SC and SSC diversity reception over correlated Nakagami-\(m\) fading channels in the presence of CCI

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Abstract: - Performance analysis of switched-and-stay combining (SSC) and selection combining (SC) diversity receivers operating over correlated Nakagami-\(m\) fading channels in the presence correlated Nakagami-\(m\) distributed co-channel interference (CCI) is presented. Novel infinite series expressions are derived for the output signal to interference ratio's (SIR’s) probability density function (PDF) and cumulative distribution function (CDF). Capitalizing on them standard performance measures criterion like outage probability (OP) and average bit error probability (ABEP) for modulation schemes such as noncoherent frequency-shift keying (NCFSK) and binary differentially phase-shift keying (BDPSK) are efficiently evaluated. In order to point out the effects of fading severity and the level of correlation on the system performances, numerically obtained results, are graphically presented and analyzed.

Key-Words: - Switched-and-Stay Combining; Selection Combining; Nakagami-\(m\) fading channels; Co-Channel Interference; Outage Probability; Average Bit-Error Probability.

1 Introduction
In wireless communication systems various techniques for reducing fading effects and influence of the cochannel interference are used [1]. Space diversity reception, based on using multiple antennas is widely considered as a very efficient technique for mitigating fading and cochannel interference (CCI) effects. Increasing channel capacity and upgrading transmission reliability without increasing transmission power and bandwidth is the main goal of these techniques. Depending on complexity restriction put on the communication system and amount of channel state information available at the receiver, there are several principal types of combining techniques and division, that can be generally performed. Combining techniques like equal gain combining (EGC) and maximal ratio combining (MRC) require all or some of the amount of the channel state information of received signal. MRC and EGC combining techniques require separate receiver chain for each branch of the diversity system, which increase its complexity. Switch and stay combining (SSC) and selection combining (SC) are the least complex diversity techniques, and can be used in conjunction with various modulation schemes.

In general case, with SSC diversity applied, the receiver selects a particular branch until its signal-to-noise ratio (SNR) drops below a predetermined threshold. When threshold is achieved, the combiner switches to another branch and stays there regardless of whether the SNR of that branch is above or below the predetermined threshold. [1-3]. Simillary, selection combining, assuming that noise power is equally distributed over branches, selects the branch with the highest signal-to-noise ratio (SNR), that is the branch with the strongest signal [1-4].

In cellular systems, where the level of the cochannel interference is sufficiently high as compared to the thermal noise, SSC selects a particular branch until its signal-to-interference ratio (SIR) drops below a predetermined SIR ratio (SIR-based switched diversity). When defined ratio
is achieved, the combiner switches to another branch and stays there regardless of SIR of that branch. In cellular systems SC selects the branch with the highest signal-to-interference ratio (SIR-based selection diversity).

This type of diversity can be measured in real time both in base stations and in mobile stations using specific SIR estimators as well as those for both analog and digital wireless systems (e.g., GSM, IS-54) [5, 6].

The fading among the channels is correlated due to insufficient antenna spacing, which is a real scenario in practical diversity systems, resulting in a degradation of the diversity gain [7]. Therefore, it is important to understand how the correlation between received signals affects the system performance. Several correlation models have been proposed and used in the literature for evaluating performance of diversity systems. The constant correlation model corresponds to a scenario with closely placed diversity antennas and circular symmetric antenna arrays [8].

There are many distributions that well describe statistics of a mobile radio signal in the mobile radio environments. It has been found experimentally, that while the Rayleigh and Rice distributions can be indeed used to model the envelope of fading channels in many cases of interest, the Nakagami-$m$ distribution offers a better fit for a wider range of fading conditions in wireless communications.

An approach to the performance analysis of SSC diversity receiver operating over correlated Ricean fading satellite channels can be found in [9, 10]. Analysis of the SSC diversity receiver operating over correlated Weibull and $\alpha$-$\mu$ fading channels in terms of outage probability (OP), average bit error probability (ABER), can be found in [11, 12]. Dual-branch SSC diversity receiver with switching decision based on SIR, operating over correlated Ricean fading channels in the presence of correlated Nakagami $m$ distributed CCI, is presented in [13]. In papers [14-16] selection diversity over Weibull and $\alpha$-$\mu$ fading channels has been analysed. In recent works [17, 18] the joint PDF and CDF of the multivariate Nakagami-$m$ and Rayleigh distributions, respectively, are developed for the case of exponential correlation. In [19] SIR based SC combining over Nakagami-$m$ fading channels in the presence of CCI was presented, but only for the dual-branch case. Moreover to the best author's knowledge, no specific analytical study of SSC and multibranches SC involving assumed constant correlated model of Nakagami-$m$ fading for both desired signal and co-channel interference, has been reported in the literature.

