to $o(NN_u^2)$.

First step: Construction of the N subsets of N_u codes

For $n = 0$ to $N-1$

The first code is initialized:

$z_0 = n$

For $k = 1$ to $N_u - 1$

The k^{th} code is chosen to minimize the maximum value of the power of the mutual interference between the selected and the remaining codes:

$$z_k = \underset{\substack{z_u \in S \\ z_u \neq z_0, \cdots, z_{k-1}}}{arg\,min} \; \underset{z_v = z_0, \cdots, z_{k-1}}{max} P_{MAI}^{z_u, z_v (d)}$$

End for

The subset built from the initial state $z_0 = n$ is stored:

$\left(S_{N_u}^{(d)}\right)_n = \{z_0, \cdots, z_{N_u-1}\}$

End For

Second step: Selection of the optimal code subset

The optimal code subset among the N available subsets is the one which minimizes the cost function:

$$S_{N_u}^{(d)} = \underset{\left(S_{N_u}^{(d)}\right)_n}{arg\,min} \; \underset{\substack{(z_u, z_v) \in \left(S_{N_u}^{(d)}\right)_n^2 \\ z_v < z_u}}{max} P_{MAI}^{z_u, z_v (d)}$$

Table I: CODE ALLOCATION ALGORITHM

It must be added the number of operations performed during the first step that builds the N subsets. Since for each of the N initial states, each iteration k consists in calculating a maximum over k values and a minimum over $N-k$ values, the complexity of this step is:

$$o(N\sum_{k=1}^{Nu-1} k(N-k)) = o(N^2 N_u^2) \qquad (18)$$

Finally, the total complexity of the iterative algorithm is equal to $o(NN_u^2)$. Compared to the algorithm proposed by [9], ours requires fewer operations to obtain the $N_u(N_u-1)/2$ power values, since it uses the approximated power instead of the exact power and the analytical equalized channel FCF instead of an estimated FCF. When using the exact expression of the mutual interference power (9), the calculation of each power value consists in the sum of (N_c/2+1) terms, after calculating the Nc/2 values of the code PCF $R_{\omega}^{zu,zv}(n)$ and the (Nc/2+ 1) values of the channel FCF $R_{Hg}(n)$. With the d^{th} order approximation (14), the sum is limited to (d + 1) terms and involves only d values for the PCF and (d + 2) values for the FCF. Furthermore, by using the analytical expressions of the channel FCF [20] and after respectively MRC, EGC and MMSE equalization, the estimation stage is eliminated and the algorithm complexity is reduced again.

Moreover the iterative nature of the algorithm makes easy and fast the reselection of codes when communications are starting or ending in the MC-CDMA system. Assume that the N subsets ($S_{Nu}^{(d)}$) n=1⋯N = {z_0 = n,…, z_{Nu-1}} built during the previous allocation of Nu codes are stored. Then the selection of a new subset of codes only requires updating these N subsets and selecting the best one in accordance with the minimum maximum criterion. For this, one iteration must be performed for every initial state z_0 in the first step of the allocation algorithm. The objective is to select the missing codes z_{Nu} when the load changes from N_u to N_u + 1 active users and to suppress the last allocated codes z_{Nu-1} for a transition from N_u to N_u – 1 users. In both cases, the second stage of the algorithm remains unchanged.

6. Results for the different equalizers with Walsh codes

In order to quantify the performance gain obtained through the proposed code allocation strategy for the different equalization techniques, the average BER of a MC-CDMA transmission has been obtained by simulation. Two different cases are considered: a system with allocation of the spreading codes by our algorithm, and a system with random allocation of the codes. Results are given versus the key parameters that influence performance including the load, the degree of correlation between the subcarriers and the signal to noise ratio. For simulation, the number of subcarriers is equal to 64 as in IEEE 802.11a/g standard, the subcarriers are QPSK modulated and the code length Nc is simply equal to Ns. In this section, we consider the family of Walsh codes.