In this paper, an approach to the performance analysis of proposed SSC and SC diversity receivers over constant correlated Nakagami-$m$ fading channels, in the presence of correlated CCI, will be presented. Novel infinite series expressions for probability density function (PDF) and cumulative distribution function (CDF) of the output SIR for SC and SSC diversity will be derived. Numerical results for important performance measures, such as OP and ABER for modulation schemes such as noncoherent frequency-shift keying (NCFSK) and binary differentially phase-shift keying (BDPSK) will be shown graphically for different system parameters in order to point out the effects of fading severity and the level of correlation on the system performances.

2 SSC diversity receiver statistics

Nakagami fading ($m$-distribution) describes multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves [2]. It provides good fits to collected data in indoor and outdoor mobile-radio environments and is used in many wireless communications applications. The desired signal received by the $i$-th antenna can be written as [20]:

$$D_i(t) = R_i e^{j \phi_i(t)} e^{j [2 \pi f_c t + \Phi(t)]},$$  \hspace{1cm} (1)

where $f_c$ is carrier frequency, $\Phi(t)$ desired information signal, $\phi_i(t)$ the random phase uniformly distributed in $[0, 2\pi]$, and $R_i(t)$, a Nakagami-$m$ distributed random amplitude process given by [2):

$$f_{R_i} (t) = \frac{2 t^{2m-1}}{\Gamma(m) \Omega^m} \exp \left( -\frac{t^2}{\Omega} \right), \quad t \geq 0  \hspace{1cm} (2)$$

where $\Gamma(*)$ is the Gamma function, $\Omega = E(t^2)$, with $E$ being the mathematical expectation operator, and $m$ is the inverse normalized variance of $t^2$, which must satisfy $m \geq 1/2$, describing the fading severity. The resultant interfering signal received by the $i$-th antenna is:

$$C_i(t) = r_i(t) e^{j \theta_i(t)} e^{j [2 \pi f_c t + \psi(t)]},$$ \hspace{1cm} (3)

where $r_i(t)$ is also Nakagami-$m$ distributed random amplitude process, $\theta_i(t)$ is the random phase, and $\psi(t)$ is the information signal. This model refers to the case of a single co-channel interferer.
Now joint distributions of pdf for both desired and interfering signal correlated envelopes for dual-branch signal combiner could be expressed by [21]:

\[
p_{h, x, (R_1, R_2)} = \left( \frac{1}{\sqrt{\pi \rho_d}} \right)^{m_d} \frac{1}{\Gamma(m_d)} \sum_{k=0}^{\infty} \frac{\Gamma(m_d + k_1 + k_2) (m_d - k_1 - k_2)^{m_d - k_1 - k_2}}{(1 - \rho_d)^{m_d - k_1 - k_2}} \times \left( \frac{1 + \sqrt{\rho_d}}{1 - \sqrt{\rho_d}} \right)^{m_d - k_1 - k_2},
\]

respectively. Like it was explained SSC selects a particular branch until its SIR drops below a predetermined SIR ratio and when defined ratio is achieved, the combiner switches to another branch and stays there regardless of SIR of that branch. SSC diversity reception model was presented at Fig. 1.

The joint PDF of \( z_1 \) and \( z_2 \) can be expressed by [22]:

\[
f_{n, z, (z_1, z_2)} = \frac{1}{4 \sqrt{z_1 z_2}} \int_0^{2 \pi} f_{h, z, (r_1, r_2)}(r_1, r_2) r_1 r_2 dr_1 dr_2.
\]

with \( S_k = \frac{\Omega_d}{\Omega_k} \) being the average SIR’s at the \( k \)-th input branch of the dual-branch combiner and :

\[
G_i = \left( \frac{1}{\sqrt{\rho_d}} \right)^m \left( \frac{1}{\sqrt{\rho_i}} \right)^m \frac{\Gamma(m_i + k_i + k_l) \Gamma(m + k_l)}{\Gamma(m_i) \Gamma(m)} \frac{1}{\sqrt{\rho_d}} \frac{1}{\sqrt{\rho_i}} \times \frac{\Gamma(m_i + m_j + k_i + k_j + k_l) \Gamma(m + m_i + k_i + k_j + k_l)}{\Gamma(m_i + k_i + k_j + k_l) \Gamma(m + m_i + k_i + k_j + k_l)} \frac{1}{\sqrt{\rho_d}} \frac{1}{\sqrt{\rho_i}} \times \frac{\Gamma(m_i + m_j + k_i + k_j + k_l)}{\Gamma(m_i + k_i + k_j + k_l) \Gamma(m + m_i + k_i + k_j + k_l) \Gamma(m + k_i + k_j + k_l) \Gamma(m + k_i)}.
\]

Let \( z_{ssc} \) represent the instantaneous SIR at the SSC output, and \( z_r \) the predetermined switching threshold for the both input branches. As it was previously explained SSC selects a particular branch \( z_r \) until its SIR drops below a \( z_{ssc} \). When this happens, the combiner switches to another branch. Following [23], the PDF of \( z_{ssc} \) is given by

\[
f_{z_{ssc}}(z) = \begin{cases} v_{ssc}(z) & z \leq z_r, \\ v_{ssc}(z) + f_{z_r}(z) & z > z_r. \end{cases}
\]

where \( v_{ssc}(z) \), according to [23], can be expressed as

\[
v_{ssc}(z) = \int_{z_r}^{z} f_{z_r}(z, z) dz,
\]

\( f_{z_r}(z, z) \) being the interfering signal correlated envelopes for dual-branch signal combiner.
Moreover, \( v_{ssc}(z) \) can be expressed as infinite series [24]:

\[
v_{ssc}(z) = \sum_{k_1,k_2=0}^{\infty} \sum_{l_1,l_2=0}^{\infty} G_{i_1} m_d m_{a_k} m_{l_i} m_{l_j} \cdot (1-\sqrt{\rho_d})^{m_{a_k}+k_1} \cdot (1-\sqrt{\rho_c})^{m_{l_i}+l_1} \times \frac{z^{m_{a_k}+k_1}}{(m_d (1-\sqrt{\rho_d}) z + m_{a_k} (1-\sqrt{\rho_d}) S_1)^{m_{a_k}+m_{a_k}+k_1+l_1}}
\]

\[
\left( \frac{m_d}{m_d (1-\sqrt{\rho_d}) z + m_{a_k} (1-\sqrt{\rho_d}) S_1} \right)^{m_{a_k}+m_{a_k}+k_1+l_1}
\cdot B
\left( \frac{m_d z}{m_d (1-\sqrt{\rho_d}) z + m_{a_k} (1-\sqrt{\rho_d}) S_1}, m_d, m_{a_k} \right)
\]

\[
\times B
\left( \frac{m_d z}{m_d (1-\sqrt{\rho_c}) S_2}, m_d + k_2, m_{a_k} + l_2 \right)
\]

(11)

with \( B(z,a,b) \) denoting the incomplete Beta function [25]. In the same manner, the \( f_z(z) \) can be expressed as:

\[
f_z(z) = \frac{m_{a_k} m_{l_i} z^{m_{a_k}+m_{l_i}}}{(m_d S_1 + m_{a_k} S_1)^{m_{a_k}+m_{l_i}}} \cdot \Gamma(m_d + m_{a_k}) \cdot \Gamma(m_{a_k}) \cdot \Gamma(m_{l_i}) \cdot \Gamma(m_{l_i})
\]

(12)

Similar to (9), the CDF of the SSC output SIR, i.e. the \( F_{ssc}(z) \) is given by [23]:

\[
F_{ssc}(z) = \begin{cases} 
F_z(z) - F_{ssc}(z), & z \leq z_r, \\
F_z(z) + F_{ssc}(z), & z > z_r.
\end{cases}
\]

(13)

By substituting (7) and (12) in

\[
F_z(z) = \int_0^1 f_z(z) dz,
\]

(15)

\( F_{ssc}(z) \), and \( f_z(z) \) can be expressed as the following infinite series, respectively:

\[
F_{ssc}(z) = \sum_{k_1,k_2=0}^{\infty} \sum_{l_1,l_2=0}^{\infty} G_{i_1} \cdot B
\left( \frac{m_d z}{m_d z + m_{a_k} S_1}, m_d, m_{a_k} \right)
\times B
\left( \frac{m_d z}{m_d z + m_{a_k} S_1}, m_d + k_2, m_{a_k} + l_2 \right)
\]

\[
\times B
\left( \frac{m_d z}{m_d z + m_{a_k} S_1}, m_d + k_2, m_{a_k} + l_2 \right)
\]

(16)

The nested infinite sum in (15) converges for any value of the parameters \( \rho_d, \rho_c, m_d, m_{a_k} \), and \( S_r \). Let us assume that CDF series (15) is truncated with \( K_i, L_i \) in the variables \( k_i \) and \( l_i \), respectively.

Then the remaining terms comprise the truncation error, \( E_r \), which can be expressed using the approach given in [26] as:

\[
|E_r| < \sum_{k_i=0}^{K_i-1} \sum_{l_i=0}^{L_i-1} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \sum_{i_3=0}^{\infty} \sum_{i_4=0}^{\infty} \sum_{i_5=0}^{\infty} \sum_{i_6=0}^{\infty} \xi
\]

\[
+ \sum_{k_i=0}^{K_i-1} \sum_{l_i=0}^{L_i-1} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \sum_{i_3=0}^{\infty} \sum_{i_4=0}^{\infty} \sum_{i_5=0}^{\infty} \sum_{i_6=0}^{\infty} \xi
\]

(18)

where \( \xi \) is given by:

\[
\xi = G_i \times B
\left( \frac{m_d z}{m_d z + m_{a_k} S_1}, m_d + k_2, m_{a_k} + l_2 \right)
\times B
\left( \frac{m_d z}{m_d z + m_{a_k} S_1}, m_d + k_2, m_{a_k} + l_2 \right)
\]

(19)

Table 1. Terms need to be summed in (15) to achieve accuracy at the 6th significant digit.