5.2 Performance versus the system load

Figure 4 represents the BER of a MC-CDMA system versus the load for $N_s\Delta f\sigma_\tau$= 0.48, Eb/N0 = 12 dB and the different equalization techniques: EGC, MRC, MMSE. For each of them, the figure shows two different curves when codes are allocated by our algorithm. They correspond to two approximation orders, the first order and the 11th order which always leads to the minimum BER. The first order approximation of the interference power is accurate enough and also provides the minimum BER in MMSE case whatever the load and in MRC and EGC cases for a load above 16. Nevertheless for a lower load, it provides a BER smaller than 8×10^{-8} with MRC and than 4×10^{-7} with EGC, so it leads to a significant improvement of the performance. Whatever the equalizer, the performance gain provided by code allocation is very important for a loading rate smaller than 1/4, is less important but still significant for a rate between 1/4 and 1/2, and strongly decreases when the rate exceeds 1/2 because two effects combine: the MAI level increases and the degree of freedom for choosing the N_u codes among the N available codes decreases. More precisely, in a low loaded system, the MRC technique which is the most sensitive to MAI offers the most significant gain. For $N_u \le N_c/G$ = 16 (here the power of two G is equal to 4 since L = 4 for $Ns\Delta f\sigma_\tau$ = 0.48), the performance remains constant equal to 2×10^{-8} when using the code sets allocated by the algorithm with the 11th order approximation. This BER value corresponds to the optimal performance obtained for a single-user transmission, because the MAI is completely canceled. For comparison, for N_u = 16, the BER obtained with randomly selected codes is equal to 2.4×10^{-3}. However, once the limit of N_c/G = 16 users is exceeded, the performance quickly degrades. Even with code allocation, the MRC technique remains very sensitive to MAI. The performance gain is lower with the MMSE technique, which gives the best performance with random code allocation. For N_u =16, the BER is equal to 3.8×10^{-6} with the allocation and to 3.7×10^{-5} with random allocation.

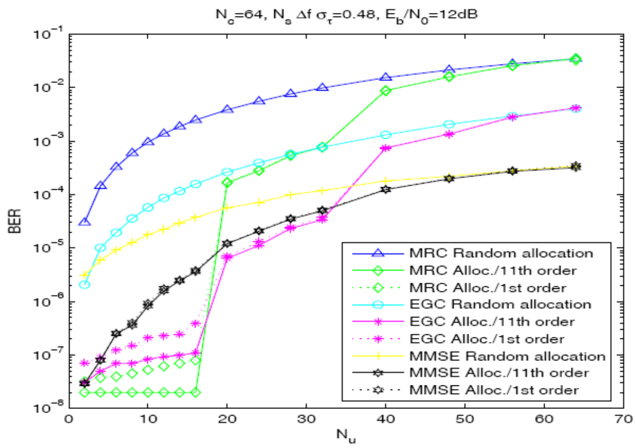

Fig. 4 BER versus the number of users N_u for the different detection techniques

Now in absolute, we can notice that the best performance is obtained when the allocation algorithm is coupled with MRC for a small load less than N_c/G, with EGC for a load between N_c/G and N_c/2, then with MMSE for a higher load.

6.1 Performance versus the subcarrier correlation degree

Figure 5 shows the BER of a quarter loaded MC-CDMA system versus $Ns\Delta f\sigma_\tau$ which characterizes the degree of correlation between subcarriers, for Eb/N0=12 dB and for the different equalization techniques. In accordance with Figure 4, our code allocation algorithm uses the first order approximation in MMSE case and the 11th order approximation with MRC and EGC to get the minimum BER.

First we can notice that whatever the value of $Ns\Delta f\sigma\tau$, the performance always benefits from using the code allocation algorithm. But the gain is more important for low values of $Ns\Delta f\sigma\tau$. For example, for $Ns\Delta f\sigma_\tau$= 0.48, the average BER is respectively equal to 2.4×10^{-3}, 1.5×10^{-4} and 3.8×10^{-5} with random allocation in MRC, EGC and MMSE cases while it drops to 2×10^{-8}, 10^{-7} and 4×10^{-6} with the proposed allocation. As previously, the most significant gain is obtained with the MRC technique because the MAI is canceled when the MC-CDMA system uses the codes provided by our algorithm for $Nu \leq Nc/G$, i.e. $G \leq Nc/Nu = 4$ (in simulations, $L \leq 4$ for $Ns\Delta f\sigma_\tau \leq 0.48$) and the performance gain is much lower with the MMSE technique.