<table>
<thead>
<tr>
<th>( S_r/z = 10 \text{dB} )</th>
<th>( S_r/z = 11 \text{dB} )</th>
<th>( m_d = 1 )</th>
<th>( m_d = 1.2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_d = 0.3 )</td>
<td>( \rho_d = 0.2 )</td>
<td>24</td>
<td>21</td>
</tr>
<tr>
<td>( \rho_d = 0.3 )</td>
<td>( \rho_d = 0.3 )</td>
<td>28</td>
<td>25</td>
</tr>
<tr>
<td>( \rho_d = 0.3 )</td>
<td>( \rho_d = 0.4 )</td>
<td>37</td>
<td>35</td>
</tr>
<tr>
<td>( \rho_d = 0.4 )</td>
<td>( \rho_d = 0.3 )</td>
<td>31</td>
<td>27</td>
</tr>
<tr>
<td>( \rho_d = 0.5 )</td>
<td>( \rho_d = 0.5 )</td>
<td>51</td>
<td>47</td>
</tr>
</tbody>
</table>
Further simplification of the previous expression could be performed by bounding and approximating the above series with the generalized hypergeometric functions by using approach presented in [18] at the expense of more mathematical rigor. In Table 1, the number of terms to be summed in order to achieve accuracy at the desired significant digit is depicted. The terms need to be summed to achieve a desired accuracy depend strongly on the correlation coefficients, $\rho_d$ and $\rho_c$. It is obvious that number of the terms increases as correlation coefficients increase. Also, when $\rho_c > \rho_d$, we need more terms for correct computation.

3 Performance analysis of SSC diversity reception

3.1 Outage probability
Since the outage probability (OP) is defined as probability that the instantaneous SIR of the system falls below a specified threshold value, it can be expressed in terms of the CDF of $z_{ssc}$, i.e. as:

$$ P_{out} = P_R(\xi < \gamma) = \int_0^\gamma \lambda(z) d\lambda = F_\xi(\gamma) \quad (20) $$

where $\gamma$ is the specified threshold value. Using (11) and (12) the $P_{out}$ performances results have been obtained. These results are presented in the Fig. 2, as the function of the normalized outage threshold (dB) for several values of $\rho_d$, $\rho_c$, $m_d$, $m_c$.

![Outage probability versus normalized outage threshold for the balanced dual-branch SSC diversity receiver and different values of $\rho_d$, $\rho_c$, $m_d$, $m_c$.](image)

Normalized outage threshold (dB) is defined as being the average SIR’s at the input branch of the balanced dual-branch switched-and-stay combiner, normalized by specified threshold value $\gamma$. Results show that as the signal and interference correlation coefficient $\rho_d$ and $\rho_c$ increase and normalized outage threshold decreases, OP increases.

3.2 Average bit error probability
The ABER ($\bar{P}_e$) can be evaluated by averaging the conditional symbol error probability for a given SIR, i.e. $P_e(z)$, over the PDF of $z_{ssc}$, i.e. $f_{ssc}(z)$ [19]:

$$ \bar{P}_e = \int_0^\infty P_e(z)f_{ssc}(z)dz \quad (21) $$

where $P_e(z)$ depends on applied modulation scheme. For a binary differentially coherent phase-shift keying (BDPSK) and no-coherent frequency shift keying (NCFSK) modulation schemes the conditional symbol error probability for a given SIR threshold can be expressed by $P_e(z) = 1/2 \exp(-\lambda z)$, where $\lambda$=1 for binary DPSK and $\lambda$=1/2 for NCFSK [27]. Hence, substituting (8) into (21) gives the following ABER expression for the considered dual-branch SSC receiver

$$ \bar{P}_e = \int_0^\infty P_e(z)f_{ssc}(z)dz + \int_{z_t}^\infty P_e(z)f_{ssc}(z)dz \quad (22) $$

Using the previously derived infinite series expressions, we present representative numerical performance evaluation results of the studied dual-branch SSC diversity receiver, such as ABER in case of two modulation schemes, DPSK and NCFSK.

Applying (22) on BDPSK and NCFSK modulation schemes, the ABER performance results have been obtained as a function of the average SIR’s at the input branches of the balanced dual-branch switched-and-stay combiner, i.e. $S_1=S_2=z_r$, for several values of $\rho_d$, $\rho_c$, $m_d$, $m_c$.

These results are plotted in Fig. 3. It’s shown that while as the signal and interference correlation coefficient, $\rho_d$ and $\rho_c$, increase and the average SIR’s at the at the input branches increases, the ABER increases at the same time. It is very interesting to observe that for lower values of $S_t$, due to the fact that the interference is comparable to desired signal ABER deteriorates more severe when the fading severity of the signal and interferers changes.
Finally, considering values from Fig. 3 better performance of BDPSK modulation scheme versus NCFSK modulation scheme are shown.