When $Ns\Delta f\sigma_\tau$ increases, i.e. when the correlation degree between subcarriers decreases, the performance gain decreases for all detection techniques, since in the extreme case of uncorrelated subcarriers, the BER performance is known to no longer depend on the codes assigned to the active users [8]. Finally, the best performance is obtained when the proposed code allocation algorithm is combined with MRC for low values of $Ns\Delta f\sigma\tau$ ($\leq$ 0.5), with EGC for values of $Ns\Delta f\sigma\tau$ between 0.5 and 1.7, and with the MMSE technique for a lower correlation between the subcarriers: $Ns\Delta f\sigma\tau \geq 1.7$.

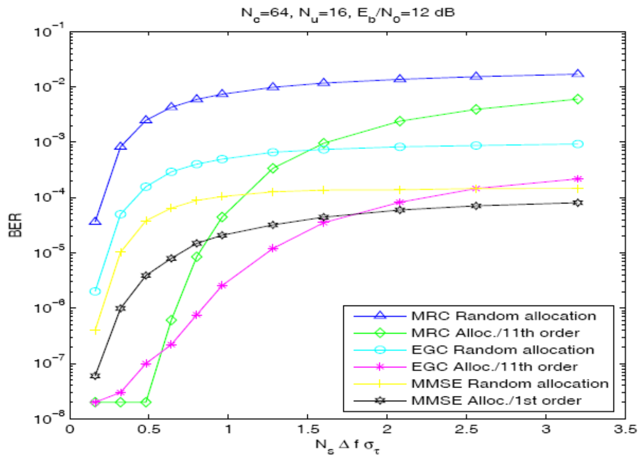

Fig. 5. BER versus $Ns\Delta f\sigma_\tau$ for the different equalization techniques

7. Conclusion

In this paper, we propose a spreading code allocation strategy for a downlink MC-CDMA system transmitting through a realistic correlated Rayleigh fading channel. The allocation problem is formulated as a minimization problem of a cost function related to MAI. The cost function is based on an approximation of the exact expressions of the MAI power for MRC, EGC and MMSE equalization in the case of a typical exponential PDP. The allocation algorithm is iterative and uses the minimum maximum criterion to select the spreading codes of the active users. Via simulation results, we provide a complete analysis of the allocation efficiency versus the key parameters including the loading rate, the degree of correlation between the faded subcarriers, the approximation order, the equalization technique and the family of codes. It shows that the proposed code allocation algorithm always improves the performance of a MC-CDMA system without additional complexity at the receiver. Using the suitable approximation order, the algorithm is able to provide the optimal code sets and consequently leads to the minimum BER.

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Hassan El Ghazi received a PhD degree in electrical engineering from Telecom Lille - UVHC, France in 2008. He obtained a M. Sc in Wireless communications from UVHC in 2004. In 2007-2009, he worked as research engineer at TeSa. His research interests are related to wireless communications, OFDM and traffic engineering.

Hamza Dahmouni received a PhD degree in Networks and Computer Science from Telecom Bretagne - ENST, France in 2007. He obtained a M. Sc in Networks from Paris-IV University in 2003, and a M. Sc in Wireless network design from Telecom SudParis - INT in 2002. In 2004-2007, he worked as research engineer at France Telecom. His research interests are related to traffic engineering and performance evaluation in heterogeneous networks.

Nawal Bounouader PHD in progress in traffic engineering from Mohammed V University, she obtained an engineering diploma of telecommunication from INPT in 2007. In 2007-2009 she worked as a project manager in Meditelecom, here research interests are related to KPI evaluation of networks and wireless system (MIMO and OFDM).