4 SC diversity receiver statistics

The performance of the multibranch SC can be carried out by considering, as in [28], the effect of only the strongest interferer, assuming that the remaining that is uncorrelated between antennas. Furthermore, $R_i(t)$, $r_i(t)$, $\Phi_i(t)$, and $\Theta_i(t)$ are assumed to be mutually independent and sufficiently high for the effect of thermal noise on system performance to be negligible (interference-limited environment) [19]. Now, due to insufficient antennae spacing, both desired and interfering signal envelopes experience the multivariate Nakagami-\(m\) fading with joint distributions. We are considering constant correlation Nakagami-\(m\) model of distribution. The constant correlation model [29] can be obtained from by setting $\Sigma_{ij} = 1$ for $i = j$ and $\Sigma_{ij} = \rho$ for $i \neq j$ in correlation matrix, for both desired signal and interference. Now joint distributions of pdf for both desired and interfering signal correlated envelopes for multi-branch signal combiner could be expressed by [12]:

$$p_{R_1,\ldots,R_n}(R_1,\ldots,R_n) = \frac{1}{\Gamma(m)} \sum_{k=0}^{\infty} \sum_{k_0=0}^{\infty} \ldots \sum_{k_n=0}^{\infty} \frac{2^k}{k! k_1! \ldots k_n!} \left( \frac{m_{n} - 1}{\Omega (n-1)} \right)^{m_{n} - 1} \left( \frac{m_1 - 1}{\Omega_1} \right)^{m_1 - 1} \ldots \left( \frac{m_n - 1}{\Omega_n} \right)^{m_n - 1} \frac{1}{\Gamma(m_{n} - 1)} \frac{1}{\Gamma(m_1 - 1)} \ldots \frac{1}{\Gamma(m_n - 1)}$$

$$\times \exp \left( - \frac{m_j R_j^2}{\Omega_j (n-1)} \right) \ldots \exp \left( - \frac{m_n R_n^2}{\Omega_n (n-1)} \right)$$

$$p_{r_1,\ldots,r_n}(r_1,\ldots,r_n) = \frac{1}{\Gamma(m)} \sum_{k=0}^{\infty} \sum_{k_0=0}^{\infty} \ldots \sum_{k_n=0}^{\infty} \frac{2^k}{k! k_1! \ldots k_n!} \left( \frac{m_{n} - 1}{\Omega (n-1)} \right)^{m_{n} - 1} \left( \frac{m_1 - 1}{\Omega_1} \right)^{m_1 - 1} \ldots \left( \frac{m_n - 1}{\Omega_n} \right)^{m_n - 1} \frac{1}{\Gamma(m_{n} - 1)} \frac{1}{\Gamma(m_1 - 1)} \ldots \frac{1}{\Gamma(m_n - 1)}$$

$$\times \exp \left( - \frac{m_j r_j^2}{\Omega_j (n-1)} \right) \ldots \exp \left( - \frac{m_n r_n^2}{\Omega_n (n-1)} \right)$$

Let $S_{\text{c}} = \Omega_{\text{c}} / \Omega_{\text{d}}$ be the average SIR’s at the k-th input branch of the multi-branch selection combiner. Joint probability density function of instantaneous values of SIR in n output branches $\lambda_k$, $k=1,\ldots,N$, as in [30],

$$\lambda = \lambda_{\text{c}} = \max(\lambda_1, \lambda_2, \ldots, \lambda_N).$$

Substituting (23) and (24) in (26), we obtain:

$$p_{R_1,\ldots,R_n}(R_1,\ldots,R_n) = \frac{1}{\Gamma(m)} \sum_{k=0}^{\infty} \sum_{k_0=0}^{\infty} \ldots \sum_{k_n=0}^{\infty} \frac{2^k}{k! k_1! \ldots k_n!} \left( \frac{m_{n} - 1}{\Omega (n-1)} \right)^{m_{n} - 1} \left( \frac{m_1 - 1}{\Omega_1} \right)^{m_1 - 1} \ldots \left( \frac{m_n - 1}{\Omega_n} \right)^{m_n - 1} \frac{1}{\Gamma(m_{n} - 1)} \frac{1}{\Gamma(m_1 - 1)} \ldots \frac{1}{\Gamma(m_n - 1)}$$

$$\times \exp \left( - \frac{m_j R_j^2}{\Omega_j (n-1)} \right) \ldots \exp \left( - \frac{m_n R_n^2}{\Omega_n (n-1)} \right)$$

$$p_{r_1,\ldots,r_n}(r_1,\ldots,r_n) = \frac{1}{\Gamma(m)} \sum_{k=0}^{\infty} \sum_{k_0=0}^{\infty} \ldots \sum_{k_n=0}^{\infty} \frac{2^k}{k! k_1! \ldots k_n!} \left( \frac{m_{n} - 1}{\Omega (n-1)} \right)^{m_{n} - 1} \left( \frac{m_1 - 1}{\Omega_1} \right)^{m_1 - 1} \ldots \left( \frac{m_n - 1}{\Omega_n} \right)^{m_n - 1} \frac{1}{\Gamma(m_{n} - 1)} \frac{1}{\Gamma(m_1 - 1)} \ldots \frac{1}{\Gamma(m_n - 1)}$$

$$\times \exp \left( - \frac{m_j r_j^2}{\Omega_j (n-1)} \right) \ldots \exp \left( - \frac{m_n r_n^2}{\Omega_n (n-1)} \right)$$

$$m_d$$ and $m_c$ are the fading severity parameters for the desired and interference signal, correspondingly. $\Omega_{\text{c}} = R_k^2$ and $\Omega_{\text{d}} = r_k^2$ are the average signal desired and interference powers at i-th branch, respectively.
with:

\[
G_i = \frac{\left(1 - \sqrt{\rho_d}\right)^{m_d} \left(1 - \sqrt{\rho_c}\right)^{m_c}}{\Gamma(m_d)\Gamma(m_c)} \\
\frac{\Gamma\left(m_d + k_i + \ldots + k_n\right)\Gamma\left(m_c + l_i + \ldots + l_n\right)}{\Gamma\left(m_d + k_i\right)\ldots\Gamma\left(m_d + k_n\right)\Gamma\left(m_c + l_i\right)\ldots\Gamma\left(m_c + l_n\right)} \\
\frac{\Gamma\left(m_d + m_c + k_i + l_i\right)\ldots\Gamma\left(m_d + m_c + k_n + l_n\right)}{\Gamma\left(m_d + l_i\right)\ldots\Gamma\left(m_c + l_i\right)k_i \ldots k_n l_i \ldots l_n!}
\]

\[
F(t) = \frac{1}{\left(1 + \left(n-1\right)\sqrt{\rho_c}\right)}
\]

\[\rho_d \leq \rho_c\]

(28)

For this case joint cumulative distribution function can be written as [19, 30]:

\[
F_{t_1, t_2, \ldots, t_n}(t_1, t_2, \ldots, t_n) = \prod_{i=1}^{n} p_{t_i}(x_i)\ dx_i
\]

(29)

Substituting expression (7) in (8), and after integration joint cumulative distribution function becomes:

\[
F_{t_1, t_2, \ldots, t_n}(t_1, t_2, \ldots, t_n) = \sum_{k_1=0}^{t_1} \sum_{k_2=0}^{t_2} \ldots \sum_{k_n=0}^{t_n} \frac{G_i}{\rho_d^{m_d+k_1} \rho_c^{m_c+k_2} \ldots \rho_c^{m_c+k_n}}
\]

\[
\times \left(\frac{t_i}{m_i\left(1-\sqrt{\rho_d}\right)S_i}\right)^{m_i+k_i} \left(\frac{t_i}{m_d\left(1-\sqrt{\rho_c}\right)S_n}\right)^{m_d+k_n}
\]

(30)

The nested infinite sum in (32), as one can see from Table 2, for triple balanced diversity case, converges for any value of the parameters \(\rho_d, \rho_c, S_1, S_2, S_3, m_d\) and \(m_c\). As is shown in this table, the number of the terms need to be summed to achieve a desired accuracy, depends strongly on the correlation coefficients \(\rho_d\) and \(\rho_c\). The number of the terms increases as correlation coefficient increases. For the special case of \(m_d=1\) and \(m_c=1\) we can evaluate expression for cdf for Rayleigh desired signal and co-channel interference.

Table 2. Terms need to be summed in (32) to achieve accuracy at the \(7^\text{th}\) significant digit presented in the brackets. We consider triple branch selection combining diversity case

<table>
<thead>
<tr>
<th>Term</th>
<th>(S_{dB}=10)</th>
<th>(m_d=1)</th>
<th>(m_c=1.2)</th>
<th>(m_c=1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>12</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>16</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>18</td>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Probability density function (PDF) of the output SIR can be obtained easily from previous expression:

\[ p_s(t) = \frac{d}{dt} F_s(t) = \sum_{l_1} \sum_{l_2} G l_1 l_2 \times \sum A_i(t) + \ldots + A_i(t) \] (33)

with:

\[ A_i(t) = \left( \frac{1}{(m_i + k_i) \cdots (m_i + k_{\gamma})} \right) \left( \frac{S_i}{t + m_i \left( 1 - \rho_d \right) S_i} \right) \] (34)

Fig. 4 shows probability density function of output signal to interference ratio for balanced \((S_1 = S_2)\) and unbalanced ratio \((S_1 \neq S_2)\) of SIR at the input of the branches and various values of correlation coefficient \(\rho_d\) and \(\rho_c\).

### 5 Performance analysis of SC diversity reception

#### 5.1 Outage probability

Outage probability versus normalized parameter \(S_i/\gamma\) for balanced and unbalanced ratio of SIR at the input of the branches and various values of correlation coefficient \(\rho_d\) and \(\rho_c\) is shown on Fig. 5. It is very interesting to observe that for lower values of \(S_i/\gamma\) (< 3dB) outage probability deteriorates when the fading severity decreases due to interference domination. For higher values of \(S_i/\gamma\) (when desired signal dominates), interference fading severity increase leads to outage probability decrease.
Fig. 6. Outage probability versus $S_i/\gamma$

The average error probability at the SC output is derived for noncoherent and coherent binary signalling.

5.2 Average bit error probability

The average error probability at the SC output is obtained for noncoherent and coherent binary signalling.

Substituting (33) in (21) numerically obtained average error probability is shown on Fig. 7 for several values of correlation coefficient and balanced (unbalanced) SIRs. Similarly, as at the SSC reception case better performance are obtained for the BDPSK modulation scheme versus NCFSK modulation scheme.

6 Appendix

In order to derive CDF of the SIR at the output of the SC diversity system with $N$ branches for the case of Nakagami-$m$ correlated fading channels in the presence of CCI we must first derive PDF of instantaneous values of SIR. Defining the instantaneous values of SIR at the $k$-th diversity branch as $\lambda_k={R_k^2/\Omega_k^2}$, PDF can be obtained as [30]:

$$p_{\lambda_1,\ldots,\lambda_n}(t_1, t_2, \ldots, t_n) = \frac{1}{2^n \sqrt{t_1 t_2 \cdots t_n}} \cdot$$

\[
\iint_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \cdots \int_{0}^{\infty} p_{R_1 R_2 \cdots R_n} \left(r_1 r_2 \cdots r_n \right) \cdot \frac{1}{\Omega_1^{2/3} \cdots \Omega_n^{2/3}} \cdot dr_1 \cdots dr_n \cdot
\]

(A.1)

Substituting (23) and (24) into (A.1), results in:

$$p_{\lambda_1,\ldots,\lambda_n}(t_1, t_2, \ldots, t_n) = \sum_{k_1, \ldots, k_n=0}^{\infty} \prod_{k=1}^{n} \frac{-m_k \lambda_k^{2}}{\Omega_k^{2/3}}$$

$$C_i \int_{0}^{\infty} r_1^{2i + 2m_1 + 2k_1 + \ldots + 2m_n + 1} e^{-\frac{m_1 \lambda_1}{\Omega_1^{1/3}}} \cdot e^{-\frac{m_2 \lambda_2}{\Omega_2^{1/3}}} \cdot dr_1 \times$$

$$\cdots \times \int_{0}^{\infty} r_n^{2i + 2m_n + 2k_n + \ldots + 2m_n + 1} e^{-\frac{m_n \lambda_n}{\Omega_n^{1/3}}} \cdot e^{-\frac{m_n \lambda_n}{\Omega_n^{1/3}}} \cdot dr_n$$

(A.2)

with:

$$C_i = 2^n (1 - \sqrt{\rho})^{n-1} (1 - \sqrt{\rho})^{n-1} \Gamma(m_1 + k_1 + \ldots + k_n) \Gamma(m_2 + k_1 + \ldots + k_n) \cdots \Gamma(m_n + k_1 + \ldots + k_n)$$

$$\frac{1}{\Gamma(m_1) \cdots \Gamma(m_n) \cdots} \frac{1}{\Gamma(k_1) \cdots \Gamma(k_1) \cdots} \frac{1}{\Gamma(k_n) \cdots \Gamma(k_n) \cdots}$$

$$\rho_d = \frac{1}{2} \cdot \frac{1}{\Omega_i^{1/3}} \cdot \frac{1}{\Omega_i^{1/3}} \cdots$$

$$\rho_c = \frac{1}{2} \cdot \frac{1}{\Omega_i^{1/3}} \cdot \frac{1}{\Omega_i^{1/3}} \cdots$$

$$m_i = \frac{1}{1 + (n-1) \sqrt{\rho_i}}$$

$$m_i = \frac{1}{1 + (n-1) \sqrt{\rho_i}}$$

$$m_i = \frac{1}{1 + (n-1) \sqrt{\rho_i}}$$

$$m_i = \frac{1}{1 + (n-1) \sqrt{\rho_i}}$$

(A.3)
Let $S_i = \Omega_d / \Omega_{ci} = $ be the average SIR’s at the $k$-th input branch of the multi-branch selection combiner. Then, the following integrals from previous expression can be presented in the form:

$$I_1 = \int_0^{+\infty} t_1^{2k_1 + 2m_1 + 2l_1 - 1} e^{-\frac{t_1}{\Omega_d (1-\sqrt{\rho_d})}} \bigg(\sum_{n=0}^{\infty} \frac{m_1 (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} \bigg)^{m_1} dt_1;$$

$$I_2 = \int_0^{+\infty} r_2^{2k_2 + 2m_2 + 2l_2 - 1} e^{-\frac{r_2}{\Omega_d (1-\sqrt{\rho_d})}} \bigg(\sum_{n=0}^{\infty} \frac{m_2 (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} \bigg)^{m_2} dr_2; \quad \text{(A.4)}$$

Now by using variable substitutions:

$$t_i = \rho_d^i \left(\lambda_i (1-\sqrt{\rho_d}) m_i + S_i (1-\sqrt{\rho_d}) m_i\right), \quad i = 1, \ldots, n$$

and well-known definition of Gamma function:

$$\Gamma(a) = \int_0^{+\infty} \exp(-t) dt \quad \text{(A.6)}$$

Finally, (A.2) can be written as

$$p_{\lambda_1,\ldots,\lambda_n} = \sum_{i=1}^{\infty} \sum_{j=0}^{\infty} G_i \left(\sum_{c=i}^{\infty} \frac{m_i (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_i\right)^{m_i} \cdot \cdots \cdot \left(\sum_{c=2}^{\infty} \frac{m_n (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_n\right)^{m_n} \cdot \left(\frac{m_k (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_k\right)^{m_k}$$

$$\cdot \left(\frac{m_l (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_l\right)^{m_l} \cdots \left(\frac{m_n (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_n\right)^{m_n} \bigg(\frac{m_i (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_i\bigg)^{m_i} \cdots$$

$$\bigg(\frac{m_n (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_n\bigg)^{m_n} \cdot \left(\frac{m_k (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_k\right)^{m_k} \cdot \left(\frac{m_l (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_l\right)^{m_l} \cdots \bigg(\frac{m_n (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_n\bigg)^{m_n} \bigg(\frac{m_i (1-\sqrt{\rho_d})}{\Omega_d (1-\sqrt{\rho_d})} S_i\bigg)^{m_i} \cdots$$

with:

$$G_i = \frac{\Gamma(m_i + k_i + \ldots + k_n) \Gamma(m_l + k_i + \ldots + k_n) \ldots \Gamma(m_n + k_i + \ldots + k_n)}{\Gamma(m_i + k_i + \ldots + k_n) \Gamma(m_l + k_i + \ldots + k_n) \ldots \Gamma(m_n + k_i + \ldots + k_n)}$$

$$I_3 = \int_0^{+\infty} \left(\frac{1}{1 + (n-1)\sqrt{\rho_d}}\right)^{n+t_i + l_i} \left(\frac{1}{1 + (n-1)\sqrt{\rho_d}}\right)^{n+t_i + l_i} \cdots$$

$$\text{(A.8)}$$

Now, CDF of output SIR could be derived from (A.9) by equating the arguments $t_1 = t_2 = t_n = \lambda$ as in [30]. By substituting, we obtain the following expression:

$$F_{\lambda_1,\ldots,\lambda_n}(t) = \sum_{i=1}^{\lambda} \sum_{j=0}^{\lambda} C_i \times J_i \times \cdots \times J_n; \quad \text{(A.10)}$$

whereas:

$$J_i = \int_0^{+\infty} \left(\frac{x_i^k}{(a + b x^n)^p}\right) \cdots \frac{m_i + k_i + \ldots + k_n}{\Gamma(m_i + k_i + \ldots + k_n)} \cdots \frac{m_n + k_n + \ldots + k_n}{\Gamma(m_n + k_n + \ldots + k_n)} \cdots$$

$$\text{(A.11)}$$

The integrals $J_i, i = 1, \ldots, n$ can easily be solved using the well-known definition of incomplete beta function [25]:

$$\int_0^{x} \frac{a^m}{n} \frac{B_z}{\Gamma(m + 1, n, p - m + 1)}; \quad z = \frac{b \lambda^r}{a + b \lambda^s}, \quad a > 0, \quad b > 0, \quad n > 0, \quad 0 < \frac{m + 1}{n} < p$$

$$\text{(A.12)}$$

Now using the famous relationship between incomplete beta and $\frac{m}{a}$ hypergeometric function:

$$B_z(a, b) = \frac{m}{a} \frac{F_1(a, 1 - b, 1 + a, z)}{\text{(A.13)}}$$

and after some straightforward manipulations we obtain CDF in the form of (32).

7 Conclusion

In this paper, the performance analysis of system with switch-and-stay and selection diversity is...
combining, based on SIR over constant correlated Nakagami-m fading channels in the presence of co-channel interference, was obtained. Correlation model was observed for evaluating performances of proposed diversity system. The complete statistics for the SSC and SC output SIR is given in the infinite series expressions form, i.e., PDF, CDF, OP. Using these new formulae, ABER was efficiently evaluated for some modulation schemes BDPSK and NCFSK. As an illustration of the mathematical formalism, numerical results of these performance criteria are presented, describing their dependence on correlation coefficient and fading severity.

References:
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